INTEGRATION OF CONTROL AND FAULT DETECTION: NOMINAL AND ROBUST DESIGN [1]

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Abstract. The integrated design of control and fault detection is studied. The result of the analysis is that it is possible to separate the design of the controller and the filter for fault detection in the case where the nominal model can be assumed to be fairly accurate. In the uncertain case, however, the design of the filter and the controller can not be separated when an optimal design is desired. For systems with significant uncertainties, there turns out to be a fundamental trade-off between the performance in the control loop and the performance in the filter

Keywords. Robust control, Robust fault detection, Separation, μ-synthesis, Model uncertainties.

1. INTRODUCTION

The integration of controller and fault detection filter design has not received much attention in the literature dealing with fault detection and isolation (FDI). The design of both nominal and robust filters for FDI has in general been considered as a separate design from design of feedback controllers. In a lot of cases, this is also in correspondence with practice, because the FDI filters are designed for existing control systems. It is therefore quite relevant to consider the FDI design case as a separate design problem as it has been in a lot of papers, see e.g. (Frank, 1990), (Patton *et al.*, 1989), (Patton and Chen, 1991), (Frank, 1996), and (Mangoubi *et al.*, 1995) to mention a few.

On the other hand, there are also cases where it is possible to implement an integrated design of both the feedback controller and the fault detection filter. The question is then: Is it optimal to make two separate designs of the feedback controller and the FDI filter? This question has not been answered unambiguously. However, there has been some indication of an answer in a few papers. A four parameter controller setup has been considered in (Nett *et al.*, 1988), (Jacobson and Nett, 1991) in connection with control and FDI. The design setup applied by Nett et al. (Nett *et al.*, 1988) resulted in an integrated design of the controller and the FDI filter. The $\mathcal{L}_1$ design approach has been applied in (Ajbar and Kantor, 1993) for an integrated controller and FDI filter design for a system with model uncertainty. The question about separation has not been considered in this paper. Some indication of when separation is possible has been given in (Tyler and Morari, 1994). In this paper the four parameter controller structure from (Nett *et al.*, 1988) has been applied. An $\mathcal{H}_2$ design of an integrated controller and FDI filter has been considered in the nominal case. It turns out that the design of the feedback controller is separated from the three other controllers in the $\mathcal{H}_2$ case. Further, a design example indicate that this is not the case when the system include model uncertainties. Recently, the nominal case has been completely analyzed in (Stoustrup and Grimble, 1996). It has been shown that there is a complete separation between the design of feedback controllers and FDI filters in the nominal case in a rather wide generality.

Following the line from (Stoustrup and Grimble, 1996), the uncertain case will be analyzed in the following.

2. DESIGN SETUP

The design setup which will be applied in this paper is illustrated in Figure 1. The setup uses the so-called stan-

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dard problem philosophy, see e.g. (Zhou *et al.*, 1995).

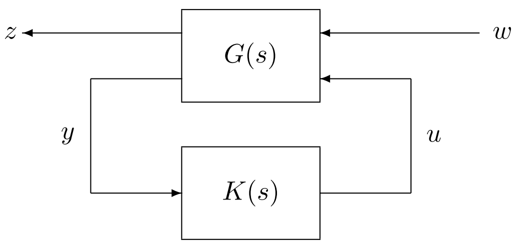

Fig. 1. Control system in standard configuration

A state space description of $G(s)$ can be formulated by:

$$G(s) = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{array}\right] \tag{1}$$

or given as transfer functions

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} G_{zw}(s) & G_{zu}(s) \\ G_{yw}(s) & G_{yu}(s) \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix} \tag{2}$$

The model $G(s)$ contains both the nominal model and weight matrices for the disturbance and performance specifications. In a standard design, a controller $K(s)$ is designed such that the closed loop is internally stable and a suitable norm of the closed loop transfer function from w to z is minimized or made smaller than a prespecified level.

