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Fault Detection for Nonlinear Systems - A Standard Problem Approach

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Abstract

The paper describes a general method for designing (nonlinear) fault detection and isolation (FDI) systems for nonlinear processes. For a rich class of nonlinear systems, a nonlinear FDI system can be designed using convex optimization procedures. The proposed method is a natural extension of methods based on the extended Kalman filter.

1 Introduction

A fault detection and isolation (FDI) system for a dynamical process, fundamentally has to trade off the risk of false alarms to the risk of undetected faults. Occurence of false alarms is largely dictated by the quality of the model of which the design of the FDI system relies, see e.g. [DG96, FD94, Pat94] and [PC96].

Consequently, false alarm will often occur for processes that are time-varying, processes that are poorly modeled due to lack of data in the modeling process, processes that have a distributed parameter nature, which is poorly captured by finite dimensional linear models.

Another class of processes where sensitive FDI designs might lead to frequent false alarms, are those processes that are subject to substantial *unknown* nonlinear dynamics. However, in the current technology, even for a process with a *known* nonlinearity, most FDI design methods lead to a situation with large probabilities of false alarms, simply due to the fact that they rely on linear methods and, hence, erroneously tend to detect the nonlinear effects as faults. Nonlinear FDI detectors has until recently only been considered in few papers in spite of the tremendous problems caused by nonlinear phenomena. Nonlinearities in connection with FDI has shortly been discussed in [Fra90] and in [PC96]. Lately, however, there has been increased interest in this issue, see e.g. [BF97, SRPF97, SPD97, PVC97, BAFK97, ESPK97].

In the survey paper by Willsky, [Wil76], it is pointed out that fault detection consists of 3 different tasks - detection/alarm, isolation and estimation. The first task consists of making a decision either that something has gone wrong/a fault has appeared or that everything is fine. The isolation task is to determine the source of the fault. The extent of the fault is estimated in the last task. An estimation of the fault will obviously both give a detection and an isolation of the faults.

In this paper we will focus on direct estimation of the faults in the nonlinear case, and we shall try to demonstrate that this problem at least in principle has a simple remedy which combines in an elegant way the ideas behind the extended Kalman filter with modern optimization based

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design techniques.

The standing assumption will be that the process in consideration is described by a nonlinear model selected from a rich class of nonlinear dynamical systems subjected to general fault types, represented as exogenous signals.

For simplicity, we shall not include disturbances or model uncertainty, and fault models are included implicitly only. However, we would like to emphasize that these inclusions are straightforward extensions which can be handled by the very same optimization methods, all based on the so called standard problem paradigm of robust control. The application of these techniques has been documented independently in a recent series of papers, see e.g. [SGN97] and the references therein.

2 Problem Formulation

We shall consider a general class of nonlinear systems as depicted in Figure 1.

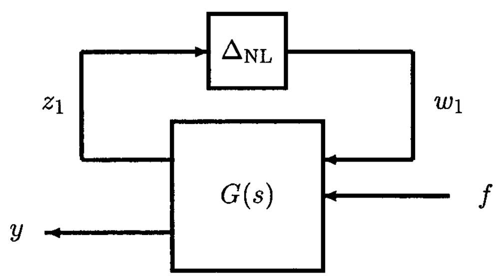


Figure 1: A class of nonlinear systems subjected to faults

In Figure 1, $G(s)$ is a linear system with two sets of (vector) inputs: w_1 and f, and Δ_{NL} represents a nonlinear - possibly dynamical - mapping. The exogenous signal f is the vector of faults to be detected and isolated by the FDI system. It is of significant importance, although not explicitly expressed in this paper, to formulate a dynamical model for the anticipated faults. In Figure 1 this dynamical model has been incorporated in $G(s)$.

The interconnection of $G(s)$ and the Δ_{NL} block represents a full nonlinear model of the dynamical process, for which we wish to design a FDI system. Usually, $G(s)$ should be thought of as the linearization of the process in some operating point.

In the extended Kalman filter, the underlying idea is to copy any nonlinear dynamics in the observer. We shall generalize this concept in terms of the fault detection architecture shown in Figure 2, although the suggested approach will not be observer based. A similar architecture was used for gain scheduling purposes in [Pac94] and for control of time varying systems in [Ran96].

