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Simultaneous Design of Controller and Fault Detector

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Abstract

In this paper we give a convenient method for integrated control and detection and isolation of sensor and actuator faults in closed-loop. We formulate the problem as a closed-loop problem and give a state-space description. Hence well known $\mathcal{H}_\infty$ design techniques can be used to solve the design problem. However, the setup is valid for any induced norm design.

1 Introduction

The issue of fault detection and control has been an active research area in the last two decades. The outcome is surveyed in [1, 2] and a more fully introduction is given in [3]. There has recently appeared a number of results using $\mathcal{H}_\infty$-optimization for obtaining a (robust) fault detection and isolation (FDI) in open-loop, see e.g. [4]. The result presented here is for closed-loop.

2 Design Methods

First let us consider the following finite dimensional linear time invariant model describing a plant

$$G: \begin{cases} \dot{x} = Ax + B_1\omega + B_2 u \\ z = C_1 x + D_{11}\omega + D_{12} u \\ y = C_2 x + D_{21}\omega \end{cases} \quad (1)$$

The setup considered is depicted in figure 1 where $G(s)$ is the transfer matrix of the plant and $K(s)$ is the controller. Sensor and actuator faults have been added as inputs and the diagnostic output (estimate) $\hat{f} \triangleq [\ f_a{}^T \quad f_s{}^T\ ]^T = u_2$ is obtained by introducing an extra to-be-controlled output with $z_2 = f - u_2$.

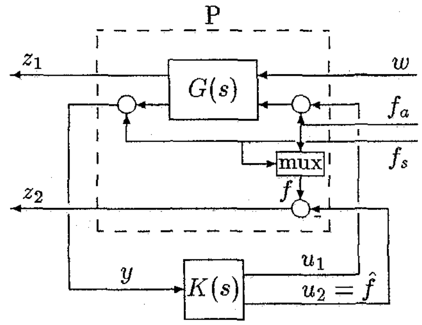

Figure 1: Control system with diagnostics

A description of the augmented system $P(s)$ reordered into state-space form is given by

$$P: \begin{cases} \dot{x} = Ax + B_1 w + [\ B_2 \quad 0\ ]f + [\ B_2 \quad 0\ ]u \\ z_1 = C_1 x + D_{11} w + [\ D_{12} \quad 0\ ]u \\ z_2 = f + [\ 0 \quad -I\ ]u \\ y = C_2 x + D_{21} w + [\ 0 \quad I\ ]f \end{cases} \quad (2)$$

where $u = [\ u_1{}^T \quad u_2{}^T\ ]^T$.

Now consider the transfer matrix from $v = [\ w^T \quad f^T\ ]^T$ to $z = [\ z_1{}^T \quad z_2{}^T\ ]^T$ of the closed-loop system in figure 1, $\mathcal{F}_\ell(P,K) = T_{zv} = \begin{bmatrix} T_{z_1 w} & T_{z_1 f} \\ T_{z_2 w} & T_{z_2 f} \end{bmatrix}$. We note for some suitable norm that: $\|T_{z_1 w}\|$ small implies disturbance attenuation, $\|T_{z_1 f}\|$ small means that undetected failures are not disastrous, $\|T_{z_2 w}\|$ small secures no false alarms, $\|T_{z_2 f}\| \to 0 \ \Rightarrow\ u_2 \to f$ i.e. a good estimate. This means we want to solve the multiobjective problem

$$\|T_{zw}\| < \gamma_1 \ \text{ and } \ \|T_{z_2 f}\| < \gamma_2 \quad (3)$$

in contrast to minimizing $\|T_{zv}\|$, which would be more conservative than (3).

However, the faults are only expected in a certain frequency region. Hence we pass the signal z_2 through a suitable filter, say $W(s)$. The new design object is then defined as $\tilde{P}(s) \triangleq \mathrm{diag}(I, W(s), I)P(s)$. The design of controller and fault detector is obtained by solving (3) with respect to $T_{zv} = \mathcal{F}_\ell(\tilde{P}, K)$.