Instead of using a standard one parameter controller as shown in Figure 1, a two parameter controller described by:

$$\begin{pmatrix} u \\ a \end{pmatrix} = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} y$$

will be used in the following in connection with fault detection. The additional output signal a from the controller is a diagnostic signal. This signal will in the following be applied to derive an estimate of faults in the controlled system.

The fault cases that need to be taken care of is e.g. fault in the sensors and/or actuators as illustrated in Figure 2, where f_a is an actuator fault and f_s is a sensor fault.

Here, the measurement used by the controller are $y = y_c + f_s$ rather than y_c and the control signals to the system are $u_c + f_a$ instead of u_c.

However, also internal faults can appear in the system $G(s)$ which we want to take care of. The setup in Figure 2 does not take care of internal faults in the system. To obtain a more general fault description, instead of specifying the faults as faults on actuators or on sensors, they will be described as faults affecting the generalized

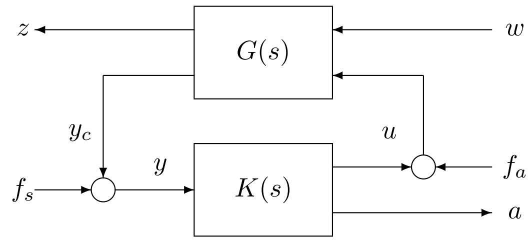

Fig. 2. Control system setup with actuator and sensor faults and diagnostic signal

system directly. Denoting the generalized fault signal be f, the open loop transfer function is given by:

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} G_{zw} & G_{zf} & G_{zu} \\ G_{yw} & G_{yf} & G_{yu} \end{pmatrix} \begin{pmatrix} w \\ f \\ u \end{pmatrix} \tag{3}$$

The special case with actuator and sensor faults can be described in the general form (3) by using:

$$\begin{aligned} f &= \begin{pmatrix} f_a \\ f_s \end{pmatrix} \\ G_{zf} &= \begin{pmatrix} G_{zu} & 0 \end{pmatrix} \\ G_{yf} &= \begin{pmatrix} 0 & I \end{pmatrix} \end{aligned}$$

For obtaining a good estimation of the individual faults, fault models are included in the generalized system as frequency weightings on the faults signals:

$$f = W_f(s)v$$

where v is a signal that is anticipated to have a flat power spectrum. The generalized setup is shown in Figure 3.

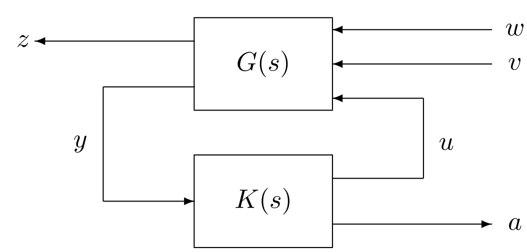

Fig. 3. Generalized setup for control and fault detection

Now we just need to formulate the design setup in Figure 3 as a standard design problem as illustrated in Figure 1. For doing this, let us define an additional output r as the fault estimation error:

$$r = f - a \tag{4}$$

This is the standard way of formulating a filter design problem in the standard problem setup, see (Zhou *et al.*, 1995). The generalized system $G_{ncf}(s)$ is then given by:

$$\begin{pmatrix} z \\ r \\ y \end{pmatrix} = G_{ncf}(s) \begin{pmatrix} w \\ v \\ u \end{pmatrix} \tag{5}$$

with

$$G_{ncf}(s) = \left(\begin{array}{cc|cc} G_{zw} & G_{zf}W_f & G_{zu} & 0 \\ 0 & W_f & 0 & -I \\ \hline G_{yw} & G_{yf}W_f & G_{yu} & 0 \end{array}\right)$$

