Proceedings of the American Control Conference
San Diego, California • June 1999

Gain-scheduling Procedures for Unstable Controllers: A Behavioral Approach

Jakob Stoustrup[1] Palle Andersen[1] Tom S. Pedersen[1] Mads E. Hangstrup[1]

Abstract

A behavioral approach to the design of gain-scheduled controllers is presented. Plant models are considered to be black-box models obtained at finitely many operating points, and it is assumed that no analytical nor empirical relations are known between physical parameters and the plant models. In this situation, it is suggested to interpolate in intermediate operating points, simply by interpolating in signal space in the behavioral formalism. Two controller architectures are suggested that lead to global stability for systems that remain close to the interpolated values.

1 Introduction

Many industrial plants have dynamics and gains that depend strongly upon one or more parameters characterizing the operating point. Several examples of fundamentally different nature can be given. Airborne objects such as airplanes and missiles have maneuvering dynamics that depend strongly upon the speed/aero-dynamic pressure. Similarly, sea-going objects such as ships and submarines have maneuvering dynamics that depend upon the speed/hydro-dynamic pressure. Process control systems such as power plants, are examples of a fundamentally different nature. The dynamical properties of process control systems - such as time constants and time delays - are often inversely proportional to the production rate [ÅW89]. In power plants, time constants and gains varies with the load.

Several papers have emerged that deal with analysis of gain-scheduled controllers [SB92], [SA90], [SA92], [LR95], [Rug91]. See also [Sha88]. Shamma and Athans ([SA90], [SA92]) seek to give rigorous mathematical justifications of well-known heuristic rules-of-thumb such as *the scheduling variable should capture the plants non-linearities* and *the scheduling variable should vary slowly.*

In [Sha88] Shamma suggested a method for computing an upper bound for the allowed rate of change of the scheduling parameter. In [SB92] this method was adopted in the development of two candidate procedures for the computation of gain-scheduling controllers with guaranteed closed-loop stability. However, no numerical examples were included. In [Mor97] the practicability of the method in [Sha88] for solving a power plant boiler control problem was examined by numerical evaluation. For this problem it was found to be extremely conservative. The same observation was made in [Sha88], where the control object was an F-8 research aircraft. Remedies were suggested but these were found to be inadequate for the particular problem.

The simplest approach for gain-scheduling is to interpolate linearly between the parameters of the fixed controllers. This approach was suggested in [SB92] using LQG controllers. Simulation studies in [Mor97] however showed, that simple linear interpolation between the parameters may lead to unrealistic trajectories of the closed-loop eigenvalues.

In the approach suggested in [Pac94] and [AG95] the primary task is to obtain/construct a plant model that can be written as an LFT between some fixed plant interconnection matrix and the scheduling variables. For this linear parameter-varying (LPV) plant model, a single LPV controller for the complete range of operating conditions is computed in one shot by solving systems of coupled LMIs. A problem with this approach is, that it may be very difficult to obtain an appropriate candidate for the interconnection matrix mentioned above. Also, if the synthesis algorithm fails, no indication is given of where the bottlenecks are.

2 Problem Formulation

In the simplest case with only two different linear controllers gain-scheduling is often achieved by a

[1]Department of Control Engineering, Aalborg University, Fr. Bajersvej 7C, DK-9220 Aalborg East, Denmark. Email: {jakob/pa/tom/meh}@control.auc.dk . URL: www.control.auc.dk/~{jakob/pa/tom/meh} .

0-7803-4990-6/99 $10.00

scheme as depicted in Figure 1.

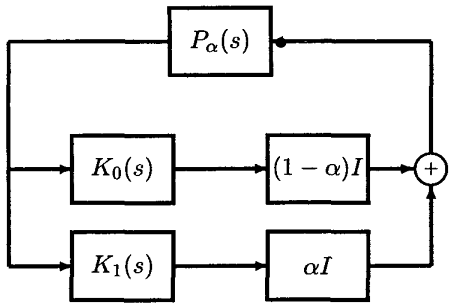

Figure 1: Control system with classical gain-scheduling

In the diagram, $P_\alpha(s)$ is the physical system parameterized by α, $K_0(s)$ and $K_1(s)$ are the multivariable controllers designed for $\alpha = 0$ and $\alpha = 1$, resp. (i.e. for $P_0(s)$ and $P_1(s)$, resp.), and αI is a gain-scheduling block. It is easily seen that the diagram implements $\alpha K_0(s) + (1-\alpha)K_1(s)$ which is a simple linear interpolation of the two controllers designed for the two extreme operating points.

