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# Multi Objective Design Techniques applied to Fault Detection and Isolation

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## Abstract

Various methods for design of fault detectors by using multi objective techniques are presented in this paper. The advantages by using multi objective design methods will be shown. The design methods will be compared to standard fault detection and isolation (FDI) design methods. The FDI problem with uncertain model dynamics is addressed as well.

## 1 Introduction

Improved availability has in recent years been an ever increasing demand of industrial control systems. A control system rarely functions continuously throughout the scheduled life cycle of the plant and controller hardware. Due to wear of mechanical and/or electrical components both actuators and sensors can fail in more or less critical ways. To improve availability it is of paramount importance to detect when faults are likely to happen, and to identify as fast as possible which faults have taken place. For safety critical processes, threats to humans and/or the environment of course play even more significant roles.

To meet such industrial needs, a number of schemes for Fault Detection and Isolation (FDI) have been put forward in the literature on automatic control. Most of the papers appearing in journals and in conference proceedings have dealt with the design of filters which monitor a process, and generate alarms when faults may have occurred.

Extensive surveys on early work on FDI can be found in [2] and in [8]. Many of these techniques are observer based, such as e.g. [4]. These methods have since been refined and extended. A more recent reference in this line of research is [3].

Motivated by a paper of Patton [7], where the design of fault detectors by using multi objective methods has shortly been discussed, this subject will be investigated in more details in this paper. It is quite obvious that potentially multi objective design methods might have advantages compared with traditional design methods for fault detectors. The reason is that the design of fault detectors is not only dealing with design of filters that detects fault signals, but also at the same time to design the filter such that noise signal signals are rejected. Further, if the system is uncertain, the filter must also be robust against model uncertainty. This results in three objectives that are included in the design of a fault detectors in the uncertain case. One way to overcome possible conservatism which occurs in some traditional FDI design methods is to use multi objective design methods which might reduce this conservatism.

The intention in this paper is not to come up with any new general multi objective design methods. Instead, we study two existing multi objective *control* design methods and simply apply them in connection with FDI problems. The first method shows how it is possible to separate the design problem for $i$ fault signals into $i$ FDI problems. The next method which will be considered in connection with FDI is a $\mu$ based method. The FDI problem is formulated as a $\mu$ optimization problem. The FDI design problem turns out to be a $\mu$ optimization problem when the system includes uncertainties. At last, a multi objective design FDI design problem formulated in [1] will be considered. In [1] an observer based approach has been applied. Here we will consider the same design problem by using a general setup and see which consequences this has for the design problem.

## 2 Problem Formulation

The problem is to detect and isolate faults, i.e. to provide an estimate $\hat{f}$ for any faults occurred. This situation is depicted in Figure 1, where $f$ represent the fault signal and disturbances are modeled as a vector of exogenous signals $d$ which enter the system.

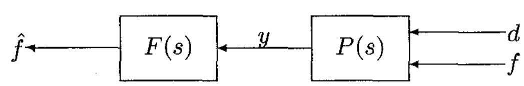


Figure 1: Fault detection and isolation configuration

For a detection and isolation filter to work correctly, it is usually necessary to build disturbance and fault models into the design procedure. In this paper, such models will be represented as coloring filters for $d$ and $f$. I.e., $d$ and $f$ are considered to be the result of filtering fictitious signals $w_d$ and $w_f$ through filters $W_d$ and $W_f$, respectively, which are diagonal rational matrices. If $w_d$ and $w_f$ are assumed to have flat power spectra, $d$ and $f$ will be frequency colored signals, with their spectra given by $W_d$ and $W_f$, respectively.