3. NOMINAL DESIGN OF FILTERS

Using the system setup in (5) and apply a two parameter controller $u = K(s)y$ we get the following closed-loop transfer function

$$\begin{pmatrix} z \\ r \end{pmatrix} = T_{ncf}(s) \begin{pmatrix} w \\ v \end{pmatrix} \tag{6}$$

with

$$T_{ncf}(s) = \begin{pmatrix} G_{zw} & G_{zf}W_f \\ 0 & W_f \end{pmatrix} + \begin{pmatrix} G_{zu} & 0 \\ 0 & -I \end{pmatrix} K(s)(I - \begin{pmatrix} G_{yu} & 0 \end{pmatrix} K(s))^{-1} \begin{pmatrix} G_{yw} & G_{yf}W_f \end{pmatrix}$$

For simplicity, assume that $G(s)$ is open loop stable (the unstable case can be dealt with as well in this methodology, but is computationally harder). Then the Youla parameterization of all stabilizing controllers can be obtained simply by making the substitution:

$$\begin{aligned} Q(s) &= K(s)(I - \begin{pmatrix} G_{yu} & 0 \end{pmatrix} K(s))^{-1} \\ K(s) &= Q(s)(I + \begin{pmatrix} G_{yu} & 0 \end{pmatrix} Q(s))^{-1} \end{aligned} \tag{7}$$

where $Q(s)$ is a stable proper transfer function. Further, let $Q(s)$ be partitioned as:

$$Q(s) = \begin{pmatrix} Q_1(s) \\ Q_2(s) \end{pmatrix} \tag{8}$$

Then we get the following equation for the closed loop transfer function T_{ncf}:

$$T_{ncf}(s) = \begin{pmatrix} G_{zw} + G_{zu}Q_1G_{yw} & G_{zf}W_f + G_{zu}Q_1G_{yf}W_f \\ -Q_2G_{yw} & W_f - Q_2G_{yf}W_f \end{pmatrix} \tag{9}$$

Note that Q_1 only appear in the first row of T_{ncf} and Q_2 only in the second row of T_{ncf}. A separation between the design of Q_1 and Q_2 has then been obtained by using a Youla parameterization. Calculating $K(s)$ directly from (7) result in the following equation:

$$\begin{aligned} K(s) &= \begin{pmatrix} Q_1(s)(I + G_{yu}Q_1(s))^{-1} \\ Q_2(s)(I + G_{yu}Q_1(s))^{-1} \end{pmatrix} \\ &= \begin{pmatrix} Q_1(s)(I + G_{yu}Q_1(s))^{-1} \\ Q_2(s)(I - G_{yu}K_1(s)) \end{pmatrix} \end{aligned} \tag{10}$$

The result indicate that also the original controller structure is separated in a design of the feedback controller $K_1(s)$ and a design of the fault detection filter $K_2(s)$. $K_2(s)$ depend of $K_1(s)$. A complete analysis can be found in (Stoustrup and Grimble, 1996).

4. ROBUST DESIGN OF FILTERS

In the two previous sections, we have looked at the interdependence of controller and filter design in the nominal case. It turned out that there exist a separation between controller design and filter design. In this section, the robustness aspect will be considered. Let us consider the setup from Figure 3, and include a model uncertainty. This setup is illustrated in Figure 4.

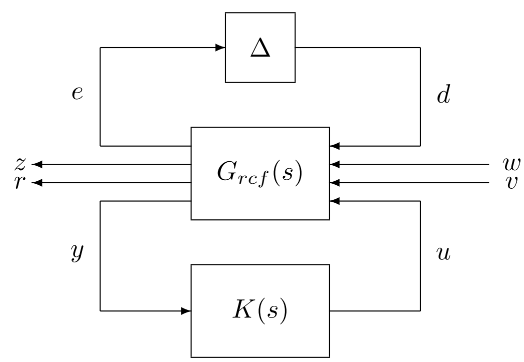

Fig. 4. Generalized setup for robust control and fault detection

It is assumed that Δ is scaled such that $\|\Delta\| \le 1 \ \forall\omega$. Further, Δ can be structured or unstructured. The transfer function from w to z define the performance for the closed-loop control system and the transfer function from v to r define the performance for the fault detection filter.