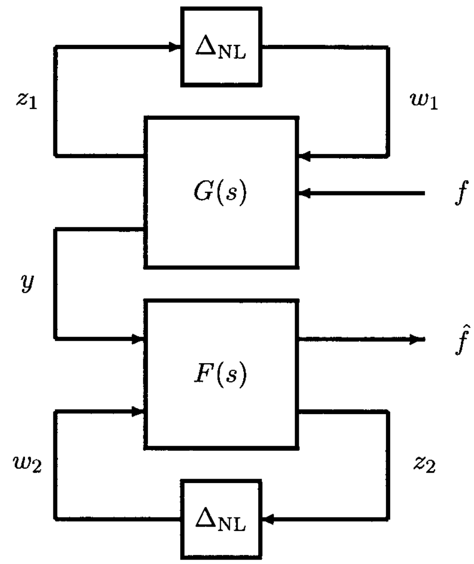


Figure 2: Fault detection for a nonlinear system

In Figure 2, the interconnection of $F(s)$ and the lower Δ_{NL} block represents the FDI system to be designed. $F(s)$ is a free linear parameter to be synthesized, whereas Δ_{NL} is simply a copy of the (known) nonlinear dynamics of the process.

The signal $\hat{f}$ is the estimate of f generated by the FDI system. By using the general setup shown in Figure 2, it is possible to handle both actuator faults, sensor faults and internal fault signals, see e.g. [SGN97].

The signal w_2 represents the response to the test signal z_2 generated by the linear part of the FDI system, $F(s)$. Hence, in analogy, in the extended Kalman filter, w_2 would be an estimate of w_1 based on the estimate z_2 of the internal signal z_1.

The nonlinearity Δ_{NL} in this setting will be assumed to be sector bounded in an $\mathcal{H}_\infty$ sense. To be more precise, we shall employ a stability argument below. The crucial assumption is then that by absorbing dynamical weights, $G(s)$ can be designed such that it is possible to infer stability of the nonlinear loop with some specific Δ_{NL} from robust stability w.r.t. the $\mathcal{H}_\infty$ unit ball. It is quite easy to describe a nonlinearity for which this is not possible *globally*, but in practice the assumption will usually hold, at least in some reasonable neighborhood of the linearization $G(s)$.

In the following we shall describe a synthesis procedure for the linear part $F(s)$ of the FDI system.

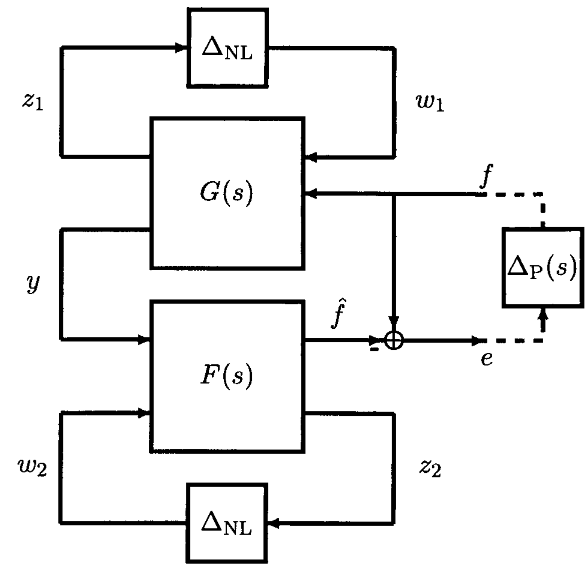


Figure 3: Introduction of a performance block

3 A Standard Problem Approach

In this section we shall first rewrite the isolation problem as a decoupling problem, and then, subsequently, transform this decoupling problem into an equivalent stability problem.

First, as in standard observer approaches, we consider the fault estimates rather than the estimates themselves:

$$e(t) = f(t) - \hat{f}(t)$$

Hence, the problem now has to been transformed to making $e(t)$ small for any (bounded) $f(t)$ or, equivalently, to bound the (nonlinear) operator gain from f to e, which we shall take to mean the $\mathcal{L}_2$-$\mathcal{L}_2$ gain. Without loss of generality (by absorbing scalings in the $G(s)$ part) we can assume that the required $\mathcal{L}_2$-$\mathcal{L}_2$ gain is unity.

The next step is to transform the $\mathcal{L}_2$-$\mathcal{L}_2$ gain requirement to a stability requirement. Sufficiency for this is readily obtained through a small gain argument, by employing the above assumption.

Indeed, in Figure 3 the $\mathcal{L}_2$-$\mathcal{L}_2$ gain from f to e is inferred to be bounded by one if robust stability holds for the system augmented with the Δ_{P} block inserted, for all $\Delta_{\mathrm{P}} \in \mathcal{H}_\infty$, $\|\Delta_{\mathrm{P}}\|_\infty < 1$.

The final step now is to reformulate the setup depicted in Figure 3 into a standard problem formulation (see e.g. [ZDG96] for a description of the standard problem). The result is shown in Figure 4.