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In order to employ methods from multi objective control theory, we reformulate the filtering problem as a control problem, where the desired filter $F(s)$ is interpreted as the controller to be designed. In this line of thinking, the control signal $u$ is the signal generated by the controller, i.e. $u = \hat{f}$. The quantity $z$ to be controlled, i.e., the quantity that is desired to be made small is the fault estimation error $z = f - \hat{f}$. The undesired effects $w$ to be compensated for are the two fictitious signals $w_d$ and $w_f$ generating $d$ and $f$:

$$w = \begin{pmatrix} w_d \\ w_f \end{pmatrix}$$

The inputs $y$ to the 'controller', i.e. the measurements to be used for generating the 'control signal', are of course the actual measurements $y$ that are available to the filter.

With these four signals we can formulate the equivalent control problem in standard form.

The abstract generalized plant $G$ is given by:

$$\begin{aligned} \begin{pmatrix} z \\ y \end{pmatrix} &= \begin{pmatrix} \begin{pmatrix} 0 & W_f \end{pmatrix} & -I \\ P\begin{pmatrix} W_d & 0 \\ 0 & W_f \end{pmatrix} & 0 \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix} \\ &= \begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix} = G \begin{pmatrix} w \\ u \end{pmatrix} \end{aligned} \tag{1}$$

The system described by (1) does not include any uncertainties. To handle the uncertain FDI case, we need to include an uncertainty description in the system setup. This can be done by extending the generalized plant $G$ with an additional external input signal $d$ and an additional output signal $e$. The connection external output $e$ is then fed back through the uncertain block $\Delta$ to the external input signal $d$,

$$d = \Delta(s)e$$

where $\Delta$ is bounded by $||\Delta|| \leq 1$ and otherwise unknown.

The resulting system is given by:

$$\begin{pmatrix} e \\ z \\ y \end{pmatrix} = \left(\begin{array}{c|cc} G_{ed} & G_{ew} & G_{eu} \\ \hline G_{zd} & G_{zw} & G_{zu} \\ G_{yd} & G_{yw} & G_{yu} \end{array}\right) \begin{pmatrix} d \\ w \\ u \end{pmatrix} \tag{2}$$

Due to space limitations, we will have to refrain from detailing the actual modeling of the remaining transfer matrices $G_{ed}$, $G_{ew}$, $G_{eu}$, $G_{zd}$, and $G_{yd}$, but simply refer to [11], where also hints on weight selections can be found.

# 3 Multi Objective FDI Design

In this section, two multi objective design methods are applied in connection with FDI design. Both advantages as well as disadvantages will be discussed.

## 3.1 A Sensitivity Multi Objective Approach

The first method applied in connection with FDI was originally developed in connection with roll damping of ships, see [10]. A complete description of the multi objective sensitivity design method can be found in [9].

The closed loop transfer function from $w$ to $z$ is given by:

$$T_{zw}(s) = G_{zw}(s) + G_{zu}(s)F(s)G_{yw}(s)$$

We partition $F$ in its rows, $P\begin{pmatrix} W_d & 0 \\ 0 & W_f \end{pmatrix}$ in its columns, and $\begin{pmatrix} 0 & W_f \end{pmatrix}$ in all its entries:

$$\begin{aligned} F(s) &= \begin{pmatrix} \mathbf{f}_1(s) \\ \vdots \\ \mathbf{f}_m(s) \end{pmatrix} \\ P(s)\begin{pmatrix} W_d(s) & 0 \\ 0 & W_f(s) \end{pmatrix} &= \begin{pmatrix} \mathbf{p}_1(s) & \cdots & \mathbf{p}_r(s) \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 0 & W_f(s) \end{pmatrix} = \begin{pmatrix} \mathbf{w}_{11}(s) & \cdots & \mathbf{w}_{1r}(s) \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{m1}(s) & \cdots & \mathbf{w}_{mr}(s) \end{pmatrix}$$

Note, that if $W_f$ is diagonal as it would usually be, only the $m$ functions $\mathbf{w}_{1,r-m+1}(s)$, $\mathbf{w}_{2,r-m+2}(s)$, ...,$\mathbf{w}_{mr}(s)$ are not identically zero in the last matrix.