The generalized system $G_{rcf}(s)$ in Figure 4 is given by:

$$\begin{pmatrix} e \\ z \\ r \\ y \end{pmatrix} = G_{rcf}(s) \begin{pmatrix} d \\ w \\ v \\ u \end{pmatrix}$$

where

$$G_{rcf}(s) = \left(\begin{array}{ccc|cc} G_{ed} & G_{ew} & G_{ef}W_f & G_{eu} & 0 \\ G_{zd} & G_{zw} & G_{zf}W_f & G_{zu} & 0 \\ 0 & 0 & W_f & 0 & -I \\ \hline G_{yd} & G_{yw} & G_{yf}W_f & G_{yu} & 0 \end{array}\right) \tag{11}$$

In comparison to the system used in Section 3, the introduction of the uncertainty block Δ changes the possible design concepts considerably, as it will be demonstrated below.

Consider Figure 5, where the Δ_p and Δ_f blocks represent performance specifications for the closed-loop transfer function and performance for the fault detection signal. Introduction of such fictitious perturbation blocks is a standard trick in μ synthesis to obtain robust performance, see e.g. (Zhou *et al.*, 1995). It is assumed that weight matrices on the performance specifications in Figure 5 are included in the generalized system $G_{rcf}(s)$.

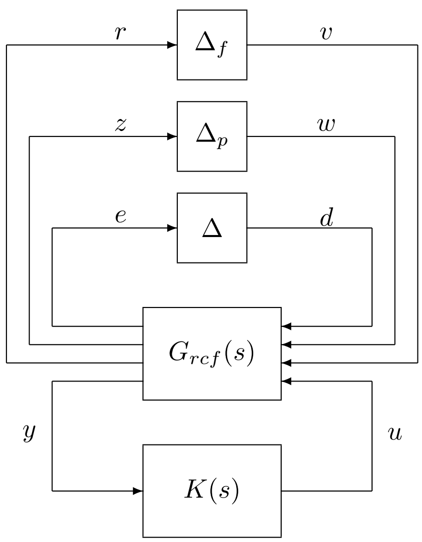

Fig. 5. Generalized setup for robust control and fault detection with performance specifications represented by fictitious perturbation blocks

Applying the same technique as in the previous two section by using a parameterization of the controllers, we get the following closed loop transfer function $T_{rcf} = \mathcal{F}_l(G_{rcf}(s), Q(s))$ with:

$$T_{rcf} = \begin{pmatrix} G_{ed} + G_{eu}Q_1G_{yd} & G_{ew} + G_{eu}Q_1G_{yw} & G_{ef}W_f + G_{eu}Q_1G_{yf}W_f \\ G_{zd} + G_{zu}Q_1G_{yd} & G_{zw} + G_{zu}Q_1G_{yw} & G_{zf}W_f + G_{zu}Q_1G_{yf}W_f \\ -Q_2G_{yd} & -Q_2G_{yw} & W_f - Q_2G_{yf}W_f \end{pmatrix} \quad (12)$$

At first glance, it seems there is again a separation between the two parameters in $Q(s)$. $Q_1(s)$ appear only in the first two rows and Q_2 appear only in the last row of T_{rcf}. However, due to the fact that the feedback loop with Δ, Δ_p, and Δ_f, is considered directly in the design process, there is no separation in this case due to the model uncertainties. This can be seen quite easily by considering a separate design of a robust feedback controller Q_1 and a robust fault detection filter Q_2 - see below.

First, let us consider the design of a robust stabilizing controller followed by a design of a nominal filter. The feedback controller design problem with respect to robust stability is represented by the following closed loop transfer function:

$$T_{rsc}(s) = G_{ed} + G_{eu}Q_1G_{yd} \quad (13)$$

The design problem is a standard $\mathcal{H}_\infty$ design problem when Δ is unstructured. Otherwise it is a μ design problem. The following design of a nominal filter is represented by the following closed loop transfer function:

$$T_{npf}(s) = \begin{pmatrix} -Q_2G_{yw} & W_f - Q_2G_{yf}W_f \end{pmatrix} \quad (14)$$

Here, there is again separation between the two designs because Q_1 appear only in (13) and Q_2 only in (14).

