Efficient Water Supply in HVAC Systems

Mohammad Komareji

Ph.D. Thesis

Section of Automation & Control
Department of Electronic Systems
Aalborg University
9220 Aalborg
DENMARK

Efficient Water Supply in HVAC Systems
Ph.D. thesis

ISBN 978-87-92328-07-6
September 2008

Preface

This thesis is submitted in partial fulfilment of the requirements for the PhD degree at Section of Automation & Control, Department of Electronic Systems, Aalborg University, Denmark. The work has been carried out in the period of three years, from September 2005 to September 2008, under supervision of Professor Jakob Stoustrup and Associate Professor Henrik Rasmussen.

The project was jointly sponsored by Danish Energy Net, Center for Embedded Software Systems, and The Faculty of Engineering and Science at Aalborg University. It was carried out as a cooperation between Aalborg University, Grundfos A/S, Exhausto A/S, and Danish Technological Institute.

Acknowledgements

I would like to express my gratitude to all people who gave me a very enjoyable time at Aalborg University and created a comfortable working environment in Section of Automation and Control, Department of Electronic Systems.

A very special thanks goes to my great academic advisors: Professor Jakob Stoustrup, an outstanding professor in mathematics and control theory, and Associate Professor Henrik Rasmussen, who has lifetime experience in control theory and its applications, from whose profound knowledge and inspiration I benefited a lot during this project.

Particular thanks to Dr. Niels Bidstrup, chief engineer in Grundfos A/S, and Finn Nielsen, project manager in Exhausto A/S, for their invaluable guidance that this project required.

Thanks also to Peter Svendsen, project manager in Danish Technological Institute, for his cooperation and patience in setting-up the HVAC system in the lab and runnig several experiments afterwards.

I am grateful to the Center for Embedded Software Systems (CISS) headed by Professor Kim Guldstrand Larsen for partial financial support of this project.

I would like to thank Professor William Bahnfleth for giving me the opportunity to stay at the Pennsyvania State University and valuable helps while I was there.

I am deeply indebted to my parents, Akbar and Maryam, for their prayers and encouragements made this accomplishment possible.

September 2008, Aalborg, Denmark
Mohammad Komareji

Abstract

Increased energy costs have brought about increased concern by building owners about the operating cost and energy budgets for buildings. This growing energy conservation consciousness has brought about many changes in the attention focused on the energy performance of buildings, particularly that of heating, ventilating, and airconditioning systems - hereafter abbreviated HVAC systems. Various types of heating, ventilating, and air-conditioning (HVAC) applications are: apartment buildings, banks, office buildings, hospitals, industrial plants, schools, restaurants, department stores, hotels, etc.

This project aims at optimal model-based control of a typical industrial HVAC system consisting of a heat rocovery wheel and a water-to-air heat exchanger. In the current HVAC system a certain amount of water circulates through the coil and the temperature of the inlet air is controlled by the amount of hot water injected to the hydronic circuit where the coil is installed. However, in the new HVAC system the water flow through the coil is manipulated as a control variable too. Thus, it will result in less energy consumption by the pump which supplies the coil as the pump speed will decrease at part load conditions.

HVAC systems are in steady state conditions more than 95% of their operating time. To that end, to derive optimality criteria a static model for the HVAC system is supposed. The objective function is composed of the electrical power for different components, encompassing fans, primary/secondary pump, tertiary pump, and air-to-air heat exchanger wheel; and a fraction of thermal power used by the HVAC system. The goals that have to be achieved by the HVAC system appear as constraints in the optimization problem. Solving the defined problem results in two optimality criteria: 1- maximum exploitation of the heat recovery wheel. 2- equality of the supply hot water flow and the water flow going through the coil.

Then the optimal model-based controller (Here Model Predictive Control 'MPC' is applied) is designed to follow the goals of the HVAC system (comfort conditions) while the optimality criteria are met. The HVAC system is splitted into two independent subsystems (the heat recovery wheel and the water-to-air heat exchanger) through an internal feedback. By selecting the right set-points and appropriate cost functions for each subsystem's controller the optimal control strategy is respected to gaurantee the minimum thermal and electrical energy consumption. Then, the optimal control strategy which was developed is adopted for implemenation in a real life HVAC system. The bypass flow problem is addressed and a controller is introdeuced to deal with this problem.

Finally, a simplified control structure is proposed for optimal control of the HVAC system. The proposed simple control algorithm can be implemented through two propotional-integral (PI) controllers. All models and control algorithms which are developed throughout this thesis have been verified experimentally.

Resumé

Øgede udgifter til energi har medført øget bekymring hos boligejere ang. driftsomkostninger og energi omkostninger for bygninger. Denne voksende energibesparelse bevidsthed har medført mange ændringer i opmærksomheden om bygningers energimæssige ydeevne, især af, varme, ventilation og aircondition systemer - herefter forkortet HVAC-systemer. Forskellige former for opvarmning, ventilation og aircondition (HVAC) ansøsystemer er: boligkomplekser, banker, kontorbygninger, sygehuse, fabrikker, skoler, restauranter, varehuse, hoteller osv.

Dette projekt omhandler optimal model-baseret kontrol af et typisk industriet HVAC system som består af et varmegenvindingshjul og en vand-til-luft-varmeveksler. I det nuværende HVAC system cirkulerer en vis mængde vand gennem varmeveksleren og temperaturen af, er styret af mængden varmt vand der injicere i vand kredsen, hvor varmeveksleren er installeret. Men i det nye HVAC system er vandet som løber gennem varmeveksleren yderemere en kontrol variable. Dette resultere i et lavere energiforbrug i varmeveksleren pumpen, der forsyner varmeveksleren med varmt vand, idet pumpens hastighed vil reduceres ved del last situationer.

HVAC-systemer er i steady state betingelser i mere end 95% af deres driftstid. Derfor anvendes til at udregne kriteriet for optimal drift. Cost funktion er sammensat af den elektriske strøm til forskellige komponenter, det omfatter bløsere, primære/sekundære pumpe, tertiær pumpe, og luft-til-luft varmeveksler, og en brøkdel af den termiske effekt der anvendes af HVAC system. De mål, der skal opfyldes af HVAC system vises som begrænsninger i optimeringsproblemet. Et løse defineret problem resulterer i to optimal udnyttelse kriterier: 1 - maksimal udnyttelse af varmegenvindingshjulet. 2 - flowet fra varmtvandsforsyningen skal være lig med flowet gennem varmeveksleren.

Herefter er den optimale model-baseret controller (Model Predictive Control 'MPC' anvendt) designet til at følge målene i HVAC system (komfort betingelser), mens de optimale udnyttelse kriterier er opfyldt. HVAC systemet er delt i to uafhængige delsystemer (varmegenvindingshjulet og vand-til-luft-varmeveksler) gennem en indre feedback. Ved at vælge de rigtige set-punkter og relevante omkostnings funktioner for hvert delsystem's controller, er den optimale strategi for kontrol overholdt ved at sikre et mindstemål af termisk og elektrisk energi forbrug. Derefter er den optimale kontrol strategi, implementeret i et industrielt HVAC system. Shunt flow problem er rettet og en controller introduceres for håndtere dette problem.

Endelig forslås en forenklet kontrol struktur, til optimal kontrol af HVAC system. Den foreslåede enkle kontrol algoritme kan gennemføres ved hjælp af to propotional-integrerende (PI) controllere. Alle modeller og kontrol algoritmer, som er udviklet i hele denne afhandling er blevet bekræftet eksperimentelt.

List of Figures

List of Tables

Contents

Chapter 1

Introduction

1.1 Project Background

This PhD project was offered at Section of Automation and Control, Department of Electronic Systems, Aalborg University. It was jointly sponsored by Danish Energy Net[1], Center for Embedded Software Systems[2], and The Faculty of Engineering and Science at Aalborg University. The project was carried out as a cooperation between Aalborg University, Grundfos A/S[3], Exhausto A/S[4], and Danish Technological Institute[5].

A pre-project [1] which was done by The Danish Technological Institute, energy and industry section, in partnership with Exhausto A/S, Grundfos A/S, and Aalborg University has documented that there is nothing in the design that prevents us from using variable flow in the heating surfaces. Thus, the project named Efficient Water Supply in Heating, Ventilating, and Air-Conditioning (HVAC) Systems was defined and developed based on the knowledge achieved in the pre-project.

[1] www.danskenergi.dk

[2] www.ciss.dk

[3] An annual production of approximately 16 million pump units makes Grundfos one of the worlds leading pump manufacturers. The pumps are manufactured by Group production companies in Brazil, China, Denmark, Finland, France, Germany, Hungary, Italy, Switzerland, Taiwan, United Kingdom and the United States. In addition to pumps and pump systems, Grundfos develops, produces and sells electric motors and high-technology electronic equipment to make the pumps intelligent, increase their capacity and minimize their power consumption. website: www.grundfos.com

[4] EXHAUSTO A/S develops, manufactures, markets, and delivers ventilation units with heat recovery, roof fans, wall fans and box fans, control devices, cooker hoods, and a variety of other ventilation components for complete ventilation systems for the professional ventilation market. Website: www.exhausto.com

[5] www.dti.dk

1.2 Motivations

The mission of a heating, ventilating, air-conditioning (HVAC) system is to deliver conditioned air to maintain thermal comfort and indoor air quality.

Literature documents direct linkages of worker performance with air temperatures without apparent effects on worker health. Many studies indicate that small (few degrees of centigrade) differences in temperatures can influence workers speed or accuracy by 2% to 20% in tasks such as typewriting, learning performance, reading speed, multiplication speed, and word memory. Thus, maintaining thermal comfort is a crucial issue.

As the price of crude oil is getting higher and higher (more than 100% price increase in less than a year), the energy consumption issue is attracting more and more attentions. Thus, the energy consumption by HVAC system is also another important issue. The consumption of energy by heating, ventilating, and air conditioning (HVAC) equipment in industrial and commercial buildings constitutes more than 50% of the world energy consumption. In spite of the advancements made in microprocessor technology and its impact on the development of new control methodologies for HVAC systems aiming at improving their energy efficiency, the process of operating HVAC equipment in commercial and industrial buildings is still an inefficient and high-energy consumption process. According to the estimations by optimal control of HVAC systems almost 100 GWh electrical energy can be saved yearly in Denmark with five millions inhabitants.

To summarize, the most desirable HVAC system is one which maintain thermal comfort and indoor air quality while consuming the minimum energy. In this project we will approach these goals by introducing new control strategies for the HVAC system.

1.3 HVAC Systems

The mission of a heating, ventilating, and air-conditioning (HVAC) system is to deliver conditioned air to maintain thermal comfort and indoor air quality. On average we spend around 90% of our whole life inside buildings. Literature documents direct linkage of worker performance with air temperatures without apparent effects on worker health. Many studies indicate that small (few degrees of centigrade) differences in temperatures can influence workers speed or accuracy by 2% to 20% in tasks such as typewriting, learning performance, reading speed, multiplication speed, and word memory. Thus, maintaining thermal comfort and as a result HVAC systems are important issues in our life.

In this chapter first the basic components of HVAC systems along with some describing equations are introduced. Then the HVAC system which will be considered in this thesis for optimal control design is presented. Finally, different hydronic circuits are expressed and the suitable hydronic circuit for the mentioned HVAC system is discussed.

1.3.1 Components of HVAC Systems

In this section basic elements of a HVAC are described. Some simple equations along with the components' descriptions are also presented which can help to build up a base for better understanding the coming chapters.

Duct

Ducts are used in HVAC systems to deliver and remove the air. These needed air flows include, for example, supply air, return air, and exhaust air.

Like modern steel food cans, at one time air ducts were often made of tin. Tin is more corrosion resistant than plain steel, but is also more expensive. With improvements in mild steel production and its galvanization to resist rust steel sheet metals has replaced tin in ducts. Ducts are commonly wrapped or lined with fiberglass thermal insulation, both to reduce heat loss or gain through the duct walls and water vapor from condensing on the exterior of the duct when the duct is carrying cooled air. Insulation, particularly duct liner, also reduces duct-borne noise. Both types of insulation reduce breakout noise through the ducts' sidewalls. In all new construction (except low-rise residential buildings), air-handling ducts and plenums installed as part of an HVAC air distribution system should be thermally insulated in accordance with section 6.2.4.2 of ASHRAE Standard 90.1. Duct insulation for new low-rise residential buildings should be in compliance with ASHRAE Standard 90.2. Existing buildings should meet the requirements of ASHRAE Standard 100.

Duct system losses are the irreversible transformation of mechanical energy into heat. The two types are losses are: friction losses and dynamic losses. Friction losses are due to fluid viscosity and are a result of momentum exchange between molecules in laminar flow (Reynolds number less than 2000) and between individual particles of adjacent fluid layers moving at different velocities in turbulent flow. Friction losses occur along the entire duct length. For fluid flow in conduits, friction loss can be calculated by the Darcy and Colebrook equation:

$$\Delta p_f = \frac{1000 f L}{D_h} \frac{\rho V^2}{2} \tag{1.1}$$

where

Δp_f = friction loss in terms of total pressure, *Pa*

f = friction factor, dimensionless

L = duct length, m

D_h = hydraulic diameter[6], mm

V = velocity, m/s

[6]The hydraulic diameter, D_h is a commonly used term when handling flow in noncircular tubes and channels. Using this

ρ = density, Kg/m^3

Dynamic losses result from flow disturbances caused by duct-mounted equipment and fittings that change air flow's path direction and/or area. These fittings include entries, exits, elbows, transitions, and junctions. If the air density is constant and there is no elevation, according to the Bernoulli equation dynamic losses are proportional to the square velocity.

Considering the recent discussion and equation (1.1) reveals that the total losses in the duct network (friction losses plus dynamic losses) are proportional to the square air flow rate:

$$\Delta p_t \propto q^2 \tag{1.2}$$

where q is the air flow[7].

Fan

Fan is an important component of the HVAC system: it creates a pressure difference and causes air flow. The electric motor is the prime mover of the fans. Fan motor power (P_f) is related to the produced fan hydraulic power through the fan efficiency factor (η_f). As we know, the fan produced hydraulic power (P_h) is proportional to the production of the pressure losses along the duct network and the air flow:

$$P_h \propto q \cdot \Delta p_t \tag{1.3}$$

Combining (1.2) and (1.3), and bearing in mind that $P_f = \eta_f P_h$, the following result can be concluded:

$$P_f \propto q^3 \tag{1.4}$$

Based on the fan operation, HVAC systems can be categorized as Constant Air Volume (CAV) and Variable Air Volume (VAV) systems.

In a CAV system, the supply air flow rate and consequently the fan speed is constant but the supply air temperature is varied to meet the thermal load of the space. In a VAV system the controller not only plays with the supply air temperature but also changes the air flow rate in accordance with the ventilation demand; ASHRAE Standard 62 requires that each building occupant receives sufficient

term one can calculate many things in the same way as for a round tube. The hydraulic diameter is calculated as:

$$D_h = 4A/P$$

where
A = duct area, mm
P = perimeter of cross section, mm

[7] $q = A \cdot V$

outdoor ventilation air to maintain his or her zone's maximum CO_2 concentration at or below 0.1%. The requirement could be met through direct measurement or by supplying a fixed quantity of outdoor outdoor ventilation air per person (10-15 l/s per person). The big advantage of VAV systems is that they conserve considerable amount of energy in comparison with CAV systems. The reason for this energy saving is quite obvious from (1.4) which indicates the dependency of the fan power consumption on the cube air flow rate. Due to this fan energy saving VAV systems are more common. However, in small buildings and residences CAV systems are often the system of choice because of simplicity, low cost and reliability.

Pipe and Valve

Pipes interconnect individual components in a hydronic circuit. Pressure drop caused by fluid friction in fully developed flows of all well-behaved (Newtonian) fluids is described by the Darcy-Weisbach equation:

$$\Delta p_f = \frac{L}{D}\,\frac{\rho V^2}{2} \tag{1.5}$$

where
Δp = pressure drop, Pa
f = friction factor, dimensionless
L = length of pipe, m
D = internal diameter of pipe, m
ρ = fluid density at mean temperature, Kg/m^3
V = average velocity, m/s

Noise, erosion, installation, and operating costs limit the maximum and minimum velocities in piping systems.

A valve regulates the flow of materials such as gases, fluidized solids and liquids by opening, closing or partially obstructing various passageways. Valves and fittings cause pressure losses greater than those caused by the pipe alone. These losses can be expressed as

$$\Delta p_v = K\rho\,\frac{V^2}{2} \tag{1.6}$$

where K is the geometry and size-dependent loss coefficient.

Combining equations (1.5) and (1.6) results in that the total pressure drop ($\Delta p_t = \Delta p_f + \Delta p_v$) through the hydronic circuit is proportional to the square fluid flow rate (Q):

$$\Delta p_t \propto Q^2 \tag{1.7}$$

Pump

A pump moves liquids or gases from lower pressure to higher and overcomes this difference by adding energy to the system. Pumps fall into two major groups: rotodynamic pumps and positive displacement pumps. Their names describe the method for moving a fluid.

Rotodynamic pump uses for example a rotating impeller to increase the velocity of a fluid. However, a positive displacement pump causes a fluid to move by trapping a fixed amount of it then forcing that trapped volume into the discharge pipe. The periodic fluid displacement results in a direct increase in pressure.

Following the similar discussion as it was mentioned about fan power consumption, it can be concluded that the pump power consumption is also proportional to the cube fluid flow rate:

$$P_p \propto Q^3 \tag{1.8}$$

Again similarly, playing with the speed of the pump will result in huge amount of pump energy saving.

Heat Exchanger

The task of a heat exchanger is to efficiently transfer heat from one medium to another, whether the media are separated by a solid wall so that they never mix, or the media are in direct contact. There are plenty of different types of heat exchangers for enormous various purposes. In the next section two kinds of heat exchangers will be described.

1.4 The HVAC System

The HVAC system which will be considered here is a typical HVAC system made by Exhausto A/S and shown in Fig 1.1. It is composed of two heat exchangers: heat recovery part and a water-to-air heat exchanger (an air coil).

In general the heat recovery part has the mission of transferring heat from the exhausted room air to the fresh sucked air. Throughout this process there can be either mixing between the exhausted and fresh air or no mixing between them at all. Here a heat recovery wheel is applied as a heat recovery part. As can be seen in Fig 1.1, there are two separate ducts for conducting the exhausted room air and the fresh outdoor air. An aluminum made wheel rotates between two ducts and recovers the thermal energy from exhausted air. It should be noted that there is no mixing between two air streams (Practically there is a little bit leakage between the two air streams). Temperature of the fresh air that leaves the heat recovery part is controllable through the wheel rotation speed manipulation.

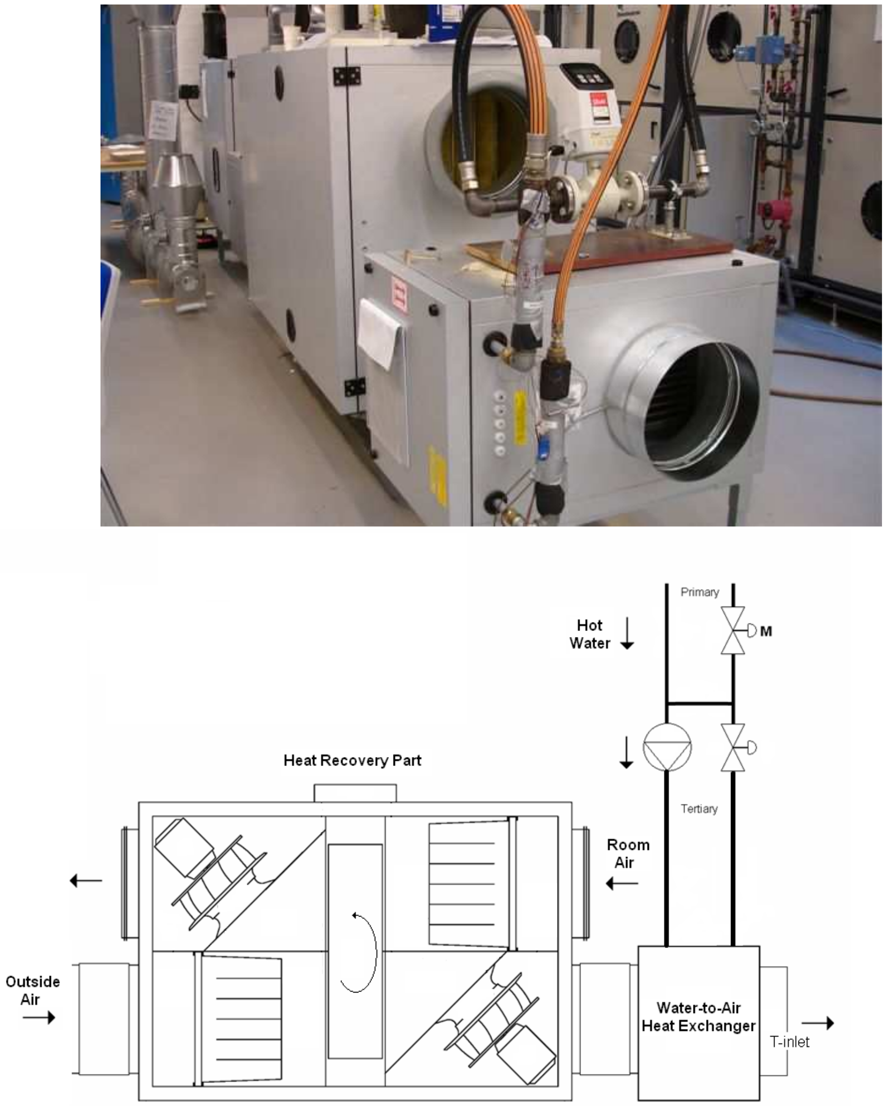

Figure 1.1: The HVAC system

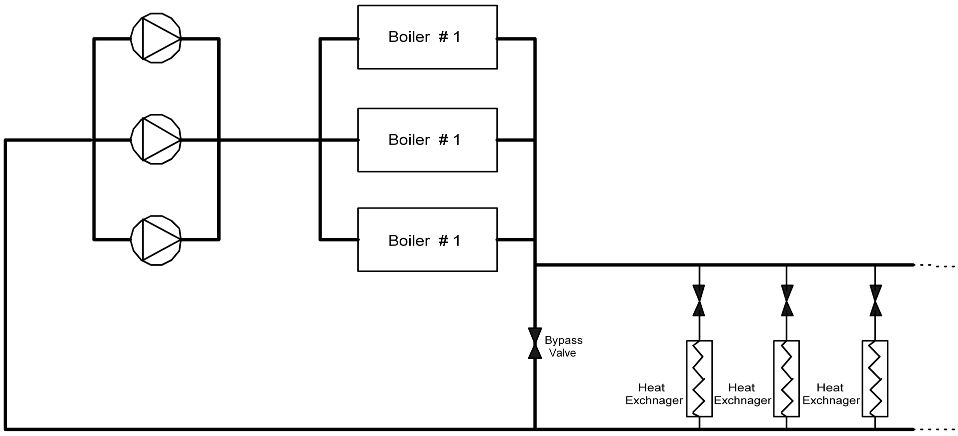

Figure 1.2: Primary-only hydronic circuit

After preliminary warming up of the fresh air, it goes through the water-to-air heat exchanger for the final heating. The main task of the water-to-air heat exchanger is to transfer thermal energy from hot water to the fresh air through the coil. The coil is connected to a hydronic circuit which supplies the hot water. The configuration of the hydronic circuit has a significant role in the control of the water-to-air heat exchanger. Thus, we will elaborate this issue in the following section.

1.5 Different Hydronic Circuit Configuration

In this section different hydronic circuit configurations will be described. Then we will argue about the most suitable hydronic circuit configuration for this project.

1.5.1 Primary-Only Hydronic Circuit

A primary-only hydronic circuit is shown in Fig 1.2. Pumps are equipped with variable speed drives (VSD) to adapt the pump speed to the required water flow. Water flow can be changed by using the control valve but using VSD is more energy-efficient. A bypass valve can be seen in the picture. In heating purposes this valve can be eliminated but when we are going to use primary-only circuit in cooling purposes we need to guarantee the minimum flow through the chillers. Therefore, in this case the bypass valve has the responsibility to maintain the minimum flow through the chillers. Advantages and disadvantages of primary-only hydronic circuits can be summarized as follows:

Advantages:

- Lower first costs: This is due to the elimination of the secondary pumps and associated fittings, vibration isolation, starters, power wiring, controls, etc. These savings are partly offset by higher costs of variable speed drives for the p-only system and the cost of the bypass valve and associated controls.

- Less space required: again due to the eliminated secondary pumps. This can result in substantial cost reductions, depending on the plant layout and space constraints.

- Reduced pump design motor power requirement and size: There are two reasons for this reduction. First, the additional fittings and devices (shut-off valves, strainers, suction diffusers, check valves, headers, etc.) required for the secondary pumps are eliminated. Second, in most cases, average pump efficiency is also higher with the p-only system.

- Lower pump energy costs: Contrary to conventional wisdom, p-only systems always use less pump energy. This is in part due to the reduced pump full-load power requirement, but mostly because the variable speed drives provide near cube-law savings for both flow through the primary circuit as well as flow through the secondary circuit.

Disadvantages:

- Bypass Control Problem: In cooling systems a minimum flow is required not to harm the chillers.

- Complexity of Control: Complex control systems are prone to failure and they need on-site trained professionals for checking and maintenance.

- Less Flexibility: If some changes happen in the demand, a new pump might be replaced. This replacement can result in expensive cost because the main pump has to be changed. To avoid this problem the pump can be selected oversized but still it may cause huge different cost.

1.5.2 Primary-Secondary Hydronic Circuit

A primary-secondary hydronic circuit is shown in Fig 1.3. As can be seen in the Fig 1.3, the configuration of the primary pumps are dedicated. However, it is not a necessary configuration. They can be used as manifolded in Fig 1.2. Also, in the primary-only configuration the pumps can be arranged as dedicated.

Primary pumps in primary-secondary configuration are often constant speed pumps and the variable speed pumps are installed in the secondary circuit to provide the required water flow in accordance with the load. The main advantage of primary-secondary hydronic circuit is its simplicity. Thus, it is easy to control and complicated control procedures is not required. Because of its simplicity, also highly skilled on-site staff is not needed.

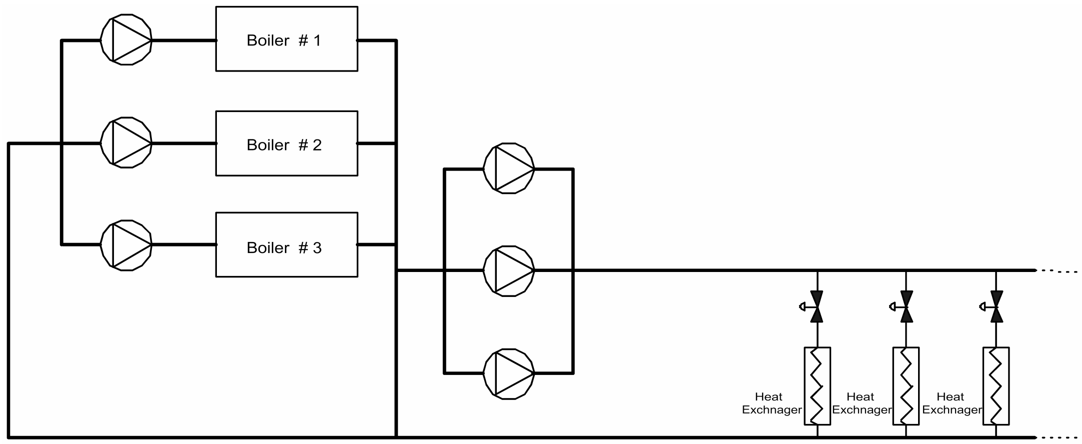

Figure 1.3: Primary-secondary hydronic circuit

1.5.3 Primary-Secondary-Tertiary Hydronic Circuit

A primary-secondary-tertiary hydronic circuit is shown in Fig 1.4. The primary-secondary-tertiary pumping makes the building, or load, loop hydraulically independent of the distribution loop and separates both from generation loop. This type of pumping system allows for blending of the water at supply temperature from the central plant with return water at each building, so each building gets a different supply temperature, which sometimes is called thermal independence from building to building. The coils in a given building can be provided with any temperature, ranging from the hot water supply temperature coming from the boiler to the chilled water supply temperature coming from the chiller evaporator. Proper design of the low-pressure-drop common pipe is essential to achieve the loops hydraulic and thermal independence. We can control the blending process at the common pipe connection between the distribution (secondary) loop and the building (tertiary) loop by using a controller. This controller must be carefully programmed to establish some priorities in determining the valve's position and the amount of recirculation that occurs. The first of these priorities is the blended supply temperature going to the building coils. Closing the valve in the secondary crossover bridge reduces the secondary flow with respect to the tertiary, or building, flow existing at that moment. This reduction forces blending of return water with supply water and decreases the secondary pump's energy cost. In hot water system, a substantial amount of blending could occur, resulting in considerable secondary flow and pumping cost decreases. This is possible because hot water coils can be selected to provide significant heat output even at low supply temperature. Chilled water systems require more careful control. With chilled water coils, excessive blending causes the supply temperature to the coils to rise above the dew point of the air passing over the coil; thus, the coil no longer can dehumidify the air. Totally, primary-secondary-tertiary pumping system has significant advantages in large central plant systems.

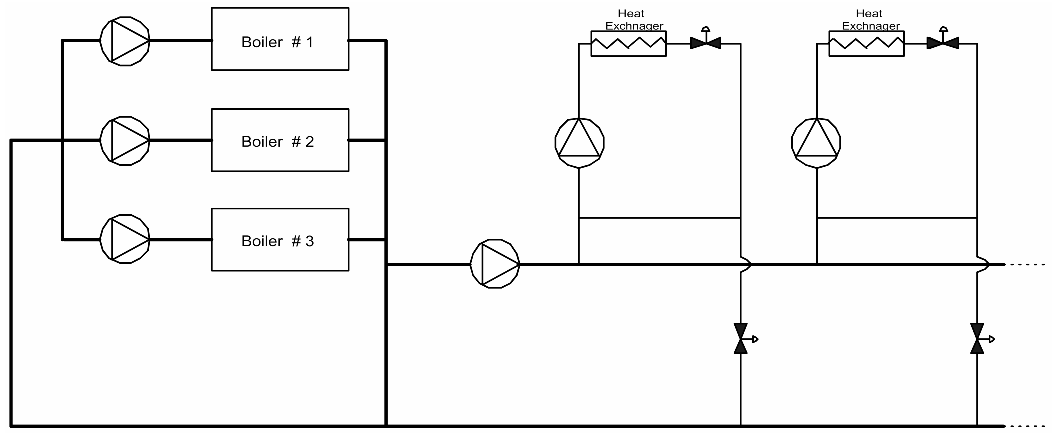

Figure 1.4: Primary-secondary-tertiary hydronic circuit

1.5.4 The Suitable Hydronic Circuit Configuration for This Project

All distributing pumps are centrally located with balancing valves and control valves installed at cooling coils, heating coils, and heat exchangers to restrict and regulate water flow by creating water pressure drops and power losses across the valves. In a system with a large number of balancing valves and control valves, total water power and energy losses across the valves can be significant. This wasted pump power and energy would be eliminated if the balancing valves and control valves were removed from the piping system. Thus, it will be objective to use a pumping system where pumps are locally located at the coils. The local pumps circulate and regulate water through the cooling coils, heating coils, and heat exchangers without balancing valves and control valves. In a local pumping system, a variable speed pump is installed at each cooling or heating coil without a centrally located pump. Pump speed and flow rate are controlled by the same controller that would otherwise regulate the control valve. The local pump will circulate and regulate water as required through the coil and the piping system, eliminating the need for control valves which eliminates the pressure drops and power losses across the valves. Pump head and power overcome only the essential piping and equipment pressure losses. The head of each pump is varied and depends on the pump location. The head is determined by summing all of the pressure losses in its flow path. Also, the local pumping system requires less horsepower than the central system at design load. Therefore, equipment costs (including pumps, motors and VSD) of the local pumping system should be lower in proportion to the reduced horsepower. Totally, the lower first cost in conjunction with lower operating cost make it desirable to select the local pumping system.

In this project to benefit the advantages of local pumping system, we suppose a variable speed pump for each water-to-air heat exchanger which regulates water flow through the coil. According to the definitions, this part will be called tertiary hydronic circuit. Depending on the size of the system and the way that hot or cold water is provided we can have primary/secondary pumps or only primary pumps. The focus of this project will be on the tertiary hydronic part. We will try to find the optimal control algorithm for this part respecting the thermal comfort standards.

1.6 Review of Previous Work

In this project, optimal model-based control of a heating, ventilating, air-conditioning (HVAC) system will be considered. First, we will derive the steady state optimality criterion for the HVAC system. Then, we will design the dynamic controllers in such a way that this optimality criterion is satisfied. This is a reasonable approach because HVAC systems are in steady state conditions around 95% of their operating time.

