GENERAL CONDITIONS FOR LOOP TRANSFER RECOVERY

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ABSTRACT.

This paper gives a general and concise formulation of the Loop Transfer Recovery (LTR) design problem. Necessary and sufficient conditions are given for obtaining LTR for non-strictly proper systems when general observer-based controllers or general output feedback controllers are applied. The connection between these two controller types is also described in the light of LTR.

1. INTRODUCTION.

The scope of this paper is to introduce a more systematic and general way of describing the recovery conditions. This will be done by considering different types of recovery errors, i.e. the differences between the desired and the obtained loop transfer functions, for both open and closed-loop transfer or sensitivity functions. The recovery error description is general, because it is not related to any specific controller type. Open-loop recovery errors was first introduced by Goodman [4] for full order observer-based controllers, and later extended to include minimal-order observer-based controllers [7]. LTR design by using full-order or minimal-order oberver-based controllers for non-strictly proper minimum phase systems has also been treated in [2].
Instead of using specific observer-based controllers, we will use the much more general Luenberger observer formulation [6] in connection with LTR-design. The Luenberger observer includes all known observer types as special cases [7]. The second controller type which will be used here is a general output feedback controller. Based on these two controller types, it is possible to give general necessary and sufficient conditions for obtaining exact recovery, i.e. a zero recovery error for non-strictly proper systems. It will be shown that the two sets of conditions for obtaining exact and asymptotic recovery, are the same for both controller types. The connection between the two controllers will also be treated in the light of LTR.
The two controllers will be introduced in section 2 and the general LTR conditions are derived in section 3 followed by a discussion in section 4.

2.0 CONTROLLER TYPES.

Let the FDLTI plant model be represented by a minimal state-space realization S(A,B,C,D):

$$\Sigma : \begin{cases} \dot{x} = Ax + Bu, & x \in \mathbb{R}^n, u \in \mathbb{R}^r \\ y = Cx + Du, & y \in \mathbb{R}^m \end{cases} \tag{1}$$

$$G(s) = C(sI - A)^{-1}B = C\Phi(s)B$$

with $n > m \geq r$ and C,B of full rank. To control the plant, a general controller is used, described by:

$$\Sigma_C : \begin{cases} \dot{z} = Hz + \alpha Gu + Ey, & z \in \mathbb{R}^r, \alpha \in \{0,1\} \\ w = Pz + Vy \end{cases} \tag{2}$$

with the transfer function:

$$C(s) = V + P(Is - H + \alpha GP)^{-1}(E - \alpha GV) \tag{3}$$

The matrices H,G,E,P and V must satisfy:

$$\begin{array}{ll} \text{(i)} & H \text{ is stable,} \\ \text{(ii)} & TA - HT = EC \\ \text{(iii)} & G = TB - ED, \\ \text{(iiii)} & F = PT + VC \\ \text{(iiiii)} & VD = DV = 0 \end{array} \tag{4}$$

where F is the is the state feedback gain called the target design. Eqs. (4) is a generalization of the Luenberger conditions for strictly proper systems [6] to non-strictly proper systems. Σ_C is the general Luenberger observer for $\alpha = 1$ and the general output feedback controller for $\alpha = 0$. The separation theorem is not valid for $\alpha = 0$, so it must further be required that :

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A+BK & BP \\ EC & H \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} \tag{5}$$

is a stable system.

3. GENERAL LTR-CONDITIONS.

The LTR design method [1,4,7,8,9] will now be applied in the sequel the for design of the general controller. First, different types of recovery errors are defined independent of the applied controller.

Definition 3.1. Let the open-loop recovery error $E_I(s)$ the sensitivity recovery error $E_{S,I}(s)$ and the input-output recovery error $E_{IO,I}(s)$ be defined as:

$$\begin{aligned} E_I(s) &= F(sI-A)^{-1}B - C(s)G(s) \\ E_{S,I}(s) &= (I-F\Phi(s)B)^{-1} - (I-C(s)G(s))^{-1} \\ E_{IO,I}(s) &= G(s)(I+F\Phi(s)B)^{-1} - G(s)(I-C(s)G(s))^{-1} \end{aligned} \tag{6}$$

These recovery errors can be rewritten into more convenient forms by using Σ_C:

Lemma 3.2.
Define:

$$M_I(s) = P(sI-H)^{-1}G \tag{7}$$

Then

$$
\begin{aligned}
E_I(s) &= M_I(s)(I+\alpha M_I(s))^{-1}(I-\alpha F\Phi B) \\
E_{S,I}(s) &= -(I-F\Phi B)^{-1}M_I(s)(I-F\Phi B+(\alpha-1)M_I(s))^{-1}(I-\alpha F\Phi B) \quad (8) \\
E_{IO}(s) &= G(s)E_{S,I}(s)
\end{aligned}
$$

Proof. The proof of Lemma 3.2 is omitted. An equivalent proof can be found in [7] for $\alpha = 1$.

Based on this result, it is possible to give necessary and sufficient conditions for achieving exact recovery, i.e. $E_I = 0$. It is clear that if one of the recovery errors in Definition 3.1 is zero, the other two recovery errors will also equal zero.

Theorem 3.3
Exact recovery is obtained if and only if one of the following equivalent conditions holds:

$$\text{(i)}\ E_{S,I} = 0,\quad \text{(ii)}\ M_I(s) = 0,\quad \text{(iii)}\ \langle H|\mathrm{Im}G\rangle \subset \ker P \qquad (9)$$

If we assume that H is non-defective all three are equivalent to:

(iiii) $Pv_i = 0 \vee w_i^T G = 0,\ i = 1,\dots,p$

where v_i and w_i^T are right and left eigenvectors, respectively, associated with the eigenvalue λ_i of H.

Proof. See [7].

Note that the conditions in Theorem 3.3 are independent of α. It follows directly by rewriting the transfer function of the controller into:

$$C(s) = (I + \alpha M_I(s))^{-1}(V + P(sI - H)^{-1}E) \qquad (10)$$

that C(s) is independent of α when exact recovery is obtained.
Each of the four equivalent conditions in Theorem 3.3 are necessary and sufficient for achieving exact recovery. Only in rather special cases, however, it is possible to achieve exact recovery with a free target design F. Therefore it is interesting to study the asymptotic recovery case, to which condition (ii) generalize, as we shall see in the sequel.
Aymptotic recovery is defined by the following.

Definition 3.4.
Asymptotic recovery is said to be achievable if and only if $\forall\varepsilon > 0$ there exist a controller $C_\varepsilon(s)$ such that:

$$\| S_{TFL}(s) - S_{I,\varepsilon}(s) \|_H < \varepsilon \qquad (11)$$

where $S_{I,\varepsilon}$ is the closed-loop sensitivity function corresponding to $C_\varepsilon(s)$ and $\|\cdot\|_H$ is any 'suitable' norm, e.g. the H_2 or H_∞-norm.

The following is an immediate consequence of Lemma 3.2.

Corollary 3.5.
Asymptotic recovery is possible if and only if $\forall\varepsilon > 0$ there exist a controller $C_\varepsilon(s)$ such that:

(i) $\|M_{I,\varepsilon}(s)\|_H < \varepsilon$
(ii) The closed loop system is stable.
where $M_{I,\varepsilon}(s)$ is the recovery matrix corresponding to $C_\varepsilon(s)$.

Note again that the asymptotic recovery condition is independent of α as in Theorem 3.3.

4. DISCUSSION.

The LTR results for non-strictly proper plants derived in this paper all concern the input-node case. The dual LTR results for the output-node can be derived in a similar way. Further, the conditions derived for obtaining exact recovery can directly be used in the discrete-time case.
We have in this paper only considered the two controller types described by $\alpha = 0$ or 1 in eq. (2), but the LTR result derived here are also valid for all values of α in the interval [0,1]. As it has been shown in sec. 3, the obtained LTR controllers will be exactly equal when exact recovery is achieved, where as the two LTR controllers will approach in the asymptotic case. Considering the controller for $\alpha = 0$ in eq. (2) is motivated by the asymptotic recovery result in [3], where it has been shown that this controller type result in smaller gains than observer-based controllers (α=1) for the same recovery level. A more systematic recovery analysis for $\alpha \in [0,1]$ in eq. (2) based on recovery conditions derived here and in [3] will be given in a forthcoming paper. The connection between LTR controllers and the unknown input observer described in [5] is also considered.

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