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Depending on the signals (energy bounded, amplitude bounded) one choose a suitable signal norm giving rise to an induced norm ($\mathcal{H}_\infty$, $\mathcal{L}_\infty$) for which various known approaches can be used to solve (3). However, the problem is inherently singular which limits the number of optimization methods of choice.

The potentially more conservative problem of minimizing $\|T_{zv}\|$ has the advantage over the multiobjective problem that this gives rise to more straightforward and simple optimization problems.

The simultaneous design of fault detection and control has the advantage that the resulting control/estimator has order n, whereas a controller design followed by adding a fault estimator using an output estimator and a filter gives order $3n$, and does not take into account that control and FDI might be contrary objectives.

Using LMIs [5] for solving the $\mathcal{H}_\infty$ disturbance attenuation problem $\|T_{zv}\|_\infty < \gamma$, give at most a design with order $n-1$ and the singular structure is handled smoothly.

Furthermore, recently there has appeared a number of methods that can handle multi objective problems directly, which would be advantageous for the present problems.

3 Example

In this section we present an example illustrating the simultaneous design of $\mathcal{H}_\infty$-control and estimation of faults in the actuators and the sensors. The system considered is stable, 4th order, and has 1 disturbance, 1 actuator, 1 output, and 2 sensors. The state-space data is given by the compact system matrix

$$G(s) = \left[\begin{array}{cccc|cc} -4.873 & 0.758 & 9.541 & 8.763 & 0.483 & 0.003854 \\ 4.673 & -5.385 & 10.94 & -435.2 & 1.498 & 9.072 \cdot 10^{-4} \\ -0.3003 & -0.6001 & -5.334 & -20.23 & 9.889 & -2.865 \cdot 10^{-5} \\ 0.03852 & 0.7446 & 0.5703 & -83.28 & 0.1282 & -13.96 \\ \hline 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 \end{array}\right]$$

The purpose is to design a combined controller and estimator such that good disturbance attenuation from w to z together with good estimate of f_a and f_s are obtained. The output is filtered by: $W(s) = \frac{10}{s+10} I_{2\times 2}$.

The derived $\mathcal{H}_\infty$ minimization problem is solved by the LMI-method and resulted in a controller of order 1 with $\|T_{zv}\|_\infty = 0.57$.

The $\mathcal{H}_\infty$ norm of the elements in T_{zv} was calculated to $\|T_{z_1 w}\|_\infty = 0.29$, $\|T_{z_1 f}\|_\infty = 0.001$, $\|T_{z_2 w}\|_\infty = 0.33$, and $\|T_{z_2 f}\|_\infty = 0.49$. The norm of $T_{z_1 w}$ and $T_{z_2 f}$ indicates that good disturbance attenuation and fault estimation could be expected. However, since $T_{z_2 f}$ is a 2×2 transfer matrix more could be said about the fault estimation when calculating the $\mathcal{H}_\infty$ norm of each element in $T_{z_2 f}$. $\|(T_{z_2 f})_{11}\|_\infty = 0.49$, $\|(T_{z_2 f})_{22}\|_\infty = 0.02$, and the off-diagonal elements are close to zero. This means that the estimates of f_a and f_s are nearly isolated, i.e. does not influence each other, and that a very fine estimate of f_s could be expected. The $\mathcal{H}_\infty$ norm of $(T_{z_2 f})_{11}$ was reached near DC and goes to zero beside DC, so the steady-state estimate of f_a is expected to be approximately 50% of the actual value.

Figure 2 shows a simulation with a sensor failure. The disturbance, w, was coloured noise with an amplitude of approximately 0.15V.

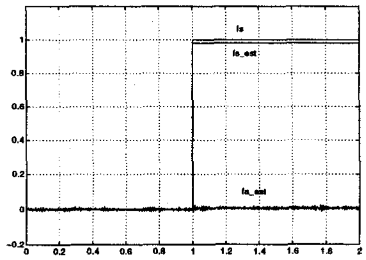

Figure 2: Actuator and sensor estimate for sensor failure

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