We review the previous works under two distinctive categories: modeling of HVAC systems and optimal control of HVAC systems.

1.6.1 Modeling of HVAC Systems

Due to our approach to solve the optimal control problem of the HVAC system, which was mentioned above, we will need both static (steady-state) model and dynamic model of the HVAC system.

The steady state models of HVAC systems are important because they can be applied to estimate energy consumptions and to optimize the performance of the system. In [11] they developed a mathematical model of a section of a building. The building model includes the effects of air exchange, conduction through walls and fenestration, solar radiation, energy storage in furniture, and internal loads from occupants and equipment. It can predict both transient and static behavior of the system. The model is modular (including six modules: external wall, internal wall, window, ceiling, floor, and air) so that they can be easily replaced with others and make the number of rooms adjustable.

[15] develops HVAC system steady-state models and validates them against the monitored data of a existing VAV system to use for energy consumption and thermal comfort calculations. The final goal of this work is to develop a supervisory layer which performs based on the two-objective genetic algorithm to optimize the operation of a HVAC system.

Steady-state models of HVAC system components are developed in [16]. Those models are interconnected to simulate the responses of the VAV system. The developed steady state model later is used to formulate the optimal control problem.

The rotary regenerator (also called the heat wheel) is an important component of energy intensive sectors, which is used in many heat recovery systems due to its high efficiency. In [17] a model of a rotary enthalpy wheel heat exchanger based on a new semi-empirical NTU correction factor method is developed. Given only two reference data points, the model is able to predict effectiveness for any balanced and unbalanced flow condition.

A model for heat wheel based on physical principles is developed in [18]. Then they analyze the temperature distribution and its variations in time and investigate how the airflow, temperature and rotational speed of the wheel influence upon the dynamic response.

[19] develops a 2D, steady state model of a rotary desiccant wheel. The model is capable of predicting steady state behavior of desiccant wheels having at the most three sections (process, purge, and regeneration).

The fundamental dimensionless groups for air-to-air energy wheels that transfer both sensible heat and water vapor can be derived from the governing nonlinear and coupled heat and moisture transfer equations. These dimensionless groups for heat and moisture transfer are found to be functions of the operating temperature and humidity of the energy wheel. Unlike heat exchangers that transfer only sensible heat, the effectiveness of energy wheels is a function of the operating temperature and humidity as has been observed by several energy wheel manufacturers and researchers. The physical meaning of the dimensionless groups and the importance of the operating condition factor are explained in [20], [21], [22], and [23].

Underwood and Crawford develop a model to predict the effects of inlet air temperature, air flow rate, and inlet water temperature during closed loop control of the outlet air temperature using water flow rate as control variable [24]. This model is characterized by two first order differential equations (one for air side and one for water side). Least squares fits are performed to identify the model parameters on the basis of a series of open loop tests.

[25] presents A new dynamic coil model. This model is developed via the exact solution of a previously unsolved partial differential equation, which governs the coil dynamics for a step change in water flow rate. This new model is the first step toward developing a future model that can accurately predict the coil dynamics for several varying coil inlet conditions expected to occur under MIMO control. The model is compared with previously published simplified PDE coil models, which used an approximation to this exact solution, and against actual measured coil dynamics. The coil model is shown to have superior performance in predicting the actual coil behavior.

1.6.2 Optimal Control of HVAC Systems

Most existing HVAC system processes are optimized at the local loop level. However, a strategy using the optimization of the individual zone air temperature set points combined with other controller set points during occupied periods could reduce further system energy use. Using a multi-objective genetic algorithm, which will permit the optimal operation of the buildings mechanical systems when installed in parallel with a buildings central control system, optimization process, the supervisory control strategy set points, such as supply air temperature, supply duct static pressure, chilled water supply temperature, minimum outdoor ventilation, reheat (or zone supply air temperature), and zone air temperatures are optimized with respect to energy use and thermal comfort [15].

In [12] an objective function which consists of costs and energy demand is defined. The limitations in the system appear as constraints to this objective function. Solving the recent optimization problem results in an optimal combination of the characteristics of the HVAC system and the control strategy. Then a sequential control is developed, tested by simulation and implemented in an existing plant. The HVAC system here consists of the following components: sorption regenerator, heat regenerator, humidifier and air heater for supply air and humidifier and air heater for return air. The supply air fan as well as the return air fan transport the air masses. The heaters are loaded by hot water. In winter the HVAC system works as a conventional air conditioning system. With the aid of the two regenerators a high level of recovery of heat and humidity is possible. In the summer the heater for supply air is out of action. The dehumidification is done by the sorption regenerator. The cooling can be achieved by an adiabatic humidification.

They show in [13] how gradient-based optimization can be used to minimize energy consumption of distributed environmental control systems without increasing occupant thermal dissatisfaction. Fuzzy rules have been generated by data from gradient optimization, showing that a fuzzy logic control scheme based on nearest neighbors approximates closely the gradient-based optimized results.

It is well known that a building's thermal mass influences thermal conditions within the space. Thermal mass is generally considered to be negative in the case of intermittent air conditioning, since the heat load tends to increase due to heat storage load. However, taking an HVAC system with heat storage tanks as an analogy, there would appear to be a possibility of storing heat in the building structure during times of non-occupancy, thus reducing equipment capacity requirements or saving running costs by utilizing cheap night-rate electricity. [14] proposes a dynamic optimization technique that minimizes objective functions such as running cost or peak energy consumption taking advantage of the recent mentioned phenomenon.

Classical HVAC control techniques such as ON/OFF controllers (thermostats) and proportional-integral-derivative (PID) controllers are still very popular because of their low commissioning cost. However, in the long run, these controllers are expensive because they operate at a very low-energy efficiency. One important factor affecting the efficiency of air conditioning systems is the fact that most HVAC systems are set to operate at design thermal loads while actual thermal loads affecting the system are time-varying. Therefore, control schemes that take into consideration time varying loads should be able to operate more efficiently and better keep comfort conditions than conventional control schemes. [26] presents a nonlinear controller for a heating, ventilating, and air conditioning (HVAC) system capable of maintaining comfort conditions under time varying thermal loads. The controller consists of a regulator and a disturbance rejection component designed using Lyapunov stability theory. The mitigation of the effect of thermal loads other than design loads on the system is due to an on-line thermal load and state estimator. The availability of the thermal load estimates allows the controller to keep comfort regardless of the thermal loads affecting the thermal space being heated or cooled. Simulation results are used to demonstrate the potential for keeping comfort and saving energy of this methodology on a variable-air-volume HVAC system operating on cooling mode. The same idea follows in [27]. The control system attempts to find an economic optimum to supply heat to the building with the use of a predictor for the indoor temperature, while maintaining a comfortable temperature in the building.

1.7 Contributions

This section presents the contributions of this thesis. This project requires lots of modeling which is the first step in model-based approaches. There are plenty different models of HVAC systems in the literature but rarely models which are useful from control point of view can be found. The major controllers have been used for HVAC systems are PID controllers because of their cheap first-cost and simplicity while in this project advanced control techniques are used. Finally, the advanced controller is simplified for commercialization purposes.

1.7.1 Contribution 1

Optimal set-point synthesis for a HVAC system applied to meet ventilation demands of a single-zone area: HVAC systems often work in their steady state regime (more than 95% of their operating time). Thus, to control the system optimally set-point optimization approach seems objective. To derive the optimality criteria static model for the HVAC system is applied. So, we define an objective (cost) function composed of all electrical and thermal power consumptions in the system. Ventilation goals and actuator limits appear as constraints in the optimization problem. Finally, the optimization problem is solved and the optimality criteria are derived [28].

This approach results in a performance that is very close to the ideal optimal operation while it has the advantage of less complication in computation and implementation.

1.7.2 Contribution 2

Developing a nonlinear dynamic model for a water-to-air heat-exchanger: In this project control inputs to the water-to-air heat exchanger are primary water flow and tertiary water flow. The output of the heat exchanger is inlet temperature. Therefore, to develop a model which is a true representative of the inputs and the output of the system the nonlinear water-to-air heat exchanger model which was proposed in [24] is extended. The proposed water-to-air heat exchanger model assumed constant temperature for the hot water supply to the coil. We include the energy balance equation of the supply hydronic circuit in the nonlinear model of the water-to-air heat exchanger to have the variable supply hot water temperature for the coil. In other hand, by doing that we will have primary water flow and tertiary water flow as control inputs to the system [29].

1.7.3 Contribution 3

Developing a gain varying model for a rotary heat recovery wheel: The temperature of the fresh air that leaves the rotary heat recovery wheel is controllable by changing the rotation speed of the wheel. Thus, for model-based control of the rotary heat recovery wheel a model which describes that relation is necessary. In the literature there are plenty of models for rotary heat recovery wheels but unfortunately none of them are useful from a control point of view. We estimate the steady state gain by benefiting from the results of the static analysis part. Then we discuss that a first order model can capture the dynamic behavior of the rotary heat recovery wheel. So, totally the model will be a first order system along with a variable gain [29].

1.7.4 Contribution 4

Design and implementation of the optimal model-based controller for the HVAC system: Dynamic model of the system is analyzed and then the system is broken into two independent subsystems (rotary heat recovery wheel and water-to-air heat exchanger). Utilizing the excellent features of the model predictive control (MPC) and introducing an internal feedback in the system the optimality criteria are met [29].

1.7.5 Contribution 5

Design and implementation of the simplified optimal controller for the HVAC system: Implicit measuring of the water flow by means of thermocouples leads us to a simplified optimal controller for the system. This control scheme consists of two PI controller. One of them controls the inlet temperature by manipulating the primary water flow while the other one tries to keep the primary and the tertiary flows close as far as possible [30].

1.7.6 Contribution 6

Experimental verifications of the new developed models and control algorithms for the HVAC system: All the new models and control algorithms which were developed throughout this thesis are verified experimentally in the Danish Technological Institute's lab on a typical HVAC system manufactured by Exhausto while hot water is pumped to the system via Grundfos new permanent magnet variable speed pump [28], [29], [30].

1.7.7 Contribution 7

A parameterization of the observer-based controllers; bumpless transfer by covariance interpolation: HVAC systems are nonlinear systems. One of the most common ways to control a nonlinear system is to linearize the nonlinear system around some specific operating points and then applying the linear control techniques. Afterwards the problem of how to switch between different linear controllers comes up.

Interpolation between two observer-based controllers is not a trivial task because the simple gain interpolation can leave the system unstable for some intermediate points. So, we have proposed an algorithm to interpolate between two observer-based controllers for a linear multivariable system such that the closed loop system remains stable throughout the interpolation. The proposed algorithm can be applied for bumpless transfer between two observer-based controllers. This algorithm has been used in bumpless transfer between two observer-based controllers which were designed based on the linearized model of the HVAC system. However, the proposed algorithm is still too naive to be applied for the real HVAC system which has nonlinear behaviors [31].

1.8 Outline of Thesis

This thesis is presented as a collection of papers. Thus, the rest of this thesis is organized as follows:

Summary of work: This chapter presents a brief description of the work that was carried out in this project. The main goal is to give a comprehensive formulation of the problem and its solution while there will be no need for the reader to go through the paper collections.

Conclusions: Conclusions, perspectives, and possible future works are discussed here.

Optimal Set-point Synthesis in HVAC Systems: This paper presents optimal set-point synthesis for the heating, ventilating, and air-conditioning (HVAC) system. The objective function is composed of the electrical power for different components, encompassing fans, primary/secondary pump, tertiary pump, and air-to-air heat exchanger wheel; and a fraction of thermal power used by the HVAC system. The goals that have to be achieved by the HVAC system appear as constraints in the optimization problem. To solve the optimization problem, a steady state model of the HVAC system is derived while different supplying hydronic circuits are studied for the water-to-air heat exchanger. Finally, the optimal set-points and the optimal supplying hydronic circuit are resulted.

Optimal Model-Based Control in HVAC Systems: This paper presents optimal model-based control of the heating, ventilating, and air-conditioning (HVAC) system. First dynamic model of the HVAC system is developed. Then the optimal control structure is designed and implemented. The HVAC system is split into two subsystems. By selecting the right set-points and appropriate cost functions for each subsystem controller the optimal control strategy is respected to guarantee the minimum thermal and electrical energy consumption. Finally, the controller is applied to control the mentioned HVAC system.

Simplified Optimal Control in HVAC Systems: This paper presents simplified optimal control of the heating, ventilating, and air-conditioning (HVAC) system. First the optimal control strategy which was developed is adopted for implementation in a real life HVAC system. Then the bypass flow problem is addressed and a controller is introduced to deal with this problem. Finally a simplified control structure is proposed for optimal control of the HVAC system.

Appendices:

- Appendix A: This section deals with the decoupling of the HVAC system.
- Appendix B (A Parameterization of The Observer-Based Controllers: Bumpless Transfer by Covariance Interpolation): This paper presents an algorithm to interpolate between two observer-based controllers for a linear multivariable system such that the closed loop system remains stable throughout the interpolation. The method interpolates between the inverse Lyapunov functions for the two original state feedbacks and between the Lyapunov functions for the two original observer gains to determine an intermediate observer-based controller. This algorithm has been used in bumpless transfer between two observer-based controllers which were designed based on the linearized model of the HVAC system. However, the proposed algorithm is still too naive to be applied for the real HVAC system which has nonlinear behaviors.
- Appendix C: This part describes the HVAC test system set-up.

Bibliography

[1] P. Svendsen and H. Andersen, ”Energy Efficient Pump Coupling in HVAC Systems”, *Technical Report*, Danish Technological Institute (Industry and Energy Section), 2005.

[2] S. Somasundaram, D.W. Winiarski, D.B. Belzer, ”Screening Analysis for EPACT-Covered Commercial HVAC and Water-Heating Equipment”, *Transaction of the ASME*, Vol. 124, June 2002, pp 116-124.

[3] J. Lin, S. Chen, and P.D. Roberts, ”Modified Algorithm for Steady-state Integrated System Optimization and Parameter Estimation”, *IEE Proceedings*, Vol. 135, March 1988.

[4] S. Gros, B. Srinivasan, and D. Bonvin, ”Static Optimization via Tracking of The Necessary Conditions of Optimality Using Neighboring Extremals”, *American Control Conference Proceeding*, June 2005, pp 251-255.

[5] N. Nassif, S. Kajl, and R. Sabourin, ”Two-Objective On-Line Optimization of Supervisory Control Strategy”, *Building Services Engineering Research and Technology*, 2004, pp 241-251.

[6] L.F.S. Larsen, ”Model Based Control of Refrigeration Ssytems”, *Ph.D. Thesis*, Aalborg University, Denmark, 2005.

[7] M. Wetter, ”Simulation Model: Air-to-air Heat Exchnager”, *Technical Report*, Lawrence Berkeley National Laboratory, 1999.

[8] N. Nassif, S. Kajl, and R. Sabourin, ”Ventilation Control Strategy Using Supply CO_2 Concentration Set Point”, *International Journal of HVAC & R Research*, 2005, pp 239-262.

[9] R.H. Green, ”An Air-Conditioning Control System Using Variable-speed Water Pumps”, *ASHRAE Transaction:Research*, part 1, 1994, pp 463-470.

[10] W.P. Bahnfleth, ”Varying Views on Variable-primary Flow”, *Chilled Water Engineering J.*, March 2004.

[11] Craig Lin, Clifford C. Federspiel, David M. Auslander, Multi-Sensor Single-Actuator Control of HVAC Systems, *Energy Systems Laboratory*, 2002.

[12] Clemens Felsmann and Gottfried Knabe, Simulation and Optimal Control Strategies of HVAC Systems, *Seventh International IBPSA Conference*, August 2001.

[13] S. Ari, I.A. Cosden, H.E. Khalifa, J.F. Dannenhoffer, P. Wilcoxen, and C. Isik, Constrained Fuzzy Logic Approximation for Indoor Comfort and Energy Optimization, *Fuzzy Information Processing Society, Annual Meeting of the North American*, June 2005.

[14] Tatsuo Nagai, Dynamic optimization technique for control of HVAC system utilizing building thermal storage, *IBPSA proceedings*, 1999.

[15] Nabil Nassif, Stanislaw Kajl, Robert Sabourin, Optimization of HVAC Control System Strategy Using Two-Objective Genetic Algorithm, *HVAC&R Research*, Vol. 11, No. 3, July 2005.

[16] G. R. Zheng and M. Zaheer-Uddin, Optimization of thermal processes in a variable air volume HVAC system, *IEEE Control Systems*, 1995.

[17] S.W. Freund, Simulation of air-to-air energy recovery systems for HVAC energy conservation in an animal housing facility, *Master thesis*, University of Wisconsin-Madison, 2003.

[18] Zhuang Wu, Roderick V.N. Melnik, Finn Borup, Model-based analysis and simulation of regenerative heat wheel, *Energy and Buildings*, 2005.

[19] Y.M. Harshe, R.P. Utikar, V.V. Ranade, D. Pahwa, Modeling of rotary desiccant wheels, *Chemical Engineering Technology*, Vol. 28, No. 12, 2005.

[20] C.J. Simonson and R.W. Besant, Energy wheel effectiveness: part I-development of dimensionless groups, *International journal of heat and mass transfer*, 1998.

[21] L.A. Sphaier, W.M. Worek, Analysis of heat and mass transfer in porous sorbents used in rotary regenerators, *International journal of heat and mass transfer*, 2004.

[22] L.Z. Zhang, J.L. Niu, Performance comparisons of desiccant wheels for air dehumidification and enthalpy recovery, *Applied thermal engineering*, March 2002.

[23] N. Ghodsipour, M. Sadrameli, Experimental and sensitivity analysis of a rotary air preheater for the flue gas heat recovery, *Applied thermal engineering*, November 2002.

[24] D.M. Underwood and R.R. Crawford, Dynamic nonlinear modeling of a hot-water-to-air heat exchanger for control applications, *ASHRAE transactions: research*, 1991.

[25] Chris C. Delnero, Dave Dreisigmeyer, Douglas C. Hittle, Peter M. Young, Charles W. Anderson, Michael L. Anderson, Exact Solution to the Governing PDE of a Hot Water to Air Finned Tube Cross Flow Heat Exchanger, *International journal of HVAC&R research*, 2004.

[26] Betzaida Arguello-Serrano and Miguel Velez-Reyes, Nonlinear Control of a Heating, Ventilating, and Air Conditioning System with Thermal Load Estimation, *IEEE transactions on control systems technology*, Vol. 7, No. 1, January 1999.

[27] Peter Lute and Dolf van Paassen, Optimal Indoor Temperature Control Using a Predictor, *IEEE Control Systems*, 1995.

[28] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, F. Nielsen, P. Svendsen, Optimal set-point synthesis in HVAC systems, American Control Conference, New York, NY, 2007.

[29] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, F. Nielsen, P. Svendsen, Optimal model-based control of HVAC systems, American Control Conference, Seattle, WA, 2008.

[30] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, F. Nielsen, P. Svendsen, Simplified optimal control in HVAC systems, Submitted for publication, 2008.

[31] J. Stoustrup, M. Komareji; A parameterization of the observer-based controllers: bumpless transfer by covariance interpolation, Submitted for publication, 2008.

Chapter 2

Summary of Work

This chapter presents a brief description of the work that was carried out in this project. The main goal is to give a comprehensive formulation of the problem and its solution while the collection of papers serves as a complimentary for further insight.

First optimality criteria based on the static model of the HVAC system are derived. It is objective to apply the static model of the system because HVAC systems are in steady state conditions more than 95% of their operating time. Then the model-based controller is designed to follow the objectives of the HVAC system while the optimality criteria are met. Finally, the control system is simplified and some practical issues are addressed.

2.1 Modeling

2.1.1 Static Modeling

The HVAC system that will be considered consists of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger. In this section the temperature efficiency of these two heat exchangers, which can be used as a steady state model of heat exchangers, will be described.

Air-to-air Heat Exchanger

The air-to-air heat exchanger is a rotary heat exchanger in aluminum, with low pressure loss (shown in Fig. 2.1). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow. Here, it is supposed that the ratio of the supply air flow to the return air flow is one. Therefore, η_{t2} will be a function of air flow (q_a), that is the same for both supply and return air, and the rotation speed of the wheel (n).

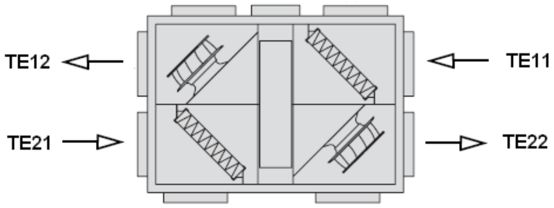

Figure 2.1: The air-to-air heat exchanger scheme

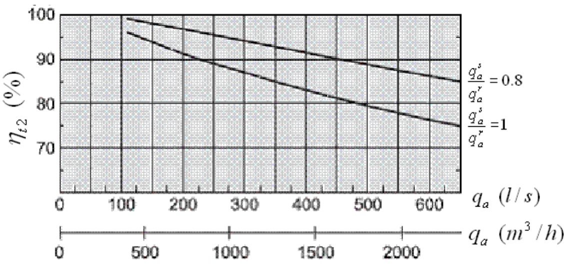

Figure 2.2: Dependency of η_{t2} on q_a while n=10 rpm; q_a^s and q_a^r represent supply air flow and return air flow, respectively.

In this context, results of testing the rotary heat exchanger that was performed according to European Standard for laboratory testing of air-to-air heat recovery devices (EN 247, EN 305, EN 306, EN 307, EN 308) will be used. This European Standard is intended to be used as a basis for testing heat recovery devices for HVAC systems, which as specified in EN 247 consist of the heat exchanger itself installed in a casing having the necessary air duct connecting elements and in some cases the fans and pumps, but without any additional components of the HVAC system.

According to results of the test, it is possible to specify η_{t2} as a multiplication of two functions. Fig. 2.2 and 2.3 illustrate these functions. Therefore, η_{t2} can be described as following:

$$\eta_{t2} = (-1.0569 \cdot 10^{-4}\, q_a + 0.9943) \cdot wrf(n) \tag{2.1}$$

Water-to-air Heat Exchanger

The water-to-air heat exchanger is shown in Fig. 2.4. As can be seen, a primary/secondary-tertiary hydronic circuit supplies the heat exchanger with hot water. The air flow that passes the hot coil is controllable by changing the speed of the fan installed in the air-to-air heat exchanger.

Here, temperature efficiency (η_{t1}) is a function of hot water flow (q_{wt}) and air flow (q_a). To obtain this function several experiments were done. Results are illustrated in Fig. 2.5. Again, it is possible to describe η_{t1} as a multiplication of two functions that the first one depends only on air flow (q_a) and the second one depends only on water flow (q_{wt}):

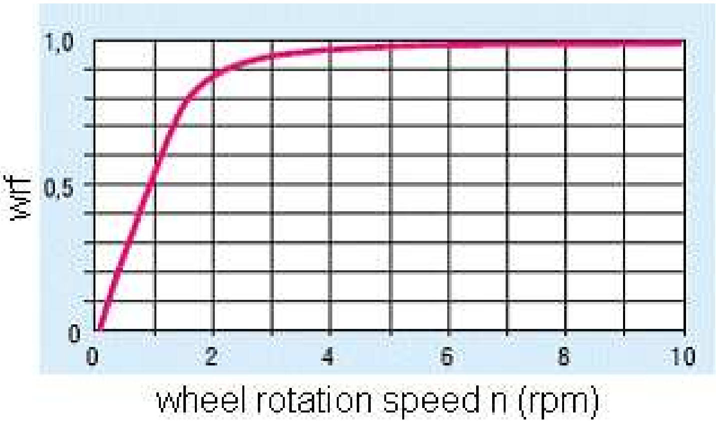

Figure 2.3: Normalized dependency of η_{t2} on n

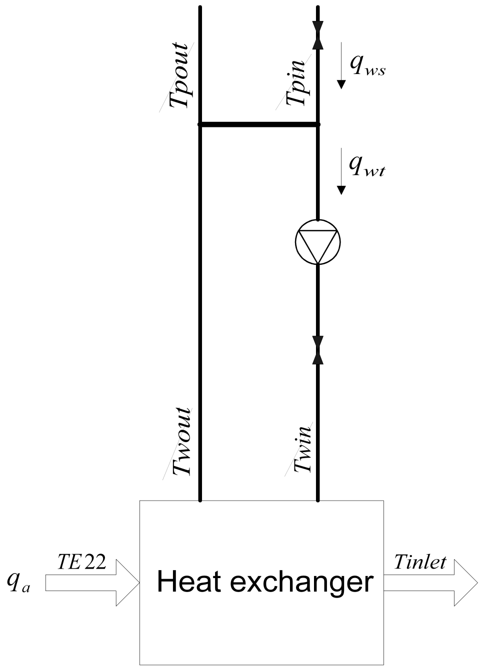

Figure 2.4: The water-to-air heat exchanger scheme

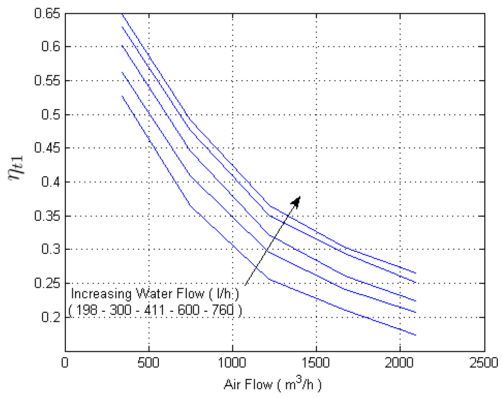

Figure 2.5: Result of experiments on water-to-air heat exchanger

$$\eta_{t1} = \frac{1}{0.3215}\,(a\,q_{wt}^4 + b\,q_{wt}^3 + c\,q_{wt}^2 + d\,q_{wt} + e)\cdot(A\,q_a^3 + B\,q_a^2 + C\,q_a + D) \tag{2.2}$$

where:

$a = -5.399\cdot 10^{-12}[1/(lit/h)^4]$ $\quad$ $b = 1.0733\cdot 10^{-8}[1/(m^3/h)^4]$

$c = -7.887\cdot 10^{-6}[1/(lit/h)^3]$ $\quad$ $d = 2.7199\cdot 10^{-3}[1/(m^3/h)^3]$

$e = 8.3711\cdot 10^{-4}[1/(lit/h)^2]$ $\quad$ $A = 1.0665\cdot 10^{-10}[1/(m^3/h)^2]$

$B = -1.643\cdot 10^{-7}[1/(lit/h)]$ $\quad$ $C = -2.880\cdot 10^{-4}[1/(m^3/h)]$

$D = 0.6927$

2.1.2 Dynamic Modeling

In this section dynamic model of HVAC system components will be developed. Then the overall nonlinear model of the HVAC system will be linearized. This linear model will be used to design the controller later.

Air-to-air Heat Exchanger

According to the earlier discussion η_{t2} can be described as following:

$$\eta_{t2} = (-1.0569\cdot 10^{-4}\,q_a + 0.9943)\cdot wrf(n) \tag{2.3}$$

As we know, η_{t2} definition is as following:

$$\eta_{t2} = \frac{TE22 - TE21}{TE11 - TE21} \tag{2.4}$$

Combining recent equations (equations 2.3 and 2.4) will result in the steady state gain for the wheel model:

$$TE22 = TE21 + wrf(n) \cdot (TE11 - TE21) \cdot (-1.0569 \cdot 10^{-4} q_a + 0.9943) \tag{2.5}$$

Fig 2.6 shows an energy wheel operating in a counter flow arrangement. Under typical operating conditions, warm air enters the tube during the supply part of the cycle and transfers energy to the matrix. This energy is then transferred from the matrix to the air during the exhaust part of the cycle. The half plane of the matrix tube is assumed impermeable and adiabatic and the bulk mean temperatures of air are used in the model. The formulation is therefore one dimensional and transient with space (x) and time (t or $\theta = w \cdot t$) as the independent variables. The governing equations for heat transfer (energy equations) in energy wheel for air and matrix include energy storage, convection, conduction based on the usual assumptions are as follows respectively:

$$\rho_a C_{pa} A_a \frac{\partial T_a}{\partial t} + U \rho_a C_{pa} A_a \frac{\partial T_a}{\partial x} + h \frac{\acute{A}_s}{L} (T_a - T_m) = 0 \tag{2.6}$$

$$\rho_m C_{pm} A_m \frac{\partial T_m}{\partial t} - h \frac{\acute{A}_s}{L} (T_m - T_a) = \frac{\partial}{\partial x} (K_m A_m \frac{\partial T_m}{\partial x}) \tag{2.7}$$

It is reasonable to suppose that the conductivity has a small share in heat transfer through the matrix. Thus, equation (2.7) can be rewritten as following:

$$\frac{\partial T_m}{\partial t} + \frac{NTU}{C_r^* P} T_m = \frac{NTU}{C_r^* P} T_a \tag{2.8}$$

Equation (2.8) shows that air temperature (T_a) can be assumed as the input for the matrix temperature (T_m) differential equation. It means the matrix temperature as a function of time (t) will perform as an output of the ordinary first order differential equation. Another point that should be emphasized is that the time constant ($\frac{C_r^* P}{NTU}$)in the differential equation is fixed. That is, the time constant depends on matrix (wheel) properties. Thus, it is a design parameter not a control parameter. It is claimed that air stream temperature has the same behavior as matrix temperature. So, The air stream temperature shows first order dynamic behavior.

As a result, the wheel behavior can be modeled by a first order transfer function. So, we will have:

$$TE22(s) = \frac{TE21}{\tau\, s + 1} + \frac{wrf(s) \cdot (TE11 - TE21)(-1.0569 \times 10^{-4} q_a + 0.9943)}{\tau\, s + 1} \tag{2.9}$$

The first part on the right side of the equation (2.9) will be treated as disturbance. That is, the transfer function from $TE22$ to wrf is as following:

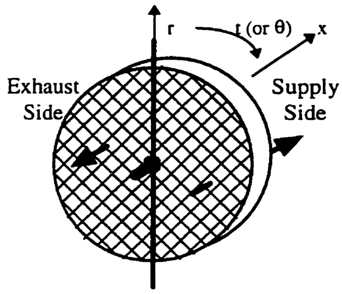

Figure 2.6: Counter flow energy wheel

$$\frac{TE22(s)}{wrf(s)} = \frac{(TE11 - TE21)(-1.0569 \times 10^{-4} q_a + 0.9943)}{\tau\, s + 1} \tag{2.10}$$

As it was discussed, the time constant (τ) is fixed and according to the experiments, it is 28.0374 seconds.

Water-to-air Heat Exchanger

Here, the nonlinear coil model that was developed by Underwood and Crawford will be applied. According to their model, the differential equations, resulted from energy balance equations, which describe the coil behavior are as follows:

$$\begin{aligned}[(-C_{pw} - b/2)\dot{m_{wt}}(t) - d/2\dot{m}_a(t) - a/2]\ T_{wout}(t) + [(C_{pw} - b/2)\dot{m_{wt}}(t) - d/2\dot{m}_a(t) - a/2]\ T_{win}(t) \\ +(b\ \dot{m_{wt}}(t) + d\ \dot{m}_a + a)\ TE22(t) = C_w \frac{d}{dt} T_{wout}(t)\end{aligned} \tag{2.11}$$

$$\begin{aligned}-\dot{m}_a(t) C_{pa} Tinlet(t) + [(C_{pa} - d)\dot{m}_a(t) - b\dot{m_{wt}}(t) - a] \cdot TE22(t) + (a/2 + b/2\dot{m_{wt}}(t) + d/2\dot{m}_a(t))T_{win}(t) \\ +(a/2 + b/2\dot{m_{wt}}(t) + d/2\dot{m}_a(t))T_{wout}(t) = C_a \frac{d}{dt} Tinlet(t)\end{aligned} \tag{2.12}$$

where a, b, d, C_{pa}, C_{pw}, C_a, and C_w are unknown parameters that have to be identified through the experiments.

The unknown parameters have identified through some experiments on the coil. Fig 2.7 show verification of the model along with identified parameters.

