ROBUST CONTROL OF MICROVIBRATIONS

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1 Introduction

Recent years have seen a dramatic increase in the stability requirements placed on satellite based payload instruments with a consequent increase in the level of vibration suppression demanded from its structure. Hence low amplitude vibrations with frequencies between 1 and 1000 Hz or so-called microvibrations - which were once neglected because the low levels of disturbance they caused on board satellites were below the tolerable level - are of critical importance. This, in turn, has prompted an increasing volume of research into the development of efficient techniques for their suppression.

In effect, microvibrations are produced by the functioning of on-board equipment (sources) such as reaction wheels, gyroscopes, thrusters, electric motors, etc which propagate through the satellite structure towards sensitive equipment (receivers) jeopardising their correct functioning. Figure 1 is a schematic diagram of this process. Vibration suppression levels required vary from case to case but are particularly demanding for micro-gravity experiments and accurately targeted optical instruments [1]. Included in this latter class are mirror pointing systems, such as those found on space telescopes [2], where small mechanical disturbances produce jitter which can result in severe blurring of the images collected by these systems.

In practice, the reduction of vibration levels on-board satellites can be attempted by action at the sources, receivers and along the vibration paths. At the sources, this action consists of attempting to minimise the amplitude of the vibrations by, for example, placing equipment on appropriate mountings. The same basic approach is employed at the receivers but with the objective of sensitivity reduction. Finally, along the vibration paths, modifications of structural elements or equipment relocation can be attempted with the aim of reducing the mechanical coupling between sources and receivers.

Passive damping technology is the basis of these approaches and is, for routine applications, often capable of providing the required levels of dynamic disturbance rejection. In particular, active control techniques are only considered for the most demanding applications. Also to apply active control schemes to suppress microvibrations requires an appropriate mathematical modelling technique i.e. it must produce computationally feasible models but still retain the essential system dynamics.

The most obvious approach to modelling the system dynamics here is to employ the finite element method (FE) due to the accuracy achievable with a sufficiently fine mesh. Use of such a mesh can, however, generate an unacceptably large computational load. Consequently FE based models are not suitable for the design of active control schemes for microvibration suppression. They can, however, be used, as here, to verify the models developed for active control analysis/design.

Alternatives to FE models can be classified into three categories. These are elastic wave based methods, energetic or variational methods, and mechanical impedance based methods. Further details of these methods can be found, for example, in [3], [4], and [5] respectively. A study of the advantages and disadvantages of each of these groups of methods was undertaken [6] and as a result it was decided to use a Lagrange-Rayleigh-Ritz (LRR) method in the research programme which forms the basis for the work reported here.

One highly attractive feature of the mathematical models developed by this procedure is that they can be immediately written in a form suitable for linear (and nonlinear) controller design studies.

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In this paper the use of linear optimal control laws augmented by Loop Transfer Recovery (LTR) is investigated. The results given are taken from the work of Aglietti et al [6], [7].

2 System Description and Modelling

Equipment on-board satellites is often mounted on lightweight panels where microvibrations have to be suppressed to achieve the required level of stability. In the work reported here, attention is restricted to one such mass loaded panel which is an acceptable compromise between problem complexity and the need to gain useful insights as to the benefits (and limitations) of linear active control schemes. A schematic diagram of the arrangement considered is shown in Figure 2 where the equipment mounted on the panel is modelled as lumped masses and the disturbances as point forces. (More recent work has extended the modelling technique beyond the assumption of lumped masses for the equipment.)

The senors and actuators for the active control scheme to be designed (and eventually implemented) are twin patches of piezoelectric material bonded onto opposite faces of the panel. The bending vibrations of the plate produce a stretching or shrinking of the patches depending on whether they are on the top or bottom of the plate as illustrated in Figure 3a. Due to the piezoelectric effect, these deformations induce an electric field perpendicular to the plate which is detected by the electrodes. The outer electrodes of the patches are electrically connected together and the plate, which is grounded, is used as the other electrode for both patches of the pair (Figure 3a). This configuration is also used for the actuator, but in this case the electric field is applied externally to produce contraction or expansion and hence a curvature of the plate.

It is important to note that the effectiveness of the piezoelectric elements, both as sensors and actuators, decreases if the wavelength of the deformations is smaller than the patch. The essential reason for this reduced effectiveness is because the signal produced in this case is partially or completely cancelled (as illustrated in Figure 3b) by the opposing field generated by the other part of the patch as it is deformed in the opposite direction. This limiting factor is especially important when attempting to control high frequency vibrations which have, of course, very short wavelengths. One possible means of increasing the effectiveness of the patches in such cases would be to increase the patch dimension but care should be exercised since this would also diminish the control authority (or action) at low frequencies.

