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An Architecture for Implementation of Multivariable Controllers

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Abstract

An architecture for implementation of multivariable controllers is presented in this paper. The architecture is based on the Youla-Jabr-Bongiorno-Kucera parameterization of all stabilizing controllers. By using this architecture for implementation of multivariable controllers, it is shown how it is possible to change from one multivariable controller to another multivariable controller online in a smooth way with guarantee for closed loop stability. This includes also the case where the controllers are unstable. Gain scheduled controllers can be implemented in this architecture.

The general architecture for smooth online changes of multivariable controllers can also handle the start up and close down of multivariable systems. Furthermore, the start up of unstable multivariable controllers can also be handled in this architecture. Finally, implementation of (unstable) controllers as a stable Q parameter in a Q-parameterized controller can also be achieved.

1 Introduction

Even for stable systems, most (post-) modern control techniques based on various optimization techniques, such as $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{L}_1/\ell_1$ norm based or μ optimization based designs tend to give unstable controllers.

The industrial use of unstable controllers has been limited. This is unfortunate, considering that for some plants, no stable controller will achieve optimality (in a mixed sensitivity sense). Moreover, for some plants, no stable controller will robustly stabilize the system. Finally, for some unstable plants - violating the interlacing property - no stable controller will stabilize even the nominal system.

The requirement of the controllers to be open-loop stable is usually known as *strong stabilization.* Recently, it has been shown that the order of a strongly stabilizing $\mathcal{H}_\infty$ controller can become unbounded as the plants approach having pole/zero cancellations [4]. Some bounds on performance for strongly stabilizing controllers can be found from [3].

Hence, there are good reasons to consider unstable multivariable controllers for several industrial applications. There are, however, some quite severe practical problems in implementing unstable controllers that are frequently overlooked or at least underemphasized in the literature on control theory.

One problem is that simply starting up an unstable controller is difficult. Many industrial plants require soft start-up procedures, where the control signal is varied gradually from off to full power. This does not work in the case of an unstable controller, since the controller need the (full) plant to stabilize itself.

Another problem is that most complex industrial applications involve some kind of gain-scheduling procedures. Gain-scheduling is usually implemented as a bank of parallel controllers where most controllers are inactive. But if unstable controllers are left inactive, their internal states will tend to infinity.

In this paper, we suggest a general framework for handling unstable controllers which can be applied both to start-up situations and to gain-scheduling implementations.

2 Controller Implementation

The following results are derived by using coprime factorication of systems and controllers. However, it is straightforward to set up state space descriptions for the derived results. A state space description of the coprime factorization for general controllers can be found in the book of Tay et al., [6].

Let us consider the following MIMO system given by:

$$G_{yu}(s) = NM^{-1} = \tilde{M}^{-1}\tilde{N}, \quad N, M, \tilde{M}, \tilde{N} \in RH_\infty \tag{1}$$

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Further, let a number of stabilizing controllers for the system G_{yu} be given by:

$$K_i(s) = U_i V_i^{-1} = \tilde{V}_i^{-1}\tilde{U}_i, \quad U_i, V_i, \tilde{U}_i, \tilde{V}_i \in RH_\infty \quad (2)$$

for $i = 0, \cdots, p$. Note that the coprime factorizations can be chosen to satisfy the double Bezout equation given by:

$$\begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} \begin{bmatrix} M & U_i \\ N & V_i \end{bmatrix} = \begin{bmatrix} M & U_i \\ N & V_i \end{bmatrix} \begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix},$$

for $i = 0, \cdots, p$.

Based on K_0, all stabilizing controllers for the system $G_{yu}(s)$ can now be described by [7]:

$$\begin{aligned} K_0(Q) &= (U_0 + MQ)(V_0 + NQ)^{-1}, \; Q \in RH_\infty \\ &= K_0 + \tilde{V}_0^{-1} Q \left(I + V_0^{-1} N Q\right)^{-1} V_0^{-1} \end{aligned} \quad (3)$$

Based on this Youla-Jabr-Bongiorno-Kucera (YJBK) parameterization of all stabilizing controllers given in (3) based on K_0, we have the following result, [1].

Theorem 1 *Let the system be given by (1) and let a number of stabilizing controllers for the system be given by (2). Then K_i, $i = 1, \cdots, p$ can be implemented as $K_0(Q_i)$ where the stable Q_i parameter is given by:*

$$Q_i = \tilde{U}_i V_0 - \tilde{V}_i U_0, \; i = 1, \cdots, p$$

or

$$Q_i = \tilde{V}_i (K_i - K_0) V_0, \; i = 1, \cdots, p$$

Proof. Follows directly by simple calculations. □

The result show how it is possible to implement a controller as a stable Q parameter based on another stabilizing controller. The result also show that it is possible to change controller online without any jumps, just by scaling the Q parameter from zero to full value in a continuous way. The closed loop system is guaranteed to be stable for all values of Q_i. This is very useful in connection with implementation of unstable controllers.