This leads to the following formula:

$$T_{zw}(s) = \begin{pmatrix} T^1_{zw}(s) \\ \vdots \\ T^m_{zw}(s) \end{pmatrix} = \begin{pmatrix} \mathbf{w}_{11}(s) - \mathbf{f}_1(s)\mathbf{p}_1(s) & \cdots & \mathbf{w}_{1r}(s) - \mathbf{f}_1(s)\mathbf{p}_r(s) \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{m1}(s) - \mathbf{f}_m(s)\mathbf{p}_1(s) & \cdots & \mathbf{w}_{mr}(s) - \mathbf{f}_m(s)\mathbf{p}_r(s) \end{pmatrix}$$

Now, the main observation is that each row of $T_{zw}$ depends on only one of the $\mathbf{f}_i$'s, $i = 1 \ldots m$. This has the important consequence that each row of $F(s)$ can be considered as a design parameter which determines the behavior of the filter associated with one specific fault. This, in turn, has two main implications:

1. The filtering for each fault can be optimized completely independent and even by different criteria.
2. The conservatism which is symptomatic for several popular optimization techniques, such as $\mathcal{H}_\infty$ and $\mathcal{H}_2$ optimization, can be greatly reduced.

This can be formulated more stringent by specializing to specific norm constraints. The following theorem gives the $\mathcal{H}_\infty$ result, but less us stress here that any norm or mix of norms can be applied.

**Theorem 1** *The following two statements are equivalent. 1. There exists a filter $F(s)$ such that*

$$||T^1_{zw}(\cdot)||_\infty < 1, \quad \ldots, \quad ||T^m_{zw}(\cdot)||_\infty < 1$$

*2. For each $i = 1 \ldots m$ there exists a filter $\mathbf{f}_i$ such that*

$$\| \begin{matrix} \mathbf{w}_{i1}(s) - \mathbf{f}_i(s)\mathbf{p}_1(s) & \cdots & \mathbf{w}_{ir}(s) - \mathbf{f}_i(s)\mathbf{p}_r(s) \end{matrix} \|_\infty < 1$$

*Moreover, in case either condition is satisfied, $\mathbf{f}_i$ can be computed (see e.g. [11]) as a solution to a standard $\mathcal{H}_\infty$ problem with the following data:*

$$G^i(s) = \begin{pmatrix} (\ \mathbf{w}_{i1}(s) & \cdots & \mathbf{w}_{ir}(s)\ ) & -1 \\ (\ \mathbf{p}_1(s) & \cdots & \mathbf{p}_r(s)\ ) & 0 \end{pmatrix} \quad (3)$$

The conservatism avoided by this simple trick can be as much as a factor equal to the square root of the number of faults.

The cost of designing the sub-filters independently, obviously is that they do no longer share a common observer structure. This means that the filter order will be much higher than if all sub-filters were computed by one (potentially conservative) optimization. On the other hand, detection devices especially for noisy systems are known to be critical to fine tuning, and therefore the high filter orders will be justified in many applications. Moreover, actual substantial benefits from applying several observers have been reported previously in the FDI literature.

In comparison to the standard approach which uses (1) directly for optimization, a method based on Theorem 1 will typically be less conservative. However, it can be argued that such a method will still be conservative, since it considers all possible cross couplings between individual faults in a worst case scenario, which might be too pessimistic for less faulty systems.

Alternatively, assuming (1) that detection is extremely important, (2) that isolation of a single fault is rather important, but (3) that the capability to distinguish between simultaneous faults is less important, a suitable filter can be found as:

$$F(s) = \begin{pmatrix} \mathbf{f}_1(s) \\ \vdots \\ \mathbf{f}_m(s) \end{pmatrix}$$

where each $\mathbf{f}_i(s)$, $i = 1 \ldots m$ is a solution to a standard $\mathcal{H}_\infty$ problem of the special form:

$$\left\| G^i_{11}(\cdot) - F(\cdot) G^i_{21}(\cdot) \right\|_\infty < 1$$

with $G^i_{11}$ and $G^i_{21}$ given by the following standard form:

$$G^i(s) = \begin{pmatrix} G^i_{11}(s) & G^i_{12}(s) \\ G^i_{21}(s) & G^i_{22}(s) \end{pmatrix}$$
$$= \begin{pmatrix} (\ 0 & \cdots & 0 & \mathbf{w}_{i,r+i-m}(s)) & -1 \\ (\ \mathbf{p}_1(s) & \cdots & \mathbf{p}_{r-m}(s) & \mathbf{p}_{r+i-m}(s)\ ) & 0 \end{pmatrix}$$