The next design case consist of a design for a feedback controller with respect to robust performance followed by a design of a nominal filter. The design problem for the feedback controller is represented by the following closed loop transfer function:

$$T_{rpc} = \begin{pmatrix} G_{ed} + G_{eu}Q_1G_{yd} & G_{ew} + G_{eu}Q_1G_{yw} \\ G_{zd} + G_{zu}Q_1G_{yd} & G_{zw} + G_{zu}Q_1G_{yw} \end{pmatrix} \quad (15)$$

This design problem is a μ design problem due to the structure in the perturbations, see Figure 6. As expected, only the feedback parameter Q_1 appear here.

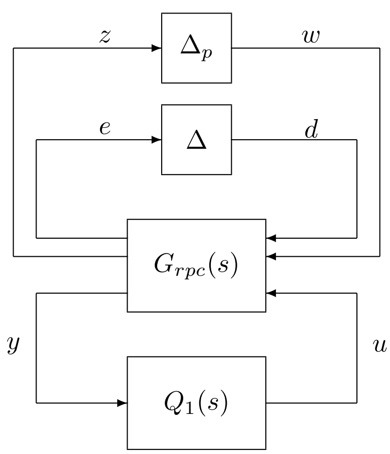

Fig. 6. Generalized setup for robust control with performance specifications

The design of the nominal filter is still given by (14). Also in this case, there is a separation between the two designs.

In the last two design cases, the filter is designed with respect to robust performance. In the first case, the design of a feedback controller is represented by (13), i.e. designed with respect to robust stability. The design problem for a filter with respect to the model uncertainty is represented by the following transfer function:

$$T_{rpf} = \begin{pmatrix} G_{ed} + G_{eu}Q_1G_{yd} & G_{ew} + G_{eu}Q_1G_{yw} \\ -Q_2G_{yd} & -Q_2G_{yw} \\ & & G_{ef}W_f + G_{eu}Q_1G_{yf}W_f \\ & & W_f - Q_2G_{yf}W_f \end{pmatrix} \quad (16)$$

The filter design problem is also a μ design problem due to the structure in the (partly fictitious) perturbations, see Figure 7.

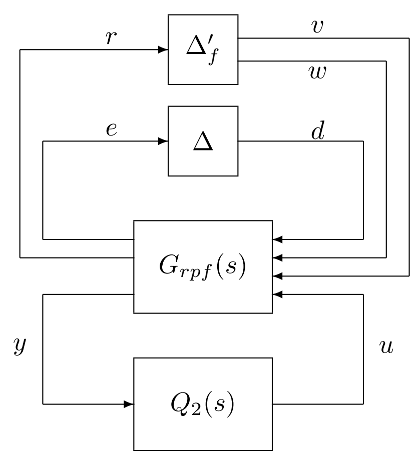

Fig. 7. Generalized setup for the design of a robust fault detection filter

From (16) we can see directly that the two designs are not separated any more. Both controllers appear in the design problem in (16). Another way to see that the two designs are coupled, is to consider the closed loop transfer function given by:

$$\begin{aligned} T_{rf}(s) &= \mathcal{F}_u(T_{rpf}, \Delta) \\ &= W_f - Q_2G_{yf}W_f - Q_2G_{yd}\Delta \\ &\quad \times(I - (G_{ed} + G_{eu}Q_1G_{yd})\Delta)^{-1} \\ &\quad \times(G_{ef}W_f + G_{eu}Q_1G_{yf}W_f) \end{aligned}$$

It is clear from the above equation that in general the design of Q_2 depends on Q_1. Generically, it will never be possible to separate the design of the two controllers due to the feedback with the uncertainty block Δ.

The last design case consist of a design of a feedback controller with respect to robust performance represented by (15) followed by the above filter design from (16). Also in this case there is no separation between the two designs.