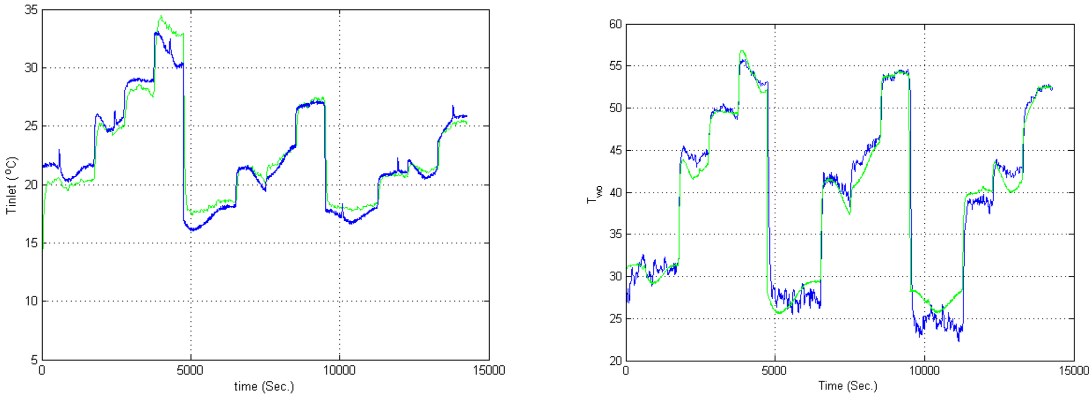

Figure 2.7: Coil model verification, blue curve: real output, green curve: simulated output

According to the hydronic circuit configuration $T_{win}(t)$ will be as following:

$$T_{win}(t) = \frac{T_{pin}(t)\ \dot{m}_{ws} + T_{wout}(t)\ (\dot{m}_{wt} - \dot{m}_{ws})}{\dot{m}_{wt}} \tag{2.13}$$

where it is supposed that $\dot{m}_{wt} \geq \dot{m}_{ws}$.

The recent formula for T_{win} should be placed in coil model (equations (2.11) and (2.12))to have the water-to-air heat exchanger model versus real inputs $\dot{m}_w$ and $\dot{m}_{wp}$. Therefore, final water-to-air heat exchanger model will be as following:

$$\begin{aligned}[k_1 - b\dot{m}_{wt} - k_2\dot{m}_{ws} + k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{wout}(t) + [k_2\dot{m}_{ws} - k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{pin}(t) \\ +[a + b\ \dot{m}_{wt} + d\ \dot{m}_a(t)]TE22 = C_w\frac{d}{dt}T_{wout}(t)\end{aligned} \tag{2.14}$$

$$\begin{aligned}-\dot{m}_a C_{pa} Tinlet(t) + [(C_{pa} - d)\dot{m}_a - b\dot{m}_{wt} - a]TE22 + [-k_1 + b\dot{m}_{wt} - b/2\dot{m}_{ws} - k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{wout}(t) \\ +[b/2\dot{m}_{ws} + k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{pin}(t) = C_a\frac{d}{dt}Tinlet(t)\end{aligned} \tag{2.15}$$

where:

$k_1 = -a - d\ \dot{m}_a(t)$

$k_2 = C_{pw} - b/2$

$k_3 = d/2\ \dot{m}_a(t) + a/2$

2.2 Optimality Criteria

The aim of this section is to find the optimality criteria for the described HVAC system. Thus, an objective function is needed to formulate the problem. The HVAC system mission can be described as constraints for the defined objective function.

2.2.1 Objective Function

The aim of this project is to have the HVAC system run while consuming minimum electrical and thermal energy. Therefore, the desired objective function is defined as following:

$$J = P_{pt} + P_{pp} + P_f + P_w + \Phi/2.5 \tag{2.16}$$

subject to:

$$q_a = q_{a_0}$$

$$Tinlet = 19$$

$$Tpout \leq 40$$

and,

$$0 \leq q_{wt} \leq 743$$

$$0 \leq q_{ws} \leq 1400$$

$$300 \leq q_a \leq 2200$$

$$0 \leq n \leq 10$$

where:

P_{pt} , P_{pp}, P_f , P_w , and Φ are tertiary pump power, primary pumping power, fan power, wheel rotation power, and thermal power, respectively.

q_{a_0} is a constant that will be determined in accordance with the required ventilation. This formulation discuss a typical HVAC system used for ventilation purposes.

Tertiary Pump Power (P_{pt})

The hydronic circuit that is used for supplying the water-to-air heat exchanger is a primary/secondary -tertiary circuit isolated from each other by a bypass pipe. The bypass pipe is a short length of full bore piping. The pressure drop across the bypass pipe is then small compared to the pressure drop in the tertiary circuit and through the supply circuit. The supply water flow (q_{ws}) is controlled by the

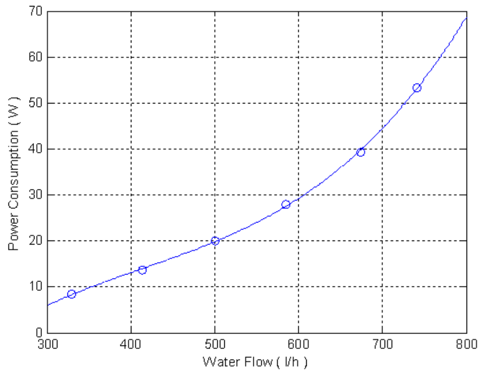

Figure 2.8: Tertiary pump power vs q_{wt}

motorized primary/secondary valve. A variable speed pump and a valve is installed in the tertiary circuit. The tertiary valve is used to set the desired maximum flow rate through the variable speed pump. By changing the speed of the tertiary pump, it is possible to sweep the interval between 330 (l/h) and 743 (l/h) for the tertiary water flow (q_{wt}). If the tertiary water flow has to be less than 330 (l/h), the pump will be pulsed. Fig. 2.8 illustrates power of the tertiary pump as a function of q_{wt}. This curve is approximated by the following polynomial:

$$P_{pt} = A_p\, q_{wt}^3 + B_p\, q_{wt}^2 + C_p\, q_{wt} + D_p \tag{2.17}$$

where:

$A_p = 5.1873 \cdot 10^{-7} [1/(lit/h)^3]$ $\quad$ $B_p = -6.4260 \cdot 10^{-4} [1/(lit/h)^2]$

$C_p = 3.2906 \cdot 10^{-1} [1/(lit/h)]$ $\quad$ $D_p = -48.8641$

When the pump is pulse width modulated, it is assumed that the power of the pump is the duty cycle fraction of the pump power while it is running at its minimum speed, i.e. it is proportional to the pump working period.

Primary Pumping Power(P_{pp})

The primary/secondary pumping power has to be measured implicitly because there is no direct access to the primary/secondary pump. Therefore, it is supposed that the efficiency of the pump in converting electrical power to hydraulic power is 50%. The curve indicating required pressure drop versus primary/secondary water flow (q_{ws}) is shown in Fig. 2.9. A second order polynomial is used to represent this curve. As we know, multiplying water flow by head results in hydraulic power. So, primary pumping power can be expressed as follows:

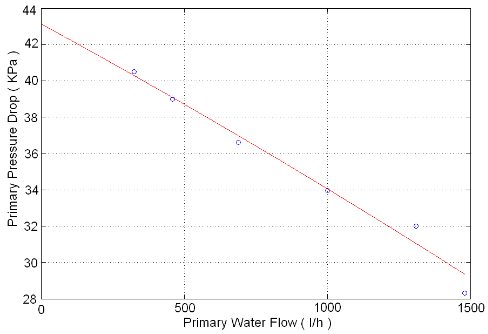

Figure 2.9: Primary pressure drop vs q_{ws}

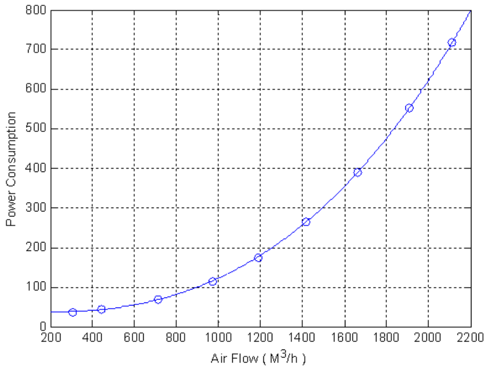

Figure 2.10: Fan power vs air flow (q_a)

$$P_{pp} = \frac{2}{3600} q_{ws} \cdot \left(-4.8131 \cdot 10^{-7} q_{ws}^2 - 8.5955 \cdot 10^{-3} q_{ws} + 43.1390\right) \tag{2.18}$$

Fan Power(P_f)

The HVAC system structure is assumed fixed during the entire work. As a result, the path for the air does not change. So, it is possible to have fan power as a function of air flow (q_a). Fig. 2.10 illustrates this function. The curve is approximated by a third order polynomial as follows:

$$P_f = A_f q_a^3 + B_f q_a^2 + C_f q_a + D_f \tag{2.19}$$

where:

$A_f = 4.7354 \cdot 10^{-8}[1/(m^3/h)^3]$ $B_f = 6.705 \cdot 10^{-5}[1/(m^3/h)^2]$

$C_f = -3.2527 \cdot 10^{-2}[1/(m^3/h)]$ $D_f = 40.3043$

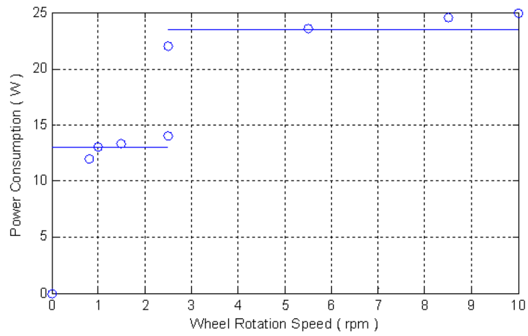

Figure 2.11: Wheel power consumption vs n

Wheel Rotation Power(P_w)

The electrical power input to the wheel as a function of rotation speed of the wheel is sketched in Fig. 2.11. The step change in the figure is due to the frequency converter used to control rotation speed of the wheel.

As Fig. 2.11 reveals, it is feasible to assume that the electrical power of the wheel is composed of three parts:

$$\begin{aligned} P_w &= 23.5\ W \qquad 10\,rpm \geq n > 2.5\,rpm \quad (1 \geq wrf > 0.9) \\ P_w &= 13\ W \qquad 2.5\,rpm \geq n > 0\,rpm \quad (0.9 \geq wrf > 0) \\ P_w &= 0\ W \qquad n = 0\,rpm \quad (wrf = 0) \end{aligned} \tag{2.20}$$

Thermal Power (Φ)

Φ is the thermal power that is being used by the water-to-air heat exchanger:

$$\Phi = \rho_w\, q_{wt}\, C_{pw}\, (Twin - Twout) \tag{2.21}$$

In steady state conditions:

$$\rho_w\, q_{wt}\, C_{pw}\, (Twin - Twout) = \rho_a\, q_a\, C_{pa}\, (Tinlet - TE22) \tag{2.22}$$

According to the definitions, we have:

$$Tinlet = TE22 + \eta_{t1}\, (Twin - TE22) \tag{2.23}$$

$$TE22 = TE21 + \eta_{t2} \ (TE11 - TE21) \tag{2.24}$$

Substituting (2.23) and (2.24) in (2.22) will result in:

$$\Phi = \rho_a \ q_a \ C_{pa} \ \eta_{t1} \ (Twin - TE21) + \rho_a \ q_a \ C_{pa} \ \eta_{t1} \ \eta_{t2} \ (TE21 - TE11) \tag{2.25}$$

This formula will be used for computing the thermal power consumption. Thermal power is divided by 2.5 in the objective function because thermal power is 2.5 time cheaper than electrical power to produce.

To obtain optimality criteria , the optimization problem that was defined has to be solved. Solving the defined optimization problem will be presented while two different cases are assumed for the hydronic circuit. In the first case it is assumed that $q_{ws} \leq q_{wt}$. In the second case we will have $q_{ws} \geq q_{wt}$. These two cases are selected because they are the most general cases.

2.2.2 Computing Optimal Set-points while $q_{ws} \leq q_{wt}$

According to the discussion so far, the optimization problem which has to be solved consists of a four-variable (q_a, q_{ws}, q_{wt}, n) objective function along with two equality constraint and two inequality constraints. Because q_a will be determined in accordance with required ventilation, actually we have to deal with a three-variable optimization problem along with an equality constraint and two inequality constraints. Thus, in the sequel by optimization problem we mean the latter statement. For convenience we will deal with $TE22$ instead of n in the procedure of solving.

Because, in this case, supply water flow (q_{ws}) is always less than or equal to the tertiary water flow (q_{wt}), mixing between the supply water flow that enters the tertiary circuit and a part of the tertiary return water flow occurs. So, the temperature of the water that enters the heat exchanger is as follows:

$$Twin = \frac{q_{ws} \ \cdot \ Tpin \ + \ (q_{wt} - q_{ws}) \ \cdot \ Twout}{q_{wt}} \tag{2.26}$$

Actually, the mission of the motorized primary/secondary valve is controlling *Twin* by changing the supply water flow (q_{ws}).

As it was mentioned, to solve the optimization problem here we have to deal with three variables q_{ws}, q_{wt}, and $TE22$. One of these variables is dependent due to equality constraint (Tinlet = 19):

$$Tinlet \ = \ (1 - \eta_{t1}) \ \cdot \ TE22 + \eta_{t1} \ \cdot \ Twin \tag{2.27}$$

As we know, the energy balance equation in a water-to-air heat exchanger is as follows:

$$\rho_w \, q_{wt} \, C_{pw} \, (Twin - Twout) = \rho_a \, q_a \, C_{pa} \, (Tinlet - TE22) \tag{2.28}$$

Substituting the two recent equations in equation (2.26) will result in a formula for q_{ws} versus q_{wt} and $TE22$:

$$q_{ws} = \frac{k \;\; \eta_{t1} \;\; q_{wt} \; (19 - TE22)}{\eta_{t1} q_{wt} (Tpin - TE22) + (q_{wt} - k\eta_{t1})(TE22 - 19)} \tag{2.29}$$

Also, in this case $q_{ws} \leq q_{wt}$; Therefore, substituting equation (2.29) in the recent inequality results in an inequality as following:

$$TE22 \geq \frac{19 - Tpin \; \eta_{t1}}{1 - \eta_{t1}} \tag{2.30}$$

Combining equations (2.27) and (2.28) results in a formula for $Twout$:

$$Twout = \frac{19q_{wt} + TE22(-q_{wt} + \eta_{t1} q_{wt} + k\eta_{t1}) - 19k\eta_{t1}}{\eta_{t1} q_{wt}} \tag{2.31}$$

In this case, $Twout$ is equal to $Tpout$ because the supply water flow is less than or equal to the tertiary water flow. Thus, the inequality constraint ($Tpout \leq 40$) can be translated into the following inequality:

$$(-q_{wt} + \eta_{t1} q_{wt} + k\eta_{t1}) \; TE22 \leq 40\eta_{t1} q_{wt} + 19k\eta_{t1} - 19q_{wt} \tag{2.32}$$

Finally, the optimization problem transferred to an objective function of two variables (q_{wt} and $TE22$) with two inequality constraints (inequalities (2.30) and (2.32)). The typical feasible region of this optimization problem is shown in Fig. 2.12 (assuming $TE21 = -12$, $Tpin = 80$ and $q_a = 2104.9$). Then optimal set-points in different conditions as a result of solving the optimization problem have been derived.

2.2.3 Computing Optimal Set-points while $q_{ws} \geq q_{wt}$

We have to deal with a three-variable optimization problem along with an equality constraint and two inequality constraints again. The only difference is the fact that supply water flow is greater than or equal to tertiary water flow. The impact of keeping supply water flow higher than or equal to the tertiary water flow on the system is that the supply tertiary water flow to the water-to-air heat exchanger will not be mixed water. So, $Twin$ will be equal to the $Tpin$. Actually, water mixing occur between the return

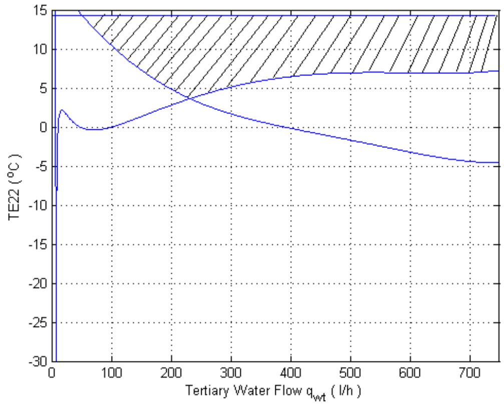

Figure 2.12: Feasible region while $q_{ws} \leq q_{wt}$ ($TE21 = -12$, $Tpin = 80$ and $q_a = 2104.9$)

tertiary water flow (q_{ws}) and the hot water passes the balance pipe ($q_{ws} - q_{wt}$). Therefore, we have:

$$Tpout = \frac{q_{wt} \cdot Twout + (q_{ws} - q_{wt}) \cdot Tpin}{q_{ws}} \tag{2.33}$$

According to the equality constraint ($Tinlet = 19$) and the fact that $Twin$ is always equal to $Tpin$, it is possible to have $TE22$ as a function of q_{wt}:

$$TE22 = \frac{19 - Tpin\ \eta_{t1}}{1 - \eta_{t1}} \tag{2.34}$$

Substituting equations (2.28) and (2.34) in equation (2.33) will result in the desired formula for $Tpout$:

$$Tpout = \frac{-(Tpin - 19)\ k\ \eta_{t1} + Tpin\ q_{ws} - Tpin\ \eta_{t1}\ q_{ws}}{(1 - \eta_{t1})\ q_{ws}} \tag{2.35}$$

Using the recent formula for $Tpout$, the inequality constraint ($Tpout \leq 40$) can be translated into the following inequality:

$$q_{ws} \leq \frac{(Tpin - 19)\ k\ \eta_{t1}}{(Tpin\ -\ 40)\ (1 - \eta_{t1})} \tag{2.36}$$

To summarize, the optimization problem transferred to an objective function of two variables (q_{wt} and q_{ws}) with two inequality constraint (inequality (2.36) and $q_{ws} \geq q_{wt}$). The typical feasible region in this case is sketched in Fig. 2.13 (supposed $TE21 = -30$, $Tpin = 60$ and $q_a = 1674.1$).

Solving the optimization problem in this case results in the same optimal values obtained in the previous case.

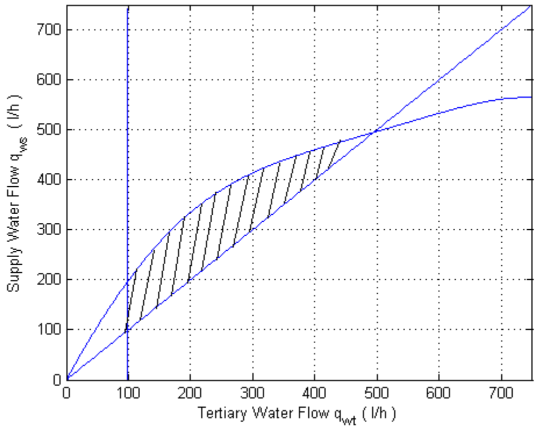

Figure 2.13: Feasible region while $q_{ws} \geq q_{wt}$ ($TE21 = -30$, $Tpin = 60$ and $q_a = 1674.1$)

2.2.4 Optimality Criteria

Solving the optimization problem in different conditions reveals that in all conditions supply water flow (q_{ws}) and tertiary water flow (q_{wt}) are equal. It also shows that using the air-to-air heat exchanger to produce required heat is cheaper than using the water-to-air heat exchanger. That is, the control strategy must be designed in such a way that maximum exploitation of the air-to-air heat exchanger is achieved. To summarize, to make the HVAC system perform optimally the control strategy has to be defined in a way that the following conditions are satisfied:

1. The maximum possible exploitation of the air-to-air heat exchanger is achieved.

2. In the steady state conditions the supply water flow (q_{ws}) must be equal to the tertiary water flow (q_{wt}). That is, it is optimal to make the system work in a way such that no water passes through the bypass pipe.

If the control strategy respects the mentioned conditions the HVAC system will perform in such a way that it will result in minimum thermal and electrical energy consumption.

2.3 Optimal Model-based Control

2.3.1 Controller Design

The mentioned HVAC system is going to be used for ventilation purposes. It means that the air flow (q_a) will be determined in accordance with the required ventilation and the inlet air temperature ($Tinlet$)

has to be kept at 19^oC. So, the optimal controller task is to track the set-point for the inlet air temperature while satisfying the conditions that were described in the control strategy section to guarantee the optimal performance of the system. The traditional way to design the control system that works in this way is applying the two-layer hierarchical control system. The lower layer performs direct regulatory control, where the aim is to maintain selected process variables at their desired set-point values, and the upper layer, known as the supervisory layer, has the task of determining the set-points of the regulatory controllers to obtain optimal steady state performance.

Looking at the linear model that was developed before reveals that the HVAC system can be split into two decoupled subsystems as follows:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3 & b_4 \\ b_1 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$

$$y = [1 \;\; 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{2.37}$$

$$T\dot{E}22 = a_6 \cdot TE22 + b_5 \cdot wrf \tag{2.38}$$

It means that control of the HVAC system can be considered as control of the air-to-air heat exchanger and control of the water-to-air heat exchanger separately. It should be noted that $TE22$ acts as disturbance for the water-to-air heat exchanger in this new formulation.

If the set-point for temperature of the fresh air that leaves the wheel ($TE22$) is defined as the set-point for temperature of the inlet air (19^oC) we will be sure that the air-to-air heat exchanger has its maximum contribution to warm up the fresh air. Thus, the first condition for optimality will be met. The second condition for optimality can be included in the cost function that will be defined for the water-to-air heat exchanger controller. Therefore, there is no need to design an explicit supervisory layer. The block diagram of the HVAC system along with the new optimal control system is illustrated in Fig 2.14.

2.3.2 Comparison of New Control System with Current Control System

Typical industrial HVAC control system is illustrated in Fig 2.15. As can be seen, two controllers, one to control the wheel and another one to control the water-to-air heat exchanger, which communicate to each other through some if-then rules are used. The main goal of this communication is to have the maximum exploitation of the wheel. So, current controllers respect the first optimality criterion but it has nothing to do to satisfy the second optimality condition. To summarize, we can say that the new control system is simpler because those two controllers are independent and the new control system is working in an optimal way. So, it meets all our expectations.

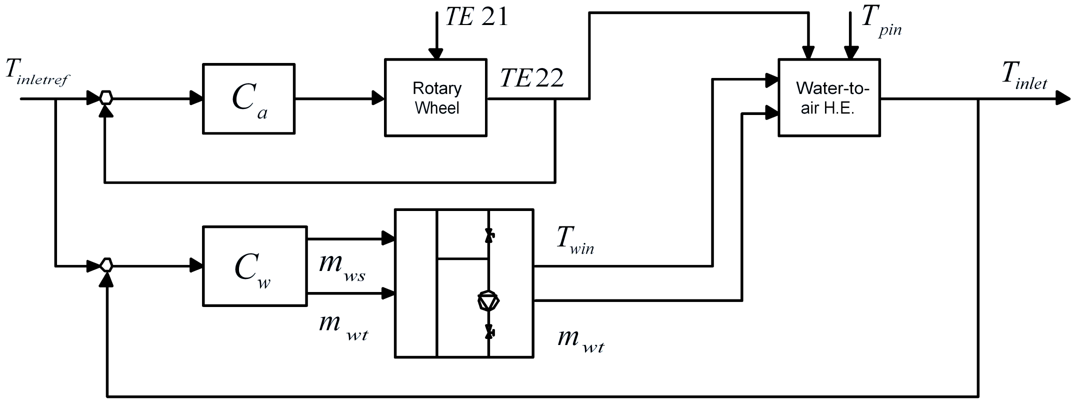

Figure 2.14: New control system

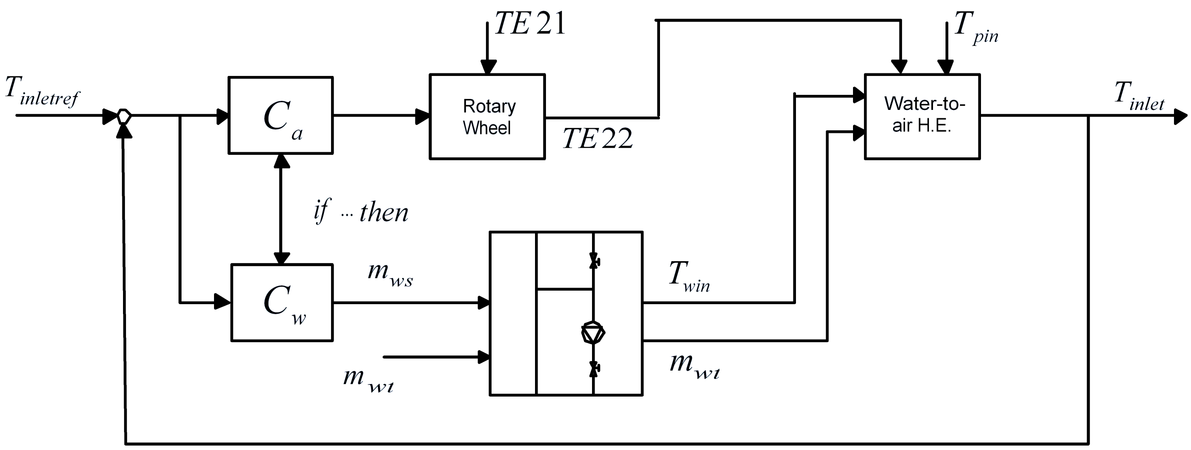

Figure 2.15: Typical industrial HVAC control system

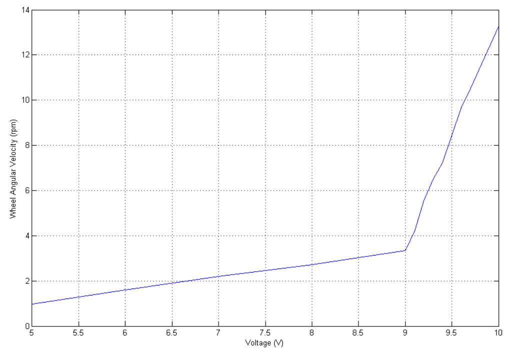

Figure 2.16: Wheel speed vs. voltage

2.3.3 Air-to-air Heat Exchanger Controller

To design controller for the rotary wheel we need to model the wheel actuators. To do so, several experiments were done. Fig 2.16 shows the relation between the voltage and the wheel speed. As can be seen in the Fig 2.16, the curve describing the relation between the voltage and the wheel speed can be approximated by two lines ($\frac{speed}{V} = 1/5$ for $0 \leq speed \leq 3$ and $\frac{speed}{V} = 10$ for $3 \leq speed \leq 10$). It should be noted that there is also a time delay varying from 6 seconds to 22 seconds while the speed of the wheel is going to change.

Fig 2.3 showed the normalized curve that describes the effect of the wheel speed on the efficiency of the wheel. This nonlinear curve also will be approximated by three lines ($\frac{wrf}{speed} = 8/15$ for $0 \leq speed \leq 1.6$, $\frac{wrf}{speed} = 4/55$ for $1.6 \leq speed \leq 3$ and $\frac{wrf}{speed} = 7/1000$ for $3 \leq speed \leq 10$).

Therefore, the rotary wheel along with actuators can be modeled as follows:

$$\frac{TE22}{V} = \frac{k(TE11 - TE21)(-1.0569 \times 10^{-4} q_a + 0.9943)}{\tau s + 1} e^{-Ts} \tag{2.39}$$

where:

$k \in \{8/75, 4/275, 7/100\}$

$6 \leq T \leq 22$

It should be noted that the outdoor air temperature ($TE21$) will perform as a disturbance through a first order system on the wheel. Thus, the model of the rotary wheel is a first order system along with varying gain, varying delay and disturbance. The input (v) is also constrained. These conditions indicate that a model predictive controller (MPC[1]) is a good choice for the control.

[1] The only advanced control methodology which has made a significant impact on industrial control engineering is predictive control. It is currently being increasingly applied in the process industry. The main reasons for its success in these applications are:

- It handles multivariable control problems naturally.

The control problem can be formulated as follows:

$$\min_{v[k/k]} \sum_{i=1}^{6} \|TE22[k+i/k]-19\|^2_{I(i)}$$

subject to:

$$0 \leq v[k/k] \leq 10 \tag{2.40}$$

The sampling time for the controller is supposed to be 15 seconds. The gain and delay for the internal model of the MPC controller are 1.7 (maximum gain) and 22 (maximum delay), respectively.

2.3.4 Water-to-air Heat Exchanger Controller

To control the water-to-air heat exchanger we have to deal with constrained inputs. We also have to penalize inputs in a way that in the steady state conditions no water passes through the the bypass pipe. So, again MPC is a good candidate for this control problem. To design the MPC controller we need to modify equation (2.37) as follows:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3+b_4 & b_4 \\ b_1+b_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} - \dot{m_{ws}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$

$$y = [1 \;\; 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{2.41}$$

where:

$a1 = -0.0352$	$a2 = 0.0310$	$a3 = 0.0564$
$a4 = -0.5961$	$a5 = 0.4833$	$b1 = 17232$
$b2 = 46628$	$b3 = 227635$	$b4 = -199119$

Thus, the control problem can be described as follows:

$$\min_{\dot{m_{ws}}[k+i/k],\dot{m_{wt}}[k+i/k]-\dot{m_{ws}}[k+i/k]} \sum_{i=1}^{6} \|Tinlet[k+i/k]-19\|^2_{I(i)}$$

- It can take account of actuator limitations.
- It allows operation closer to constraints (compared with conventional control), which frequently leads to more profitable operation. Remarkably short pay-back periods have been reported.
- Control update rates are relatively low in these applications, so that there is plenty of time for the necessary on-line computations.

$$+\sum_{i=0}^{1} \|\dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]\|^2_{(0.2\times I(i))}$$

subject to:

$$0 \leq \dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k] \leq 350 \tag{2.42}$$

The first term in the cost function represents the set-point tracking and the second term is the representative of the optimality condition (no flow in the bypass pipe). Unfortunately, the controller does not have a good performance because of the oscillations around the set-point. To deal with this problem the two following candidates are proposed:

- First Candidate:

$$\min_{\dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]} \sum_{i=1}^{6} \|Tinlet[k+i/k] - 19\|^2_{I(i)}$$

$$+\sum_{i=0}^{1} \|\dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]\|^2_{(0.2\times I(i))} + \sum_{i=0}^{1} \|\dot{m_{ws}}[k+i/k]\|^2_{(0.1\times I(i))}$$

subject to:

$$0 \leq \dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k] \leq 350 \tag{2.43}$$

- Second Candidate:

$$\min_{\dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]} \sum_{i=1}^{6} \|Tinlet[k+i/k] - 19\|^2_{I(i)}$$

$$+\sum_{i=0}^{1} \|\dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]\|^2_{(0.2\times I(i))} + \sum_{i=0}^{1} \|\Delta\dot{m_{ws}}[k+i/k]\|^2_{(0.1\times I(i))}$$

subject to:

$$0 \leq \dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k] \leq 350 \tag{2.44}$$

Both candidates show satisfactory results.
Analyzing of the results should be divided into two separate issues as follows:

- The first issue deals with actuator modeling of the water-to-air heat exchanger (primary valve which controls the supply water flow q_{ws} and the variable speed pump which has control over the tertiary water flow q_{wt}).

  The primary valve modeled as a simple gain. Due to the sample time of 15 seconds the dynamic behavior and the delay of the valve is not important.

When the zero voltage is applied to the pump, it will not shut down. So, the pump will keep running at the minimum speed and there will be a minimum tertiary flow. If we want to achieve a tertiary flow less than the minimum flow we have to apply a pulse-modulated voltage signal to the pump. Pulsing of the pump will cause some problems:

1. Short-circuit in the bypass pipe when the pump stops (delivering hot water to the return supply water that is not acceptable in hydronic systems)
2. Possible change in supply tertiary water temperature because of pump starting and stopping
3. Possible oscillations around the set-point ($Tinlet = Tref^oC$) due to pump pulsing

According to the above problems that the pump pulsing results in, applying a pulse-modulated voltage signal cannot be a good solution for the real life needs without bringing severe costs to the installation. Thus, a simple and fine real life solution can be a combination of the new control strategy and the current control strategy. That is, in the area that the applied voltage to the pump is not zero the new control strategy will be used but when the applied voltage to the pump is zero and less thermal energy is required the current control strategy will perform.

Both controllers successfully follow the mentioned hybrid strategy while the set-point is perfectly tracked.

- The second issue is about the flow in the bypass pipe. Fig 2.17 illustrates the four water temperatures around the bypass pipe (supply primary water temperature ($Tpin$), supply tertiary water temperature ($Twin$), return primary water temperature ($Tpout$), and return tertiary water temperature ($Twout$)). Comparing $Tpin$ and $Twin$ shows switching between the new control strategy and the Exhausto control strategy and then switching between the Exhausto control strategy and the new control strategy. Comparing $Tpout$ and $Twout$ reveals that apart from the time that the controller follows the Exhausto control strategy there is always a short circuit. That is, the return primary water is warmed up. This is prohibited by the Copenhagen Building Regulations. Next section will be dedicated to dealing with this problem.

2.4 Bypass Flow Problem

It was explained that the controllers in the previous section showed good performance. However, they had a severe problem: bypass flow problem. This problem is more destructive when the primary water flow is more than the tertiary water flow. In the recent case not only the controller stays away from the optimal performance but also it violates the constraints (It is not allowed to warm up the water returning to the boiler). Thus, it is vital to solve this problem.