Moreover, the above result can also be applied in connection with implementation of unstable controllers for a stable system, where no other stabilizing controller are implemented. We have the following result.

Lemma 2 *Let $K_u = U_u V_u^{-1} = \tilde{V}_u^{-1}\tilde{U}_u$, $U_i, V_i, \tilde{U}_i, \tilde{V}_i \in RH_\infty$ be an unstable controller for a stable system $G_{yu}(s) = NM^{-1} = \tilde{M}^{-1}\tilde{N}$, $N, M, \tilde{M}, \tilde{N} \in RH_\infty$. The unstable controller can then be implemented as*

$$K_u = K(Q_u) = MQ_u \left(I + \tilde{M} N Q_u\right)^{-1} \tilde{M}$$

where

$$Q_u = \tilde{U}_u V = \tilde{V}_u K_u V = \tilde{U}_u \tilde{M}^{-1}$$

where V and $\tilde{V}$ satisfies the Bezout equations:

$$\tilde{V} M = I, \quad \tilde{M} V = I$$

Proof. The proof is omitted. □

It is quite easy to show that the implementation of an unstable controller given in [5] is equivalent with the above implementation based on the YJBK parameterization. Let the controller from Lemma 2 be given by:

$$K_0(Q) = MQ_u(I + \tilde{M}NQ_u)^{-1}\tilde{M}$$

with

$$Q_u = \alpha \tilde{U}_u V, \quad \alpha \in [0, \; 1]$$

which gives

$$\begin{aligned} K_0(Q, \alpha) &= \alpha M \tilde{U}_u (I + \alpha N \tilde{U}_u)^{-1} \\ &= K_u, \text{ for } \alpha = 1 \end{aligned}$$

The controller given in [5] is given by:

$$\begin{aligned} K(\alpha) &= \alpha \left(I + \alpha K_u G_{yu} (I - K_u G_{yu})^{-1}\right)^{-1} \\ &\quad \times (I - K_u G_{yu})^{-1} K_u \end{aligned}$$

which can be rewritten into

$$\begin{aligned} K(\alpha) &= \alpha (I - K_u G_{yu})^{-1} \\ &\quad \times \left(I + \alpha K_u G_{yu} (I - K_u G_{yu})^{-1}\right)^{-1} K_u \\ &= \alpha M \tilde{V}_u \left(I + \alpha K_u N \tilde{V}_u\right)^{-1} K_u \\ &= \alpha M \tilde{V}_u \left(I + \alpha \tilde{V}_u^{-1} \tilde{U}_u N \tilde{V}_u\right)^{-1} \tilde{V}_u^{-1} \tilde{U}_u \\ &= \alpha M \left(I + \alpha \tilde{U}_u N\right)^{-1} \tilde{U}_u \\ &= \alpha M \tilde{U}_u (I + \alpha N \tilde{U}_u)^{-1} \end{aligned}$$

which shows that the two implementations are identical.

The result in Theorem 1 gives an implementation of a multivariable controller as a specific stable Q parameter in a parameterization of all stabilizing controllers. Theorem 1 gives one way to change the applied controller from K_0 to K_i online in closed loop and also in a way such that the closed loop system is stable for all applied controllers. Further, we do not necessary need to be limited to the use of two controllers given by K_0 and K_i. It is not only possible to change the controller from K_0 to one of the p controllers given by K_i, it is also possible to change the controller K_i to K_j, $i, j = 1, \cdots, p$, $i \neq j$. In the case where we want to change the applied controller between all p (or a subset) stabilizing controllers, we get the following result.

Theorem 3 *Let the system $G_{yu}(s)$ be given by (1) and let p stabilizing controllers for the system be given by (2). Further, let the controllers be implemented as:*

$$\begin{aligned} K_i = K_0(Q) &= K_0 + \tilde{V}_0^{-1} Q_i (I + V_0^{-1} N Q_i)^{-1} V_0^{-1}, \\ & Q_i \in RH_\infty,\ i = 1, \cdots, p \end{aligned}$$

with Q_i given by

$$Q_i = \tilde{U}_i V_0 - \tilde{V}_i U_0,\ i = 1, \cdots, p$$

Let a linear combination of the Q_i parameters be given by

$$Q = \sum_{i=1}^{p} \alpha_i Q_i.$$

with $\sum_{i=1}^{p} \alpha_i = 1$. Then the resulting controller K is independent of K_0 and is given by

$$K(Q) = \left(\sum_{i=1}^{p} \alpha_i \tilde{V}_i \right)^{-1} \sum_{i=1}^{p} \alpha_i \tilde{U}_i$$

