Starting up Unstable Multivariable Controllers Safely*

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Abstract

The problem of superimposing a multivariable controller on a running plant is considered. A simple but effective controller architecture is suggested which allows the transition from a conventional controller to a full multivariable controller to take place in a continuous way. This architecture allows for unstable controllers to be handled in a reliable way. Moreover, bandwidth properties can be tuned separately. This architecture can be extended to a tool for implementing gain scheduled controllers in the same fashion.

Keywords: Startup strategies; Unstable controllers;

1 Introduction

In several industrial environments as e.g. power plants, installing a full multivariable controller is difficult due to safety requirements. Often the starting point is a plant which is already controlled by several single loop controllers, for instance of PI or PID type. The new controller is then introduced in parallel, varying the control signal continuously by a tuning procedure.

Usually, this is implemented simply by applying a controller of the form $\alpha K(s)$ where $\alpha \in [0; 1]$ and $K(s)$ is the new multivariable controller. To start up the multivariable compensator, the parameter α is varied slowly from 0 to 1, while the closed loop behavior is being monitored, manually or automatically. For $\alpha = 0$, only the inner loops are active, but as α approaches 1, the multivariable compensator is taking over control of the plant.

This procedure works well in many applications, but it has some pitfalls as well, the most significant being that the procedure requires the controller to be stable. This is obvious since control signal from an unstable controller will always diverge when α is small.

In part due to this limitation, the industrial use of unstable controllers have been limited. This is unfortunate, considering that

- for some plants, no stable controller will achieve

optimality (in a mixed sensitivity sense)

- for some plants, no stable controller will robustly stabilize the system
- for some plants, no stable controller will stabilize even the nominal system

The requirement of the controller to be open-loop stable is usually known as *strong stabilization*. Recently, it has been shown that the order of a strongly stabilizing $\mathcal{H}_\infty$ controller can become unbounded as poles and zeros approach [SS86]. Some bounds on performance for strongly stabilizing controllers can be found from [OMK91].

In this paper we will suggest a general architecture for starting up a multivariable compensator, which is not required to be stable. Nevertheless, stability is guaranteed throughout the tuning procedure.

2 A General Architecture for Startup Procedures

A simple but effective way to overcome the difficulties in the traditional approach to multivariable controller startup is depicted in Figure 1. Here, $P(s)$ is the phys-
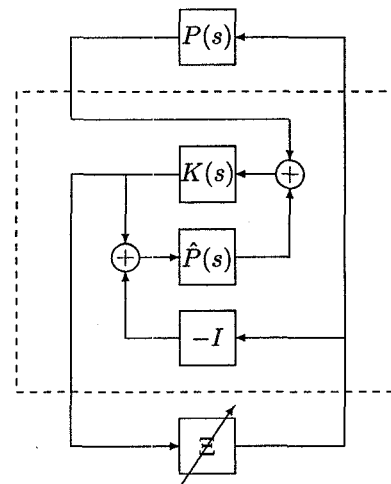

Figure 1: General architecture with tuning

ical plant including inner control loops, $K(s)$ is the (potentially unstable) controller, and $\hat{P}(s)$ is a model of the physical plant. To preserve simplicity in this presentation, we shall assume that $\hat{P}(s) \equiv P(s)$ below, although it should be emphasized that robustness considerations are readily introduced in the suggested

architecture. Finally Ξ is a square 'tunable' block, which will play the role played by α in the classical tuning approach.

We can now formulate the main properties of the suggested tuning architecture. The proof is omitted due to space limitation.

Theorem 1 *Consider the system in Figure 1. Assume that $\hat{P}(s) \equiv P(s)$ is a stable transfer matrix, and that $K(s)$ is an internally stabilizing controller for $P(s)$. Then the following properties hold.*

1. *The system is internally stable for any choice of Ξ as a stable transfer matrix*

2. *For $\Xi = 0$, the (new) controller is disabled*

3. *For $\Xi = I$, the overall control structure is equivalent to the desired multivariable controller $\hat{K}(s)$*

4. *The overall controller structure is a linear fractional transformation $K_\Xi = \mathcal{F}_\ell(G, \Xi)$, where*

$$G(s) = \begin{pmatrix} 0 & I \\ K(I - \hat{P}K)^{-1} & -K\hat{P}(I - K\hat{P})^{-1} \end{pmatrix}$$

From an input-output point of view, the overall controller is equivalent to a multivariable controller with the following transfer function:

$$\Xi(I - KP(I - \Xi))^{-1}K$$

3 Tuning the Bandwidth

The simplest possible choice of Ξ is of course

$$\Xi = \alpha I$$

where α is a scalar that is varied slowly from 0 to 1.

However, for plants with potential robustness problems related to unknown high frequency behavior, a better approach is to choose

$$\Xi(s) = \alpha \frac{1}{1 + s\tau} I \qquad (1)$$

where α is a scalar that is varied slowly from 0 to 1, and τ is a scalar that is varied slowly from ∞ to 0.

The authors recommend the following tuning procedure:

1. choose $\tau = \tau_0$ where $\left(0; \frac{1}{\tau_0}\right)$ is a "trusted" frequency range

2. increase α slowly from 0 to 1

3. decrease τ slowly from τ_0 to 0.

The intuition behind this procedure is that if instability is encountered, this is much more likely at high frequencies. Hence the quality of the control is improved somewhat by selecting both a large value of α

and τ. At this point the control signal is low pass filtered. The subsequent tuning of τ can then continue until a reasonable bandwidth is achieved. Throughout this process it is possible to evaluate the quality of the model continuously.

Even if $K(s)$ does not stabilize $P(s)$, the existence of a suitable τ_0 is guaranteed if the plant behavior is well known at low frequencies. Using a homotopy argument, this can be formalized in the following result which we state without proof.

Proposition 2 *Assume that P and $\hat{P}$ are both linear, time-invariant finite-dimensional, stable systems, and that $P(0) = \hat{P}(0)$. Then for any $\varepsilon > 0$, there exists a $\tau_0(\varepsilon)$ such that every K_Ξ with Ξ as in (1) satisfying*

$$\alpha \in (0; 1 - \varepsilon) , \qquad \tau \in (\tau_0(\varepsilon); \infty)$$

internally stabilizes the closed loop system depicted in Figure 1.

4 Conclusion

Introducing a multivariable controller in an outer loop with the requirement of a continuous transition from the classical controller to the multivariable controller does not necessarily imply that the multivariable controller needs to be stable.

In this paper we have provided a general architecture for tuning controllers which are allowed to be open-loop unstable. This can be advantageous both in terms of performance and (robust) stability, for a number of applications.

The method can also be applied to a range of similar problems which arise for gain scheduled controllers. This is achievable by introduction of a model for each operational mode.

The suggested approach has only few disadvantages. One, admittedly, of some significance is the high controller order, which is implied from having a model of the plant included (in addition to what might already be represented in K if K is model based) in the controller. However, it is believed that the methodology can still be useful even for high order systems with the use of model order reduction techniques. The stabilizing effect can in many cases be achieved by a very simplified model of the plant.

References

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