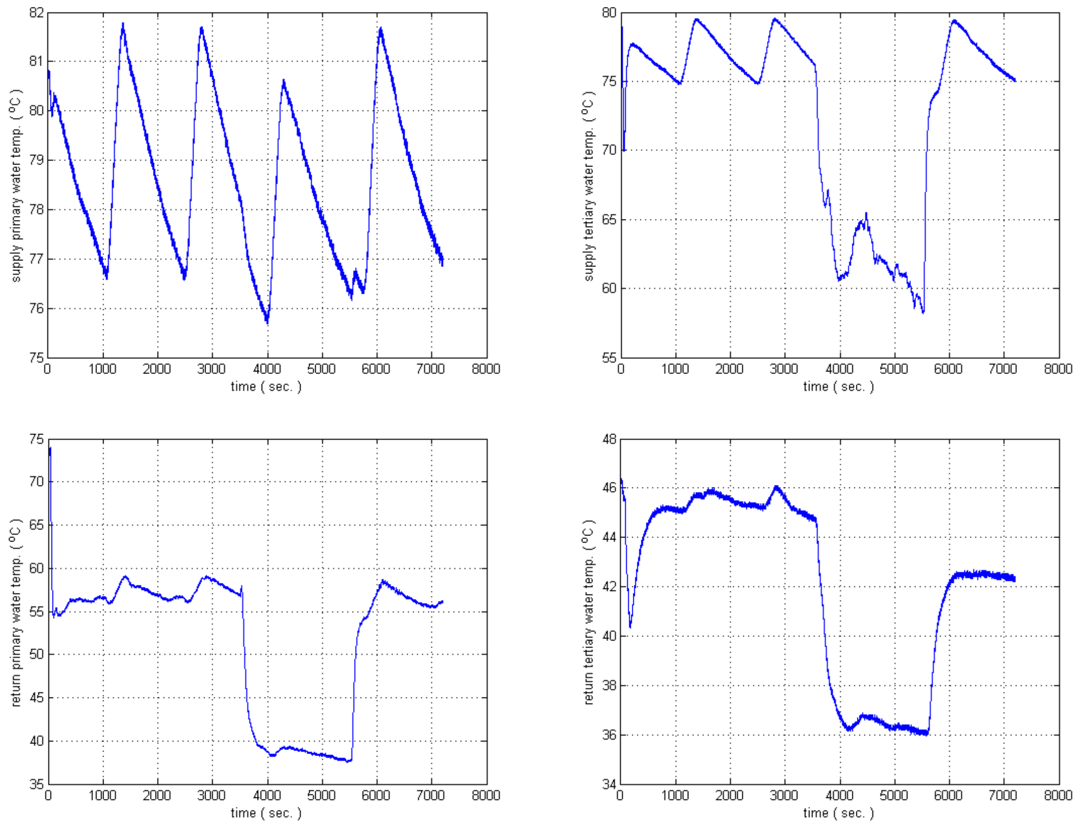

Figure 2.17: Four temperatures around the bypass pipe

2.4.1 Measuring The Bypass Flow

It is not reasonable to measure the bypass flow through a flow-meter in real life. However, we can measure the bypass flow implicitly through thermocouples. This way of measurement is acceptable due to the cheap price of thermocouples.

First we have to define the bypass flow as a quantity with direction. Consequently, we will always treat bypass flow as a difference between the primary water flow and the tertiary water flow ($q_{ws} - q_{wt}$). That is, when the primary flow is greater than the tertiary flow the bypass flow will have a positive sign and when the primary flow is less than the tertiary flow a negative sign will accompany the bypass flow. We consider two cases now:

- Negative Bypass Flow

  When there is a negative bypass flow the return primary water temperature ($Tpout$) and the return tertiary water temperature ($Twout$) are equal. Although, the supply primary water temperature ($Tpin$) is always greater than the supply tertiary water temperature ($Twin$). The difference between two recent temperatures is proportional to the ratio of the primary water flow and the tertiary water flow.

  The energy balance equation for the supply water side will result in the following equation:

  $$(q_{wt} - q_{ws})\, Twout \;+\; q_{ws}\, Tpin \;=\; q_{wt}\, Twin \tag{2.45}$$

  By rearranging the above equation we will have:

  $$\frac{q_{wt}}{q_{ws}} \;=\; \frac{Tpin \;-\; Twout}{Twin \;-\; Twout} \tag{2.46}$$

  We subtract 1 from both side of the equation. So,

  $$\frac{q_{wt}}{q_{ws}} - 1 \;=\; \frac{Tpin \;-\; Twin}{Twin \;-\; Twout} \tag{2.47}$$

- Positive Bypass Flow

  The story of the negative bypass flow is similar to the story of the positive bypass flow. Thus, When there is a positive bypass flow the supply primary water temperature ($Tpin$) and the supply tertiary water temperature($Twin$) are equal. Although, the return primary water temperature ($Tpout$) is always greater than the supply tertiary water temperature ($Twout$). Again, the difference between two recent temperatures is proportional to the ratio of the primary water flow and the tertiary water flow.

The energy balance equation for the return water side will result in the following equation:

$$(q_{ws} - q_{wt})\, Tpin \,+\, q_{wt}\, Twout \,=\, q_{ws}\, Tpout \tag{2.48}$$

By rearranging the above equation we will have:

$$\frac{q_{wt}}{q_{ws}} = \frac{Tpin - Tpout}{Tpin - Twout} \tag{2.49}$$

We subtract 1 from both sides of the equation. So,

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{Twout - Tpout}{Tpin - Twout} \tag{2.50}$$

The recent equation can be rewritten as

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{Twout - Tpout}{Twin - Twout} \tag{2.51}$$

According to the above discussion and combining equations 2.47 and 2.51 we will have:

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{(Tpin - Twin) + (Twout - Tpout)}{Twin - Twout} \tag{2.52}$$

or equivalently:

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{(Tpin + Twout) - (Twin + Tpout)}{Twin - Twout} \tag{2.53}$$

So, by measuring the four temperatures ($Tpin$, $Tpout$, $Twin$, and $Twout$) and using the recent formula there will be enough information for a controller to make the bypass flow approach zero.

2.4.2 Slow Bypass Compensation

As it was mentioned before, it is vital to control the bypass flow. In the previous section designing controllers without considering this problem was discussed. Fig 2.18 introduces a new structure for the control system. In the new structure to design the MPC controller the same procedures as it was explained have to be followed. That is, the MPC controller is the controller from the previous section. To deal with the bypass problem, a bypass compensator has been added to the control system. The bypass compensator is much slower than the main controller. So, the main controller and the bypass compensator are decoupled in time domain.

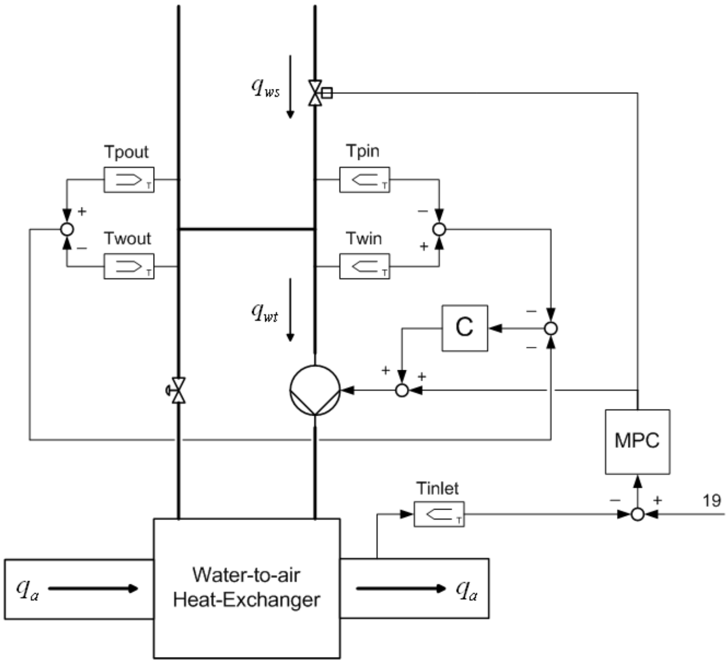

Figure 2.18: MPC controller along with bypass compensator

2.4.3 Simplified Optimal Control System Scheme

Fig 2.19 shows the simplified optimal control structure. One PI controller determines the primary water flow (q_{ws}) through the information from the inlet temperature feedback. To design this controller the linearized model from the primary water flow to the inlet temperature has been used. Tertiary water flow (q_{wt}) is controlled by a PI controller which tries to keep the tertiary water flow close to the primary water flow. The variable speed pump acts as an actuator to control the tertiary water flow. Because the variable speed pump is much faster than the primary valve which acts as an actuator to control the primary water flow, two controllers are decoupled in time domain again.

2.4.4 Optimal Solution when Applying Improperly Dimensioned Coil Applied

Fig. 2.20 shows the return primary water temperature while the simplified optimal controller was applied to the water-to-air heat exchanger. Considering the plot reveals that in some points the return primary water temperature is higher than 40^oC. It is so because of the inappropriate dimensioning of the coil. The coil is designed for 60^oC forward water temperature and 40^oC return water temperature. Thus, in some extreme situations the controller cannot keep the return water temperature less than 40^oC while controlling the inlet temperature. If it is not possible to apply a pump which is appropriately dimensioned the remedy can be forcing the controller C_2 to mix more. It can be embedded in the simplified optimal

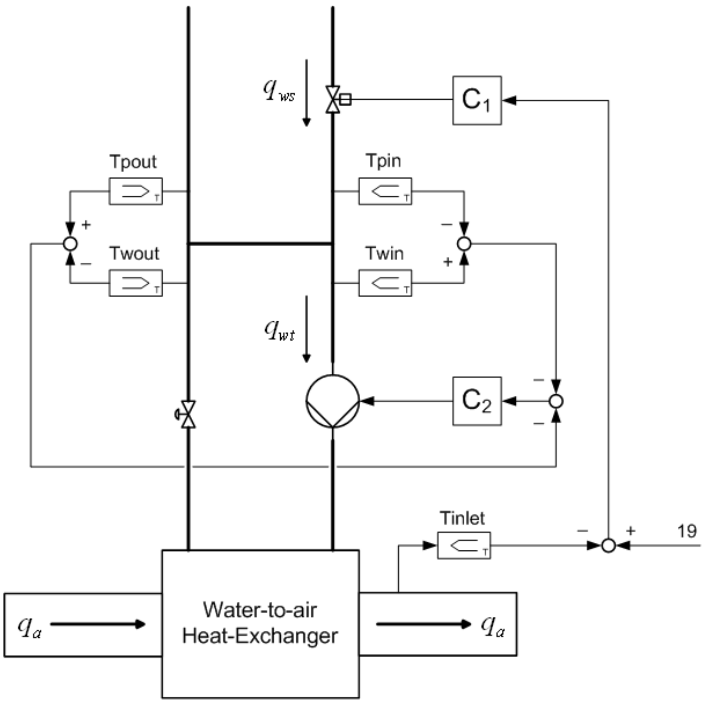

Figure 2.19: Simplified optimal control scheme

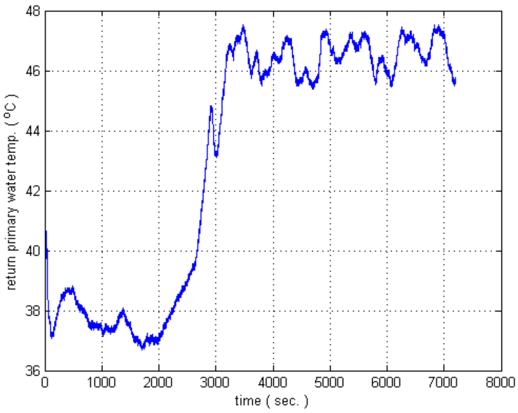

Figure 2.20: Return primary water temperature

controller as follows:

When the mixing happens the energy balance equation for the supply water side will result in the following equation:

$$(q_{wt} - q_{ws})\ Twout\ +\ q_{ws}\ Tpin\ =\ q_{wt}\ Twin \tag{2.54}$$

By rearranging the above equation we will have:

$$\frac{q_{wt}}{q_{ws}}\ =\ \frac{Tpin\ -\ Twout}{Twin\ -\ Twout} \tag{2.55}$$

Now we need not to keep the primary water flow and tertiary water flow to meet the optimality criteria. We have to mix more to lower the return primary water temperature. As a result, we subtract 1+x from both side of the equation:

$$\frac{q_{wt}}{q_{ws}}\ -(1+x)\ =\ \frac{Tpin\ -\ Twin}{Twin\ -\ Twout}+\frac{\alpha\ f_1}{Twin\ -\ Twout} \tag{2.56}$$

where

$x = \frac{\alpha\ f_1}{Twin\ -\ Twout}$ and $f_1 = \begin{cases} Tpout - 40 & if \quad Tpout > 40 \\ 0 & if \quad Tpout \leq 40 \end{cases}$

For the return water side there is no need of manipulation in the equations. Therefore, the feedback can be introduced as following:

$$\frac{q_{wt}}{q_{ws}}\ -(1+x)\ =\ \frac{(Tpin\ +\ Twout)\ -\ (Twin\ +\ Tpout)+\alpha\ f_1}{Twin\ -\ Twout} \tag{2.57}$$

We expect that in some extreme situations even when the pump runs in full speed still having the return primary water temperature less than 40^oC is impossible.

Fig. 2.21 shows the optimal trajectory of the controller when the coil is not selected properly:

- Region 1 represents the situation that the variable speed pump runs at its minimum speed and to control the inlet temperature the controller has to follow the mixing strategy.
- Region 2 is the representative of the situation that the controller follow the optimality criteria.
- Region 3 follows when the return primary water temperature exceeds 40^oC. Thus, the controller starts to mix more to keep the return primary water temperature less than 40^oC.
- Region 4 happens when the tertiary variable speed pump needs to run at its full speed to lower the return primary water temperature. We can expect that the return primary water temperature sometimes goes above 40^oC in this region.

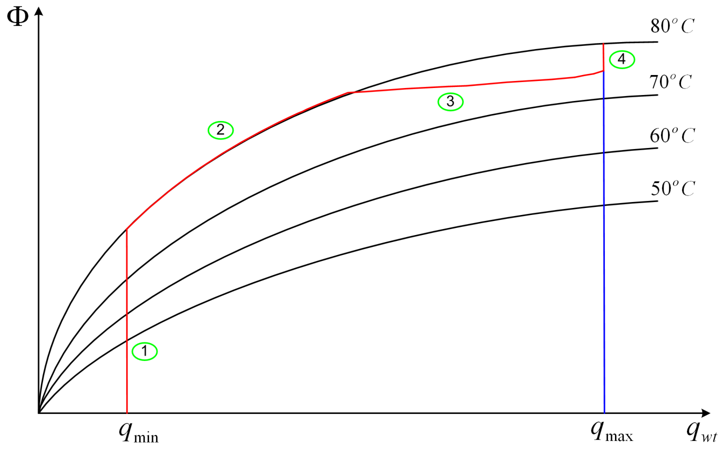

Figure 2.21: Optimal trajectory

Chapter 3

Conclusions and Future Work

3.1 Conclusions

The goal of this project was to deal with optimal model-based control of a HVAC system. The difference between the HVAC system which was applied in this project with other HVAC system was using variable water flow through the coil. As a result in the new HVAC system the water flow through the coil could be manipulated as a control variable. It resulted in less energy consumption by the pump which supplied the coil.

HVAC systems are in steady state conditions more than 95% of their operating time. Thus, the optimal control strategy that we chose in this project was based on the static model of the HVAC system. The objective function was composed of the electrical power for different components, encompassing fans, primary/secondary pump, tertiary pump, and air-to-air heat exchanger wheel; and a fraction of thermal power used by the HVAC system. The objectives that had to be achieved by the HVAC system appeared as constraints in the optimization problem. Solving the optimization problem resulted in two optimality criteria:

- The maximum advantage of the rotary wheel had to be taken. That is, the water-to-air heat exchanger has to be applied for warming up the fresh air when the rotary wheel is not capable of doing that on its own.
- No water should pass through the bypass pipe (the pipe which is used to decouple the tertiary hydronic circuit from the supply (primary/secondary) hydronic circuit).

Then the optimal model-based controller was designed to follow the objectives of the HVAC system while the optimality criteria were satisfied. Lack of good models from control point of view in the literatures made us to develop our own models for the HVAC system:

- We extended the nonlinear model which had proposed by Underwood and Crawford for the coil to model the water-to-air heat exchanger.
- For model-based control of the rotary heat recovery wheel we needed a model that described the relation between the rotation speed of the wheel and the temperature of the fresh air which was leaving the heat exchanger. We estimated the steady state gain by benefiting from the results of the static analysis part. Then we discussed that a first order model could capture the dynamic behavior of the rotary heat recovery wheel. So, totally the model would be a first order system along with a variable gain.

The next step we took was analyzing the HVAC system model and designing the controller. Thus, the HVAC system was broken into two independent subsystems. Utilizing the excellent features of the model predictive control (MPC) and introducing an internal feedback in the system the optimality criteria were met.

To deal with the problem of the bypass flow which came up during the design of the controller for the HVAC system led us to a simple optimal control structure for the HVAC system. Finally, we addressed some pratical issues.

3.2 Future Work

In Denmark radiators are used as a source for warming up the room and HVAC systems are applied as a tool for desired ventilation. In future work including the interaction between the radiators and the HVAC system in developing the optimal control strategy can lead to more comprehensive optimality criteria. It means that HVAC system can also get involved in supplying heat to the room and the optimality criteria must determine the HVAC system's share in that task.

Another interesting point can be inclusion of different HVAC modules which are supplied by the same boiler in the control problem. There are several interesting control problems which have to be solved in this configuration. One of the most important problems in this case is the oscillation problem. The problem stems from the nonlinear behavior of HVAC systems. Therefore, dealing with this problem requires sophisticated modeling and applying advanced control techniques.

Sometimes during the start-up or low heat demand time the heat coil can get frosted and it will cause damages to the system. As a result, developing an anti-frost control for the system is neccessary. Then, integration of the anti-frost control algorithm and the optimal control algorithm will be a very interesting problem.

In this project we focused on the heating problem. In future work including the cooling problem as a task for the HVAC system and extending the optimality criteria for that system also looks a charming problem as the chillers need some specific cares.

Chapter 4

Optimal Set-point Synthesis in HVAC Systems

M. Komareji [1] , J. Stoustrup [2] , H. Rasmussen [3] , N. Bidstrup [4] , P. Svendsen [5] , F. Nielsen [6]

Abstract

This paper presents optimal set-point synthesis for a heating, ventilating, and air-conditioning (HVAC) system. This HVAC system is made of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger. The objective function is composed of the electrical power for different components, encompassing fans, primary/secondary pump, tertiary pump, and air-to-air heat exchanger wheel; and a fraction of thermal power used by the HVAC system. The goals that have to be achieved by the HVAC system appear as constraints in the optimization problem. To solve the optimization problem, a steady state model of the HVAC system is derived while different supplying hydronic circuits are studied for the water-to-air heat exchanger. Finally, the optimal set-points and the optimal supplying hydronic circuit are resulted.

[1]Mohammad Komareji is a PhD student in The Department of Control and Automation, Institute of Electronic Systems, Aalborg University, Aalborg, Denmark; komareji@es.aau.dk

[2]Jakob Stoustrup is with Aalborg University as a Professor in The Department of Control and Automation; jakob@es.aau.dk

[3]Henrik Rasmussen is with Aalborg University as an Associate Professor in The Department of Control and Automation; hr@es.aau.dk

[4]Niels Bidstrup is with Grundfos Management A/S as a Chief Engineer, Ph.D.; nbidstrup@grundfos.com

[5]Peter Svendsen is with Danish Technological Institute (DTI) as a Project Manager; Peter.Svendsen@teknologisk.dk

[6]Finn Nielsen is with Exhausto A/S as a Project Manager; FNI@exhausto.dk

4.1 Introduction

Classical HVAC control techniques such as the ON/OFF controllers (thermostats) and the proportional-integral-derivative (PID) controllers are still very popular because of their low cost. However, in the long run, these controllers are expensive because they operate at a non-optimal efficiency. So, there is a high potential to apply advanced control methods to save large amount of energy. For example, by optimal control of HVAC systems almost 100 GWh energy can be saved yearly in Denmark (five million inhabitants) [2].

A common method used to maintain an industrial plant at its optimal operating condition is to calculate optimal values of feedback controller set-points, employing a steady-state mathematical model of the process [3], [4]. Steady-state optimization of an industrial process often considers that the overall control is performed within a two-layer hierarchical structure. The lower layer performs direct regulatory control, where the aim is to maintain selected process variables at their desired set-point values, and the upper layer, known as the supervisory layer, has the task of determining the set-points of the regulatory controllers to obtain optimal steady-state performance.

This kind of two layer control strategy has been applied before to a cooling system and a refrigeration system and has shown great results [5], [6]. Implementing the supervisory layer through genetic algorithms in the cooling system case showed saving energy by 19.5%. In the refrigeration system case it was proved that by using this control configuration it was possible to derive the set-points close to the optimum and thus reduce the energy consumption with up to 20%.

In this paper, the supervisory layer of the overall control of a HVAC system is considered. In Section 4.2, the HVAC system used in analysis is described. Section 4.3 presents formulation of the problem. Determination of optimal set-points through solving the defined problem is presented in Section 4.4. Section 4.5 presents conclusions and final comments.

4.2 The HVAC System Description

The HVAC system that will be considered consists of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger. In this section the temperature efficiency of these two heat exchangers, which can be used as a steady state model of heat exchangers, will be described.

4.2.1 The Air-to-air Heat Exchanger

The air-to-air heat exchanger is a rotary heat exchanger in aluminum, with low pressure loss (shown in Fig. 4.1). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow. Here, it is supposed that the ratio of the supply air flow to

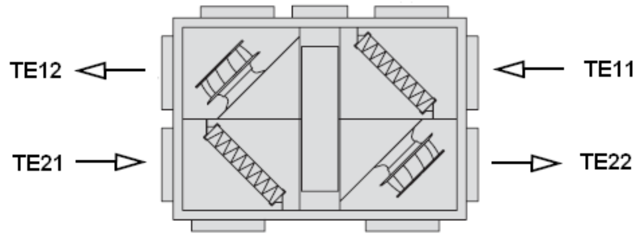

Figure 4.1: The Air-to-air heat exchanger scheme

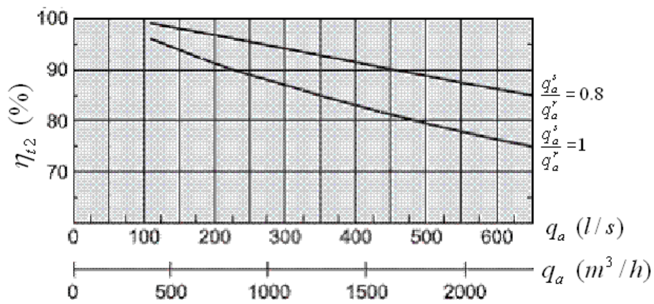

Figure 4.2: Dependency of η_{t2} on q_a while n=10 rpm; q_a^s and q_a^r represent supply air flow and return air flow, respectively.

the return air flow is one. Therefore, η_{t2} will be a function of air flow (q_a) [7] , that is the same for both supply and return air, and the rotation speed of the wheel (n).

In this context, results of testing the rotary heat exchanger that was performed according to European Standard for laboratory testing of air-to-air heat recovery devices (EN 247, EN 305, EN 306, EN 307, EN 308) will be used.This European Standard is intended to be used as a basis for testing heat recovery devices for HVAC systems, which as specified in EN 247 consist of the heat exchanger itself installed in a casing having the necessary air duct connecting elements and in some cases the fans and pumps, but without any additional components of the HVAC system.

According to results of the test, it is possible to specify η_{t2} as a multiplication of two functions. Fig. 4.2 and 4.3 illustrate these functions. Therefore, η_{t2} can be described as following:

$$\eta_{t2} = (-1.0569 \cdot 10^{-4}\, q_a + 0.9943) \cdot wrf(n) \tag{4.1}$$

4.2.2 The Water-to-air Heat Exchanger

The water-to-air heat exchanger is shown in Fig. 4.4. As can be seen, a primary/secondary-tertiary hydronic circuit supplies the heat exchanger with hot water. The air flow that passes the hot coil is controllable by changing the speed of the fan installed in the air-to-air heat exchanger.

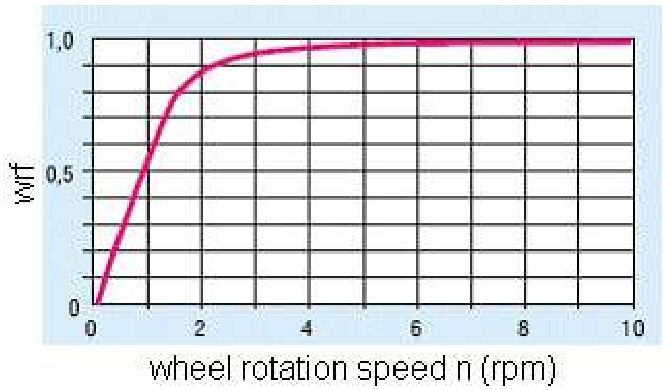

Figure 4.3: Normalized dependency of η_{t2} on n

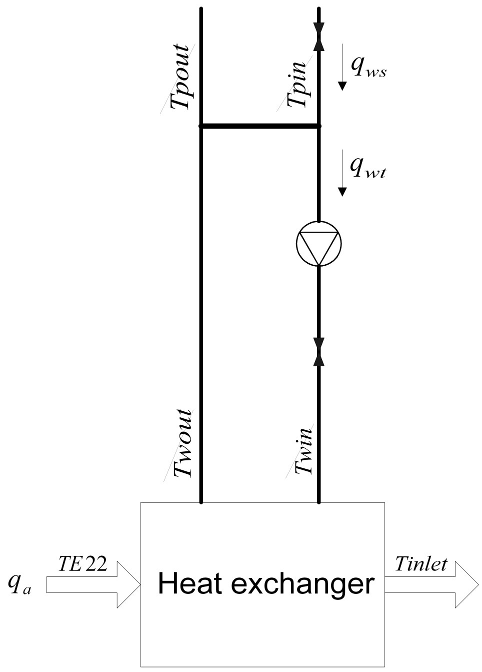

Figure 4.4: The water-to-air heat exchanger scheme

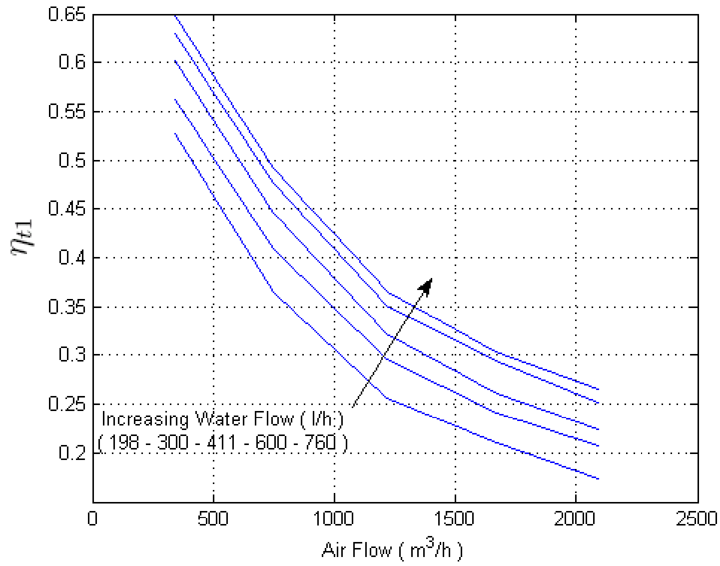

Figure 4.5: Result of experiments on water-to-air heat exchanger

Here, temperature efficiency (η_{t1}) is a function of hot water flow (q_{wt}) and air flow (q_a). To obtain this function several experiments were done. Results are illustrated in Fig. 4.5. Again, it is possible to describe η_{t1} as a multiplication of two functions that the first one depends only on air flow (q_a) and the second one depends only on water flow (q_{wt}):

$$\eta_{t1} = \frac{1}{0.3215}\,(a\;q_{wt}^4 + b\;q_{wt}^3 + c\;q_{wt}^2 + d\;q_{wt} + e)\cdot(A\;q_a^3 + B\;q_a^2 + C\;q_a + D) \qquad (4.2)$$

where:

$a = -5.399 \cdot 10^{-12}$	$A = 1.0665 \cdot 10^{-10}$
$b = 1.0733 \cdot 10^{-8}$	$B = -1.643 \cdot 10^{-7}$
$c = -7.887 \cdot 10^{-6}$	$C = -2.880 \cdot 10^{-4}$
$d = 2.7199 \cdot 10^{-3}$	$D = 0.6927$
$e = 8.3711 \cdot 10^{-4}$	

4.3 Problem Formulation

As it was mentioned, the aim of this paper is to find the optimal set-points for the described HVAC system. Thus, an objective function is needed to formulate the problem. The HVAC system mission can be described as constraints for the defined objective function.

4.3.1 Objective Function

The desired objective function is defined as following:

$$J = P_{pt} + P_{pp} + P_f + P_w + \Phi/2.5 \tag{4.3}$$

subject to:

$$q_a = q_{a_0}$$

$$Tinlet = 19$$

$$Tpout \leq 40$$

and,

$$0 \leq q_{wt} \leq 743$$

$$0 \leq q_{ws} \leq 1400$$

$$300 \leq q_a \leq 2200$$

$$0 \leq n \leq 10$$

where:

P_{pt} , P_{pp}, P_f , P_w , and Φ are tertiary pump power, primary pumping power, fan power, wheel rotation power, and thermal power, respectively.

q_{a_0} is a constant that will be determined in accordance with the required ventilation [8]. This formulation discuss a typical HVAC system used for ventilation purposes.

4.3.2 Tertiary Pump Power (P_{pt})

The hydronic circuit that is used for supplying the water-to-air heat exchanger is a primary or secondary -tertiary circuit isolated from each other by a bypass pipe. The bypass pipe is a short length of full bore piping. The pressure drop across the bypass pipe is then small compared to the pressure drop in the tertiary circuit and through the supply circuit [9]. The supply water flow (q_{ws}) is controlled by the motorized primary/secondary valve. A variable speed pump and a valve is installed in the tertiary circuit. The tertiary valve is used to set the desired maximum flow rate through the variable speed pump. By changing the speed of the tertiary pump, it is possible to sweep the interval between 330 (l/h) and 743 (l/h) for the tertiary water flow (q_{wt}). If the tertiary water flow has to be less than 330 (l/h), the pump will be pulsed. Fig. 4.6 illustrates power of the tertiary pump as a function of q_{wt}. This curve is approximated by the following polynomial:

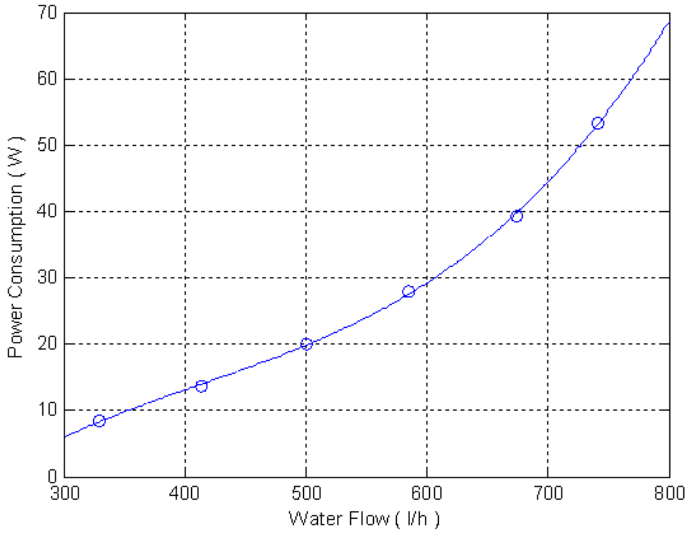

Figure 4.6: Tertiary pump power vs q_{wt}

$$P_{pt} = A_p\, q_{wt}^3 + B_p\, q_{wt}^2 + C_p\, q_{wt} + D_p \tag{4.4}$$

where:

$A_p = 5.1873 \cdot 10^{-7}$ $B_p = -6.4260 \cdot 10^{-4}$

$C_p = 3.2906 \cdot 10^{-1}$ $D_p = -48.8641$

When the pump is pulse width modulated, it is assumed that the power of the pump is the duty cycle fraction of the pump power while it is running at its minimum speed, i.e. it is proportional to the pump working period.