Remark 1 It is important to note that the final controller is independent of K_0. The reason is that it is assumed that the scaling parameters α_i satisfy $\sum_{i=1}^{p} \alpha_i = 1$. There is actually no real need in the method requiring the scaling parameters α_i to sum to 1. However, if they do not satisfy this condition, the final controller will also be a function of K_0. It should also be pointed out that the scaling parameters need not to be positive, negative values can be allowed without any closed loop stability problems.

Proof. The proof is omitted. □

Using the complete description of the controller $K(s)$ given in Theorem 3 as a feedback controller, it is interesting to give an explicit equation for the closed loop system. Such an explicit description of the closed loop system can be applied in connection with the tuning of the controller, i.e. the selection of the α vector, such that the closed loop system is optimized with respect to the operating point.

Let the complete open loop system be described by:

$$\begin{bmatrix} e \\ y \end{bmatrix} = G \begin{bmatrix} d \\ u \end{bmatrix}$$

where d is an external input vector, u is the control input vector, e is the external output signal to be controlled and y is the measurement vector. The transfer function G is given by

$$G = \begin{bmatrix} G_{ed} & G_{eu} \\ G_{yd} & G_{yu} \end{bmatrix} \tag{4}$$

with $G_{yu} = MN^{-1}$.

The closed loop system from d to e, $T_{ed}(s)$, is then given by

$$T_{ed}(s) = \mathcal{F}_l(G, K) = G_{ed} + G_{eu} K (I - G_{yu} K)^{-1} G_{yd} \tag{5}$$

We can now give an explicit description of the closed loop system T_{ed} when the controller $K(Q)$ given in Theorem 3 is applied.

Theorem 4 *Let the closed loop transfer function be given by (5). Further, let the stabilizing controller $K(Q)$ be given by*

$$K(Q) = \left(\sum_{i=1}^{p} \alpha_i \tilde{V}_i \right)^{-1} \sum_{i=1}^{p} \alpha_i \tilde{U}_i$$

with $\alpha_i \geq 0$, $\sum_{i=1}^{p} \alpha_i = 1$. Then the closed loop transfer function T_{ed} is given by:

$$T_{ed}(s) = G_{ed} + G_{eu} M \left(\sum_{i=1}^{p} \alpha_i \tilde{U}_i \right) G_{yd}$$

Proof. The proof is omitted. □

There is one important thing to note in connection with the factorization of system and controllers. This deals with the case when we want a state space description of the system and the applied controllers. It is not possible to apply the standard state space description in the case when observer based controllers are applied. If this is done, we will get a factorization of G which will depend on the applied controller. Instead the more general state space description of given in [6]. The only drawback with this method is that the order of some of the involved matrices will increase.

3 System Variation

Until now, it has been assumed that there was no descripiency between the model for the dynamic system to be controlled and the real system, which will of course not in general be the case. Variations or modeling errors in the dynamic system will shortly be considered in the following.

As in the controller case, it is possible to give a parameterization, in terms of a stable parameter, of all systems stabilized by a given controller. Let us consider the system $G_{yu,0}(s)$ given by (1) and a controller K_0 with a coprime factorization given by (2). Is is still assumed that the coprime factors satisfies the double Bezout equation. Then all systems $G_{yu}(S)$ stabilized by K_0 is given by, [2, 6]:

$$\begin{aligned} G_{yu}(S) &= (N_0 + V_0 S)(M_0 + U_0 S), \quad S \in \mathcal{RH}_\infty \\ &= G_{yu,0} + \tilde{M}_0^{-1} S (I + M_0^{-1} U_0 S)^{-1} M_0^{-1} \end{aligned} \tag{6}$$

where S is denoted the dual YJBK parameter.