The use of the LRR procedure to develop a mathematical model of the arrangement of Figure 2 is detailed in [6]. Before accepting such a model as a realistic basis for controller design/evaluation studies, it is necessary to establish its validity for such a purpose. In this research programme, this is undertaken by comparing the results produced by a mathematical model for various structural and input/output (or actuator/sensor) configurations against those produced using standard FE implemented by standard software (ANSYS).

In our FE work, only the elastic characteristics of the patches were directly modelled, and the piezoelectric effect was simulated by extrapolating in two dimensions the mono-dimensional theory reported in [8]. Three different classes of tests are performed in each case:

1. comparison of the the output displacement produced by a corresponding input voltage (i.e. 1 volt) at the patch acting as a sensor, where in the FE analysis for these tests the applied voltage was replaced by a line moment along the patch edges,

2. the displacement response of the plate (with the lumped masses added) in response to point forces, and

3. comparison of the output (i.e. voltages) at the sensor(s) in response to point force(s).

During these tests the voltages at the sensor were evaluated from the curvature of the plate at the edges of the patches. In depth analysis of the results obtained confirmed that the modelling technique developed here is a viable alternative to other approaches with the added advantage of being a suitable basis for active control studies. A detailed treatment of these tests can again be found in [6], [7].

3 Control System Design and Evaluation

The final form of the mathematical model is:

$$\begin{aligned} \dot{x} &= A\,x + B_v\,v_a + B_f\,f \\ v_s &= C_v\,x \\ w_{out} &= C_w\,x \end{aligned} \tag{1}$$

where

$$x = \begin{bmatrix} \sigma \\ \dot{\sigma} \end{bmatrix}, A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_s \end{bmatrix}, B_v = \begin{bmatrix} 0 \\ M^{-1}K_a^T \end{bmatrix} \tag{2}$$

and

$$B_f = \begin{bmatrix} 0 \\ M^{-1}s_f^T \end{bmatrix}, C_v = \begin{bmatrix} -K_b^{-1}K_c & 0 \end{bmatrix}, C_w = \begin{bmatrix} s_{out}^T & 0 \end{bmatrix} \tag{3}$$

In (1)-(3) (see [6] for the details) the column vector σ contains the modal co-ordinates and its dimension is determined by the detailed modelling strategy employed, w_{out} is the output displacement, s_{out} is the vector of mode shapes evaluated at the output location, v_s is the sub-vector of voltages at the sensors, and the matrices which define A, B_v, B_f, C_v and C_w again result from the detailed modelling procedure.

Given this state space model, it is possible to begin investigating the potential of active control schemes in this general area. In this paper, the control objective is to minimise the displacement at a specified point on the panel in the presence of point force disturbances acting at other location(s) on the panel. The control strategy employed is linear optimal control with a quadratic performance index augmented by LTR (for the necessary theoretical background see, for example, [9] and the relevant cited references).

The form of LTR employed here requires the definition of a Target Feedback Loop (TFL) which is then recovered through an asymptotic design. Basically this consists of defining a compensator whose parameters are then adjusted to asymptotically recover the TFL. Here we have followed the well known two step procedure for recovery at the input of a square plant and its dual for recovery at the plant output. The actual designs were undertaken in MATLAB, and Figure 4 shows the resulting closed loop system for recovery at the plant output where L is the Kalman filter gain matrix defined by appropriate selection of the covariance matrices W and V. The design is then completed by solving a standard linear quadratic regulator (LQR) problem with state and control weighting matrices Q and R respectively.

In effect, this design method places some of the Kalman filter poles (eigenvalues) at the zeros of the plant with the remainder allowed to become 'arbitrarily fast'. Hence it relies on the 'cancellation' of some of the plant dynamics (in particular, the zeros) by the filter dynamics and is only guaranteed to work with non-minimum phase plants. If the plant has right-half plane zeros then the LTR procedure may still work provided these zeros lie beyond the desired operating bandwidth of the system.

The general problem area arising here is characterised by non-colocated actuators and sensors. This, in turn, means that the resulting system transfer function (or transfer function matrix in the multivariable case) could well be non-minimum phase. Below we report one case in detail where LTR design is still possible to a meaningful degree.