4.3.3 Primary Pumping Power(P_{pp})

The primary/secondary pumping power has to be measured implicitly because there is no direct access to the primary/secondary pump. Therefore, it is supposed that the efficiency of the pump in converting electrical power to hydraulic power is 50%. The curve indicating required pressure drop versus primary/secondary water flow (q_{ws}) is shown in Fig. 4.7. A second order polynomial is used to represent this curve. As we know, multiplying water flow by head results in hydraulic power. So, primary pumping power can be expressed as follows:

$$P_{pp} = \frac{2}{3600}\, q_{ws} \cdot \left(-4.8131 \cdot 10^{-7}\, q_{ws}^2 - 8.5955 \cdot 10^{-3}\, q_{ws} + 43.1390\right) \tag{4.5}$$

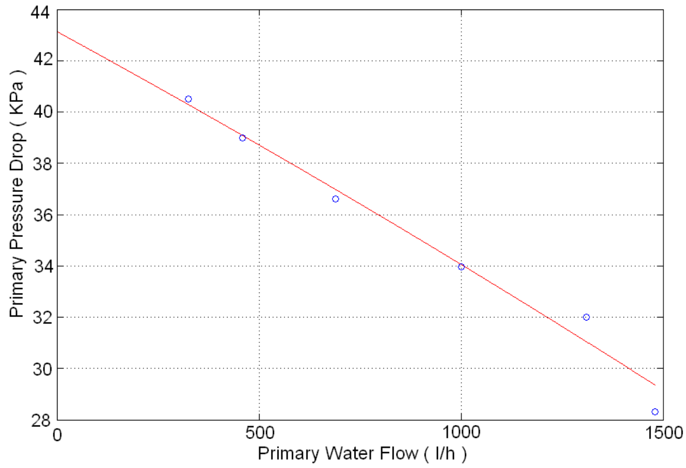

Figure 4.7: Primary pressure drop vs q_{ws}

4.3.4 Fan Power(P_f)

The HVAC system structure is assumed fixed during the entire work. As a result, the path for the air does not change. So, it is possible to have fan power as a function of air flow (q_a). Fig. 4.8 illustrates this function. The curve is approximated by a third order polynomial as follows:

$$P_f = A_f\, q_a^3 + B_f\, q_a^2 + C_f\, q_a + D_f \tag{4.6}$$

where:

$A_f = 4.7354 \cdot 10^{-8}$ $\qquad B_f = 6.705 \cdot 10^{-5}$

$C_f = -3.2527 \cdot 10^{-2}$ $\qquad D_f = 40.3043$

4.3.5 Wheel Rotation Power(P_w)

The electrical power input to the wheel as a function of rotation speed of the wheel is sketched in Fig. 4.9. The step change in the figure is due to the frequency converter used to control rotation speed of the wheel.

As Fig. 4.9 reveals, it is feasible to assume that the electrical power of the wheel is composed of three parts:

$$\begin{array}{lcc} P_w = 23.5\ \ W & 10\,rpm \geq n > 2.5\,rpm & (1 \geq wrf > 0.9) \\ P_w = 13\ \ W & 2.5\,rpm \geq n > 0\,rpm & (0.9 \geq wrf > 0) \\ P_w = 0\ \ W & n = 0\,rpm & (wrf = 0) \end{array} \tag{4.7}$$

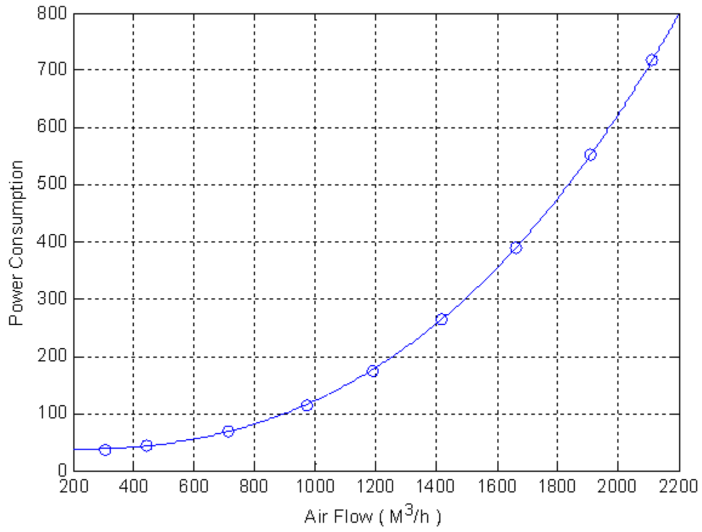

Figure 4.8: Fan power vs air flow (q_a)

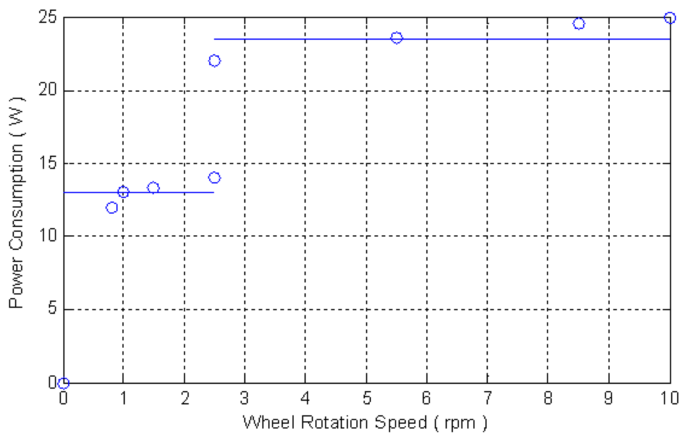

Figure 4.9: Wheel power consumption vs n

4.3.6 Thermal Power (Φ)

Φ is the thermal power that is being used by the water-to-air heat exchanger:

$$\Phi = \rho_w \, q_{wt} \, C_w \, (Twin - Twout) \tag{4.8}$$

In steady state conditions:

$$\rho_w \, q_{wt} \, C_w \, (Twin - Twout) = \rho_a \, q_a \, C_a \, (Tinlet - TE22) \tag{4.9}$$

According to the definitions, we have:

$$Tinlet = TE22 + \eta_{t1} \, (Twin - TE22) \tag{4.10}$$

$$TE22 = TE21 + \eta_{t2} \, (TE11 - TE21) \tag{4.11}$$

Substituting (4.10) and (4.11) in (4.9) will result in:

$$\Phi = \rho_a \, q_a \, C_a \, \eta_{t1} \, (Twin - TE21) + \rho_a \, q_a \, C_a \, \eta_{t1} \, \eta_{t2} \, (TE21 - TE11) \tag{4.12}$$

This formula will be used for computing the thermal power consumption. Thermal power is divided by 2.5 in the objective function because thermal power is 2.5 time cheaper than electrical power according to building regulations in Denmark.

4.4 Determining Optimal Set-points

To obtain optimal set-points, the optimization problem that was defined in the previous section has to be solved.

In this section solving the defined optimization problem is presented while two different cases are assumed for the hydronic circuit. In the first case it is assumed that $q_{ws} \leq q_{wt}$. In the second case we will have $q_{ws} \geq q_{wt}$. These two cases are selected because they are the most general cases.

4.4.1 Computing Optimal Set-points While $q_{ws} \leq q_{wt}$

According to the discussion so far, the optimization problem consists of a four-variable (q_a, q_{ws}, q_{wt}, n) objective function along with two equality constraint and two inequality constraints. Because q_a will be determined in accordance with required ventilation, actually we have to deal with a three-variable optimization problem along with an equality constraint and two inequality constraints. Thus, in the sequel by optimization problem we mean the latter statement. For convenience we will deal with $TE22$ instead of n in the procedure of solving.

Because, in this case, supply water flow (q_{ws}) is always less than or equal to the tertiary water flow (q_{wt}), mixing between the supply water flow that enters the tertiary circuit and a part of the tertiary return water flow occurs. So, the temperature of the water that enters the heat exchanger is as follows:

$$Twin = \frac{q_{ws} \cdot Tpin + (q_{wt} - q_{ws}) \cdot Twout}{q_{wt}} \tag{4.13}$$

Actually, the mission of the motorized primary/secondary valve is controlling $Twin$ by changing the supply water flow (q_{ws}).

As it was mentioned, to solve the optimization problem here we have to deal with three variables q_{ws}, q_{wt}, and $TE22$. One of these variables is dependent due to equality constraint (Tinlet = 19):

$$Tinlet = (1 - \eta_{t1}) \cdot TE22 + \eta_{t1} \cdot Twin \tag{4.14}$$

As we know, the energy balance equation in a water-to-air heat exchanger is as follows:

$$\rho_w \, q_{wt} \, C_w \, (Twin - Twout) = \rho_a \, q_a \, C_a \, (Tinlet - TE22) \tag{4.15}$$

Substituting the two recent equations in equation (2.26) will result in a formula for q_{ws} versus q_{wt} and $TE22$:

$$q_{ws} = \frac{k \;\; \eta_{t1} \;\; q_{wt} \; (19 - TE22)}{\eta_{t1} q_{wt} (Tpin - TE22) + (q_{wt} - k\eta_{t1})(TE22 - 19)} \tag{4.16}$$

Also, in this case $q_{ws} \leq q_{wt}$; Therefore, substituting equation (4.16) in the recent inequality results in an inequality as following:

$$TE22 \geq \frac{19 - Tpin \; \eta_{t1}}{1 - \eta_{t1}} \tag{4.17}$$

Combining equations (4.14) and (4.15) results in a formula for $Twout$:

$$Twout = \frac{19q_{wt} + TE22(-q_{wt} + \eta_{t1} q_{wt} + k\eta_{t1}) - 19k\eta_{t1}}{\eta_{t1} q_{wt}} \tag{4.18}$$

In this case, $Twout$ is equal to $Tpout$ because the supply water flow is less than or equal to the tertiary water flow. Thus, the inequality constraint ($Tpout \leq 40$) can be translated into the following inequality:

$$(-q_{wt} + \eta_{t1} q_{wt} + k\eta_{t1}) \; TE22 \leq 40\eta_{t1} q_{wt} + 19k\eta_{t1} - 19q_{wt} \tag{4.19}$$

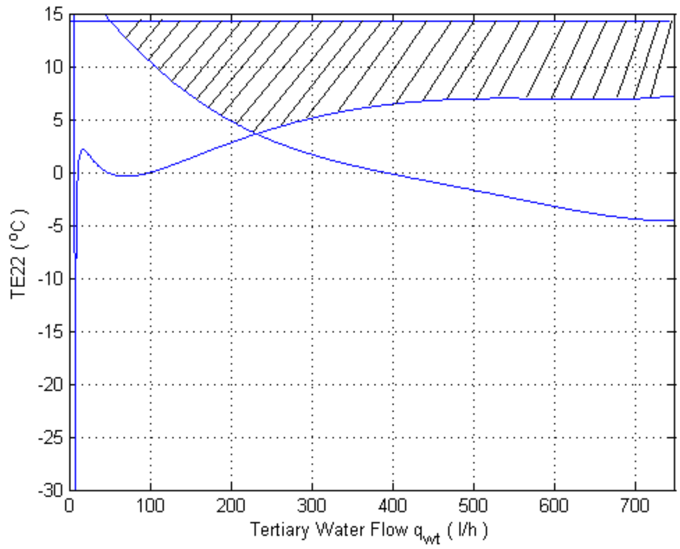

Figure 4.10: Feasible region while $q_{ws} \leq q_{wt}$ ($TE21 = -12$, $Tpin = 80$ and $q_a = 2104.9$)

Finally, the optimization problem transferred to an objective function of two variables (q_{wt} and $TE22$) with two inequality constraints (inequalities (4.17) and (4.19)). The typical feasible region of this optimization problem is shown in Fig. 4.10 (assuming $TE21 = -12$, $Tpin = 80$ and $q_a = 2104.9$). Optimal set-points in different conditions as a result of solving the optimization problem can be found in Table 4.1.

4.4.2 Computing Optimal Set-points While $q_{ws} \geq q_{wt}$

We have to deal with a three-variable optimization problem along with an equality constraint and two inequality constraints again. The only difference is the fact that supply water flow is greater than or equal to tertiary water flow. The impact of keeping supply water flow higher than or equal to the tertiary water flow on the system is that the supply tertiary water flow to the water-to-air heatexchanger will not be mixed water. So, $Twin$ will be equal to the $Tpin$. Actually, water mixing occur between the return tertiary water flow (q_{ws}) and the hot water passes the balance pipe ($q_{ws} - q_{wt}$). Therefore, we have:

$$Tpout = \frac{q_{wt} \cdot Twout + (q_{ws} - q_{wt}) \cdot Tpin}{q_{ws}} \tag{4.20}$$

According to the equality constraint ($Tinlet = 19$) and the fact that $Twin$ is always equal to $Tpin$, it is possible to have $TE22$ as a function of q_{wt}:

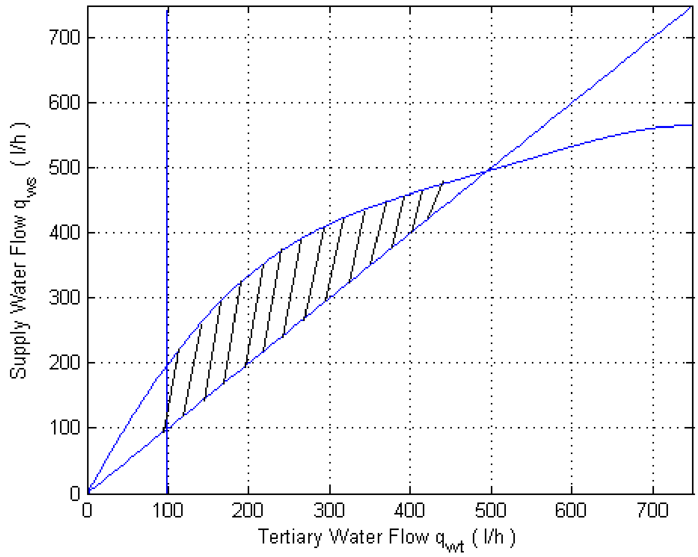

Figure 4.11: Feasible region while $q_{ws} \geq q_{wt}$ ($TE21 = -30$, $Tpin = 60$ and $q_a = 1674.1$)

$$TE22 = \frac{19 - Tpin\ \eta_{t1}}{1 - \eta_{t1}} \tag{4.21}$$

Substituting equations (4.15) and (4.21) in equation (4.20) will result in the desired formula for $Tpout$:

$$Tpout = \frac{-(Tpin - 19)\ k\ \eta_{t1} + Tpin\ q_{ws} - Tpin\ \eta_{t1}\ q_{ws}}{(1 - \eta_{t1})\ q_{ws}} \tag{4.22}$$

Using the recent formula for $Tpout$, the inequality constraint ($Tpout \leq 40$) can be translated into the following inequality:

$$q_{ws} \leq \frac{(Tpin - 19)\ k\ \eta_{t1}}{(Tpin\ -\ 40)\ (1 - \eta_{t1})} \tag{4.23}$$

To summarize, the optimization problem transferred to an objective function of two variables (q_{wt} and q_{ws}) with two inequality constraint (inequality (4.23) and $q_{ws} \geq q_{wt}$). The typical feasible region in this case is sketched in Fig. 4.11 (supposed $TE21 = -30$, $Tpin = 60$ and $q_a = 1674.1$).

Solving the optimization problem in this case results in the same optimal values obtained in the previous case. That is, Table 4.1 represents optimal set-points also in this case.

4.4.3 Consideration of Results

Regarding results of solving the optimization problem in different conditions (Table 4.1) reveals that in all conditions supply water flow (q_{ws}) and tertiary water flow (q_{wt}) are equal. Therefore, from energy point of view the optimal hydronic circuit for supplying water-to-air heat exchanger is variable-primary flow circuit [10]. It also shows that using air-to-air heat exchanger to produce required heat is cheaper than using water-to-air heat exchanger. That is, the control strategy must be designed in such a way that maximum exploitation of the air-to-air heat exchanger is achieved.

4.5 Conclusions

Optimal set-point synthesis for a HVAC system was presented in this paper. The HVAC system was a typical HVAC system consisted of an air-to-air heat exchanger and a water-to-air heat exchanger. To derive the optimal set-points, an objective function composed of electrical power of different components in the HVAC system and a fraction of thermal power used by the system was defined. The goals defined for the HVAC system were treated as constraints to the objective function. Finally, the defined optimization problem was solved using the steady state model of the system. Analysis of the obtained results revealed that in all conditions supply water flow was equal to tertiary water flow. Thus, the varying-primary flow system was the optimal hydronic circuit to supply the water-to-air heat exchanger. The synthesis done here can be applied as a supervisory layer for the two layer control of the HVAC system to make the system work at its optimal set-points.

Table 4.1: Optimal Set-points in Different Conditions While $q_{ws} \leq q_{wt}$

Air Flow (q_a)	Set-points	$Tpin$ = 80 $TE21$ = -12	80 −20	80 −30	70 −12	70 −20	70 −30	60 −12	60 −20	60 −30
2104.9	q_{wt}	51.6	73.9	104.7	62.7	90.8	131.4	80.2	118.6	179.0
	q_{ws}	51.6	73.9	104.7	62.7	90.8	131.4	80.2	118.6	179.0
	wrf	1	1	1	1	1	1	1	1	1
	$TE22$	14.2	12.4	10.1	14.2	12.4	10.1	14.2	12.4	10.1
	$Twout$	24.2	26.1	28.8	24.2	26.2	29.2	24.2	26.5	30.1
1674.1	q_{wt}	29.6	44	62.8	35.8	53.3	76.5	45	67.6	98.3
	q_{ws}	29.6	44	62.8	35.8	53.3	76.5	45	67.6	98.3
	wrf	1	1	1	1	1	1	1	1	1
	$TE22$	15.8	14.3	12.5	15.8	14.3	12.5	15.8	14.3	12.5
	$Twout$	27.9	28.9	30.2	26.9	27.9	29.2	25.8	26.8	28.2
979.9	q_{wt}	4	9.3	15.9	4.8	11.2	19.1	6.1	14	23.9
	q_{ws}	4	9.3	15.9	4.8	11.2	19.1	6.1	14	23.9
	wrf	1	1	1	1	1	1	1	1	1
	$TE22$	18.3	17.4	16.3	18.3	17.4	16.3	18.3	17.4	16.3
	$Twout$	29.7	31.8	32.5	28.5	29.9	30.5	27.0	28	28.4
308.3	q_{wt}	0	0	0	0	0	0	0	0	0
	q_{ws}	0	0	0	0	0	0	0	0	0
	wrf	0.94	0.96	0.98	0.94	0.96	0.98	0.94	0.96	0.98

Bibliography

[1] S. Somasundaram, D.W. Winiarski, D.B. Belzer, ”Screening Analysis for EPACT-Covered Commercial HVAC and Water-Heating Equipment”, *Transaction of the ASME*, Vol. 124, June 2002, pp 116-124.

[2] P. Svendsen and H. Andersen, ”Energy Efficient Pump Coupling in HVAC Systems”, *Technical Report*, Danish Technological Institute (Industry and Energy Section), 2005.

[3] J. Lin, S. Chen, and P.D. Roberts, ”Modified Algorithm for Steady-state Integrated System Optimization and Parameter Estimation”, *IEE Proceedings*, Vol. 135, March 1988.

[4] S. Gros, B. Srinivasan, and D. Bonvin, ”Static Optimization via Tracking of The Necessary Conditions of Optimality Using Neighboring Extremals”, *American Control Conference Proceeding*, June 2005, pp 251-255.

[5] N. Nassif, S. Kajl, and R. Sabourin, ”Two-Objective On-Line Optimization of Supervisory Control Strategy”, *Building Services Engineering Research and Technology*, 2004, pp 241-251.

[6] L.F.S. Larsen, ”Model Based Control of Refrigeration Ssytems”, *Ph.D. Thesis*, Aalborg University, Denmark, 2005.

[7] M. Wetter, ”Simulation Model: Air-to-air Heat Exchnager”, *Technical Report*, Lawrence Berkeley National Laboratory, 1999.

[8] N. Nassif, S. Kajl, and R. Sabourin, ”Ventilation Control Strategy Using Supply CO_2 Concentration Set Point”, *International Journal of HVAC & R Research*, 2005, pp 239-262.

[9] R.H. Green, ”An Air-Conditioning Control System Using Variable-speed Water Pumps”, *ASHRAE Transaction:Research*, part 1, 1994, pp 463-470.

[10] W.P. Bahnfleth, ”Varying Views on Variable-primary Flow”, *Chilled Water Engineering J.*, March 2004.

Chapter 5

Optimal Model-Based Control in HVAC Systems

M. Komareji[1] , J. Stoustrup[2] , H. Rasmussen[3] , N. Bidstrup[4] , P. Svendsen[5] , F. Nielsen[6]

Abstract

This paper presents optimal model-based control of a heating, ventilating, and air-conditioning (HVAC) system. This HVAC system is made of two heat exchangers: an air-to-air heat exchanger (a rotary wheel heat recovery) and a water-to-air heat exchanger. First dynamic model of the HVAC system is developed. Then the optimal control structure is designed and implemented. The HVAC system is split into two subsystems. By selecting the right set-points and appropriate cost functions for each subsystem controller the optimal control strategy is respected to guarantee the minimum thermal and electrical energy consumption. Finally, the controller is applied to control the mentioned HVAC system and the results show that the expected goals are fulfilled.

[1]Mohammad Komareji is a PhD student in The Department of Control and Automation, Institute of Electronic Systems, Aalborg University, Aalborg, Denmark; komareji@es.aau.dk

[2]Jakob Stoustrup is with Aalborg University as a Professor in The Department of Control and Automation; jakob@es.aau.dk

[3]Henrik Rasmussen is with Aalborg University as an Associate Professor in The Department of Control and Automation; hr@es.aau.dk

[4]Niels Bidstrup is with Grundfos Management A/S as a Chief Engineer, Ph.D.; nbidstrup@grundfos.com

[5]Peter Svendsen is with Danish Technological Institute (DTI) as a Project Manager; Peter.Svendsen@teknologisk.dk

[6]Finn Nielsen is with Exhausto A/S as a Project Manager; FNI@exhausto.dk

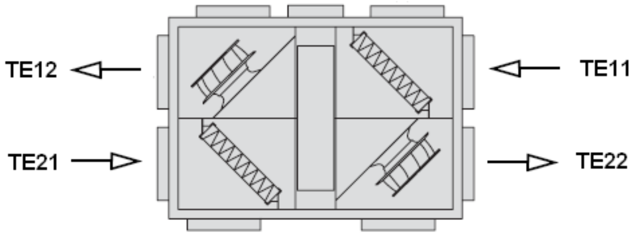

Figure 5.1: The Air-to-air heat exchanger scheme

5.1 Introduction

The consumption of energy by heating, ventilating, and air conditioning (HVAC) equipment in industrial and commercial buildings constitutes a great part of the world energy consumption [1]. In spite of the advancements made in microprocessor technology and its impact on the development of new control methodologies for HVAC systems aiming at improving their energy efficiency, the process of operating HVAC equipment in commercial and industrial buildings is still an inefficient and high-energy consumption process.

It has been estimated that by optimal control of HVAC systems almost 100 GWh energy can be saved yearly in Denmark (five million inhabitants) [2]. It shows that a huge amount of energy can be saved and according to the current energy prices it will be reasonable to invest a little bit more in the first cost of HVAC systems.

In this paper, an integrated control system is developed. That is, in the proposed control system there is no need for an explicit supervisory layer to make the system work in its optimal conditions. The optimal control strategy that has been developed in [2] is implemented here. So, the controller follows the optimal control strategy while it tracks the set-point. In Section 5.2, the dynamic model of the HVAC system is described. The controller design is presented in Section 5.3. Finally, the results of applying the proposed control system is shown in Section 5.4.

5.2 Dynamic Modeling

The HVAC system that will be considered consists of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger. In this section these components will be described and their dynamic models will be developed. Finally the overall nonlinear model of the HVAC system will be linearized. This linear model will be used to design the controller later.

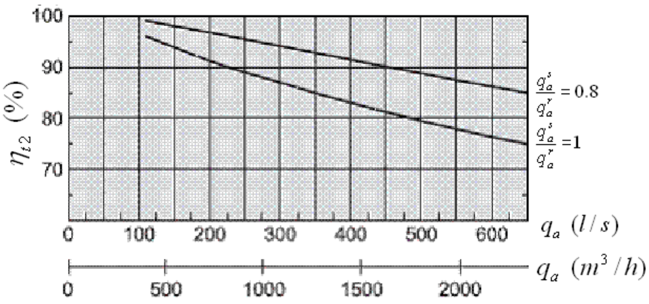

Figure 5.2: Dependency of η_{t2} on q_a while n=10 rpm; q_a^s and q_a^r represent supply air flow and return air flow, respectively.

5.2.1 Air-to-air Heat Exchanger

The air-to-air heat exchanger is a rotary heat exchanger in aluminum, with low pressure loss (shown in Fig. 5.1). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow.

Steady State Gain Determination

Here, it is supposed that the ratio of the supply air flow to the return air flow is one. Therefore, η_{t2} will be a function of air flow (q_a), that is the same for both supply and return air, and the rotation speed of the wheel (n). In this context, results of testing the rotary heat exchanger that was performed according to European Standard for laboratory testing of air-to-air heat recovery devices (EN 247, EN 305, EN 306, EN 307, EN 308) will be used. According to results of the test, it is possible to specify η_{t2} as a multiplication of two functions. Fig. 5.2 and 5.3 illustrate these functions [2]. Therefore, η_{t2} can be described as following:

$$\eta_{t2} = (-1.0569 \cdot 10^{-4}\, q_a + 0.9943) \cdot wrf(n) \tag{5.1}$$

As we know, η_{t2} definition is as following:

$$\eta_{t2} = \frac{TE22 - TE21}{TE11 - TE21} \tag{5.2}$$

Combining recent equations (equations 5.1 and 5.2) will result in the steady state gain for the wheel model:

$$TE22 = TE21 + wrf(n) \cdot (TE11 - TE21) \cdot (-1.0569 \cdot 10^{-4} q_a + 0.9943) \tag{5.3}$$

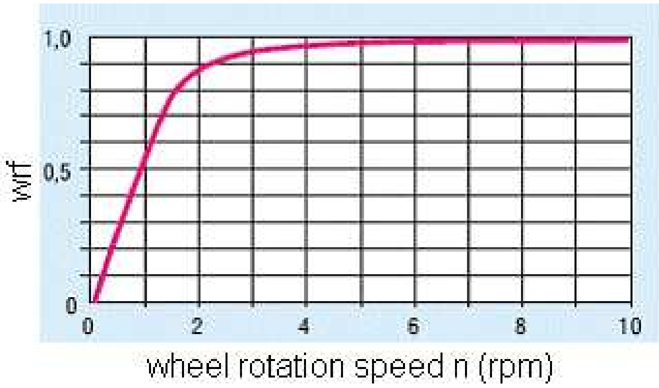

Figure 5.3: Normalized dependency of η_{t2} on n

Dynamic Behavior

Fig 5.4 shows an energy wheel operating in a counter flow arrangement. Under typical operating conditions, warm air enters the tube during the supply part of the cycle and transfers energy to the matrix. This energy is then transferred from the matrix to the air during the exhaust part of the cycle. The half plane of the matrix tube is assumed impermeable and adiabatic and the bulk mean temperatures of air are used in the model. The formulation is therefore one dimensional and transient with space (x) and time (t or $\theta = w \cdot t$) as the independent variables. The governing equations for heat transfer (energy equations) in energy wheel for air and matrix include energy storage, convection, conduction based on the usual assumptions are as follows respectively:

$$\rho_a C_{pa} A_a \frac{\partial T_a}{\partial t} + U \rho_a C_{pa} A_a \frac{\partial T_a}{\partial x} + h \frac{\acute{A}_s}{L}(T_a - T_m) = 0 \tag{5.4}$$

$$\rho_m C_{pm} A_m \frac{\partial T_m}{\partial t} - h \frac{\acute{A}_s}{L}(T_m - T_a) = \frac{\partial}{\partial x}(K_m A_m \frac{\partial T_m}{\partial x}) \tag{5.5}$$

It is reasonable to suppose that the conductivity has a Small share in heat transfer through the matrix [3]. Thus, equation (5.5) can be rewritten as following:

$$\frac{\partial T_m}{\partial t} + \frac{NTU}{C_r^* P} T_m = \frac{NTU}{C_r^* P} T_a \tag{5.6}$$

Equation (5.6) shows that air temperature (T_a) can be assumed as the input for the matrix temperature (T_m) differential equation. It means the matrix temperature as a function of time (t) will perform as an output of the ordinary first order differential equation. Another point that should be emphasized is that the time constant ($\frac{C_r^* P}{NTU}$)in the differential equation is fixed. That is, the time constant depends on matrix (wheel) properties. Thus, it is a design parameter not a control parameter. It is claimed that air stream

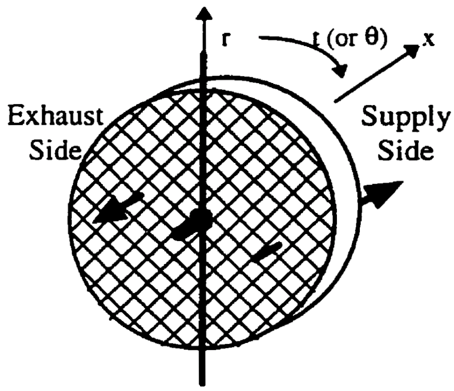

Figure 5.4: Counter flow energy wheel

temperature has the same behavior as matrix temperature [4]. So, The air stream temperature shows first order dynamic behavior.

Dynamic Model of The Air-to-air Heat Exchanger

According to our earlier debate the wheel behavior can be modeled by a first order transfer function. So, we will have:

$$TE22(s) = \frac{TE21}{\tau s+1} + \frac{wrf(s)\cdot(TE11-TE21)(-1.0569\times 10^{-4}q_a+0.9943)}{\tau s+1} \tag{5.7}$$

The first part on the right side of the equation (5.7) will be treated as disturbance. That is, the transfer function from $TE22$ to wrf is as following:

$$\frac{TE22(s)}{wrf(s)} = \frac{(TE11-TE21)(-1.0569\times 10^{-4}q_a+0.9943)}{\tau s+1} \tag{5.8}$$

As it was discussed, the time constant (τ) is fixed and according to the experiments, it is 28.0374 seconds.

5.2.2 Water-to-air Heat Exchanger

The water-to-air heat exchanger is shown in Fig. 5.5. As can be seen, a primary or secondary -tertiary hydronic circuit supplies the heat exchanger with hot water. The air flow that passes the hot coil is controllable by changing the speed of the fan installed in the air-to-air heat exchanger.

The hydronic circuit that is used for supplying the water-to-air heat exchanger is a primary or secondary-tertiary circuit isolated from each other by a bypass pipe. The supply water flow (q_{ws}) is

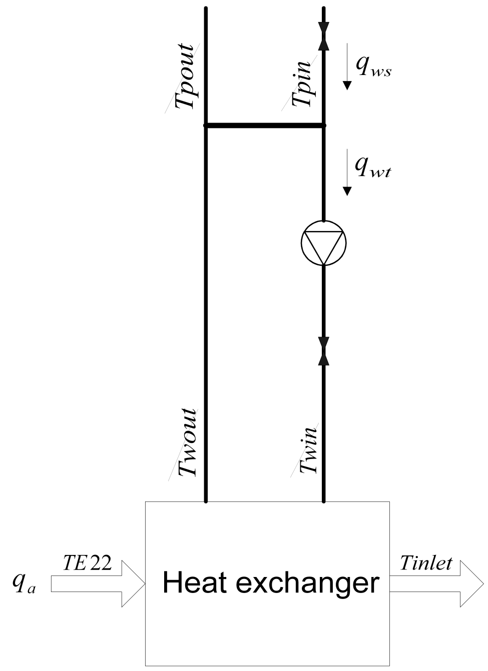

Figure 5.5: The water-to-air heat exchanger scheme

controlled by the motorized primary/secondary valve. A variable speed pump and a valve is installed in the tertiary circuit. The tertiary valve is used to set the desired maximum flow rate through the variable speed pump. By changing the speed of the tertiary pump, it is possible to sweep the desired interval for the tertiary water flow (q_{wt}).