In the latter approach, all sources of noise are assumed potentially to influence every fault estimate, but only one fault is assumed to appear at the time. A filter built from this method will almost always detect faults and isolate single faults, but it might isolate simultaneous faults incorrectly. In contrast, the approach in Theorem 1 can handle several faults at a time, and will isolate those correctly, but might give conservative filters, in case this is not expected to happen, i.e. there might for example be more false alarms, since the weights representing noise might have to be decreased in order for the optimization to succeed.

In either case, however, better results should be expected than would result from optimizing $G$ in (1) directly.

### 3.2 A $\mu$ Formulation of the FDI Problem

In this section, a $\mu$ formulation of the FDI design problem will shortly be given in the uncertain case. When the system given by (2) includes uncertainties, it is quite simple to formulate the design problem as a $\mu$ optimization problem. This will shortly be done in the following.

Let the performance specification for the filter design be specifications on the transfer functions from $w_d$ to the estimation error signal $z$ and from the fault signal $w_f$ to the estimation error signal $z$. By closing the loop from $z$ to $w_d$ and $w_f$ by a fictive perturbation block $\Delta'_f$, where $\|\Delta'_f\| \leq 1$, it is possible to handle the performance specifications in the same way as uncertain blocks in the system, see [11]. This is shown in Figure 2.

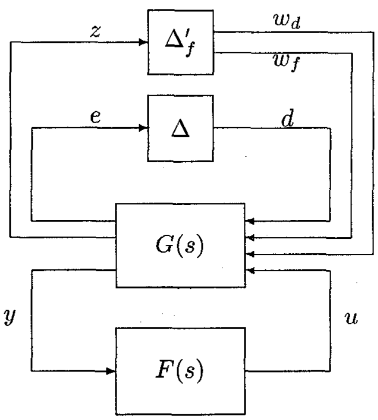


Figure 2: Generalized setup for the design of a robust fault detection filter

The design of the filter $F(s)$ can now be done using the standard $\mu$ optimization method described in e.g. [11].

It should be pointed out here, that the $\mu$ optimization method will not in general give an "optimal" let alone a closed form solution. However, in comparison to *any initial* filter, the $\mu$ method will by using scaling matrices take care of the structure in the perturbation matrix and hence *always improve* performance. The quantitative improvement will depend strongly on the application and, of course, on the initial filter (the $\mu$ toolboxes' built-in initial filter is an $\mathcal{H}_\infty$ filter, which can often be improved upon as argued above).

It should be pointed out here that there is a minor difference between $\mu$ optimization for feedback controllers and $\mu$ optimization for filters. In the feedback case, the controller will be designed such that it satisfy the design conditions as well as possible and at the same time reduce the influence of uncertainties on the closed loop system. In the filter case, it is not possible to reduce the influence of uncertainties on the output, because there is no feedback to instrument this. Instead, a $\mu$ optimization of a FDI filter will minimize the influence from the uncertainties on the fault estimate and optimize the filter with respect to the other design conditions.

# 4 An Observer based Multi Objective Approach

A multi objective approach to FDI design has been described in [1] and in [7]. The approach is based on a FDI observer combined with a residual weighting matrix. Four different design indices are setup for the design of the observer gain and the residual weighting factor. It turns out that the design problem is a multi objective design problem. This design problem is solved by using eigenstructure assignment together with a genetic algorithm. The final design method turns out to be an iterative algorithm.

