Roll Damping by Rudder Control - A New $\mathcal{H}_\infty$ Approach*

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Abstract

Roll damping and simultaneous course steering by rudder control is a challenging problem where a key factor is roll damping performance in waves. Roll is a decisive factor for the operation of ships, both due to comfort of crew and passengers and due to requirements from cargo or on-board equipment. In the paper, roll damping and steering performance requirements are described and the controller design problem is formulated in an $\mathcal{H}_\infty$ framework. It is shown that the design problem includes zeros on the imaginary axis which cannot be dealt with in standard $\mathcal{H}_\infty$ theory. The paper explains the physical origin of the zeros and contributes with new results that can handle problems with zeros on the imaginary axis directly, without using approximations in the $\mathcal{H}_\infty$ design. Finally, the results of controller design using the new method are analysed, and the controller performance is discussed using an existing LQ design as comparison.

1 Introduction

A ship rudder is primarily used to create torques to turn the ship - alter its course - but, at the same time, roll torques are created. This second effect from the rudder can be utillized to obtain damping of roll motion simultaneously with control of the ship course. When using the rudder for both tasks, some physical obstracles need to be considered. When a ship goes into a turn it always obtains a certain roll angle. If it is prevented to heel - the naval' expression for steady roll angle - turning of the ship could not be obtained either. However, in the initial phase of a turn, the force from the rudder makes the ship roll opposite to the static state field. The nature of this problem is hence single input-multi output and a non minimum phase relation exists in the rudder to roll angle dynamics. Performance requirements to the control system includes that damping of roll is effective in the frequency range of natural and wave induced roll, but the disturbance this makes to the ship heading must be limited. For these reasons, roll damping by rudder control is not a straightforward control problem. Several design issues have been solved, and Rudder Roll Damping (RRD) systems have become increasingly popular in recent years. Commercial reasons include the cost-effectiveness of this approach compared with fin stabilizer solutions and the possibility of applying the RRD concept on existing vessels.

RRD design issues have been discussed in a number of papers. The first experiments were reported by [BWB83]. Theoretic LQ results were derived [vAvNLvdK87]. Systems were designed and implemented [BHA89, KS89], $\mathcal{H}_\infty$ controllers were investigated [KJG89], and robustness properties of LQ based RRD were investigated [BC93].

Despite the progress, the effectiveness of RRD controls has been debated. Some results from full scale evaluation on vessels indicate very satisfactory results showing 50-70 % roll reduction [BHA89, KS89, Llo75]. Others indicate much less effectiveness in certain cases, and for some ships the physical properties have been such that traditional RRD designs could not be used at all. This has caused renewed research interest where robustness considerations and improvements in design methods are key issues.

In this paper we investigate the design of $\mathcal{H}_\infty$ controllers for the full single input-multi output RRD control problem. A key obstacle is the existence of an imaginary zero in the multivariable system which prevents existing $\mathcal{H}_\infty$ theory to handle the RRD problem without approximations. This obstacle is dealt with and we establish results that enable design without the approximations normally needed. The properties of the design are illustrated with theoretical data for a multipurpose naval vessel and the performance is compared with that of an existing LQ design.

2 Problem Formulation

The mathematical model for the part of the system to be controlled is a 5th order state space equation for $x_s(t)$ with waves considered as an output disturbance.

$$y_{ship} = C_s x_s + y_w \tag{1}$$

A linear model of the ship is given by, [BHA89, BC93]:

$$\begin{aligned} \dot{x}_s &= A_s x_s + B_s u_s \\ y_s &= C_s x_s \end{aligned} \tag{2}$$

where the state is $x_s = [\ v \quad r \quad \Psi \quad p \quad \theta\ ]^T$ (sway vel., turn rate, heading, roll rate, roll angle). The three matrices in (2) are given in Appendix A.