Dynamic Model of The Coil

Here, the nonlinear coil model that was developed by Underwood and Crawford [5] will be applied. According to their model, the differential equations, resulted from energy balance equations, which describe the coil behavior are as follows:

$$[(-C_{pw}-b/2)\dot{m_{wt}}(t)-d/2\dot{m}_a(t)-a/2]\ T_{wout}(t)+[(C_{pw}-b/2)\dot{m_{wt}}(t)-d/2\dot{m}_a(t)-a/2]\ T_{win}(t)$$
$$+(b\ \dot{m_{wt}}(t)+d\ \dot{m}_a+a)\ TE22(t)=C_w\frac{d}{dt}T_{wout}(t) \tag{5.9}$$

$$-\dot{m}_a(t)C_{pa}Tinlet(t)+[(C_{pa}-d)\dot{m}_a(t)-b\dot{m_{wt}}(t)-a]\cdot TE22(t)+(a/2+b/2\dot{m_{wt}}(t)+d/2\dot{m}_a(t))T_{win}(t)$$

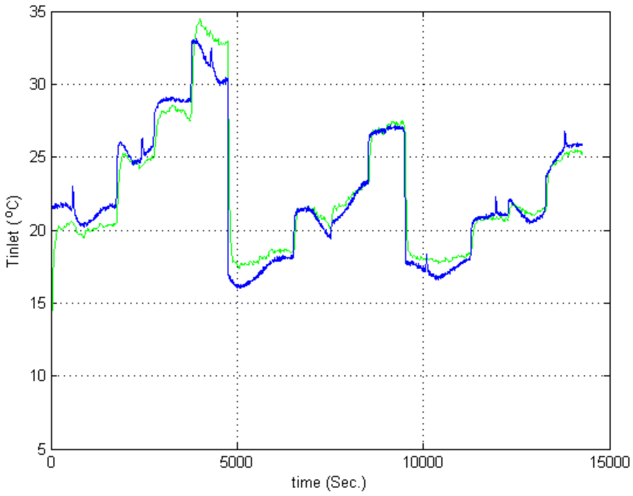

Figure 5.6: Coil model verification, blue curve: real output, green curve: simulated output

$$+(a/2+b/2\dot{m_{wt}}(t)+d/2\dot{m}_a(t))T_{wout}(t)=C_a\frac{d}{dt}Tinlet(t) \tag{5.10}$$

where a, b, d, C_{pa}, C_{pw}, C_a, and C_w are unknown parameters that have to be identified through the experiments.

The unknown parameters have identified through some experiments on the coil. Fig 5.6 show verification of the model along with identified parameters.

Dynamic Model of The Water-to-air Heat Exchanger

According to the hydronic circuit configuration $T_{win}(t)$ will be as following:

$$T_{win}(t)=\frac{T_{pin}(t)\ \dot{m}_{ws}+T_{wout}(t)\ (\dot{m_{wt}}-\dot{m}_{ws})}{\dot{m_{wt}}} \tag{5.11}$$

where it is supposed that $\dot{m_{wt}} \geq \dot{m_{ws}}$.

The recent formula for T_{win} should be placed in coil model (equations (5.9) and (5.10))to have the water-to-air heat exchanger model versus real inputs $\dot{m}_w$ and $\dot{m_{wp}}$. Therefore, final water-to-air heat exchanger model will be as following:

$$[k_1-b\dot{m}_{wt}-k_2\dot{m}_{ws}+k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{wout}(t)+[k_2\dot{m}_{ws}-k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{pin}(t)$$

$$+[a+b\ \dot{m_{wt}}+d\ \dot{m}_a(t)]TE22=C_w\frac{d}{dt}T_{wout}(t) \tag{5.12}$$

$$-\dot{m}_a C_{pa} Tinlet(t) + [(C_{pa} - d)\dot{m}_a - b\dot{m_{wt}} - a]TE22 + [-k_1 + b\dot{m_{wt}} - b/2\dot{m}_{ws} - k_3 \frac{\dot{m}_{ws}}{\dot{m_{wt}}}]T_{wout}(t)$$

$$+[b/2\dot{m}_{ws} + k_3 \frac{\dot{m}_{ws}}{\dot{m_{wt}}}]T_{pin}(t) = C_a \frac{d}{dt} Tinlet(t) \tag{5.13}$$

where:
$k_1 = -a - d\ \dot{m}_a(t)$
$k_2 = C_{pw} - b/2$
$k_3 = d/2\ \dot{m}_a(t) + a/2$

5.2.3 Linearization of The Nonlinear HVAC System Model

The model of the whole HVAC system consists of the air-to-air and water-to-air heat exchanger models which were described by equations (5.8), (5.12) and (5.13). The nonlinear model of the HVAC system can be described as following:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \\ T\dot{E}22 \end{bmatrix} = f(Tinlet, Twout, TE22, \dot{m_{ws}}, \dot{m_{wt}}, wrf) \tag{5.14}$$

where: $Twout$, $Tinlet$, and $TE22$ are states of the HVAC system. $\dot{m_{ws}}$, $\dot{m_{wt}}$, and wrf are inputs of the HVAC system.

The linearized model of the HVAC system will have the following shape:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \\ T\dot{E}22 \end{bmatrix} = \begin{bmatrix} a_4 & a_3 & a_5 \\ 0 & a_1 & a_2 \\ 0 & 0 & a_6 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \\ TE22 \end{bmatrix} + \begin{bmatrix} b_3 & b_4 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & b_5 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} \\ wrf \end{bmatrix}$$

$$y = [1\ \ 0\ \ 0] \cdot \begin{bmatrix} Tinlet \\ Twout \\ TE22 \end{bmatrix} \tag{5.15}$$

where:
$a_1 = (-a - b\ \dot{m_{wt}} - d\ \dot{m}_a - C_{pw}\ \dot{m_{ws}} + b/2\ \dot{m_{ws}} + a/2\ \frac{\dot{m_{ws}}}{\dot{m_{wt}}} + d/2\ \frac{\dot{m_{ws}}\ \dot{m}_a}{\dot{m_{wt}}})/C_w$
$a_2 = (a + b\ \dot{m_{wt}} + d\ \dot{m}_a)/C_w$
$a_3 = (a + b\ \dot{m_{wt}} + d\ \dot{m}_a - b/2\ \dot{m_{ws}} - a/2\ \frac{\dot{m_{ws}}}{\dot{m_{wt}}} - d/2\ \frac{\dot{m_{ws}}\ \dot{m}_a}{\dot{m_{wt}}})/C_a$
$a_4 = -C_{pa}\ \dot{m}_a/C_a$
$a_5 = (C_{pa}\ \dot{m}_a - a - b\ \dot{m_{wt}} - d\ \dot{m}_a)/C_a$

$a_6 = -1/\tau$
$b_1 = (C_{pw} - a/2\ \frac{1}{\dot{m_{wt}}} - b/2 - d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_w\ Tpin + (-C_{pw} + a/2\ \frac{1}{\dot{m_{wt}}} + b/2 + d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_w\ Twout$
$b_2 = (a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} + d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_w\ Tpin + (-b - a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} - d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_w\ Twout + b\ TE22$
$b_3 = (a/2\ \frac{1}{\dot{m_{wt}}} + b/2 + d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_a\ Tpin + (-a/2\ \frac{1}{\dot{m_{wt}}} - b/2 - d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_a\ Twout$
$b_4 = (-a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} - d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_a\ Tpin + (b + a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} + d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_a\ Twout - b\ TE22$
$b_5 = \frac{1}{\tau}\ (-1.0569 \times 10^{-4}\ \frac{\dot{m_a}}{\rho_a} + 0.9943)\ (TE11 - TE21)$
This linear model will be used in the control section to design the controller.

5.3 Optimal Model-based Control

5.3.1 Control Strategy

It is shown that to make the system perform optimally the control strategy has to be defined in a way that the following conditions are satisfied:

1. The maximum possible exploitation of the air-to-air heat exchanger is achieved.

2. In the steady state conditions supply water flow (q_{ws}) must be equal to the tertiary water flow (q_{wt}). That is, it is optimal to make the system work in a way that no water passes through the bypass pipe. It should be noted that it is not possible to eliminate the bypass pipe because it makes the tertiary hydronic circuit hydraulically decoupled and it is necessary to keep the bypass to remove fast disturbances.

If the control strategy respects the mentioned conditions the HVAC system will perform in such a way that it will result in minimum thermal and electrical energy consumption [2].

5.3.2 Controller Design

The mentioned HVAC system is going to be used for ventilation purposes. It means that the air flow (q_a) will be determined in accordance with the required ventilation and the inlet air temperature ($Tinlet$) has to be kept at 19^oC. So, the optimal controller task is to track the set-point for the inlet air temperature while satisfying the conditions that were described in the control strategy section to guarantee the optimal performance of the system. The traditional way to design the control system that works in this way is applying the two-layer hierarchical control system. The lower layer performs direct regulatory control, where the aim is to maintain selected process variables at their desired set-point values, and the upper layer, known as the supervisory layer, has the task of determining the set points of the regulatory controllers to obtain optimal steady state performance.

Looking at the linear model that was developed before reveals that the HVAC system can be split into two decoupled subsystems as follows:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3 & b_4 \\ b_1 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$

$$y = [1 \;\; 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{5.16}$$

$$T\dot{E}22 = a_6 \cdot TE22 + b_5 \cdot wrf \tag{5.17}$$

It means that control of the HVAC system can be considered as control of the air-to-air heat exchanger and control of the water-to-air heat exchanger separately. It should be noted that $TE22$ acts as disturbance for the water-to-air heat exchanger in this new formulation.

If the set-point for temperature of the fresh air that leaves the wheel ($TE22$) is defined as the set-point for temperature of the inlet air (19^oC) we will be sure that the air-to-air heat exchanger has its maximum contribution to warm up the fresh air. Thus, the first condition for optimality will be met. The second condition for optimality can be included in the cost function that will be defined for the water-to-air heat exchanger controller. Therefore, there is no need to design an explicit supervisory layer.

Air-to-air Heat Exchanger Controller

To design controller for the rotary wheel we need to model the wheel actuators. To do so, several experiments were done. Fig 5.7 shows the relation between the voltage and the wheel speed. As can be seen in the Fig 5.7, the curve describing the relation between the voltage and the wheel speed can be approximated by two lines ($\frac{speed}{V} = 1/5$ for $0 \leq speed \leq 3$ and $\frac{speed}{V} = 10$ for $3 \leq speed \leq 10$). It should be noted that there is also a time delay varying from 6 seconds to 22 seconds while the speed of the wheel is going to change.

Fig 5.3 showed the normalized curve that describes the effect of the wheel speed on the efficiency of the wheel. This nonlinear curve also will be approximated by three lines ($\frac{wrf}{speed} = 8/15$ for $0 \leq speed \leq 1.6$, $\frac{wrf}{speed} = 4/55$ for $1.6 \leq speed \leq 3$ and $\frac{wrf}{speed} = 7/1000$ for $3 \leq speed \leq 10$).

Therefore, the rotary wheel along with actuators can be modeled as follows:

$$\frac{TE22}{V} = \frac{k(TE11 - TE21)(-1.0569 \times 10^{-4} q_a + 0.9943)}{\tau s + 1} e^{-Ts} \tag{5.18}$$

where:
$k \in \{8/75, 4/275, 7/100\}$
$6 \leq T \leq 22$

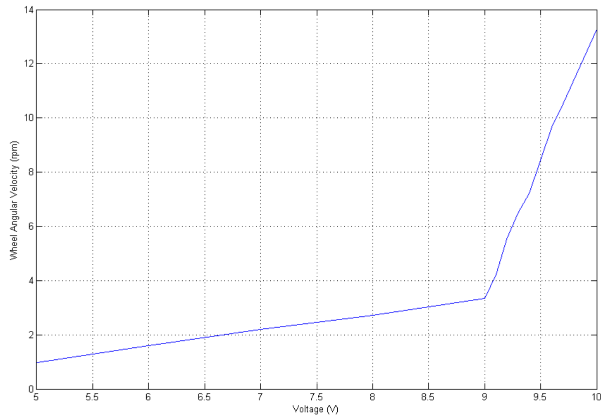

Figure 5.7: Wheel speed vs. voltage

It should be noted that the outdoor air temperature ($TE21$) will perform as disturbance through a first order system on the wheel. Thus, the model of the rotary wheel is a first order system along with varying gain, varying delay and disturbance. The input (v) is also constrained. These conditions remark that model predictive controller (MPC) is a good choice for the control.

The control problem can be formulated as follows:

$$\min_{v[k/k]} \sum_{i=1}^{6} \|TE22[k+i/k]-19\|^2_{I(i)}$$

subject to:

$$0 \le v[k/k] \le 10 \tag{5.19}$$

Sampling time for the controller is supposed to be 15 seconds. The gain and delay for internal model of the MPC controller are 1.7 and 22, respectively.

Water-to-air Heat Exchanger Controller

To control the water-to-air heat exchanger we have to deal with constrained inputs. We also have to penalize inputs in a way that in the steady state conditions no water passes through the the bypass pipe. So, again MPC is a good candidate for this control problem. To design the MPC controller we need to modify equation (5.16) as follows:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3+b_4 & b_4 \\ b_1+b_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} - \dot{m_{ws}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$

$$y = [1 \;\; 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{5.20}$$

where:

$$a1 = -0.0352, a2 = 0.0310, a3 = 0.0564$$

$$a4 = -0.5961, a5 = 0.4833, b1 = 17232$$

$$b2 = 46628, b3 = 227635, b4 = -199119$$

Thus, the control problem can be described as follows:

$$\min_{\dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]} \sum_{i=1}^{6} \|Tinlet[k+i/k] - 19\|^2_{I(i)}$$

$$+ \sum_{i=0}^{1} \|\dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]\|^2_{(0.2 \times I(i))}$$

subject to:

$$0 \leq \dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k] \leq 250 \tag{5.21}$$

The variable speed pump that is installed in the tertiary hydronic circuit will provide the required(q_{wt}). According to the pump affinity laws we have:

$$N = \frac{N_0}{q_{wt0}} \cdot q_{wt} \tag{5.22}$$

It means that by adding a gain it is possible to model the pump. Here the gain is 11.02 ($\frac{N_0}{q_{wt0}}$).

A valve will control the supply water flow (q_{ws}). Here the sampling time for the controller is 15 seconds too. Thus, transient behavior of the valve is not important and it can be modeled as a single gain. The valve has nonlinear characteristic curve in steady state conditions. So, an average value for this gain is selected. The controller is robust enough to tolerate this approximation.

5.4 Results

Fig 5.8 and 5.9 shows the result of applying the designed control system to the HVAC system. It reveals that the controller keeps perfect tracking of the set-point. At time 660 sec. a step disturbance adds to the supply hot water temperature and the temperature drops from 80^oC to 75^oC. As can be seen, the control system compensates for this disturbance and can track the set-point again. At time 1090 sec. a step disturbance adds to the outdoor air temperature and the temperature rises 5^oC. Here also the control

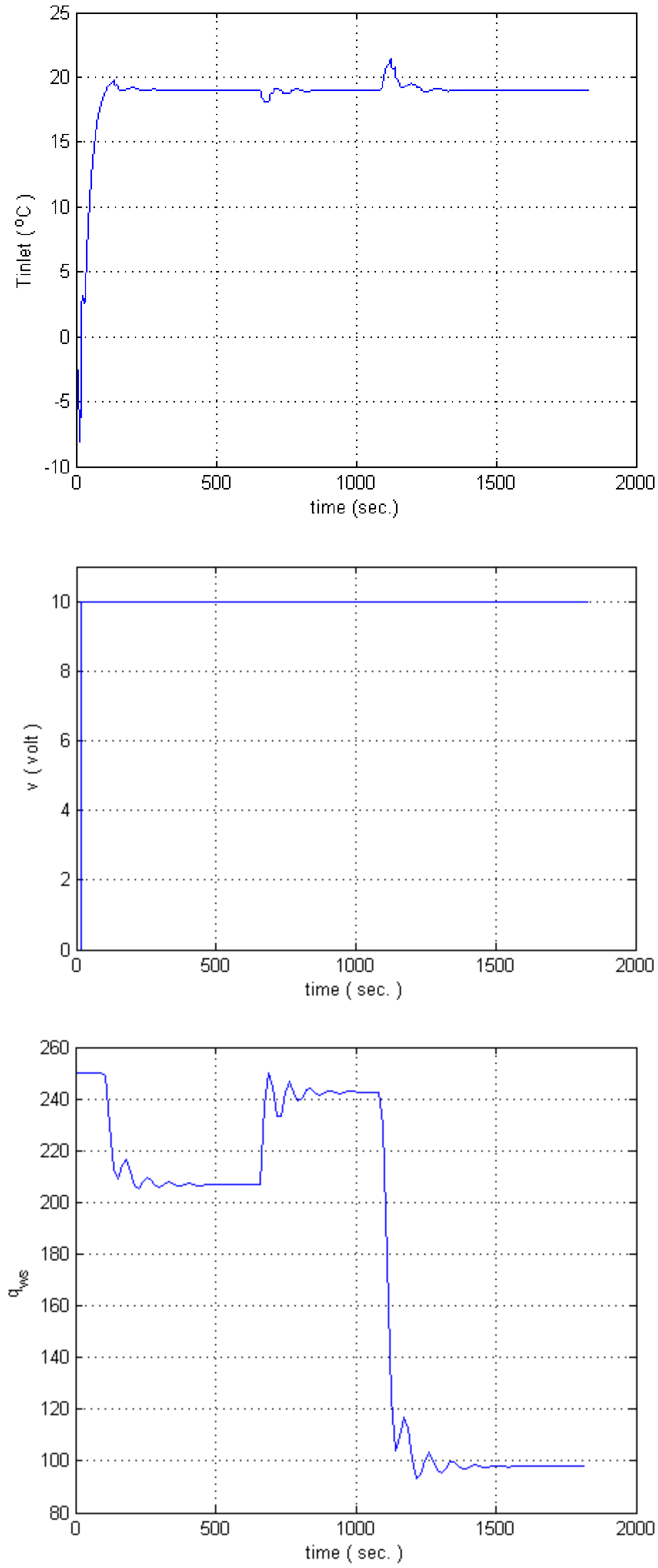

Figure 5.8: The controller performance (a)

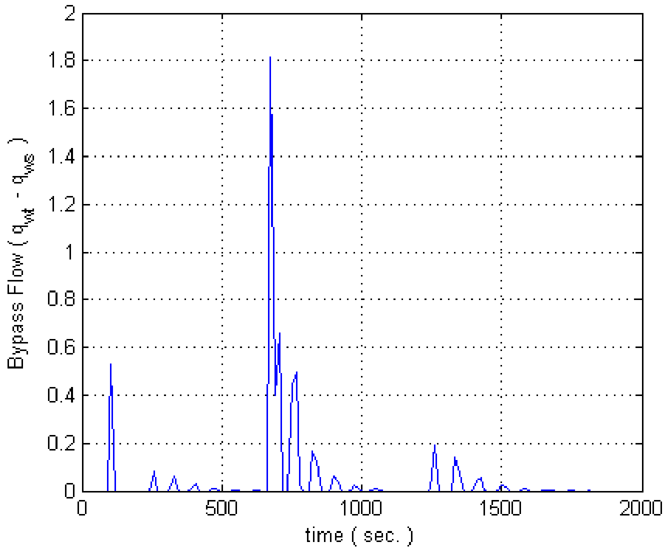

Figure 5.9: The controller performance (b)

system show perfect compensation and tracking.

5.5 Conclusions

Optimal model-based control for a heating, ventilating, and air-conditioning (HVAC) system was presented in this paper. The HVAC system was a typical HVAC system consisted of an air-to-air heat exchanger and a water-to-air heat exchanger. Dynamic model of the system was developed through dynamic modeling of different components of the system. Derived nonlinear model was linearized to design the controllers. The HVAC system was split into two subsystems and the set-points and cost functions for each subsystem controller were defined in a way that optimal control strategy which had been proposed in [2] was followed. The results of applying the developed control system showed that the system respected optimal control policy while it had the perfect tracking of the set point for the inlet air temperature.

5.6 Acknowledgments

The authors would like to appreciate financial supports and contributions of Danish Energy Net, Aalborg University, Center for Embedded Software Systems (CISS), Grundfos Management A/S, Exhausto A/S, and Danish Technological Institute (DTI) to this project.

Bibliography

[1] S. Somasundaram, D.W. Winiarski, and D.B. Belzer, Screening Analysis for EPACT-Covered Commercial HVAC and Water-Heating Equipment, *Transaction of the ASME*, Vol 124, June 2002, pp 116-124.

[2] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, P. Svendsen, and F. Nielsen, Optimal Set-point Synthesis in HVAC Systems, *American Control Conference*, New York, NY, 2007, pp 5076-5081.

[3] C. J. Simonson, *Heat and Moisture Transfer in Energy Wheels*, Ph.D. Thesis, University of Saskatchewan, Saskatoon, Canada, 1998.

[4] O. O. Abe, *Effectiveness of Energy Wheels from Transient Measurements*, M.S. Thesis, University of Saskatchewan, Saskatoon, Canada, 2005.

[5] D.M. Underwood and P.R. Crawford, Dynamic nonlinear modeling of a hot-water-to-air heat exchanger for control applications, *ASHRAE Transactions: Research*, 1991, pp. 149-155.

Chapter 6

Simplified Optimal Control in HVAC Systems

M. Komareji [1] , J. Stoustrup [2] , H. Rasmussen [3] , N. Bidstrup [4] , P. Svendsen [5] , F. Nielsen [6]

Abstract

This paper presents simplified optimal control of a heating, ventilating, and air-conditioning (HVAC) system. This HVAC system is a typical one composed of two heat exchangers: an air-to-air heat exchanger (a rotary wheel heat recovery) and a water-to-air heat exchanger. First the optimal control strategy which was developed in [1] is adopted for implementation in a real life HVAC system. Then the bypass flow problem is addressed and a controller is introduced to deal with this problem. Finally a simplified control structure is proposed for optimal control of the HVAC system. The results of implementing the simplified optimal controller show all control objectives are met.

[1]Mohammad Komareji is a PhD student in The Department of Control and Automation, Institute of Electronic Systems, Aalborg University, Aalborg, Denmark; komareji@es.aau.dk

[2]Jakob Stoustrup is with Aalborg University as a Professor in The Department of Control and Automation; jakob@es.aau.dk

[3]Henrik Rasmussen is with Aalborg University as an Associate Professor in The Department of Control and Automation; hr@es.aau.dk

[4]Niels Bidstrup is with Grundfos Management A/S as a Chief Engineer, Ph.D.; nbidstrup@grundfos.com

[5]Peter Svendsen is with Danish Technological Institute (DTI) as a Project Manager; Peter.Svendsen@teknologisk.dk

[6]Finn Nielsen is with Exhausto A/S as a Project Manager; FNI@exhausto.dk

6.1 Introduction

A great part of the produced energy in the world is consumed by heating, ventilating, and air conditioning (HVAC) systems. Due to the extremely high fuel oil price and the shortage of energy supply efficient control of HVAC systems is getting more and more attention. Optimal control of HVAC systems in Denmark (five million inhabitants) can result in saving of up to 100 GWh energy per year [1]. Regarding the recent figure makes it easy to imagine how much energy can be saved yearly by optimal control of the HVAC systems all over the world.

Maintaining thermal comfort and energy efficiency are two primary goals in the development of control modules for HVAC systems. Furthermore, control modules have to perform in such a way to guarantee that the operation of the HVAC systems do not violate any building regulations. For example, according to the Copenhagen Building Regulations there must be at least 40 degrees of cooling of the delivered hot water through the heating systems. Thus, developing an optimal control strategy for HVAC systems is a constrained optimization problem. Having derived an optimal control strategy, the dynamic controller has to be designed to meet the optimality criteria while satisfying the thermal comfort conditions. The final step is to adopt and simplify the controller for the real life systems.

In Section 6.2, the HVAC system is briefly described. The practical optimal control strategy and the implementation of the optimal controller are presented in Section 6.3. Section 6.4 discusses the bypass problem and the way to deal with that. Section 6.5 presents the simplified optimal control structure and the implementation results. Finally Section 6.6 explains the energy saving aspects.

6.2 The HVAC System Explanation

The considered HVAC system is a typical HVAC system composed of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger.

The air-to-air heat exchanger is a rotary enthalpy wheel which plays the heat recovery role (illustrated in Fig. 6.1). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow.

Fig. 6.2 shows the water-to-air heat exchanger. A variable speed pump supplies hot water to the coil. The speed change of the variable speed pump provides the mean to control the tertiary flow. The primary/secondary flow is controlled by a motorized valve. Tertiary circuit is hydraulically decoupled from the primary/secondary circuit through the bypass pipe.

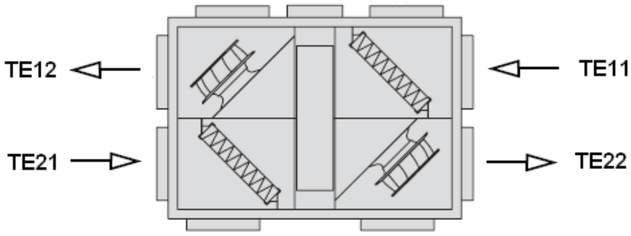

Figure 6.1: The air-to-air heat exchanger scheme

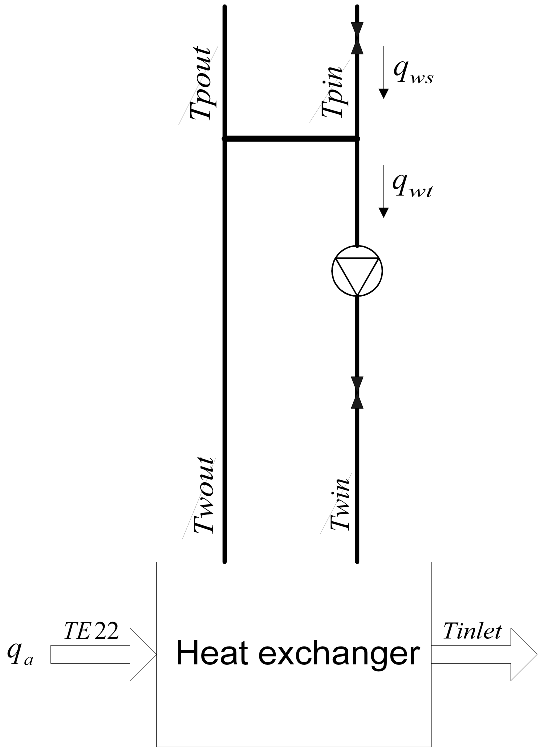

Figure 6.2: The water-to-air heat exchanger scheme

6.3 Optimal Model-based Control

This section first briefly reviews the optimal control strategy and the optimal dynamic control of the HVAC system which were developed in [1] and [2]. Then the result of the implementation of the optimal controller is presented.

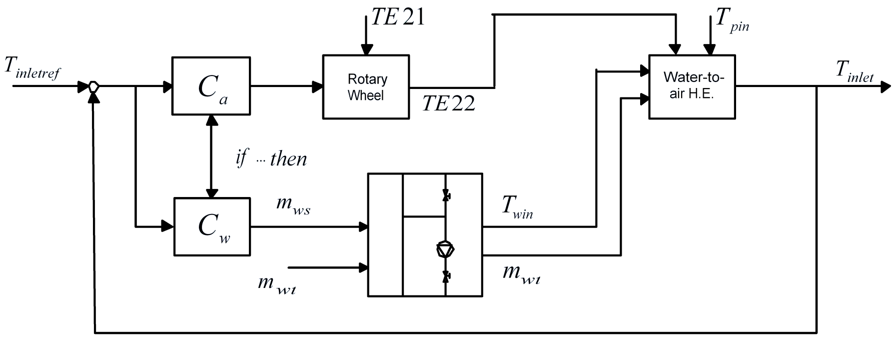

Figure 6.3: The current control scheme

6.3.1 Optimal Control Strategy

Many industrial processes like HVAC systems work in their steady state conditions most of the operation time. In this situation the main task of the controller is to reject the disturbances which act upon the process. To optimize the performance of this kind of systems, applying the steady state model of the process in the optimality analysis is objective. Following this approach for the optimality analysis of the mentioned HVAC system results in two criteria that have to be respected by the controller to guarantee optimal performance of the system [1]:

1. The maximum possible exploitation of the air-to-air heat exchanger has to be achieved.

2. In the steady state conditions supply water flow (q_{ws}) must be equal to the tertiary water flow (q_{wt}). That is, it is optimal to make the system work in a way that no water passes through the bypass pipe. It should be noted that it is not possible to eliminate the bypass pipe because it makes the tertiary hydronic circuit hydraulically decoupled and it is necessary to keep the bypass to remove fast disturbances.

It should be reminded that here optimal performance means minimum thermal and electrical energy consumption by the HVAC system while maintaining the thermal comfort.

6.3.2 Optimal Dynamic Control

The dynamic modeling and the controller design procedure of the HVAC system are described in [2]. Fig. 6.3 and 6.4 show the current control scheme and the optimal control scheme, respectively.

The current system is equipped with a constant speed pump. So, the tertiary water flow (q_{wt}) is not controllable and has to be set to its maximum value to meet the maximum heat demand by the system. The inlet air temperature is controlled by the motorized primary valve. Two controllers, the

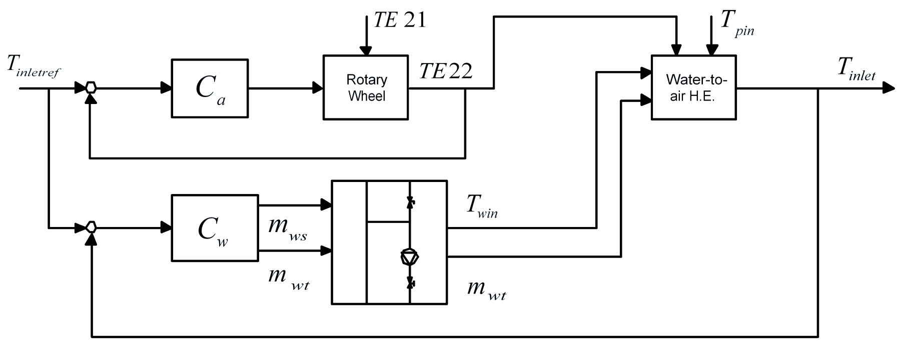

Figure 6.4: The optimal control scheme

heat recovery wheel controller and the water-to-air heat exchanger controller, communicate with one another through some if-then rules to exploit the heat recovery wheel as much as possible. As a result, the first optimality criterion is fulfilled; however, there is no chance for the satisfaction of the second optimality criterion.

The proposed optimal controller in [2] meets both optimality criteria. Moreover, the optimal controller is simpler because two controllers, the heat recovery wheel controller and the water-to-air heat exchanger controller, are completely independent.

The rest of the paper is dedicated to point out some issues about controlling the water-to-air heat exchanger.

6.3.3 Controller Implementation and Results

When zero voltage is applied to the variable speed pump, the pump will keep running at a specified minimum speed and will result in non-zero (minimum) tertiary flow (q_{wt}). Therefore, to reach the tertiary flow which is less than the minimum flow the pump has to be pulsed. Pulsing of the pump will cause some problems:

- Short-circuit in the bypass pipe when the pump stops (delivering hot water to the return supply water that is not acceptable in hydronic systems)
- Possible change in supply tertiary water temperature because of pump starts and stops
- Possible oscillations around the set-point ($Tinlet = T_{ref}$) due to pump pulsing

Due to the above troubles pulsing the pump is not a good solution. Thus, a simple and practical solution is to follow the mixing control strategy when the applied voltage to the pump is zero and there is less demand for the heat. The mixing control strategy stands for the control strategy where the tertiary

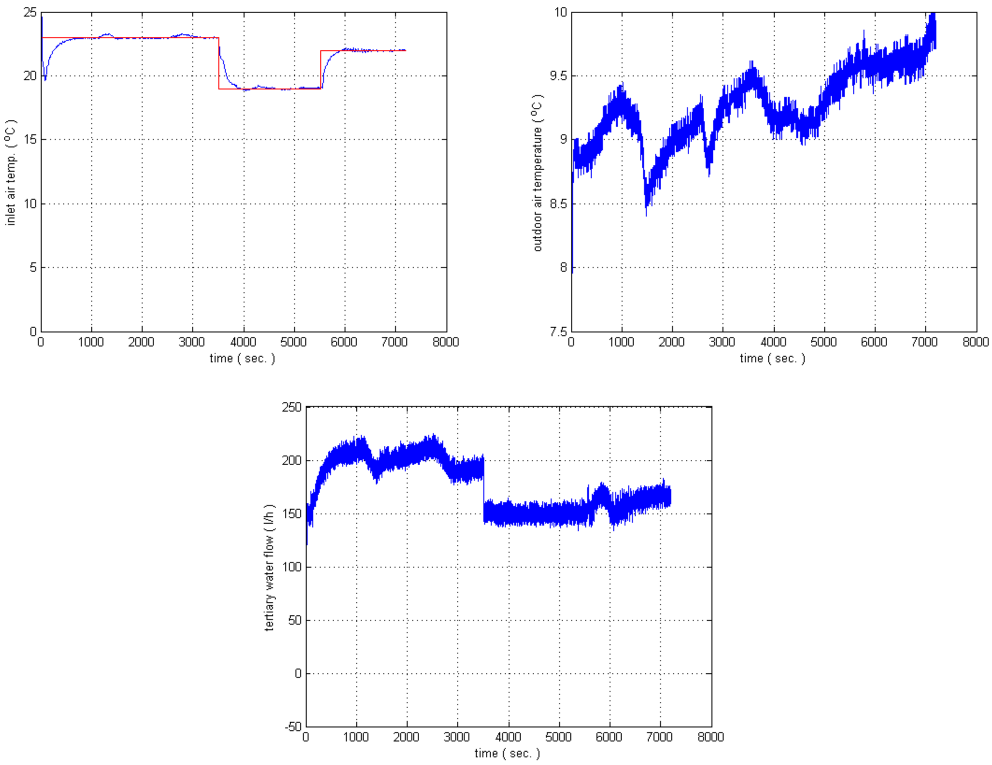

Figure 6.5: The implementation result of the practical optimal controller (a)

flow (q_{wt}) is constant and the inlet air temperature is controlled by changing the supply water flow (q_{ws}) by means of motorized primary valve.