We will in the following apply the same multi objective design problem as in [1]. However, we will not apply an observer based approach to the multi objective design problem as in [1]. Instead, a formulation where a general FDI filter as in Section 2 and 3 will be applied. The connection between the observer based approach in [1] and a general filter approach has been considered in [5].

Let us consider a slight modification of the FDI configuration given in Figure 1 by adding an input noise $\zeta$ signal and a sensor noise signal $\eta$ to the system. The FDI configuration is described by the following transfer functions:

$$y = \eta + G_{yd}d + G_{yf}f + G_{y\zeta}\zeta$$

The four performance indices setup in [1] are as follows:

- The smallest gain from the fault signal $f$ to the estimated fault signal $\hat{f}$ should be maximized.
- The maximal gain from the disturbance signal $d$ to the estimated fault signal $\hat{f}$ must be minimized.
- The effect from the measurement noise $\eta$ on the estimated fault signal $\hat{f}$ must be minimized.
- An almost perfect estimation of the fault signal $f$ is wanted to be obtained in steady state.

Let us consider the first performance index, i.e. maximize the smallest gain from the fault signal $f$ to the estimated fault signal $\hat{f}$. Instead of considering the estimated signal direct, we can use the estimation error as shown in Section 2. The advantages by using the estimation error, $z = f - \hat{f}$, is that the optimization problem turn out to be a minimization of the maximal gain of the transfer function from the fault signal $f$ to the fault estimation error $z$ given by:

$$z = f - \hat{f} = (I - F(s)G_{yf}(s))f \tag{4}$$

which is much easier to handle in the design of the FDI filter.

The first design index $J_1$ is the given by:

$$J_1(F) = \sup_{\omega\in[\omega_1,\omega_2]} \sigma_{max}((I - FG_{yf})(j\omega)) \tag{5}$$

where $\omega \in [\omega_1, \omega_2]$ is the frequency range where the index need to be optimized in.

The second performance index dealing with rejection disturbance from the estimated fault signal, can directly be applied in this general setup. The performance index is given by:

$$J_2(F) = \sup_{\omega\in[\omega_1,\omega_2]} \sigma_{max}(FG_{yd}(j\omega)) \tag{6}$$

The third performance index is related to the measurement noise rejection. Again, the index from the observer based approach in [1] can also in this case be applied directly in this setup. The performance index becomes:

$$J_3(F) = \sup_{\omega\in[\omega_1,\omega_2]} \sigma_{max}(F(j\omega)) \tag{7}$$

The last performance index dealing with the steady state performance of the FDI filter from [1] was formulated as an indirect design constraint on the observer gain. The index applied in [1] is a static index which must be minimized. The result is that the observer gain will increase with the result that the effect from the disturbance is reduced in steady state. Instead of formulating the performance index as a static condition, the condition can be included in the first three performance indices by adding a proper weight function to reflect the steady state conditions. First, let us include weight functions in the three performance indices in (5), (6) and (7).

For obtaining a good design of a FDI filter which will satisfied the design conditions reflected in the three performance indices in (5), (6) and (7), weight functions need to be included in the indices. The weight functions need to reflect the frequency range where the different objectives are important. The weighted performance indices are given by:

$$\begin{array}{rcl} J_1(F) & = & \sup_{\omega\in[0,\infty]} \sigma_{max}(W_1(I - FG_{yf})(j\omega)) \\ J_2(F) & = & \sup_{\omega\in[0,\infty]} \sigma_{max}(W_2FG_{yd}(j\omega)) \\ J_3(F) & = & \sup_{\omega\in[0,\infty]} \sigma_{max}(W_3F(j\omega)) \end{array} \tag{8}$$

The weight function $W_1(s)$ must reflect the frequency range wherein the fault signal appears or the frequency range wherein the fault signal is going to be estimated. $W_1(s)$ will in general be a low pass weight or a band pass weight. The weight function $W_2(s)$ in the second index $J_2(F)$ must reflect the frequency dependent effect of the disturbance $d$ at the estimated fault signal. This weight function will normally be a high pass weight. The last weight function should reflect the frequency dependency from the measurement noise on the estimated fault signal.