This procedure works well in many applications, but it has some pitfalls as well, the most significant being that the procedure requires both controllers to be stable. This is obvious since the control signal from an unstable controller $K_1(s)$ will always diverge when α is small and vice versa.

Basically due to this limitation, the industrial use of unstable controllers has been limited. This is unfortunate, considering that

- for some plants, no stable controller will achieve optimality (in a mixed sensitivity sense)
- for some plants, no stable controller will robustly stabilize the system
- for some plants, no stable controller will stabilize even the nominal system

The requirement of the controllers to be open-loop stable is usually known as *strong stabilization*. Recently, it has been shown that the order of a strongly stabilizing $\mathcal{H}_\infty$ controller can become unbounded as poles and zeros approach [SS86]. Some bounds on performance for strongly stabilizing controllers can be found from [OMK91].

In this paper we will suggest a general architecture for gain-scheduling between compensators, which are not required to be stable, although stability is guaranteed throughout the gain-scheduling procedure, assuming that gain-scheduling is performed sufficiently dense in the operation region.

The main idea of the paper is to introduce gain-scheduling procedures based on models obtained under several operating conditions *without relying on any specific parameterization or realization.* This is motivated by applications where the models contain no natural parameter for gain-scheduling, for example if models are obtained in a number of operating points using black-box system identification techniques.

It is obvious that different state space realizations yield different gain-scheduling systems, if the intermediate models are simply interpolated between state-space parameters.

In such cases, see e.g. [Mor97], for instance interpolating between controllable canonical forms might fail even to stabilize the system at intermediate points, even if the controllers stabilize the identified models at the experimental operating points. This does of course not mean that there might not be some realization for which state space parameter interpolation achieves good results. However, it is not at all clear given two black-box models, which state space realizations would make best sense for either one, if they were to be interpolated.

Therefore, we shall take an approach which directly interpolates in the space of measured signals. This relates to the literature on behavioral systems theory, see e.g. [Wil91]. Indeed, let m linearizations of a system under m different operating conditions be modeled by m triples:

$$\Sigma_i = (T, W, \mathcal{B}_i)\ , \quad i = 0, \ldots, m-1$$

where $T \subset \mathcal{R}$ is the time-axis, W the signal space, and $\mathcal{B}_i$, $i = 0, \ldots, m-1$ subsets of W^T called the *behavior* of the system Σ_i, $i = 0, \ldots, m-1$. The behavior $\mathcal{B}_i$ consists of those trajectories $w : T \to W$ which satisfy the laws of the system Σ_i.

From a behavioral point of view, there is only one natural way to interpolate between several operating conditions, if no analytic relation is known between parameters describing the operating conditions and the individual behaviors, namely to consider linear interpolations between the behaviors themselves:

$$\Sigma_\alpha = (T, W, \mathcal{B}_\alpha)\ , \quad \mathcal{B}_\alpha = \sum_{i=0}^{m-1} \alpha_i \mathcal{B}_i\ , \quad \alpha_i \geq 0\ , \quad \sum_{i=0}^{m-1} \alpha_i = 1 \tag{1}$$

The convex combination of behaviors should be understood in a pointwise sense:

$$\sum_{i=0}^{m-1} \alpha_i \mathcal{B}_i = \left\{ \sum_{i=0}^{m-1} \alpha_i w_i(\cdot) \; : \; w_i \in \mathcal{B}_i \,, \quad i = 0, \ldots, m-1 \right\}$$

It is straightforward to show that for any imposed input/output structure on a set of variables in a behavioral description $w(\cdot) = \begin{pmatrix} u(\cdot) \\ y(\cdot) \end{pmatrix}$, the frequency domain description corresponding to (1) becomes:

$$P_\alpha = \sum_{i=0}^{m-1} \alpha_i P_i \tag{2}$$

where $P_i(\cdot), i = 0, \ldots, m-1$ is the transfer function associated with each of the m operating points.

Whereas the 'real' system structure always remain unknown based on the information assumed available, the gain-scheduled controllers should at least stabilize *some* intermediate system structure. In the sequel of the paper, we shall restrict our attention to systems of the form (2).