The example considered is a simply supported rectangular plate with a lumped mass mounted on it together with two pairs of piezoelectric patches, which form the actuator and sensor respectively for the control system to be designed. This case is, in effect, illustrated by Figure 2 and the source of the disturbance is taken as a harmonic point force of 1 N amplitude acting perpendicular to the panel. Here the first 36 mode shapes of the bare panel (6 in each direction) were taken as Ritz functions (to model the displacement field) in the modelling and controller design/evaluation studies. (Again see [6] for the plate dimensions and the construction of (1)-(3)).

As expected, this system is non-minimum phase and hence, at best, only a limited degree of LTR is possible. Iterating through the procedure described above showed that this was indeed the case for weighting matrices of the form

$$W = B_f B_f^T, \ V = I, \ Q = C_w^T C_w + qI, \ R = I \tag{4}$$

and varying the scalar q upwards from zero to approximately 10^{-2}. Given these weighting matrix structures and the allowable range of q, extensive simulation studies were undertaken to assess the performance characteristics of the designs possible in this particular case. One key objective of these studies was to get some indication of controller performance in the presence of model uncertainty. The means of doing this was to simulate the controller designed on a model with a given number of Ritz functions against models constructed with higher numbers of these functions included.

Figure 5 is an example of the test results and shows a comparison of the displacement response at the centre of the plate without control (continuous line) and with a controller designed using the LTR procedure described above (dashed line). The controller in this case was designed using a model constructed with the first 4 Ritz functions included and the closed loop performance assessed against a model with the first 6 Ritz functions included. This clearly shows that a good level of damping has been introduced into the system.

The presentation of this paper will

- discuss this analysis/tests in greater detail; and
- outline the results of some more recent work and directions for short to medium term further research.

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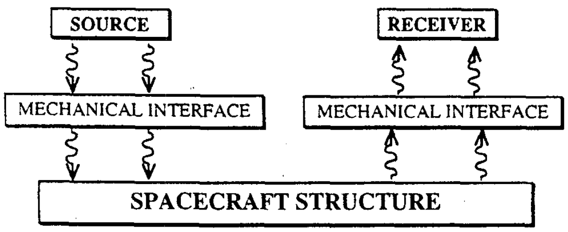


Fig. 1. Microvibration propagation path.

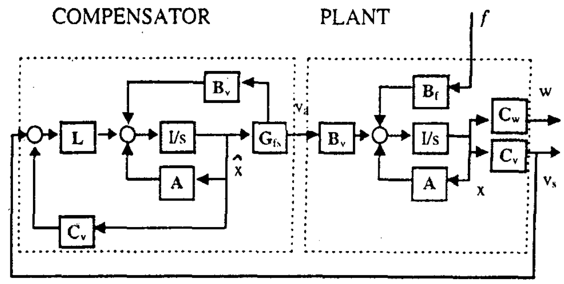


Fig. 4. Block diagram of the actively controlled system.

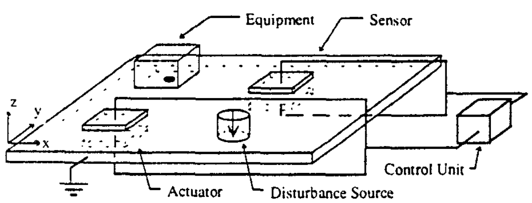


Fig. 2. Model layout.

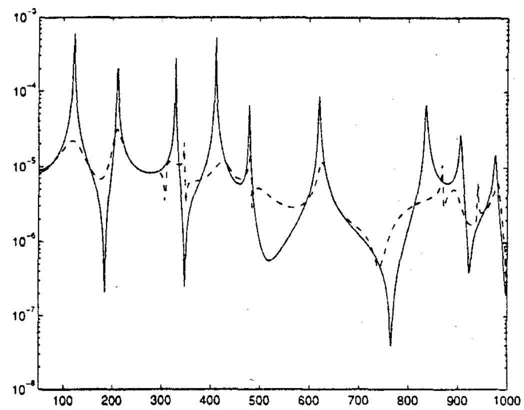
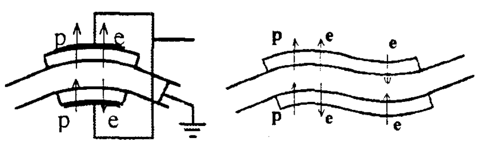


Fig. 5. Frequency response to point force
Solid line - Uncontrolled system
Dashed line - Controlled system



a) b)

Fig. 3. Patch section view during deformation
a) deformation wavelength longer then patch
b) deformation wavelength equal or shorter then patch

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