The last performance index from [1] must also be reflected in the selection of weight functions. Instead of using a static condition as done in [1] in connection with an observer based approach, a PI observer could have been used instead. A PI observer based FDI approach has been considered in [6]. In this general setup, another weight function must be added to the transfer function for the estimated fault error. For obtaining a very small steady state error, the weight function $W_4$ need to have a very high gain at low frequencies. The result of such a weight function is that the the FDI filter will include (almost) an integral term. The weighted estimation fault error is then given by:

$$\begin{aligned} z &= W_4(s)e \\ &= W_4(s)((I - F(s)G_{yf}(s))f - F(s)G_{yd}d - F(s)\eta) \end{aligned} \tag{9}$$

(note that the noise at the input $\zeta$ has been removed in (9)).

Depending on the optimization method we want to use, the FDI design problem is either a standard design problem or a multi objective design problem. In the case where we want to optimize the the $\mathcal{H}_\infty$ norm of the transfer function from the external input signals to the estimated fault error signal, we will just get a standard design problem. The problem is given by:

$$\min_F \left\| W_4 \begin{pmatrix} W_1(I - FG_{yf}) \\ W_2FG_{yd} \\ W_3F \end{pmatrix} (j\omega) \right\|_\infty \tag{10}$$

The problem in (10) can be handled by using a standard $\mathcal{H}_\infty$ method directly and there will not be any conservatism in the design. However, compared with the design method derived in [1], the FDI filter will have a higher order when a standard method $\mathcal{H}_\infty$ method is applied directly. The order of the filter will be $n + n_{w_1} + n_{w_2} + n_{w_3} + n_{w_4}$ where $n$ is the order of the system and $n_{w_i}$ is the order of weight function $W_i$. The order of the observer derived in [1] is $n$. On the other hand, applying an $\mathcal{H}_\infty$ method is a one step method without any iterations. It should be mentioned here that in connection with an $\mathcal{H}_\infty$ optimization of (10) it is also possible to put constraint on the controller order and also constraint on where the poles must be placed. In this case, an LMI approach should be applied.

Other design methods as $\mathcal{H}_2$ or $\mathcal{L}_1$ can also be applied for the the optimization of the setup given by (9) without obtaining a multi objective design problem.

On the other hand, if we want to combine optimization of different norms of the involved transfer functions, a multi objective design problem will appear.

## 5 Discussion

The two multi objective methods considered in Section 3 are two methods derived from design methods for robust feedback controllers. The first method gives a separation of the design FDI filters into a number of FDI filters. The next design method is based on the $\mu$ approach. It turns out that when the system includes uncertainties, the FDI design problem can be formulated as a $\mu$ optimization problem.

The second part of this paper deals with a multi objective FDI design problem from [1]. An observer based FDI approach has been applied in [1], whereas we have used a general FDI setup in this paper. When the problem is considered by using an observer based FDI approach, the design problem ends up as a multi objective design problem, whereas this is not necessarily the case when a general setup is applied. When we apply the general setup, the design problem might be a multi objective design problem if we want to mix different norms in the optimization of the FDI filter.

This design problem shows clearly that the selection of setup in connection with the following FDI filter design is quite important. In one formulation we get a complicated multi objective design problem whereas we just get a standard optimal design problem if we use another.

This shows also one of the drawbacks by using the observer based setup which has been quite popular in the FDI literature. The observer based FDI approach is very good when we only want to design a FDI filter under just a few design conditions. Further, a good knowledge about the system makes it possible to use design methods as e.g. eigenstructure assignment which will very often give a better result than using an optimal method as e.g. $\mathcal{H}_\infty$ optimization. Moreover, the filter order will not increase when an observer based approach is applied. However, when the observer based approach is applied in connection with design problem with more than two or three design conditions, the design of the observer gain gets very complicated.

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