Disturbance Forms

Wave disturbances cannot be modeled as a state space disturbance as forces - moments in (2). The reason is that wave forces act over the entire hull and the coefficients in a state space description would be frequency dependant. Calculation of wave induced motions is instead done as response functions from strip theory, or they may be measured. The result is that wave disturbances are characterized in a vector $y_w = [v, r, \Psi, p, \Phi]_w$. The relation between wave height, ξ_w and hull motions in y_w are complex. They depend on wave

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length. λ, wave direction relative to the ship, χ, and encounter frequency, ω_e. To a first order approximation, wave motions are linear, and we can therefore obtain the motion of the hull as a superposition of the wave induced motion and that created by rudder activity.

The reduction ratio of a motion, i.e., the ratio between the uncontrolled and controlled response, is a key indicator for control quality in waves. For RRD, and the reduction function for roll damping is the crucial factor. The mean square of each component of the motion vector $y_{ship}(t)$ is determined by the powerspectrum of wave amplitude, $G_{\zeta\zeta}$ and the wave response operator, $WRO_{y_i\zeta_i}$ as

$$E\left\{y^2_{ship,i}(t)\right\} = \frac{1}{\pi}\int_0^\infty \left|\frac{y_{ship,i}(\omega_e)}{y_{w,i}(\omega_e)}\right|^2 |WRO_{y_i\zeta_i}(\omega)|^2\, G_{\zeta\zeta}(\omega)d\omega; i = 1, 2, ..., 5 \tag{3}$$

The reduction ratio for each of the motions is

$$rr_i(\omega_e) = \left[\frac{y_{ship}(\omega_e)}{y_w(\omega_e)}\right]_i ; i = 1, 2, ..., 5 \tag{4}$$

Efficient roll damping is obtained when $|rr_5(\omega_e)|$ is well below 1 over the range of frequencies, 0.7 to 1.1 rad/sec, where natural roll and wave induced motions occur. Requirements to roll damping performance are most convenient specified in terms of the shape of the $|rr_5(\omega_e)|$ function at different values of ship speed. A maximum value of wave height needs also to be specified to check the linearity range for the rudder servosystem.

Robust control is achieved if the required value of $|rr_5(\omega_e)|$ is met regardless of changes in ship speed, loading conditions, hydrodynamic parameters or other coefficients in the equations of motion.

The basic performance problem is therefore, by nature, an $\mathcal{H}_\infty$ problem. The wave motion is an output disturbance and the roll reduction function is the sensitivity function of the closed loop control problem. The inverse of the $\mathcal{H}_\infty$ design weight function are shown as the dotted lines in figures 2 - 5.

Steering Performance

While there is a quite consice performance requirement to roll damping, steering properties are more vaguely expressed. There are two main requirements to steering performance. One is that wave motions in r and Ψ should not cause rudder fluctuation at wave frequencies. The reason is that noticeable propulsion losses occur if the rudder fluctuates too heavily and the rudder servo mechanics gets worn. A second is that the ship heading should be maintained despite steady state or low frequency disturbances, e.g., from wind. These performance requirements can be expressed in a $\mathcal{H}_\infty$ design weight function. The inverse of the selected weight function is shown as the dotted line in figure 6.

The $\mathcal{H}_\infty$ Design Setup

For the design of the robust controller, the design specifications for the turn rate and for the angle rate are given above by three two-by-two weight matrices W_i, W_o and W_S.

All weight functions are described in state space form:

$$\begin{array}{rcccc} \dot{x}_\cdot & = & A_\cdot x & + & B_\cdot u_\cdot \\ y_\cdot & = & C_\cdot x & + & D_\cdot u_\cdot \end{array} \tag{5}$$

The weight matrix W_i is placed at the external input w. The weight matrix W_o is placed at the external output z, and the weight matrix W_S is placed between the two, see below.

The reason for having three weight matrices is that the problem addressed in this paper is actually not a standard $\mathcal{H}_\infty$ control problem, but rather a *multiobjective* $\mathcal{H}_\infty$ control problem, since we have specifications for two, independent sensitivity functions.

In the mainstream tradition, multiobjective $\mathcal{H}_\infty$ problems are treated by putting up a standard $\mathcal{H}_\infty$ problem, having the specified transfer functions in the diagonal. This approach, however, in general can lead to very conservative designs, since the off-diagonal functions contribute with large weights in the optimization.