For completeness, it should be stated that however intuitive the representation (1) might seem, there are significant properties possessed by some real systems that are not readily captured by this representation. As an example could be mentioned a highly parameter dependent resonance frequency.

3 Main Results

The following results can easily be generalized to hold for an arbitrary number of systems and controllers, but for simplicity, we shall state them in the case of two systems only. In this case (2) specializes to:

$$P_\alpha(s) = (1 - \alpha) P_0(s) + \alpha P_1(s) \,, \quad \alpha \in (0, 1)$$

We shall introduce two different architectures. The first architecture is the most general, but it has a large complexity in the number of initial plant models. The second architecture is simpler and has a lower complexity in the number of initial plants.

3.1 A general architecture for gain-scheduling

Our first result shows that by introducing the two known plant models internally in the controller, it is possible to obtain internal stability for all interpolated values of the two original systems.

Theorem 1 *Assume that $P_i(s)$, $i = 0, 1$ are two (open loop) internally stable systems, and $K_i(s)$, $i = 0, 1$ are two (possibly unstable) controllers, such that $K_i(s)$ stabilize $P_i(s)$, $i = 0, 1$, and the closed loop systems are well-posed. Consider the following class of systems*

$$P_\alpha(s) = (1 - \alpha) P_0(s) + \alpha P_1(s) \,, \quad \alpha \in (0, 1)$$

and define

$$M_\alpha(s) = \begin{pmatrix} 0 & (1-\alpha)I & \alpha I \\ I & \left(1 - (1-\alpha)^2\right) P_0(s) - \alpha(1-\alpha) P_1(s) & -\alpha(1-\alpha) P_0(s) - \alpha P_1(s) \\ I & -(1-\alpha)^2 P_0(s) - \alpha(1-\alpha) P_1(s) & -\alpha(1-\alpha) P_0(s) + \left(1 - \alpha^2\right) P_1(s) \end{pmatrix}$$

and

$$\bar{K}(s) = \begin{pmatrix} K_0(s) & 0 \\ 0 & K_1(s) \end{pmatrix}$$

Then the following controller

$$K_\alpha(s) = \mathcal{F}_\ell(M_\alpha(s), \bar{K}(s)) \tag{3}$$

is internally stabilizing for any $P_\alpha(s)$, $\alpha \in (0, 1)$. Moreover, $K_\alpha(s)$ can be written:

$$K_\alpha(s) = Q_\alpha(s) \left(I + P_\alpha(s) Q_\alpha(s) \right)^{-1} \tag{4}$$

with:

$$Q_\alpha(s) = (1 - \alpha) Q_0(s) + \alpha Q_1(s) \,,$$

$$Q_i(s) = K_i(s) \left(I - P_i(s) K_i(s) \right)^{-1} \,, \quad i = 0, 1$$

A possible implementation for $K_\alpha(s)$ is shown in Figure 2.

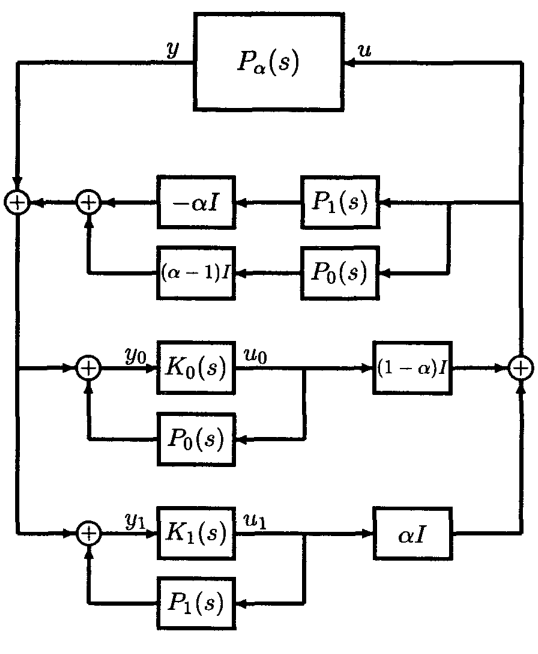

Figure 2: General gain-scheduling structure for unstable controllers

Proof. The equivalence between (3) and (4) follows from trivial algebra (Schur inversion etc.). Since $P_\alpha(s) = (1-\alpha)P_0(s) + \alpha P_1(s)$ is open loop stable for any value of α, the set of all stabilizing controllers is parametrized by (see [Kuc75, YJB79])

$$K = Q(s)\left(I + P_\alpha(s)Q(s)\right)^{-1}$$

where $Q \in \mathcal{RH}_\infty$, and the feedback loop is well-posed. Since (4) is already in this form, with $Q_\alpha \in \mathcal{RH}_\infty$ by stability and well-posedness of $Q_i(s)$, $i = 1, 2$, due to the assumptions on $K_i(s)$, $i = 1, 2$, stability and well-posedness is immediate.