Based on this parameterization of all systems stabilized by a given controller, we can give the dual result of Theorem 1. We then have the following result, [2]:

Theorem 5 *Let a stabilizing controller K_0 be given by (2) for a number of systems $G_{yu,i} = N_i M_i^{-1}$ given by (1). Then $G_{yu,i}$, $i = 1, \cdots, p$ can be implemented as $G_{yu,0}(S_i)$ where the stable S_i parameter is given by:*

$$S_i = \tilde{N}_i M_0 - \tilde{M}_i N_0, \ i = 1, \cdots, p$$

or

$$S_i = \tilde{M}_i (G_{yu,i} - G_{yu,0}) M_0, \ i = 1, \cdots, p$$

Proof. The proof of Theorem 5 follows directly the proof of Theorem 1 and is therefore omitted. □

As in the case with parameterization of all controllers, the closed loop transfer function will be an affine function of the dual YJBK parameter S. The connection between the dual YJBK parameter S and different system descriptions have been considered in [2]. Here, let us consider the general case, where the uncertain is described by an LFT of a nominal system and a block, Δ, that include the system variation,

$$G_{yu}(\Delta) = \mathcal{F}_u(G_{unc}, \Delta) \tag{7}$$

where

$$G_{unc} = \begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{pmatrix}$$

Note, that for $\Delta = 0$, $G_{yu}(\Delta) = G_{yu}$. This description of system variation as a function of the parameter Δ is very useful, especially in connection with the design of robust controllers for uncertain systems. In this case, Δ is in general unknown, apart from it being known to be stable, upper bounded by a scalar function and possibly the structure of Δ might be known. However, let us assume that the Δ parameter is known, which makes it possible to calculate S as function of Δ. The relation between S and Δ is given in the following theorem.

Theorem 6 *Let a stabilizing controller K for the system (1) be given. Further, let all systems stabilized by K be given by $G_{yu}(S)$ where S is the dual YJBK parameter. Moreover, let the system be described by $G_{yu}(\Delta)$ as function of the parameter Δ. It is further assumed that Δ is not destabilizing the closed loop system. Then $G_{yu}(S)$ and $G_{yu}(\Delta)$ is identical if and only if the stable dual YJBK parameter S is selected as:*

$$S(\Delta) = T_{3,\Delta} \Delta (I - T_{1,\Delta} \Delta)^{-1} T_{2,\Delta} \tag{8}$$

where $T_{i,\Delta} \in \mathcal{RH}_\infty$ is given by

$$\begin{aligned} T_{1,\Delta} &= G_{zw} + G_{zu} U \tilde{M} G_{yw} \\ T_{2,\Delta} &= G_{zu} M \\ T_{3,\Delta} &= \tilde{M} G_{yw} \end{aligned}$$

Proof. The proof is omitted. □

It is here important to note that the two transfer functions are identical if the transfer functions between control input u and measurement output y are considered. They will in general be different if other inputs/outputs are considered. Another important thing to note is that $S(\Delta)$ is stable as long as Δ does not make the closed loop unstable, i.e. $(I - T_1 \Delta)^{-1}$ is stable, which was required. Therefore S will always be stable.

Assume that G_{unc} has the following state space realization:

$$G_{unc} = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{array} \right]$$

and assume that a stabilizing observer based feedback controller is given by

$$K(s) = \left[\begin{array}{c|c} A + B_u F + H C_y + H D_{yu} F & -H \\ \hline F & 0 \end{array} \right]$$

where F is a stabilizing state feedback gain such that $A + B_u F$ is stable and H is a stabilizing observer gain such that $A + HC_y$ is stable. The state space realization of $T_{i,\Delta}$ is given by (note that $T_{4,\Delta} = 0$):

$$T_\Delta = \begin{pmatrix} T_{1,\Delta} & T_{2,\Delta} \\ T_{3,\Delta} & T_{4,\Delta} \end{pmatrix} = \left[\begin{array}{cc|cc} A + B_u F & -HC_y & -HD_{yw} & B_u \\ 0 & A + HC_2 & B_w + HD_{yw} & 0 \\ \hline C_z + D_{zu} F & C_z & D_{zw} & D_{zu} \\ 0 & C_y & D_{yw} & 0 \end{array} \right]$$

4 Controller Changes in Uncertain Systems

Based on the above section, it is now possible to consider the more realistic case of controller change of uncertain systems. Let the uncertain system be described by G_S given by:

$$G_S(s) = \begin{bmatrix} G_{ed} & G_{eu} & G_{eq} \\ G_{yd} & G_{yu} & G_{yq} \\ G_{rd} & G_{ru} & G_{rq} \end{bmatrix} \tag{9}$$

where $r \in \mathcal{R}^p$ is the input vector to S and $q \in \mathcal{R}^s$ is the output vector from S, i.e. $q = Sr$.