It is possible, however, to avoid this conservatism, by applying a general weight selection scheme, which combines additive and multiplicative weightings. Unfortunately, details cannot be given in this manuscript due to space limitations. In short, the weight selection scheme involves the following three weights. $W_S(\cdot)$ is a weight matrix, which has to overbound the diagonal transfer functions, in this paper by 1 since we are dealing with sensitivities, and the off-diagonal functions, by using the linear fractional transformation structure. $W_S(\cdot)$ depend on plant data only. $W_i(\cdot)$ and $W_o(\cdot)$ are matrices that depend both on plant and specification data. The diagonal of the product $W_i(\cdot)W_o(\cdot)$, however, depend on the specifications only, i.e., $[W_i(\cdot)W_o(\cdot)]_{11}$ and $[W_i(\cdot)W_o(\cdot)]_{22}$ are the inverses of the specified sensitivities, we wish to achieve.

With the design specifications described above, we get the following standard design problem:

$$\begin{array}{rcccccc} \dot{x} & = & Ax & + & B_1 w & + & B_2 u \\ z & = & C_1 x & + & D_{11} w & + & D_{12} u \\ y & = & C_2 x & + & D_{21} w & + & D_{22} u \end{array} \tag{6}$$

where the state vector is

$$x = \begin{bmatrix} x_s & x_{wi} & x_{wo} & x_{ws} \end{bmatrix}^T.$$

The 9 matrices in the standard problem are given by:

$$\begin{array}{rcl} A & = & \begin{bmatrix} A_s & 0 & 0 & 0 \\ 0 & A_{wi} & 0 & 0 \\ & B_{wo}\bar{C}_2 & A_{wo} & 0 \\ 0 & 0 & 0 & A_{ws} \end{bmatrix} \\ B_1 & = & \begin{bmatrix} 0 \\ B_{wi} \\ B_{wo}D_{wi} \\ B_{ws} \end{bmatrix}, B_2 = \begin{bmatrix} B_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ C_2 & = & \begin{bmatrix} C_s & C_{Cwi} & 0 & 0 \end{bmatrix} \\ & =: & \begin{bmatrix} C_2^1 & 0 & 0 \\ C_2^2 & 0 & 0 \\ C_2^3 & 0 & 0 \\ C_2^4 & 0 & 0 \end{bmatrix} \bar{C}_2 = \begin{bmatrix} C_2^4 \\ C_2^1 \end{bmatrix} \\ C_1 & = & \begin{bmatrix} D_{wo}\bar{C}_2 & C_{wo} & C_{ws} \end{bmatrix} \\ D_{11} & = & [D_{wo}D_{wi} + D_{ws}] \\ D_{12} & = & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ D_{21} & = & \begin{bmatrix} 0 & D_{w2i} \\ 0 & 0 \\ 0 & 0 \\ D_{w1i} & 0 \end{bmatrix}, D_{22} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \tag{7}$$

Note that the direct term D_{11} is not zero as required in the following $\mathcal{H}_\infty$ controller design. This is always the case when the design specification is an output sensitivity function. The direct term can, though, be removed very easily by using a loopshifting method from [Sto92]. The loopshifted system is given by:

$$\begin{array}{rcccccc} \dot{x} & = & A_{ls}x & + & B_{ls,1}w & + & B_{ls,2}u \\ z & = & C_{ls,1}x & & & + & D_{ls,12}u \\ y & = & C_{ls,2}x & + & D_{ls,21}w & & \end{array} \tag{8}$$

where the new matrices are given by:

$$\begin{array}{rcl} A_{ls} & = & A - \gamma^{-1}B_1 D_{11}^T C_1 \\ B_{ls,1} & = & -B_1(I - \gamma^{-2}D_{11}^T D_{11})^{-1/2} \\ B_{ls,2} & = & B_2 - \gamma^{-2}B_1 D_{11}^T D_{12} \\ C_{ls,1} & = & \gamma^{-1}(I - \gamma^{-2}D_{11}D_{11}^T)^{-1/2}C_1 \\ C_{ls,2} & = & C_2 - \gamma^{-2}D_{21}D_{11}^T C_1 \\ D_{ls,12} & = & \gamma^{-1}(I - \gamma^{-2}D_{11}D_{11}^T)^{-1/2}D_{12} \\ D_{ls,21} & = & D_{21}(I - \gamma^{-2}D_{11}^T D_{11})^{-1/2} \end{array} \tag{9}$$

where γ is the selected $\mathcal{H}_\infty$ norm for the closed loop system. The connection between the two systems in (6) and (8) is given in the following lemma, based on [Sto92]:

Lemma 1 *Let a transfer function K of appropriate dimensions be given. Then the following two statements are equivalent*

1. *K is an internally stabilizing controller for the original system (6) which makes the closed loop $\mathcal{H}_\infty$ norm from w to z smaller than* 1
2. *K is an internally stabilizing controller for the loopshifted system (8) which makes the closed loop $\mathcal{H}_\infty$ norm from w to z smaller than* 1

3 $\mathcal{H}_\infty$ Design with Zeros on the Imaginary Axis

In the previous section we derived a model of the form

$$\begin{array}{rcccccc} \dot{x} & = & Ax & + & B_1 w & + & B_2 u \\ z & = & C_1 x & & & + & D_{12}u \\ y & = & C_2 x & + & D_{21}w & & \end{array} \tag{10}$$

Unfortunately, the derived model does not satisfy the standard assumptions [DGKF89]. Two assumptions which are violated for the model obtained in Section 2 in the approach of [DGKF89] are the regularity assumptions, i.e. that D_{12} and D_{21} must have full column and row ranks, respectively. To overcome this problem we shall take off from the approach of [Sto92].

However, one further assumption is violated for the ship model which is common to [DGKF89] and [Sto92]. In either approach it is assumed that neither of the two subsystems given by the quadruples (A, B_1, C_2, D_{21}) and (A, B_2, C_1, D_{12}) have invariant zeros on the imaginary axis. The reason is that the states Ψ influence neither the other states nor the output z. This always gives reason to an invariant zero in origin. This is a consequence of the performance requirement that has a weight function on the turn rate only, and the weight function has finite gain at zero frequency. To overcome this design problem we shall provide some generalizations in the following.

We shall characterize suboptimality in terms of a certain feasibility set Γ.

Definition 2 *The* feasibility set Γ *is defined as the set of positive numbers γ for which there exist* **unique** *positive semidefinite matrices P and Q such that*

1. $F_\gamma(P) :=$
$$\begin{bmatrix} A'P + PA + C_1'C_1 + \gamma^{-2}PB_1B_1'P & PB_2 + C_1'D_{12} \\ B_2'P + D_{12}'C_1 & D_{12}'D_{12} \end{bmatrix} =: \begin{bmatrix} C_{1P}' \\ D_{12P}' \end{bmatrix} \begin{bmatrix} C_{1P} & D_{12P} \end{bmatrix} \geq 0$$
2. $G_\gamma(Q) :=$
$$\begin{bmatrix} AQ + QA' + B_1B_1' + \gamma^{-2}QC_1'C_1Q & QC_2' + B_1D_{21}' \\ C_2Q + D_{21}B_1' & D_{21}D_{21}' \end{bmatrix} =: \begin{bmatrix} B_{1Q} \\ D_{21Q} \end{bmatrix} \begin{bmatrix} B_{1Q}' & D_{21Q}' \end{bmatrix} \geq 0$$
3. $\operatorname{rank}\begin{bmatrix} C_{1P} & D_{12P} \end{bmatrix} = \operatorname{rank}_{\mathcal{R}(s)}\left[C_1(sI - A)^{-1}B_2 + D_{12}\right]$
4. $\operatorname{rank}\begin{bmatrix} B_{1Q} \\ D_{21Q} \end{bmatrix} = \operatorname{rank}_{\mathcal{R}(s)}\left[C_2(sI - A)^{-1}B_1 + D_{21}\right]$
5. $\operatorname{rank}\begin{bmatrix} A + \gamma^{-2}B_1B_1'P - s_0 I & B_2 \\ C_{1P} & D_{12P} \end{bmatrix} = n + \operatorname{rank}_{\mathcal{R}(s)}\left[C_1(sI - A)^{-1}B_2 + D_{12}\right], \forall s_0 \in \mathbb{C}^+$
6. $\operatorname{rank}\begin{bmatrix} A + \gamma^{-2}QC_1'C_1 - s_0 I & B_{1Q} \\ C_2 & D_{21Q} \end{bmatrix} = n + \operatorname{rank}_{\mathcal{R}(s)}\left[C_2(sI - A)^{-1}B_1 + D_{21}\right], \forall s_0 \in \mathbb{C}^+$
7. $\rho(PQ) < \gamma^2$