4.1 *Summary of Optimal Design Techniques*

Depending on whether robustness is considered important in the design of the control loop and/or of the filter or not, six classes of design methodologies can be identified. These classes are characterized in Table 1.

Remark 1. The entry for robust performance of the filter subject to a nominal design of the controller, presumes that the control loop will actually be stable, since the filter obviously can not stabilize an unstable control loop. Moreover, in this case there does not exist any optimization method, which directly allows to handle the coupling and gives the controller and filter in one design step. The suggested method is a reasonable suboptimal approach.

Remark 2. The $\mathcal{H}_\infty$ optimizations for filter designs listed in Table 1 is based on an assumption that Δ_f is unstructured. More realistically, Δ_f will have a block diagonal structure, corresponding to individual faults. It is straightforward to incorporate this into the design.

As a main observation regarding the case with considerable uncertainties, there are fundamental trade-offs involved with the design of the control loop and the fault detection filter. In particular the following four transfer functions are significant:

(1) making $\|T_{zw}\|_\infty$ small implies good disturbance rejection and robustness, i.e. the original control objectives are achieved
(2) making $\|T_{zv}\|_\infty$ small implies that undetected faults do not cause disastrous effects
(3) making $\|T_{rw}\|_\infty$ small implies that disturbances are not readily interpreted as faults, i.e. the risk of false alarms is reduced
(4) making $\|T_{rv}\|_\infty$ small implies that the alarm signal a becomes a good estimate of potential faults

In the uncertain case, each of these four transfer functions have to be traded off against one another. This is obtained by selecting weighting matrices appropriately.

5. CONCLUSION

The integration of feedback controller and fault detection filter design has been considered for systems with and without model uncertainties. It turns out that the design of the feedback controller and the fault detection filter can be separated in the nominal case. In the uncertain case, however, optimal functionality can not be obtained by separate designs of a robust controller and a robust filter.

	Nominal performance of filter	**Robust performance of filter**
Nominal performance of controller	(1) Separate designs (2) $\mathcal{H}_\infty$ optimizations - see Remark 2 (3) Eqn. (9) (optimize each row independently)	(1) Coupled design - see Remark 1 (2) $\mathcal{H}_\infty$ (controller) and μ (filter) optimizations (3) Eqn. (9) ($\mathcal{H}_\infty$ opt. of first row) and Eqn. (16) (Find Q_2 by μ opt.)
Robust stability of controller	(1) Separate designs (2) $\mathcal{H}_\infty$ optimizations - see Remark 2 (3) Eqn. (13) and Eqn. (14)	(1) Coupled design (2) μ optimization (3) Eqn. (16)
Robust performance of controller	(1) Separate designs (2) μ (controller) and $\mathcal{H}_\infty$ (filter) optimizations - see Remark 2 (3) Eqn. (15) (μ opt.) and Eqn. (14) ($\mathcal{H}_\infty$ opt.)	(1) Coupled design (2) μ optimization (3) Eqn. (12)

Table 1. The six possible design classes. For each class, Item (1) tells whether the design of controller and filter separates or not, Item (2) specifies the method of optimization required, and Item (3) specifies the relevant equations to be used for the optimization.

In spite of the separation in the nominal case, it can still sometimes be a good idea to do an integrated design of both the feedback controller and the fault detection filter, because it is possible e.g. to use the same observer for both the feedback controller and the filter. This has been done in (Kilsgaard *et al.*, 1996), where a 4th order SIMO system has been considered. An $\mathcal{H}_\infty$ design based on LMI has been carried out. The result was an integrated feedback controller and fault detection filter of order 1. The four parameter case has been considered in (Stoustrup *et al.*, 1997).

In the uncertain case, it is of paramount importance to trade off carefully: control performance, effects of undetected faults, risk of false alarms, and quality of fault detection. In most cases, this requires μ synthesis methods or other methods that allow for structured uncertainties.

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