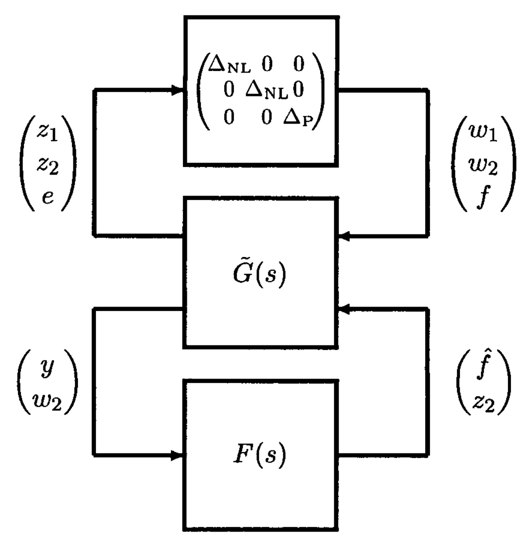


Figure 4: Standard problem formulation

In Figure 4, stability subject to any *linear* operator valued entries of Δ_{NL}, $\|\Delta_{\mathrm{NL}}\|_\infty < 1$, and of Δ_{P}, $\|\Delta_{\mathrm{P}}\|_\infty < 1$ implies the normalized nonlinear operator gain from fault vector f to fault estimation error $e = f - \hat{f}$ to be bounded by unity. This follows from a small gain argument along with the assumption of the nonlinearity.

The relationship between $\tilde{G}(s)$ in Figure 4 and $G(s)$ in Figure 3 is given by:

$$\begin{pmatrix} z_1 \\ z_2 \\ e \\ \hline y \\ w_2 \end{pmatrix} = \tilde{G}(s) \begin{pmatrix} w_1 \\ w_2 \\ f \\ \hline \hat{f} \\ z_2 \end{pmatrix}$$

$$= \left(\begin{array}{c|cc} \begin{pmatrix} I & 0 \end{pmatrix} G(s) \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \end{pmatrix} & 0 & 0 \\ 0 \quad 0 \quad 0 & 0 & I \\ 0 \quad 0 \quad I & -I & 0 \\ \hline \begin{pmatrix} 0 & I \end{pmatrix} G(s) \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \end{pmatrix} & 0 & 0 \\ 0 \quad I \quad 0 & 0 & 0 \end{array}\right) \begin{pmatrix} w_1 \\ w_2 \\ f \\ \hline \hat{f} \\ z_2 \end{pmatrix}$$

or

$$\begin{pmatrix} \tilde{z} \\ \tilde{y} \end{pmatrix} = \tilde{G}(s) \begin{pmatrix} \tilde{w} \\ \tilde{u} \end{pmatrix}$$

which means that

$$\begin{aligned} \tilde{G}(s) = & \left(\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & I & -I & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{array}\right) \\ & + \begin{pmatrix} I & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & I \\ 0 & 0 \end{pmatrix} G(s) \left(\begin{array}{ccc|cc} I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{array}\right) \end{aligned} \qquad (1)$$

or

$$\begin{aligned} \tilde{G}(s) &= \begin{pmatrix} \tilde{G}_{\tilde{z}\tilde{w}}(s) & \tilde{G}_{\tilde{z}\tilde{u}}(s) \\ \tilde{G}_{\tilde{y}\tilde{w}}(s) & \tilde{G}_{\tilde{y}\tilde{u}}(s) \end{pmatrix} \\ &= \left(\begin{array}{ccc|cc} G_{z_1 w_1}(s) & 0 & G_{z_1 f}(s) & 0 & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & I & -I & 0 \\ \hline G_{y w_1}(s) & 0 & G_{y f}(s) & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{array}\right) \end{aligned}$$

where

$$G(s) = \begin{pmatrix} G_{z_1 w_1}(s) & G_{z_1 f}(s) \\ G_{y w_1}(s) & G_{y f}(s) \end{pmatrix}$$

In particular we note, that if the state space representation of $G(s)$:

$$\begin{pmatrix} \dot{x} \\ \hline z_1 \\ \hline y \end{pmatrix} = \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right) \begin{pmatrix} x \\ \hline w_1 \\ f \end{pmatrix}$$

is of order n, so is the state space representation of $\tilde{G}(s)$:

$$\begin{pmatrix} \dot{x} \\ \hline z_1 \\ z_2 \\ e \\ \hline y \\ w_2 \end{pmatrix} = \left(\begin{array}{c|ccc|cc} A & B_1 & 0 & B_2 & 0 & 0 \\ \hline C_1 & D_{11} & 0 & D_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & I & -I & 0 \\ \hline C_2 & D_{21} & 0 & D_{22} & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{array}\right) \begin{pmatrix} x \\ \hline w_1 \\ w_2 \\ f \\ \hline \hat{f} \\ z_2 \end{pmatrix}$$

which implies that the crucial optimization in the sequel will involve a system of the same order as the original system data.