The proposed controller in [2] also shows satisfactory results while following the practical optimal control strategy. Fig. 6.5 and 6.6 illustrate the result of implementing the controller on the HVAC system. Fig. 6.5 shows the controller perfectly tracks the set-points. Looking at the the tertiary flow curve reveals that when the set-point changes from 23^oC to 19^oC, the controller switches from the optimal control strategy to the mixing control strategy and when the set-point changes from 19^oC to 22^oC the switching from mixing control strategy to optimal control strategy happens. The switches between the two different control strategy can be inferred by analyzing the temperature measurements shown in Fig. 6.6 too. The controller also shows good performance in the sense of disturbance rejection. The disturbances from outdoor air temperature (shown in Fig. 6.5) and the primary supply water temperature (shown in Fig. 6.6) are perfectly compensated.

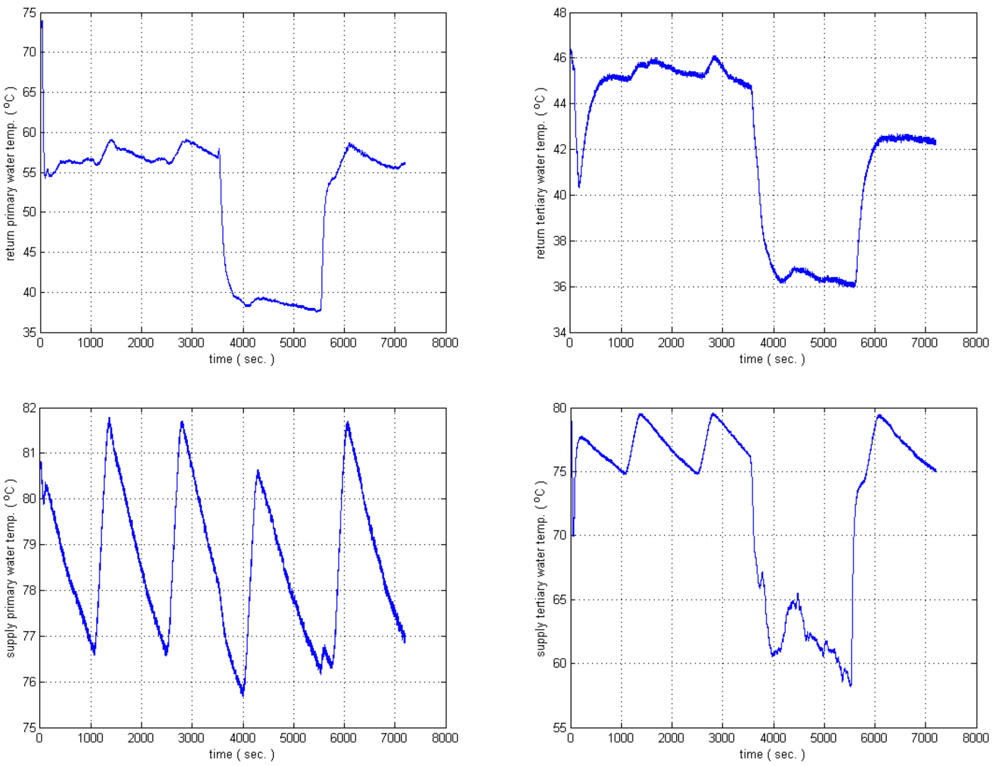

Figure 6.6: The implementation result of the practical optimal controller (b)

6.4 Bypass Flow Problem

In this section the bypass flow is defined as a quantity with direction and the implicit measuring of the bypass flow is presented. Then the way to deal with this problem is discussed.

6.4.1 Measuring The Bypass Flow

It is not reasonable to measure the bypass flow through a flow-meter in real life HVAC systems. However, we can measure the bypass flow implicitly through thermocouples. This way of measurement is acceptable due to the cheap price of thermocouples.

First the bypass flow has to be defined as a quantity with direction. Consequently, we will always treat bypass flow as a difference between the supply water flow and the tertiary water flow ($q_{ws} - q_{wt}$). That is, when the primary flow is greater than the tertiary flow the bypass flow will have a positive sign and when the primary flow is less than the tertiary flow a negative sign will accompany the bypass flow. These two cases are considered as follows:

- Negative Bypass Flow

  When there is a ngetive bypass flow the return primary water temperature ($Tpout$) and the return tertiary water temperature($Twout$) are equal. However, the supply primary water temperature ($Tpin$) is always greater than the supply tertiary water temperature ($Twin$). The difference between two recent temperatures is proportional to the ratio of the primary water flow and the tertiary water flow.

  The energy balance equation for the supply water side will result in the following equation:

$$(q_{wt} - q_{ws})\, Twout \;+\; q_{ws}\, Tpin \;=\; q_{wt}\, Twin \tag{6.1}$$

  By rearranging the above equation we will have:

$$\frac{q_{wt}}{q_{ws}} = \frac{Tpin \;-\; Twout}{Twin \;-\; Twout} \tag{6.2}$$

  We subtract 1 from both side of the equation. So,

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{Tpin \;-\; Twin}{Twin \;-\; Twout} \tag{6.3}$$

- Positive Bypass Flow

The story of the negative bypass flow is similar to the one of the positive bypass flow. Thus, When there is a positive bypass flow the supply primary water temperature ($Tpin$) and the supply tertiary water temperature($Twin$) are equal. Nevertheless, the return primary water temperature ($Tpout$) is always greater than the supply tertiary water temperature ($Twout$). Again, the difference between two recent temperatures is proportional to the ratio of the primary water flow and the tertiary water flow.

The energy balance equation for the return water side will result in the following equation:

$$(q_{ws} - q_{wt})\, Tpin \, + \, q_{wt}\, Twout \, = \, q_{ws}\, Tpout \tag{6.4}$$

By rearranging the above equation we will have:

$$\frac{q_{wt}}{q_{ws}} = \frac{Tpin \, - \, Tpout}{Tpin \, - \, Twout} \tag{6.5}$$

We subtract 1 from both side of the equation. So,

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{Twout \, - \, Tpout}{Tpin \, - \, Twout} \tag{6.6}$$

The recent equation can be rewritten as

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{Twout \, - \, Tpout}{Twin \, - \, Twout} \tag{6.7}$$

According to the above discussion and combining equations 6.3 and 6.7 we will have:

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{(Tpin \, - \, Twin) \, + \, (Twout \, - \, Tpout)}{Twin \, - \, Twout} \tag{6.8}$$

or equivalently:

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{(Tpin \, - \, Twout) \, - \, (Twin \, + \, Tpout)}{Twin \, - \, Twout} \tag{6.9}$$

To avoid singularity in the recent equation it should be used as the following equation:

$$\frac{q_{wt}}{q_{ws}} - 1 = \frac{(Tpin \, - \, Twout) \, - \, (Twin \, + \, Tpout)}{|Twin \, - \, Twout - 1| \, + \, 1} \tag{6.10}$$

So, by measuring the four temperatures ($Tpin$, $Tpout$, $Twin$, and $Twout$) and using the recent formula there will be enough information for a controller to manipulate the bypass flow.

Remark: Those four thermocouples which measure $Tpin$, $Tpout$, $Twin$, and $Twout$ should be installed sufficiently far from the bypass pipe.

6.4.2 Problem Definition

As it was mentioned, Fig. 6.6 illustrates the four water temperatures around the bypass pipe (supply primary water temperature ($Tpin$), supply tertiary water temperature ($Twin$), return primary water temperature ($Tpout$), and return tertiary water temperature ($Twout$)). Comparing $Tpout$ and $Twout$ reveals that apart from the time that the controller follows the mixing control strategy there is always a small short circuit (positive bypass flow). That is, the return supply water is warmed up. In this case, not only the controller stays away from the optimal performance but also it violates the constraints (Copenhagen Building Regulations). Thus, it is vital to solve the bypass flow problem.

6.4.3 Bypass Flow Compensation

The control system structure along with the bypass flow compensator is shown in Fig. 6.7. In this control structure the main controller (the MPC controller) is as same as before and the bypass compensator which is slower than the MPC controller deals with the bypass flow problem. According to the equation 6.10 a simple PI controller can be applied as the bypass flow compensator.

Remark: The PI controller which controls the bypass flow has to be along with an anti-windup module because for some time that the main controller follows the mixing control strategy there will be accumulation of the bypass flow error.

6.5 Simplified Optimal Control Structure

Fig 6.8 shows the simplified optimal control structure. One PI controller (C_1) determines the primary water flow (q_{ws}) through the information from the inlet temperature feedback. To design this controller the linearized model from the primary water flow to the inlet temperature [2] has been used. Tertiary water flow (q_{wt}) is controlled by a PI controller (C_2) which tries to keep the tertiary water flow close to the primary water flow. Actually C_2 is as same as the bypass compensator but C_2 is a fast controller here. The variable speed pump acts as an actuator to control the tertiary water flow. Because the variable speed pump is much faster than the primary valve which acts as an actuator to control the primary water flow, the two controllers are decoupled in time domain again. Here also for the same reason that was mentioned before the pump compensator has to be equipped with the anti-windup module.

The results of applying the recent control system to the HVAC system is shown in Fig 6.9 and 6.10. As can be seen, the control system has perfect tracking of the set-point. The control system also shows

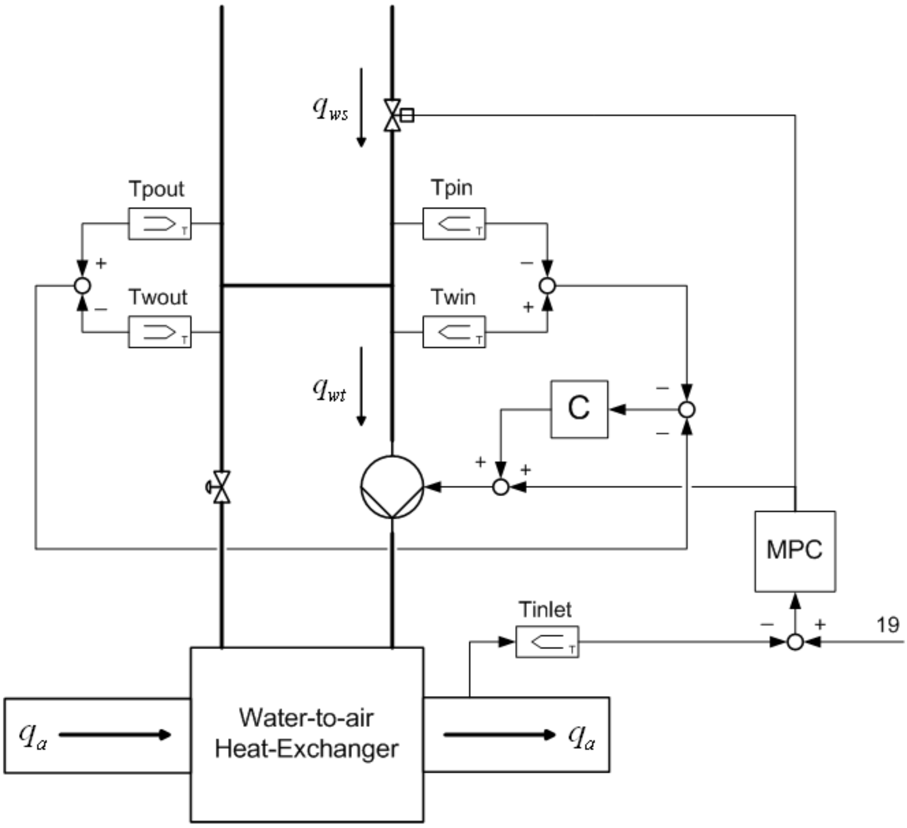

Figure 6.7: The control system along with the bypass compensator

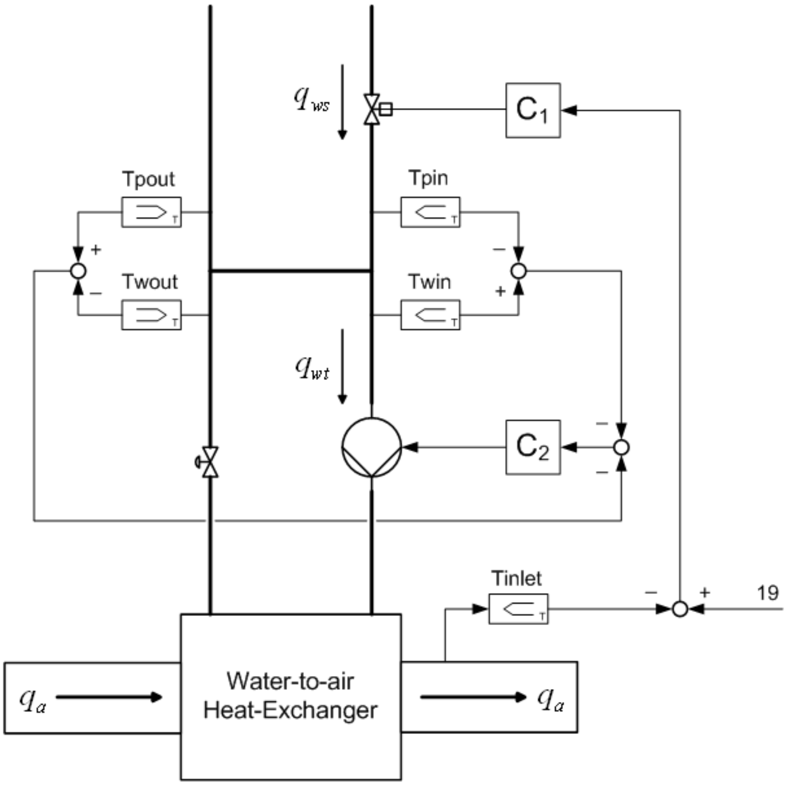

Figure 6.8: Simplified optimal control scheme

good disturbance rejection (disturbances from outdoor air temperature and the hot water temperature have been successfully rejected). Either the tertiary water flow curve or the water temperature curves around the bypass pipe obviously reveal the switch between two control strategies.

6.6 Energy Saving Aspects

The energy consumption of the pump is proportional to the third power of the flow. Mixing control strategy which requires constant tertiary water flow will impose continuous circulation of 300 l/h of water through the coil. However, the optimal control strategy plays with the tertiary water flow as the heating demand (thermal load) changes. The tertiary water flow curve in Fig. 6.9 illustrates this fact. Therefore, applying the optimal control strategy instead of the mixing control strategy will save up to 82% energy consumption of the tertiary pump. The recent energy saving figure calculation is based on the Blue Angel profile (used in the German energy labing scheme). The Europump Study Classification of Circulators examined circulator load profiles and concluded the profile to be appropriate for the Europump circulator classification scheme; in addition this profile was deemed valid for all EU member

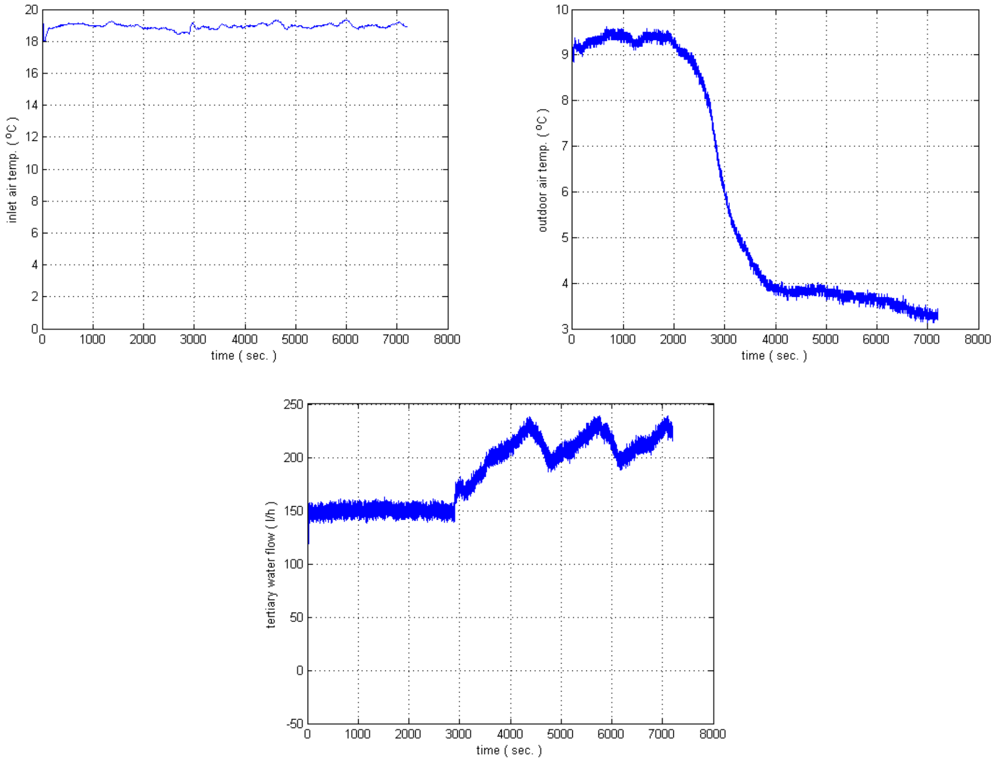

Figure 6.9: Results of applying simplified optimal control system(a)

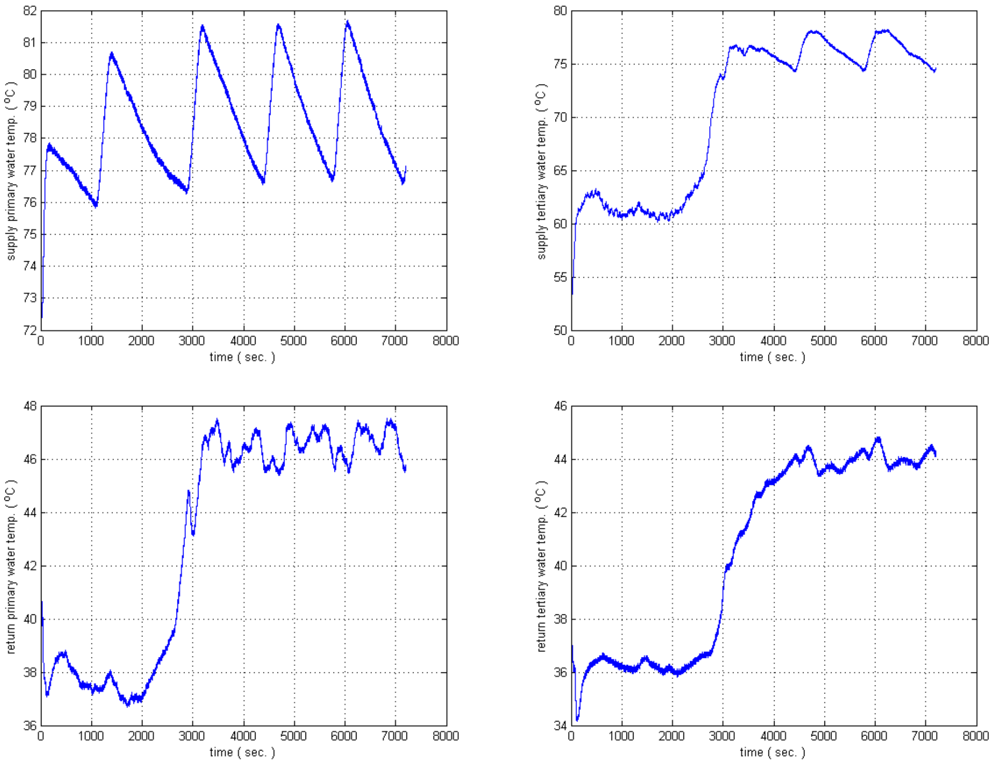

Figure 6.10: Results of applying simplified optimal control system(b)

states.

6.7 Conclusions

Simplified optimal control for a heating, ventilating, and air-conditioning (HVAC) system was presented in this paper. The optimal model-based controller which had been developed in [2] was implemented while following the practical optimal control strategy. Going through the bypass flow problem and its solution led to a simplified optimal controller. Implementation results for the recent controller showed fulfillment of the control goals.

6.8 Acknowledgments

The authors would like to thank financial supports and contributions of Danish Energy Net, Aalborg University, Center for Embedded Software Systems (CISS), Grundfos Management A/S, Exhausto A/S, and Danish Technological Institute (DTI) to this project.

Bibliography

[1] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, P. Svendsen, and F. Nielsen, Optimal Set-point Synthesis in HVAC Systems, *American Control Conference*, New York, NY, 2007, pp 5076-5081.

[2] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, P. Svendsen, and F. Nielsen, Optimal Model-Based Control in HVAC Systems, *American Control Conference,* Seattle, WA, 2008.

Nomenclature

q_a	inlet or outlet air flow (m^3/h)
$TE21$	outdoor air temperature (^oC)
$TE22$	temperature of outdoor air after heat recovery (^oC)
$TE11$	room air temperature (^oC)
$TE12$	temperature of room air after heat recovery (^oC)
q_{wt}	water flow of the tertiary circuit (l/h)
q_{ws}	water flow of the supply (primary/secondary) circuit (l/h)
$Twin$	tertiary supply water temperature (^oC)
$Twout$	tertiary return water temperature (^oC)
$Tinlet$	temperature of the supply air (^oC)
$Tpin$	primary/secondary supply water temperature (^oC)
$Tpout$	primary/secondary return water temperature (^oC)
η_{t1}	water-to-air heat exchanger temperature efficiency ($\eta_{t1} = \frac{Tinlet-TE22}{Twin-TE22}$)
η_{t2}	air-to-air heat exchanger temperature efficiency ($\eta_{t2} = \frac{TE22-TE21}{TE11-TE21}$)
ρ_w	water mass density (Kg/m^3)
C_{pw}	water specific heat ($J/Kg\ ^oC$)
ρ_a	air mass density (Kg/m^3)
C_{pa}	air specific heat ($J/Kg\ ^oC$)
wrf	wheel rotation factor ($1 \geq wrf \geq 0$)
n	rotation speed of the wheel ($10\ rpm \geq n \geq 0\ rpm$)
$\acute{A}_s$	heat transfer surface area of one tube (m^2)
U	mean air velocity in the tube (m/s)
$T_m(x,t)$	matrix temperature (^oC)
$T_a(x,t)$	air temperature (^oC)

h	convective heat transfer coefficient ($W/m^2\ {}^oK$)
L	wheel length (m)
A_m	cross sectional area of one tube of matrix (m^2)
A_a	cross sectional area of one tube for air (m^2)
K_m	matrix thermal conductivity ($W/m\ {}^oK$)
P	exposion time (half of the period) ($sec.$)
M_m	total matrix mass (Kg)
$\dot{m}_a$	air mass flow rate (Kg/h)
A_s	heat transfer surface area on the supply or exhaust side (m^2)
C_{pm}	matrix specific heat ($J/Kg\ {}^oC$)
C_r^*	$\frac{M_m\ C_{pm}\ n}{\dot{m}_a\ C_{pa}}$
NTU	$\frac{h\ A_s}{\dot{m}_a\ C_{pa}}$
C_{pc}	specific heat of the coil ($J/Kg\ {}^oC$)
$\dot{m}_{ws}$	supply water mass flow rate (Kg/h)
$\dot{m}_{wt}$	tertiary water mass flow rate (Kg/h)
m_{cw}	effective mass of the region of the coil at an average temperature equal to outlet water temperature (Kg)
m_{ca}	effective mass of the region of the coil at an average temperature equal to outlet air temperature (Kg)
C_w	$C_{pc}\ m_{cw}$ ($J/{}^oC$)
C_a	$C_{pc}\ m_{ca}$ ($J/{}^oC$)
N	pump speed (rpm)
k	$k = 1000 \cdot \frac{\rho_a\ C_a\ q_a}{\rho_w\ C_w}$

Appendices

Appendix A

Decoupling of the HVAC System

As it was mentioned, the control of the HVAC system can be broken into the control of two subsystems: heat recovery wheel (air-to-air heat exchanger) and water-to-air heat exchanger. The following equations describe those two subsystems:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3 & b_4 \\ b_1 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$

$$y = [1 \;\; 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{A.1}$$

$$T\dot{E}22 = a_6 \cdot TE22 + b_5 \cdot wrf \tag{A.2}$$

where equation (A.1) represents the water-to-air heat exchanger model and equation (A.2) represents the heat recovery wheel model.

Moreover,

$$a1 = -0.0352\,, \quad a2 = 0.0310\,, \quad a3 = 0.0564$$

$$a4 = -0.5961\,, \quad a5 = 0.4833\,, \quad b1 = 17232$$

$$b2 = 46628\,, \quad b3 = 227635\,, \quad b4 = -199119$$

The linearized water-to-air heat exchanger model is a two-input single-output system. However, a system can be decoupled if it has a square transfer matrix. So, we have to add another output to the system. We choose the state *Twout* (return tertiary water temperature) to be the second output for the system. The new system model is as following:

$$\begin{bmatrix} \frac{94913.9426 \quad (s+0.08029)}{(s+0.5962) \quad (s+0.05885)} & \frac{-58047.9815(s+0.05037)}{(s+0.5962) \quad (s+0.05885)} \\ \frac{32755.2056}{(s+0.05885)} & \frac{7933.4996}{(s+0.05885)} \end{bmatrix} \tag{A.3}$$

Now we suppose that a decoupling filter (DF) is applied to the system. So, the resulted system has to be diagonal. It can be explained as follows:

$$\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} DF_{11}(s) & DF_{12}(s) \\ DF_{21}(s) & DF_{22}(s) \end{bmatrix} = \begin{bmatrix} DP_{11}(s) & 0 \\ 0 & DP_{22}(s) \end{bmatrix} \tag{A.4}$$

Then it will result in the following equations:

$$P_{11}(s)\, DF_{12}(s) \;+\; P_{12}(s)\, DF_{22}(s) \;= 0 \tag{A.5}$$

$$P_{21}(s)\, DF_{11}(s) \;+\; P_{22}(s)\, DF_{21}(s) \;= 0 \tag{A.6}$$

and consequently:

$$\frac{DF_{11}(s)}{DF_{21}(s)} = -\frac{P_{22}(s)}{P_{21}(s)} \quad ; \quad \frac{DF_{12}(s)}{DF_{22}(s)} = -\frac{P_{12}(s)}{P_{11}(s)} \tag{A.7}$$

Considering equations A.3 and A.7 reveals that $\frac{P_{22}(s)}{P_{21}(s)}$ is a pure gain and also $\frac{P_{12}(s)}{P_{11}(s)}$ can be approximated as a pure gain. Therefore, the coupling filter will be a constant matrix.

Fig. A.1 shows the structure for decoupling control of the water-to-air heat exchanger. The output which has to be controlled is the inlet temperature ($Tinlet$). So, we can control tertiary water flow (q_{wt}) in such a way that the second optimality criterion is met. As a result, we choose the controller C_1 as a simple gain which results in equal primary and tertiary water flow at the design point. Having C_1 is necessary because two differnt actuators control the primary water flow and the tertiary water flow (primary valve and the variable speed pump, respectively). Then the controller C_2 can be a PI controller which has to be designed based on the $DP_{11}(s)$ to manipulate primary water flow in order to have the inlet temperature follow the set-point (19^oC). Controller C_3 is a PI controller which will try to keep the primary and the tertiary water flow close to one another when the system slips away from the design point.

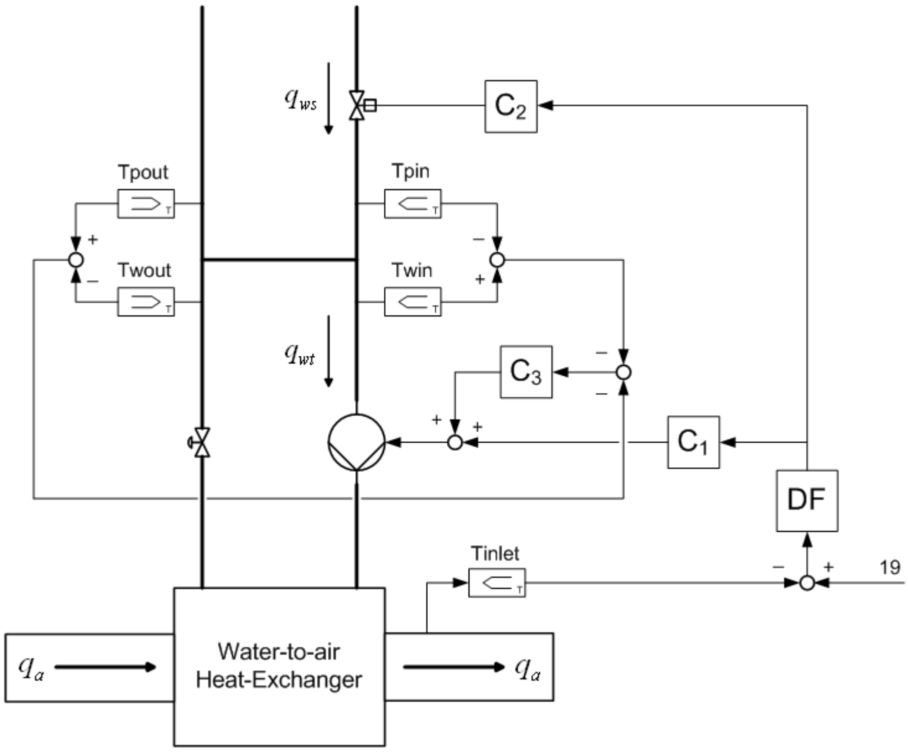

Figure A.1: Decoupling control for water-to-air heat exchanger along with bypass compensation

Appendix B

A Parameterization of The Observer-Based Controllers

A Parameterization of Observer-Based Controllers: Bumpless Transfer by Covariance Interpolation

Jakob Stoustrup[1] and Mohammad Komareji[2]

Abstract

This paper presents an algorithm to interpolate between two observer-based controllers for a linear multivarible system such that the closed loop system remains stable throughout the interpolation. The method interpolates between the inverse Lyapunov functions for the two original state feedbacks and between the Lyapunov functions for the two original observer gains to determine an intermediate observer-based controller.

B.1 Introduction

Observer-based controllers play a significant role in the industry because processing lines do not generally contain adequate number of sensors to measure all the state variables and even in some cases it

[1]Jakob Stoustrup is with Aalborg University as a Professsor in The Department of Control and Automation; `jakob@es.aau.dk`

[2]Mohammad Komareji is a PhD student in The Department of Control and Automation, Institute of Electronic Systems, Aalborg University, Aalborg, Denmark; `komareji@es.aau.dk`

may not be possible to get sensor information. Since introducing the modern control theory lots of different methods for designing observer-based controllers such as pole-placement methods, optimization methods, LMI approaches and $\cdots$ have been developed.

Ideally, one would like to design a controller that is both fast and has good measurement noise rejection properties. Clearly this is not possible, as increasing the bandwith of the closed loop system will also make the system more sensetive to measurement noise [1]. Then the option is to design two distinct controllers: Controller K_1 which has low closed loop bandwith and is therefore not very sensetive to noise but exhibits a slow response. Controller K_2 which has high bandwith and is therefore fast but very sensetive to noise. Another reason to design two controllers for a certain plant can be associated with actuator saturation [8]. Also achieving some predefined output properties in the system performance can lead to follow the scheduled controllers approach. Having designed the two controllers, the next issue which has to be addressed is how to switch between these two controllers. In many systems jumps in the input to the system are not desirable. Thus, finding a smooth way to switch between the two controllers comes up as a crucial problem.

One important step in actual gain scheduling involves implementing the family of linear controllers such that the controller coefficient (gains) are varied (scheduled) according to the current value of the scheduling variable, also called scheduling signal that may be either exogenous signal or endogenous signal with respect to the plant [9]. Various issues arise here. The issue about the obsever-based controllers here is that the simple gain interpolation technique which usually works well leaves the closed loop system unstable for some intermediate points if applied to interpolate between two observer based controllers.

This paper presents an algorithm for interpolation between two observer-based controllers, designed to control a linear multivarible system, which renders the closed loop system stable for all values of the interpolation parameter. The family of the observer-based controllers which will be introduced here can help the designer to achieve a safe bumpless transfer between two obsever-based controllers to reach the control objectives. Finally, two numerical examples illustrate our claims.

B.2 Preliminaries

The following notations are used in this paper. X^* indicates the transpose for X which is either a matrix or a vector. $X < 0$ ($X > 0$) means that X is symmeric and negetive definite (positive definite). $Re(X)$ denotes the real part of a complex number. Finally, I stands for an identity matrix with appropriate dimension.