□

3.2 An average controller structure

In the above, we have assumed that controllers K_i, $i = 0, 1$ have been prespecified for the original models P_i, $i = 0, 1$. If this is not the case, in some cases, there might still be a simple way to compute a controller that works for all P_α, $0 < \alpha < 1$, even if P_0, P_1 or both are unstable.

One possible controller that works for a certain class of systems, is depicted in Figure 3, and is

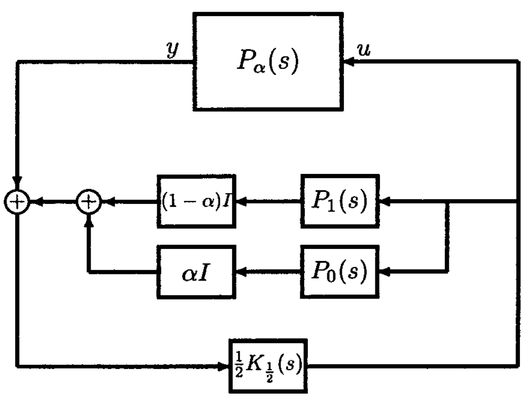

Figure 3: Gain-Scheduling structure based on controller for average plant

based on the simple observation that

$$P_\alpha(s) + P_{1-\alpha}(s) = 2P_{\frac{1}{2}} = P_0 + P_1$$

Hence, if $K_{\frac{1}{2}}$ is a controller that stabilizes $P_{\frac{1}{2}}$, then $\frac{1}{2}K_{\frac{1}{2}}$ stabilizes $P_\alpha(s) + P_{1-\alpha}(s)$ for all values of α. But this in turn implies that by adding $P_{1-\alpha}$ as a feed-forward term, the resulting controller stabilizes all $P_\alpha(s)$. More precisely, we have:

Theorem 2 *Assume that $K_{\frac{1}{2}}(s)$ is a controller that internally stabilizes the system $\frac{1}{2}(P_0(s) + P_1(s))$. Then the controller*

$$K_\alpha(s) = \mathcal{F}_\ell\left(M_\alpha(s), \frac{1}{2}K_{\frac{1}{2}}(s)\right)$$

where

$$M_\alpha(s) = \begin{pmatrix} 0 & I \\ I & \alpha P_0(s) + (1-\alpha)P_1(s) \end{pmatrix}$$

internally stabilizes P_α for $0 < \alpha < 1$.

In fact Theorem 2 could be extended beyond the open interval, if any unstable pole/zero cancellations at $\alpha = 0$ or $\alpha = 1$ would be admissible (otherwise the plant would not be stabilizable for these values at all).

Comparing Theorem 1 to Theorem 2, at first glance it seems likely that there would exist a choice of $K_{\frac{1}{2}}$ in Theorem 2 that would recover Theorem 1 as a special case. This, however, is not the case, since it is easy to verify that $K_{\frac{1}{2}}$ would have to depend on α. Hence, the two results are fundamentally different in nature.

4 A Simulation Example

The results given above establish conditions for stability of gain-scheduling controllers with the given architectures. If the gain-scheduling is based