A key role in the design used in this paper is the following result which describes suboptimality of the $\mathcal{H}_\infty$ standard problem (10) without assumptions on zeros of any kind. The result is a generalization of results found in [Sto92].

Theorem 3 *There exists an internally stabilizing controller for the system (10) which makes the closed loop $\mathcal{H}_\infty$ norm from w to z smaller than γ* if and only if

1. *(C_2, A, B_2) is detectable and stabilizable*
2. *$\gamma > \gamma_{\text{opt}}$, where $\gamma_{\text{opt}} = \sup\{\gamma : \gamma \notin \Gamma\}$*

Proof. The proof of Theorem 3 is technically involved, and will not be given in detail here, although it is a straightforward combination of the methods in [Sto92, Sch90, Sch92a, Sch92b]. Following [Sto92] we prove that $\gamma < \gamma_{\text{opt}}$ implies that the interpolation constraints are not satisfied. Moreover, assuming nonuniqueness of the matrix inequality solutions lead to the conclusion that the $\mathcal{H}_\infty$ constraint is assumed with nonstrict inequality only, using the interpolation constraints. Hence, no less γ can be obtained. Sufficiency follows by applying a cheap control argument and pursuing the line of [Sch90, Sch92a, Sch92b]. □

Remark 1 In the case of standard assumptions [DGKF89, Sto92] the set Γ will be simply connected, and will just equal the set of suboptimal $\mathcal{H}_\infty$ performance specifications. In contrast, in the face of invariant zeros on the imaginary axis Γ will in general neither equal the suboptimality set nor be simply connected. For certain, discrete values of γ the conditions in Definition 2 might have nonunique solutions. In that case the set of suboptimal γ's will equal the component of connectivity for Γ which contains infinity.

By the method in [DGKF89] an explicit controller formula can be given in terms of the two Riccati solutions. This is not the case in our more general setting. Instead, we shall proceed by computing the following auxiliary system based on the two matrices P and Q as given by Definition 2.

$$\begin{array}{rcl} \dot{x} & = & A_{PQ}x + B_{1PQ}w + B_{2PQ}u \\ z & = & C_{1PQ}x \qquad\qquad\quad + D_{12P}u \\ y & = & C_{2PQ}x + D_{21Q}w \end{array} \tag{11}$$

where the matrices are given by

$$A_{PQ} = T(A + QA'P + \gamma^{-2}B_1B_1'P + \gamma^{-2}QC_1'C_1)T$$

$$B_{2PQ} = T(B_2 + \gamma^{-2}QC_1'D_{12}),\ C_{2PQ} = (C_2 + \gamma^{-2}D_{21}B_1'P)T$$

$$B_{1PQ} = TB_{1Q},\ C_{1PQ} = C_{1P}T$$

$$T = (I - \gamma^{-2}QP)^{-1/2}$$

C_{1P} and D_{12P} are given by Definition 2(1); B_{1Q} and D_{21Q} are given by Definition 2(2).

The transformed system (11) is 'almost' minimum phase in the following sense

Proposition 4 *The two systems described by the quadruples* $(A_{PQ}, B_{1PQ}, C_{2PQ}, D_{21Q})$ *and* $(A_{PQ}, B_{2PQ}, C_{1PQ}, D_{12Q})$ *have invariant zeros in* $\mathbb{C}^+$.