Consider the open loop system

$$\dot{x} = Ax + Bu\,, \quad y = Cx + Du$$

then:

- The system is aysmptotically stable if all eigen values of A satisfy $Re(\lambda) < 0$ [4].
- A matrix A is Hurwits if and only if for any given positive definite symmetric matrix Q there exists a positive definite matrix P that satisfies the Lyapunov function [3]:

$$PA + A^*P = -Q\,, \quad Q = Q^*$$

or equivalently

$$PA + A^*P < 0$$

- The recent criterion can be written as

$$AP^{-1} + P^{-1}A^* < 0$$

Proof. We have to multiply $PA + A^*P$ by P^{-1} from right and left to get the new criterion[3]. □

B.3 Main Results

Throughout this paper we will assume that (A,B) is controllable and (C,A) is observable. It should be noted that a slightly weaker results can also result even if (A,B) and (C,A) are only stabilizable and detectable.

J. Bertram in 1959 was perhaps the first to realize that if a given system realization was state controllable, then any desired characteristic polynomial could be obtained by state-variable feedback [5]. Since then state feedback and static output feedback have been two of the most researched and written about issues in modern control theory. There is, of course, a long history of gain scheduling in applications too. However, bumpless transfer (soft switching) between two state feedbacks need some precise considerations because the gain interpolation of gain scheduled state feedbacks can leave the closed loop system unstable for the intermediate points. The following lemma presents an algorithm for interpolating between two state feedbacks while the closed loop system remains stable for all intermediate points.

[3] M and N are two arbitrary positive definite matrices. Then we have the following properties [2]:

1. M is invertible and its inverse (M^{-1}) is also positive definite.
2. If $r > 0$ is a real number then rM is positive definite.
3. The sum $M + N$ and the products MNM and NMN are also positive definite.

Lemma 1. *Consider the following control system:*

$$\dot{x} = Ax + Bu$$

and assume that $u = F_0\, x$ and $u = F_1\, x$ are both stabilizing state feedback laws, with Lyapunov functions:

$$V_0(x) = x^*\, X_0\, x \quad \text{and} \quad V_1(x) = x^*\, X_1\, x$$

respectively, with $X_i > 0, i = 0,1$. Then, a family of state feedback gains $F(\alpha)$ which stabilizes the system for every α, $0 \leq \alpha \leq 1$ is given by:

$$F(\alpha) = \mathscr{F}_\ell(J_F, \alpha I) \tag{B.1}$$

where

$$J_F = \begin{pmatrix} F_0 & (F_1 - F_0)X \\ I & I - X \end{pmatrix}, \quad X = X_1^{-1} X_0$$

Furthermore, $F(\alpha)$ satisfies $F(0) = F_0$ and $F(1) = F_1$.

Proof. Defining $Y_0 = X_0^{-1}$ and $Y_1 = X_1^{-1}$, we can rewrite the Lyapunov inequalities corresponding to $V_0(x)$ and $V_1(x)$ as:

$$Q_0 := (A + BF_0)Y_0 + Y_0(A + BF_0)^* < 0$$

and

$$Q_1 := (A + BF_1)Y_1 + Y_1(A + BF_1)^* < 0$$

respectively. We will demonstrate, that the matrix valued function

$$Y(\alpha) = (1 - \alpha)Y_0 + \alpha Y_1$$

which is positive definite for $\alpha \in (0;1)$, satisfies

$$(A + BF(\alpha))Y(\alpha) + Y(\alpha)(A + BF(\alpha))^* < 0$$

for all $\alpha \in (0;1)$. To that end, we observe that:

$$
\begin{aligned}
&(A+BF(\alpha))Y(\alpha) \\
&\quad = \left(A+B\left(F_0+\alpha(F_1-F_0)X(I-\alpha(I-X))^{-1}\right)\right)((1-\alpha)Y_0+\alpha Y_1) \\
&\quad = \left(A+BF_0+\alpha B(F_1-F_0)Y_1Y_0^{-1}\left(I-\alpha\left(I-Y_1Y_0^{-1}\right)\right)^{-1}\right)((1-\alpha)Y_0+\alpha Y_1) \\
&\quad = \left(A+BF_0+\alpha B(F_1-F_0)Y_1\left((1-\alpha)Y_0+\alpha Y_1\right)^{-1}\right)((1-\alpha)Y_0+\alpha Y_1) \\
&\quad = (A+BF_0)\left((1-\alpha)Y_0+\alpha Y_1\right)+\alpha B(F_1-F_0)Y_1 \\
&\quad = (1-\alpha)\left(A+BF_0\right)Y_0+\alpha\left(A+BF_1\right)Y_1
\end{aligned}
$$

from which we conclude that:

$$(A+BF(\alpha))Y(\alpha)+Y(\alpha)(A+BF(\alpha))^* \quad = (1-\alpha)Q_0+\alpha Q_1 < 0, \qquad \forall \alpha \in (0;1)$$

which establishes the proof.

From the last argument, note that in the special case $Q_0 = Q_1$, which is often obtainable, the proposed feedback will actually remain stable for *all* α, not just for $\alpha \in (0;1)$. □

Note also that if there is a common Lyapunov function for the both state feedback controllers the Lemma 1 interpolation reduces to the simple gain interpolation.

In most practical applications, the system states are not completely accessible and all the designer knows are the outpupts and the inputs. Hence, the estimation of the system states is often necessary to realize some specific design objectives. The important issue in designing the observer gain (L) is to have $A+LC$ as a stable system. Thus, the critical point in bumpless transfer between two observers is the stability of $A+LC$. The coming lemma expresses an algorithm for interpolating between two observers while the stability of $A+LC$ is guaranteed.

Lemma 2. *Let L_0 and L_1 be two different Luenberger observer gains for the following system:*

$$\dot{x} = Ax+Bu\,, \quad y = Cx+Du$$

and suppose that

$$V_0(x) = x^*\,Z_0\,x \quad \textit{and} \quad V_1(x) = x^*\,Z_1\,x$$

are the corresponding Lyapunov functions to $A+L_0C$ and $A+L_1C$, respectively, with $Z_i > 0$, $i = 0,1$. Then a family of observer gains $L(\beta)$, $0 \leq \beta \leq 1$ is given by:

$$L(\beta) = \mathscr{F}_\ell(J_L, \beta I) \tag{B.2}$$

where

$$J_L = \begin{pmatrix} L_0 & I \\ Z(L_1 - L_0) & I - Z \end{pmatrix}, \quad Z = Z_0^{-1} Z_1$$

Moreover, $L(\beta)$ satisfies $L(0) = L_0$ and $L(1) = L_1$.

Proof. The intermediate points admit the Lyapunov function given by

$$Z(\beta) = (1-\beta)\, Z_0 + \beta\, Z_1$$

To verify the above the claim, we have to show that

$$Z(\beta)(A + L(\beta)C) + (A + L(\beta)C)^* Z(\beta) < 0$$

The first term in left side of the Lyapunov inequality can be rewritten as:

$$\begin{aligned} &Z(\beta)(A + L(\beta)C) \\ &\quad = ((1-\beta)Z_0 + \beta Z_1)\left(A + \left(L_0 + \beta(I - \beta(I - Z))^{-1} Z(L_1 - L_0)\right)C\right) \\ &\quad = ((1-\beta)Z_0 + \beta Z_1)\left(A + L_0 C + \beta(I - \beta(I - Z_0^{-1} Z_1))^{-1} Z_0^{-1} Z_1 (L_1 - L_0) C\right) \\ &\quad = ((1-\beta)Z_0 + \beta Z_1)\left(A + L_0 C + \beta((1-\beta)Z_0 + \beta Z_1))^{-1} Z_1 (L_1 - L_0) C\right) \\ &\quad = (1-\beta)Z_0\,(A + L_0 C) + \beta Z_1\,(A + L_1 C) \end{aligned}$$

So, we can conclude:

$$\begin{aligned} &Z(\beta)(A + L(\beta)C) + (A + L(\beta)C)^* Z(\beta) \\ &\quad = (1-\beta)(Z_0(A + L_0 C) + (A + L_0 C)^* Z_0) + \beta(Z_1(A + L_1 C) + (A + L_1 C)^* Z_1) \end{aligned}$$

According to the assumptions Z_0 and Z_1 are Lyapunov functions for $A + L_0 C$ and $A + L_1 C$, respectively. Thus, we have:

$$Z_0(A + L_0 C) + (A + L_0 C)^* Z_0 < 0$$

and

$$Z_1(A + L_1 C) + (A + L_1 C)^* Z_1 < 0$$

Then the proof is immediate. □

According to the separation principle the problem of desigining an observer-based controller can be broken into two separate parts: observer design and state feedback design. This approach facillitates the design procedure. Lemma 1 presented an algorithm for interpolation between two state feedbacks while satisfying the stability critorion. Similar algorithm was described in lemma 2 for observers. Combining the results from two recent lemmas leads to an algorithm for bumpless transfer between two observer-based controllers.

Theorem 1. *Consider two observer-based controllers*

$$K_0 = \left(\begin{array}{c|c} A+BF_0+L_0C+L_0DF & -L_0 \\ \hline F_0 & 0 \end{array}\right)$$

and

$$K_1 = \left(\begin{array}{c|c} A+BF_1+L_1C+L_1DF & -L_1 \\ \hline F_1 & 0 \end{array}\right)$$

for the minimal system

$$\dot{x} = Ax+Bu\,, \quad y = Cx+Du$$

.

which have been already designed [6].

Then a family of observer-based controllers for the mentioned system is denoted as

$$K(\gamma) = \mathscr{F}_\ell(J_K, \gamma I)\,, \quad 0 \le \gamma \le 1 \tag{B.3}$$

where

$$J_K = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

$$M_{11} = \left(\begin{array}{c|c} A+BF_0+L_0DF_0+L_0C & -L_0 \\ \hline F_0 & 0 \end{array}\right)$$

$$M_{12} = \begin{pmatrix} (L_0D+B)\;(F_1+F_0)\;X & I \\ (F_1-F_0)\;X & 0 \end{pmatrix}$$

$$M_{21} = \begin{pmatrix} I & 0 \\ Z\;(L_1-L_0)\;(C+DF_0) & -Z\;(L_1-L_0) \end{pmatrix}$$

$$M_{22} = \begin{pmatrix} I-X & 0 \\ Z\;(L_1-L_0)\;D\;(F_1-F_0)\;X & I-Z \end{pmatrix}$$

X and Z are as those defined in (B.1) and (B.2).

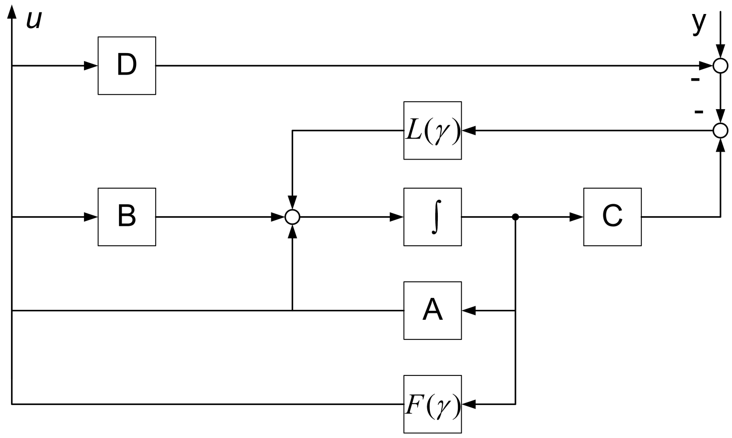

Figure B.1: The family of observer-based controllers introduced by theorem 1

Also, $K(\gamma)$ *satisfies* $K(0) = K_0$ *and* $K(1) = K_1$.

Proof. Fig. B.1 shows the family of observer-based controllers presented by theorem 1 (equation (B.3) is the LFT representation of the illustrated block diagram). Applying the principle of separation and then results in Lemma 1 and 2, the proof is immediate. □

It is interesting to see that if there is a common Lyapunov function for the closed loop system composed of the plant and the family of observer-based controllers the interpolation reduces to the simple gain interpolation. Furthermore, the closed loop system is stable for any γ (not only $0 \leq \gamma \leq 1$) and any rate of switching [1]. Otherwise, in general case which was addressed in theorem 1 we assume that the scheduling variable is slow enough not to cause stability problems.

B.4 Numerical Examples

Example 1: This example illustrates the fact that the gain interpolation between two stabilizing observer-based controller can cause instability for some intermediate points. However, it is shown that the algorithm proposed by theorem 1 does not have this deficiency.

Consider the following third order system,

$$A = \begin{pmatrix} -0.597 & -0.038 & 0.832 \\ 1.636 & -0.121 & 0.068 \\ -0.334 & -0.968 & -0.311 \end{pmatrix}, \; B = \begin{pmatrix} -0.638 \\ 0.091 \\ 0.363 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \; D = 0$$

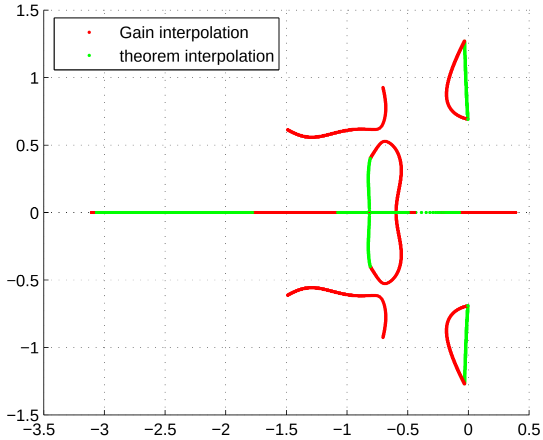

Figure B.2: Eigen Value Plot of The Closed Loop System in Example 1 where Gain Interpolation (red curve) and Theorem Interpolation (green curve) of Observer-Based Controllers are applied.

This system is unstable with eigen values of -1.3195 and $0.1453 \pm 1.0314i$. Then two different observer-based controllers have been designed for stabilizing the system:

$$K_0 = \left(\begin{array}{ccc|c} -1.863 & -1.995 & -0.393 & 1.353 \\ 0.762 & -0.896 & -0.811 & 0.861 \\ -1.75 & -1.992 & -1.751 & 1.367 \\ \hline -0.135 & 0.947 & -0.200 & 0 \end{array}\right)$$

and

$$K_1 = \left(\begin{array}{ccc|c} -1.152 & -1.103 & -0.834 & 0.916 \\ 1.033 & -0.651 & -0.376 & 0.551 \\ -1.441 & -1.784 & -0.785 & 0.901 \\ \hline -0.567 & 0.233 & 1.175 & 0 \end{array}\right)$$

Fig. B.2 illustartes the eigen value plot of the closed loop system where the gain interpolation and the interpolation proposed by the recent theorem for observer-based controllers are applied for bumpless transfer between the two designed controllers. The plot reveals that the naive gain interpolation of the controllers fails to maitain the stability of the closed loop system while the interpolation appeared in the recent theorem renders the closed loop system stable for all $0 \leq \gamma \leq 1$.

Example 2: In this example we will show the bumpless transfer between two state feedbacks designed to meet different objectives in a HVAC system applying the algorithm described by theorem 1.

We consider here the control of the inlet air temperature of a ventilation system (a water-to-air heat

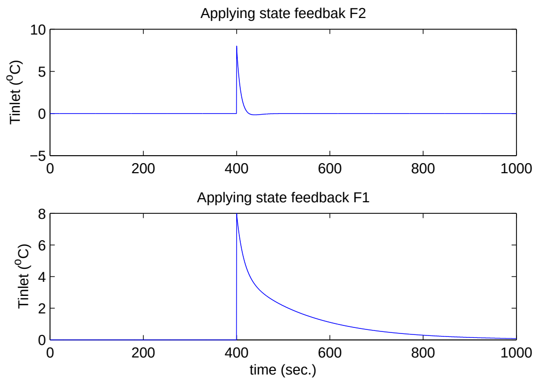

Figure B.3: inlet temperature while applying state feedbacks F1 and F2

exchanger). In accordance with the linearized model of a water-to-air heat exchanger described in [7] the linear model from primary (supply) water flow ($\dot{m}_{ws}$) to inlet air tempertaure ($Tinlet$) can be explained as following:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3 \\ b_1 \end{bmatrix} \cdot \dot{m}_{ws}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix}$$

where

$$a1 = -0.0352\,, \quad a3 = 0.0564\,, \quad a4 = -0.5961$$

$$b1 = 17232\,, \quad b3 = 227635$$

and $Twout$ represents the temperature of the water that leaves the coil. Two state feedbacks $F1 = \begin{bmatrix} -0.2464 \times 10^{-5} & 0.0155 \times 10^{-5} \end{bmatrix}$ and $F2 = \begin{bmatrix} -0.2103 \times 10^{-5} & 0.0421 \times 10^{-5} \end{bmatrix}$ are designed for this system. Fig. B.3 illustrates the system output ($Tinlet$) while state feddbacks $F1$ and $F2$ are applied to remove the step disturbance occuring at $400sec.$. The response resulted from applying of $F1$ is slow but no overshoot happens. Howere, applying $F2$ results in a faster response with overshoot. The fact is that the overshoot in the response is not desirable because in the real system it cuases some oscillations which damps very slowly.

To overcome the problem of designing a fast controller with no overshoot we combine the two state feedbacks: When the output of the system ($Tinlet$) is more than (1^oC) away from the set-point $F2$ will the the active controller but when the system output reaches the bound of ($\pm 1^oC$) from the set-

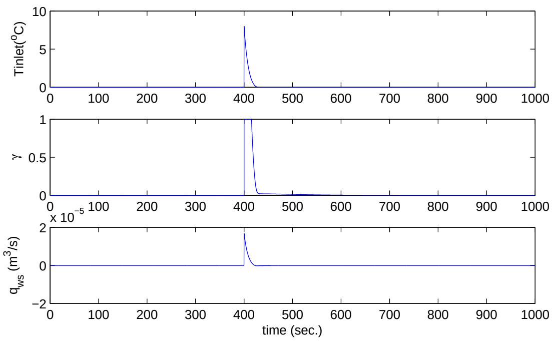

Figure B.4: inelt temperature, scheduling parameter (γ), and the control input when a family of state feedbacks presented by theorem 1 acting upon the HVAC system

point a bumpless transfer, applying the algorithm described in theorem 1, from $F2$ to $F1$ happens (γ is scheduled in accordance with the distance from the set-point). Fig. B.4 shows the result of applying the recent control strategy to remove a step disturbance occuring at $400sec.$. As can be seen, we have a fast response with no overshoot. Therefore, the recent control strategy meets the control objectives.

B.5 Conclusions

In this paper an algorithm to interpolate between two observer-based controllers was presented. The proposed algorithm guaranteed the stability of the closed loop system for the intermediate points. At the end, two numerical examples were presented. The first example showed that the naive gain interpolation failed to maitain the stability of the closed loop system while the thoerem 1 algorithm worked perfectly to keep the closed loop system stable. The second example illustrated the application of the proposed interpolation algorithm to bumpless transfer between two observer-based controllers.

References to Appendix B

[1] Joao P. Hespanha, A. Stephen Morse, Switching between stabilizing controllers, *Automatica*, 38, pp 1905-1917, 2002.

[2] Rajendra Bhatia, Positive Definite Matrices, *Princeton Series in Applied Mathematics*, 2007.

[3] Hassan K. Khalil, Nonlinear Systems (third edition), *Prentice Hall*, 2002.

[4] Katshuhiko Ogata, Modern Control Engineering (third edition), *Prentice-Hall*, 1997.

[5] R.E. Kalman, P. Flab, M.A. Arbib, Topics in Mathematical System Theory, *McGraw-Hill*, 1969.

[6] Kemin Zhou, John C. Doyle, Keith Glover, Robust and Optimal Control, *Prentice Hall*, 1996.

[7] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, P. Svendsen, and F. Nielsen, Optimal Model-Based Control in HVAC Systems, *American Control Conference*, Seattle, WA, June 2008, pp 1443-1448.

[8] Solmaz Sajjadi-Kia, Faryar Jabbari, Use of Scheduling for Anti-windup Controller Design, *American Control Conference*, New York City, NY, July 2007.

[9] Wilson J. Rugh, Jeff S. Shamma, Research on Gain Scheduling, *Automatica*, 2000, vol 36, pp 1401-1425.

Appendix C

HVAC test system set-up

The HVAC test system shown in Fig. C.1 and manufactured by Exhausto A/S is composed of two heat exchangers: Heat recovery wheel and water-to-air heat exchanger.

C.1 Heat Recovery Wheel

The air-to-air heat exchanger is a rotary heat exchanger in aluminium, with low pressure loss (shown in Fig. C.2). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow. Here, it is supposed that the ratio of the supply air flow to the return air flow is one. It means that the HVAC system will not change the pressure of the zone which is connected to. On the other hand,it removes air from the zone as much as air it adds to the zone.

Figure C.1: HVAC test system

Figure C.2: Heat recovery wheel

In this project, results of testing the rotary heat exchanger that was performed according to European Standard for laboratory testing of air-to-air heat recovery devices (EN 247, EN 305, EN 306, EN 307, EN 308) have been used. This European Standard is intended to be used as a basis for testing heat recovery devices for HVAC systems, which as specified in EN 247 consist of the heat exchanger itself installed in a casing having the necessary air duct connecting elements and in some cases the fans and pumps, but without any additional components of the HVAC system.

The temperature of the fresh air which is supplied to the room is controllable by manipulating the rotation speed of the wheel. The faster the rotation speed, the higher the supply air temperature.

C.2 Air Flow Measurement

Airflow measurement techniques are necessary to determine the most basic of indoor air quality questions: 'Is there enough fresh air to provide a healthy environment for the occupants of the building?' In HVAC systems the most common ways of measuring the air flow are based on either the pressure difference measuring or the first law of thermodynamics. For the HVAC test system we have applied the former method.

The pressure difference measuring method is based on the following equation:

$$q = A \cdot \sqrt{\frac{2(P_2 - P_1)}{\rho_a}}$$

where
q: air flow
A: cross area
P_x: absolute pressure

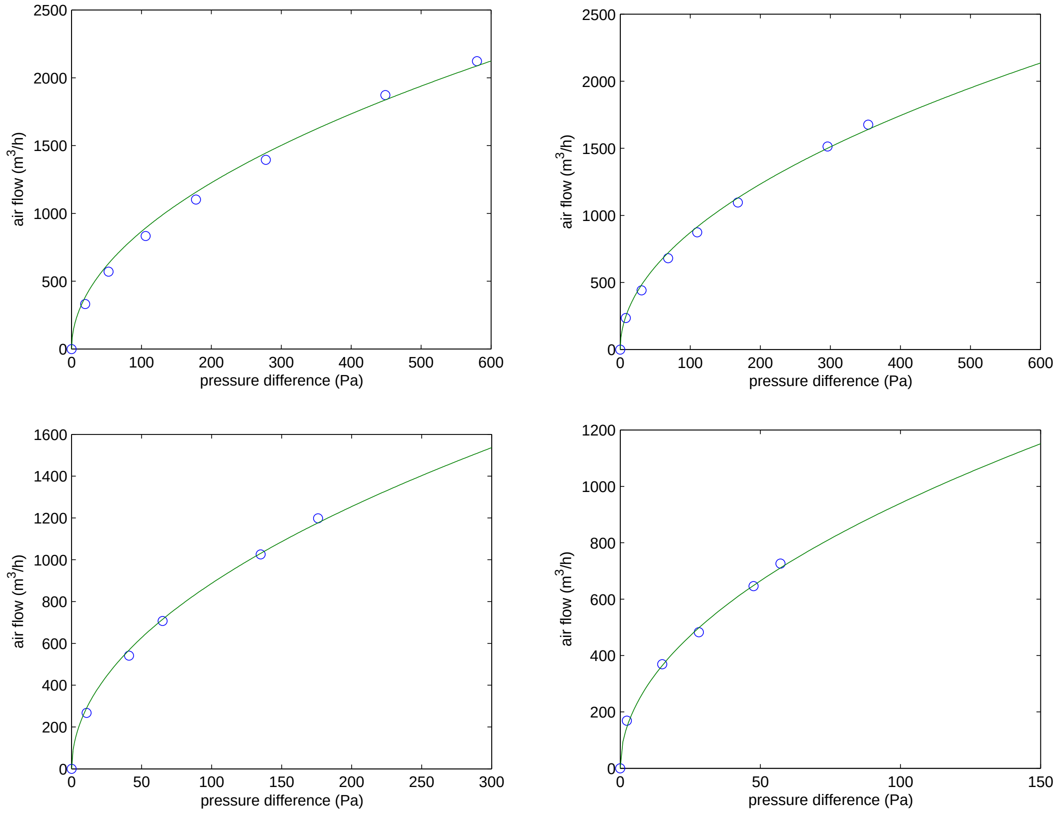

Figure C.3: inlet air flow vs. pressure difference(fan voltage: 10 V, 8 V, 6 V and 4 V)

ρ_a: density of the air

In the HVAC test system changing the fan speed will change the characteristic curve dispalying the hydraulic circuit. Hence, the air flow not only depends on the pressure difference measurements but also depends on the fan voltage. Fig. C.3 and Fig. C.4 illustrate the relation between the pressure difference and the air flow for the inlet circuit and the outlet circuit, respectively while the fan voltage has been kept constatnt at 10 V, 8 V, 6 V, and 4 V. It is necessary to have separate measurements for the inlet air flow and the outlet air flow because they have different characteristic curves (inlet air flow goes through the water-to-air heat exchanger but the outlet air flow does not). To measure the air flow when the fan voltage is an intermediate point the linear interpolation between two nearest curves has been used.

Fig. C.5 and Fig. C.6 show the final curves for measuring the inlet air flow and the outlet air flow which are functions of pressure difference ($q = K_x\sqrt{P_2 - P_1}$, assuming the constant air density) and the fan voltage, respectively.

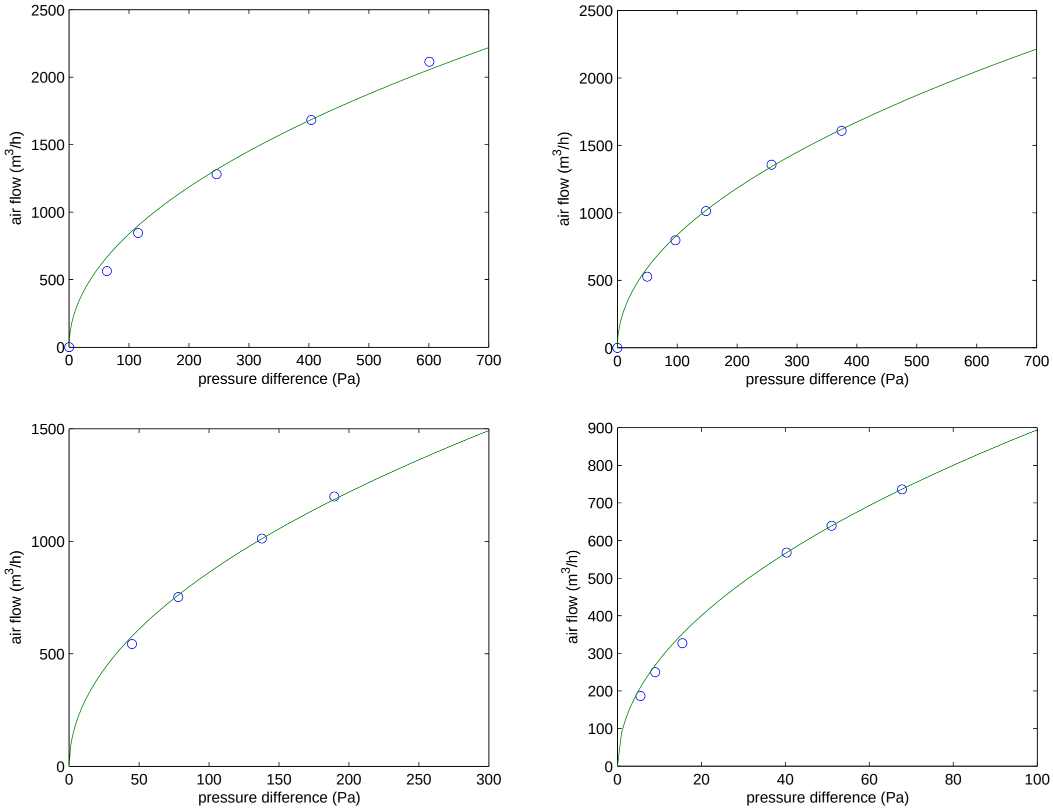

Figure C.4: outlet air flow vs. pressure difference (fan voltage: 10 V, 8 V, 6 V and 4 V)

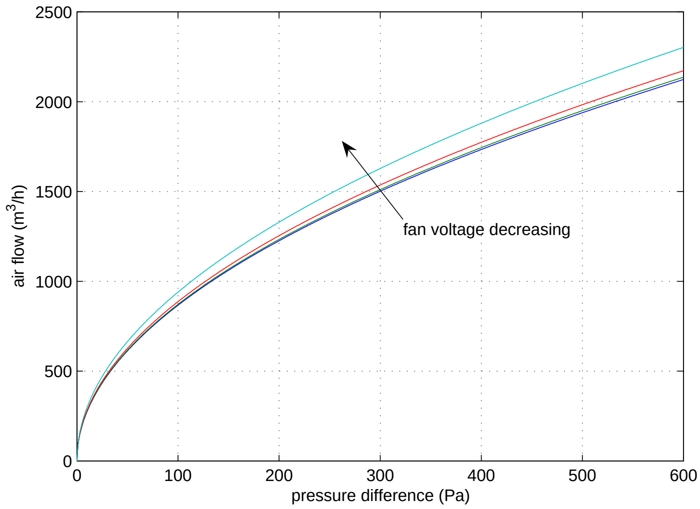

Figure C.5: inlet air flow curves

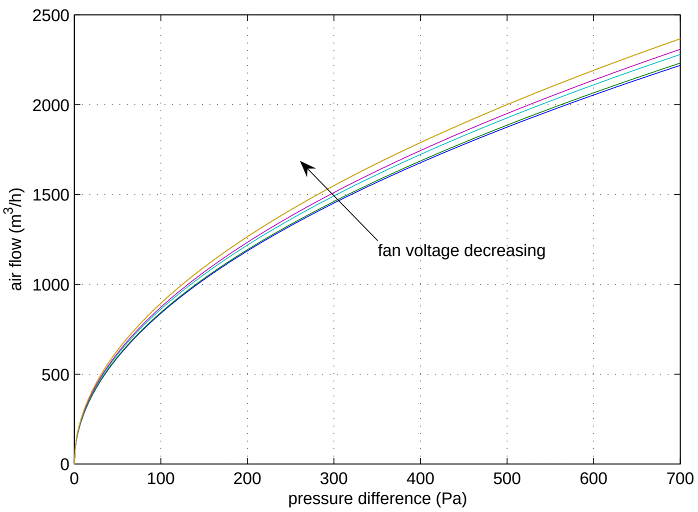

Figure C.6: outlet air flow curves

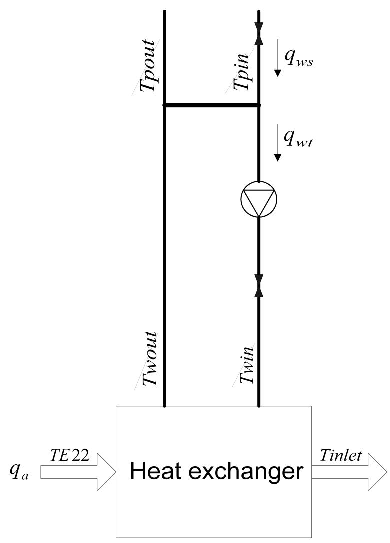

Figure C.7: water-to-air heat exchanger

C.3 Water-to-air Heat Exchanger

The water-to-air heat exchanger is shown in Fig. C.7. We can see in the figure that a bypass pipe is decoupling the tertiary circuit from the supply circuit. The hot water is supplied to the coil by Grundfos MAGNA 25-60 pump. A Siemens motorized valve control the flow of the supply hot water. On the other hand, it controls the amount of hot water which is injected to the tertiary circuit. The differnce between the HVAC test system and the current Exhasuto HVAC systems is the installation of the variable speed pump in the tertiary circuit. In the current Exhausto HVAC systems the tertiary water flow is constant (a constant speed pump is installed in the tertiary circuit) and the inlet temperature is controlled by the primary motorized valve. However, the new set-up in the test system enables us to examine control algorithm which plays with the tertiary water flow as well as a control input too.

C.4 Control Interface Module

The control interface module establishes an interface between the HVAC test system and the user. It is comprised by both hardware and software. The software is installed on the host PC, where the SIMULINK control diagram is compiled into an executable file, which is downloaded to the target PC through a LAN (or Internet and LAN). The target PC is equipped with two PCI National Instruments DAQ (PCI-6703 and PCI-6031E) which are the input card and the output card to connect target PC to the HVAC test system.