4 Optimization

The desired filter $F(s)$ can be found directly by μ-optimization w.r.t. the following structure of the singular value:

$$\begin{pmatrix} \Delta_{\mathrm{NL}} & 0 & 0 \\ 0 & \Delta_{\mathrm{NL}} & 0 \\ 0 & 0 & \Delta_{\mathrm{P}} \end{pmatrix} \qquad (2)$$

i.e. with one repeated full complex block and with one nonrepeated full complex block.

Our main result states that a nonlinear fault detection system can be computed by first finding a linear filter by solving a μ problem for a linear system structure.

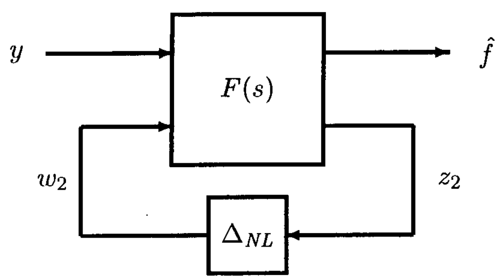
The resulting filter solves the FDI problem according to the following result:

Theorem 1 *Assume that the system* $\tilde{G}(s) = \begin{pmatrix} \tilde{G}_{\tilde{z}\tilde{w}}(s) & \tilde{G}_{\tilde{z}\tilde{u}}(s) \\ \tilde{G}_{\tilde{y}\tilde{w}}(s) & \tilde{G}_{\tilde{y}\tilde{u}}(s) \end{pmatrix}$ *and the linear filter* $F(s)$

satisfies

$$\left\| \tilde{G}_{\tilde{z}\tilde{w}}(\cdot) + \tilde{G}_{\tilde{z}\tilde{u}}(\cdot)F(\cdot)\tilde{G}_{\tilde{y}\tilde{w}}(\cdot) \right\|_{\mu} < \gamma \qquad (3)$$

w.r.t. the uncertainty structure (2), then the $\mathcal{L}_2$-$\mathcal{L}_2$ *operator gain from fault* f *to fault estimation error* $e = f - \hat{f}$ *when applying the FDI system:*



is bounded by γ *as well.*

A fault detection system based on Theorem 1 can be computed by the D-K algorithm. Although (3) is a model matching problem, the solution to this is obtained by applying standard D-K iterations to the system $\tilde{G}(s)$, since $\tilde{G}_{\tilde{y}\tilde{u}} \equiv 0$. Hence,

$$\begin{aligned}&\tilde{G}_{\tilde{z}\tilde{w}}(\cdot) + \tilde{G}_{\tilde{z}\tilde{u}}(\cdot)F(\cdot)\left(I - \tilde{G}_{\tilde{y}\tilde{u}}(\cdot)\right)^{-1}\tilde{G}_{\tilde{y}\tilde{w}}(\cdot)\\ &= \tilde{G}_{\tilde{z}\tilde{w}}(\cdot) + \tilde{G}_{\tilde{z}\tilde{u}}(\cdot)F(\cdot)\tilde{G}_{\tilde{y}\tilde{w}}(\cdot)\end{aligned}$$

Alternatively, using multiplier theory, a solution based on linear matrix inequalities can be given, which is omitted here due to space limitations.

5 Conclusion

In this paper an optimization filter synthesis procedure has been proposed for design of a fault detection and isolation system for a class of nonlinear systems.

The designed FDI system is nonlinear itself, where the nonlinearity in similarity to the extended Kalman filter is copied from the (known) nonlinearity in the process itself, although the suggested FDI system architecture is not explicitly observer based.

In spite of the fact that the resulting FDI design is nonlinear, the involved optimization only requires linear theory, and hence, the computational problems are no harder than those involved with linear optimization based filter or control theory. To compute the linear part of the FDI system, either μ optimization or optimization based on Linear Matrix Inequalities (LMI's) using multiplier techniques can be chosen. Furthermore, in the LMI formulation, convex optimization can be applied, which means that filters can be designed fast and efficiently.

The presented algorithm does not explicitly include exogenous disturbances or (linear or nonlinear) model uncertainty. However, handling these issues in the presented framework is straightforward, and has been described elsewhere.

Faults are represented as exogenous (vector) signals, where each component of the vector is isolated by the estimator, and the approach allows treatment of actuator faults, sensor faults, as well as internal faults.

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