The $\mathcal{H}_\infty$ design in Section 4 is heavily based on the following observation

Theorem 5 *Assume that* $\gamma > \gamma_{\rm opt}$ *and compute the transformed system (11). Let a transfer function* K *of appropriate dimensions be given. Then the following two statements are equivalent*

1. K *is an internally stabilizing controller for the original system (10) which makes the closed loop* $\mathcal{H}_\infty$ *norm from* w *to* z *smaller than* γ

2. K *is an internally stabilizing controller for the transformed system (11) which makes the closed loop* $\mathcal{H}_\infty$ *norm from* w *to* z *smaller than* γ

The significance of Theorem 5 in conjunction with Proposition 4 is that the problem of finding a controller for the original system can be replaced with finding a controller for the transformed system which is much easier, since this problem does not have zeros in the (open) right half plane.

4 Design Results

In both the LQ design and the $\mathcal{H}_\infty$ design, we have used gain scheduling, so the controller is optimal with respect to the ship speed.

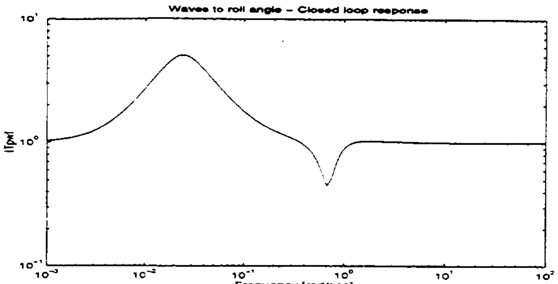

Figure 1: LQ design - Reduction of $|T_{pw}|$ at frequencies around $\omega = 0.7$ rad/sec.

An LQ Design

The results of a nominal design for a naval multirole vessel [BC93] are here used for comparing an LQ design, similar to one in actual operation on a series of ships, with the $\mathcal{H}_\infty$ approach described here. The controller is not a genuine LQ design, because sway velocity could not be estimated with sufficient accuracy. Instead, pole placement similar to that of LQ design was obtained using available state estimates. The details of the design can be found in [BHA89].

The LQ controller uses feedback from filtered turn rate and heading, i.e. the states r and Ψ not disturbed by wave motion, and measured roll rate and roll angle, i.e., p and Φ including wave motion. The LQ controller was speed scaled to obtain closed loop behaviour similar to that of the open loop system. Details can be found in the reference.

The LQ controller was:

$$\delta_{steering} = (0, -l_r\left(\frac{U_{design}}{U_{actual}}\right), -l_\Psi, 0, 0); \tag{12}$$

$$\delta_{roll} = (0, 0, 0, -l_p\left(\frac{U_{design}}{U_{actual}}\right)^2, -l_\Phi\left(\frac{U_{design}}{U_{actual}}\right)^2) \tag{13}$$

In a seaway, waves will generate roll motion, and assessment of total performance will require the wave response operators for both p and Φ, and integration of the wave spectrum times the response operator and output disturbance sensitivity function of the closed loop RRD control. This requires fairly complex information about the ship and seaway. A simpler, yet sufficient performance indicator for our purpose is the $|rr_r|$ function that shows roll damping over frequency. The performance of the LQ controller is illustrated in figure 1. Roll damping is 0.5 as required around 0.9 rad/sec. in the nominal design, but the interval where this is obtained is narrow.

The $\mathcal{H}_\infty$ Controller

Based on the formulated standard problem in Section 2 and the $\mathcal{H}_\infty$ results given in Section 3, we are able to design an internally stabilizing $\mathcal{H}_\infty$ controller which makes the $\mathcal{H}_\infty$ norm of the closed loop transfer function from w to z smaller than γ, where γ is a sufficiently large, positive number. In the following, γ has been selected to 1.1 times the optimal value of γ.

In figures 2-6, the result of the $\mathcal{H}_\infty$ design are shown for the ship speed $u = 9.0$ m/s. The solid lines in the figures are the closed-loop amplitudes and the dotted lines are the inverse

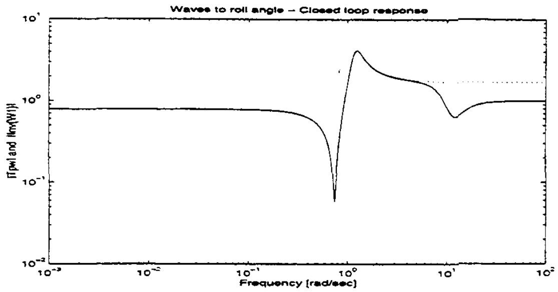

Figure 2: $\mathcal{H}_\infty$ design - Reduction of $|T_{pw}|$ around $\omega = 0.7$ rad/sec.