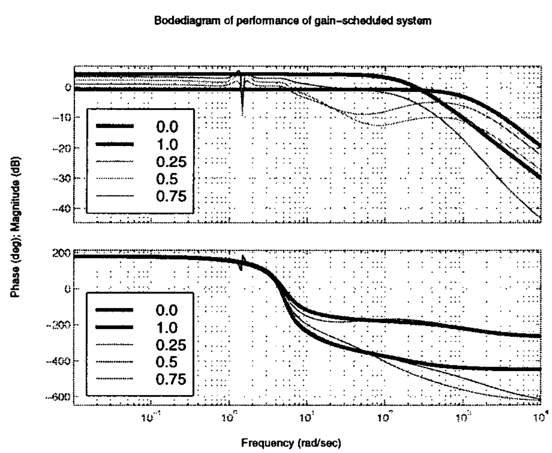

Figure 4: *Performance of gain-scheduled controller at the endpoints and at three intermediate operating points*

on systems which are close it is also expected that the resulting performance will be close to what has been obtained for the individual systems. This will be illustrated using two second order systems. The systems have a 2×2 structure with one disturbance input, one control input, one performance output and one measured output. Controllers minimizing the $\mathcal{H}_\infty$ norm of the transfer function from disturbance to performance output for the individual systems have been designed. The systems have been chosen as (random) stable systems that happen to have unstable $\mathcal{H}_\infty$ controllers. The results in Figure 4 show that the gain-scheduling controller has performance similar to the individual systems.

5 Conclusions

The problem of designing gain-scheduled controllers has been addressed for systems based on models for finitely many operating points, and with no inter-model behavior a priori given.

The authors believe this problem to be of significant industrial relevance, since gain-scheduling is often used for complex processes where modeling by first principles is unlikely to succeed or impractical.

The results given in this paper fulfills a first requirement: at least *some* intermediate class of systems to the known models are guaranteed to be stabilized.

A case study of the methods suggested in this paper is reported in [Han98], where they have been applied to a power plant control system.

References

[AG95] P. Apkarian and P. Gahinet. A convex characterization of gain scheduled $\mathcal{H}_\infty$ controllers. *IEEE Transactions on Automatic Control*, 40:853–864, 1995.

[ÅW89] K. J. Åström and B. Wittenmark. *Adaptive Control.* Addison-Wesley, 1989.

[Han98] M. E. Hangstrup. *Strategies for Industrial Multivariable Control - with Application to Power Plant Control.* Ph.D. thesis, Department of Control Engineering, Aalborg University, doc.no. D-1998-4283, 1998.

[Kuc75] V. Kucera. Stability of discrete linear feedback systems. In *Proceedings of the 6th IFAC World Congress*, Boston, MA, 1975. Paper 44.1.

[LR95] D.A. Lawrence and W.J. Rugh. Gain scheduling dynamic linear controllers for a nonlinear plant. *AUT*, 31:381–390, 1995.

[Mor97] J. H. Mortensen. *Kontrolstrategi til frigørelse af kraftværkers reguleringsevne - Hovedrapport.* Industrial Ph.D. thesis (main report) (in danish), Department of Control Engineering, Aalborg University, doc.no. D-97-4199, 1997.

[OMK91] Y. Ohta, H. Maeda, and S. Kodama. Unit interpolation in $\mathcal{H}_\infty$: bounds of norm and degree of interpolants. *Systems & Control Letters*, 17:251–256, 1991.

[Pac94] A. Packard. Gain scheduling via linear fractional transformation. *Systems & Control Letters*, 22:79–92, 1994.

[Rug91] W.J. Rugh. Analytical framework for gain scheduling. *IEEE Control Systems*, 11:79–84, 1991.

[SA90] J.S. Shamma and M. Athans. Analysis of gain scheduled control for nonlinear plants. *IEEE Transactions on Automatic Control*, 35:898–907, 1990.

[SA92] J.S. Shamma and M. Athans. Gain scheduling: Potential hazards and possible remedies. *IEEE Control Systems*, 12:101–107, 1992.

[SB92] S. M. Shahruz and S. Behtash. Design of controllers for linear parameter-varying systems by the gain scheduling technique. *Journal of Mathematical Analysis and Applications*, 168:195–217, 1992.

[Sha88] J. S. Shamma. *Analysis and design of gain-scheduled control systems (Ph.D. thesis).* Department of Mechanical Engineering, MIT, 1988.

[SS86] M.C. Smith and K.P. Sondergeld. On the order of stable compensators. *Automatica*, 22:127–129, 1986.

[Wil91] J.C. Willems. Paradigms and puzzles in the theory of dynamical systems. *IEEE Transactions on Automatic Control*, 36(3):259–294, march 1991.

[YJB79] D.C. Youla, H.A. Jabr, and J.J. Bongiorno. Modern Wiener-Hopf design of optimal controllers, part II. *IEEE Transactions on Automatic Control*, 21:319–338, 1979.