of the respective weight function multiplied with γ. For satisfying the design specifications, the inverse of the weight function must be over the closed-loop transfer functions for all frequencies.
It can be seen directly from the figures, that the hard bound to satisfy is the specification for the roll angle. The reason is that the transfer function from control input to roll angle has a nonminimum phase zero at $z = 0.915$. Hence, the corresponding output sensitivity $S(\cdot)$ will satisfy a nontrivial Bode integral sensitivity bound. To obtain a reasonable design, the weight matrices has to satisfy the Bode bound themselves. In respect to space limitations we cannot survey the systematic procedures to take these interpolation constraints into account. However, in figure 2 and figure 3 we have demonstrated two extreme possibilities. Figure 2 shows a design which imitates the LQ "narrow valley" behavior, but with a much deeper valley (0.05 in contrast to 0.47). Figure 3 demonstrates a design which suppresses all frequencies below $\omega = 1$. In both cases, the price of the very tight achievements is paid in form of an amplification peak at higher frequencies in accordance with the above mentioned "water bed effect". Such designs can of course be used in quiet mid-sea cruise only or under other special circumstances where waves are know to contain no high frequency components
The design results shown in figures 4 - 5 are obtained by using more realistic design specifications. The design specification for the fransfer function from waves to roll angle is a reduction of 50% in the frequency range from .6 rad/sec to 1.1 rad/sec. This design specification has been obtained for the designed $\mathcal{H}_\infty$ controllers. In comparison with the LQ design in figure 1, the $\mathcal{H}_\infty$ design satisfies the the design over a broader range of frequencies.
In figure 6, the amplitude of the closed loop transfer function from waves to turn rate is shown. The design specification is that the transfer function must be reduced by a factor 10 at low frequencies, which is satisfied.

5 Conclusion

A design problem for robust control of rudder-roll damping has been discussed.
Since the problem specifications were posed in frequency domain, an $\mathcal{H}_\infty$ design was a natural selection. An $\mathcal{H}_\infty$ controller was calculated by virtue of a new singular $\mathcal{H}_\infty$ approach and compared with a previous LQ like design.
As a design tool, the $\mathcal{H}_\infty$ method was fast and very direct, since no additional fine tuning was necessary on top of the weightings which were immediate from the specifications. It turned out that the hard bound to satisfy was the specification for the roll angle. The specifications could easily be met at the specified frequency range, but the transfer function need to blow up in some other frequency ranges for satisfying the Bode integral sensitivity bound. This trade off is the only part of the algorithm, where the designer might need to iterate a little to achieve the "nicest" results. The roll angle amplification at very low frequencies is, e.g. not desired due to the large hill angle created by low frequencies wind loads. Basically the preference between the designs in figures 2-5 is a matter of taste and/or cruise conditions.
In short, a comparison between the $\mathcal{H}_\infty$ and the LQ controller shows that the frequency fit of the $\mathcal{H}_\infty$ controller is significantly better at the cost of complexity. The LQ controller amplifies waves in a low frequency range, whereas the $\mathcal{H}_\infty$ controller (figure 3) rolls off to a very low level at low frequencies.

A Ship Model

The matrices for the linear ship model in (2) are given by:

$$\begin{aligned} A_s &= T^{-1}E^{-1}FT \\ B_s &= T^{-1}E^{-1}G \end{aligned} \tag{14}$$

where E, F and G are given by, [BHA89, BC93]:

$$E = \begin{bmatrix} m - Y_{\dot v} & m x_G - Y_{\dot r} & -m z_G - Y_{\dot p} & 0 & 0 \\ m x_G - N_{\dot v} & I_{zz} - N_{\dot r} & -N_{\dot p} & 0 & 0 \\ -m z_G - K_{\dot v} & -K_{\dot r} & I_{xx} - K_{\dot p} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} & & & & 0 \\ F_1 & F_2 & F_3 & F_4 & 0 \\ & & & & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix} = \begin{bmatrix} UY_{uv} & U(-m + Y_{ur}) & Y_p + UY_{up} \\ UN_{uv} & U(N_{ur} - m x_G) & N_p + UN_{up} \\ UK_{uv} & U(K_{ur} + m z_G) & K_p + UK_{up} \end{bmatrix}$$

$$F_4 = \begin{bmatrix} Y_\Phi + U^2 Y_{\Phi uu} \\ N_\Phi + U^2 N_{\Phi uu} \\ -gmGM + U^2 K_{\Phi uu} \end{bmatrix}, G = \begin{bmatrix} U^2 Y_{\delta uu} \\ l_{\delta x} U^2 Y_{\delta uu} \\ -l_{\delta z} U^2 Y_{\delta uu} \\ 0 \\ 0 \end{bmatrix}$$

$$C_s = \begin{bmatrix} 0 & I \end{bmatrix} \tag{15}$$

and T is given by

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{16}$$

such that $x = Tx_s$ The values of the constants in the matrices can be found in [BC93].

References

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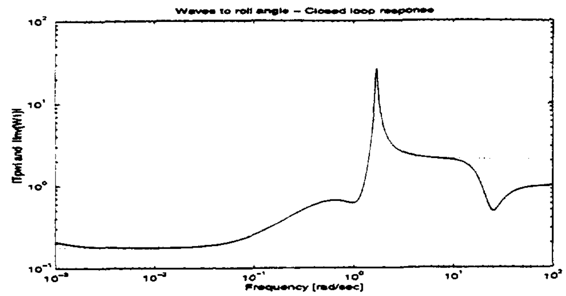

Figure 3: $\mathcal{H}_\infty$ design - Reduction of $|T_{pw}|$ for all frequencies below $\omega = 1.0$ rad/sec.

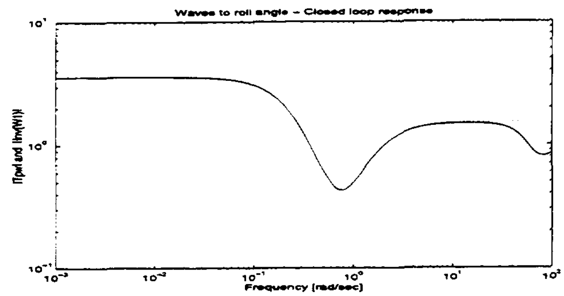

Figure 4: $\mathcal{H}_\infty$ design - Reduction of $|T_{pw}|$ in the frequency range [0.6 rad/sec. 1.1 rad/sec.] to 0.4 and a limitation of the amplitude at high frequencies.

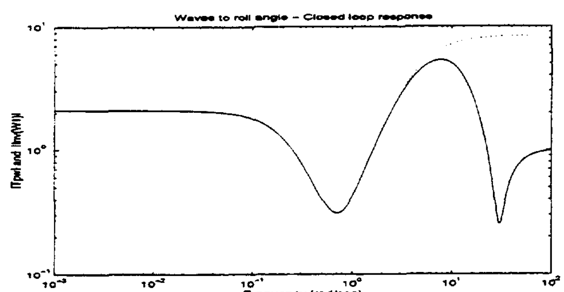

Figure 5: $\mathcal{H}_\infty$ design - Reduction of $|T_{pw}|$ in the frequency range [0.6 rad/sec. 1.1 rad/sec.] to 0.4 and a limitation of the amplitude at low frequencies.

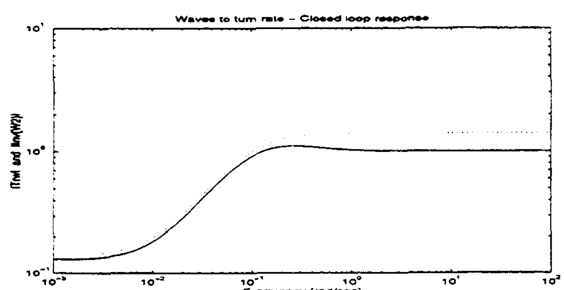

Figure 6: $\mathcal{H}_\infty$ design - Reduction of $|T_{rw}|$ at low frequencies.

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