Closing the open loop from q to r by using the relation $q = Sr$ gives the following realization of $G_S(S)$:

$$G_S(S) = \begin{bmatrix} G_{ed}(S) & G_{eu}(S) \\ G_{yd}(S) & G_{yu}(S) \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} G_{ed}(S) &= G_{ed} + G_{eq} S (I + M^{-1} U S)^{-1} G_{rd} \\ G_{eu}(S) &= G_{eu} + G_{eq} S (I + M^{-1} U S)^{-1} M^{-1} \\ G_{yd}(S) &= G_{yd} + \tilde{M}^{-1} S (I + M^{-1} U S)^{-1} G_{rd} \\ G_{yu}(S) &= G_{yu} + \tilde{M}^{-1} S (I + M^{-1} U S)^{-1} M^{-1} \end{aligned}$$

Based on the equation for $G_S(S)$ given by (10), we are now able to give the following result.

Theorem 7 *Let the open loop transfer function for $G_S(S)$ be given by (10). Let an internal stabilizing feedback controller be given by $K(Q,s)$. Then the closed loop transfer function from the external input d to the external output e is given by*

$$\begin{aligned} T_{ed}(S,Q) = G_{ed} - G_{eu}MG_{rd} \\ +(G_{eu}(M+US)+G_{eq}S) \\ \times(I-QS)^{-1}((\tilde{U}+Q\tilde{M})G_{yd}+G_{rd}) \end{aligned}$$

Proof. See [2]. □

In [6], it is shown that the stability of the closed loop transfer function which involves both Q and S, requires that the nominal feedback system is stable and that Q is stabilized $(I-QS)^{-1}$.

As a direct consequence of Theorem 7, we get the closed loop transfer functions for $S=0$ (parameterization of all stabilizing controllers for a nominal system) and for $Q=0$ (parameterization of all systems stabilized by a given controller). The two transfer functions are given by:

$$\begin{aligned} T_{ed}(Q) &= G_{ed}+G_{eu}U\tilde{M}G_{yd}+G_{eu}MQ\tilde{M}G_{yd} \\ &= T_1+T_{2,Q}QT_{3,Q} \\ T_{ed}(S) &= G_{ed}+G_{eu}U\tilde{M}G_{yd} \\ &\quad +(G_{eu}U+G_{eq})S(\tilde{U}G_{yd}+G_{rd}) \\ &= T_1+T_{2,S}ST_{3,S} \end{aligned}$$

5 Conclusion

Aspects of using parameterizations in connection with implementation of multivariable controllers have been considered. It has been shown how it is possible to apply the YJBK-parameterization with advantage in a number of cases.

First of all, by using the YJBK-parameterization, it is possible to switch between controllers in a stable way. If the controller is changed directly, i.e. $K=\alpha K_0+(1-\alpha)K_1,\ \alpha\in[0,1]$, there is no guarantee that the controller is a stabilizing controller for $\alpha\neq 0,1$. This lack of closed-loop stability is removed by using a parameterization in connection with the controller implementation. Furthermore, it is also possible to optimize a controller given as a combination of a number of pre-designed controllers. This optimization can if desirable be done on-line.

Another important issue is implementation of unstable controllers. Again, by using the YJBK-parameterization, it has been shown how unstable controllers can be implemented by using only stable transfer functions. This is especially important in connection with starting up unstable controllers.

The transients in the response is another important issue. This issue has not been investigated in this paper in detail. However, from the closed loop transfer function, we can see that the controller parameter will be changed in a smooth way and might therefore avoid some of the transients in the response that will normally appear if the controller is changed directly.

The dual YJBK-parameterization has also shortly been considered, i.e. the parameterization of all systems stabilized by a given controller. Based on these two parameterizations, the closed loop system has been considered where both the YJBK and the dual YJBK parameterization was applied. In this case, the closed loop transfer function will not be an affine function of Q and S. This will make the optimization of the Q parameter much more complicated, and is an issue for further research.

References

[1] J.B. Moore, K. Glover, and A. Telford. All stabilizing controllers as frequency shaped state estimate feedback. *IEEE Transactions on Automatic Control*, 35(2):203–208, 1990.

[2] H.H. Niemann and J.B. Moore. On the Youla parameterization. *Submitted for publication*, 1998.

[3] Y. Ohta, H. Maeda, and S. Kodama. Unit interpolation in $\mathcal{H}_\infty$: bounds of norm and degree of interpolants. *Systems & Control Letters*, 17:251–256, 1991.

[4] M.C. Smith and K.P. Sondergeld. On the order of stable compensators. *Automatica*, 22:127–129, 1986.

[5] J. Stoustrup and H.H. Niemann. Starting up unstable multivariable controllers safely. In *Proceedings of the IEEE Conference on Decision and Control*, pages 1437–1438, San Diego, CA, USA, 1997.

[6] T.T. Tay, I.M.Y. Mareels, and J.B. Moore. *High performance control.* Birkhäuser, 1997.

[7] K. Zhou, J.C. Doyle, and K. Glover. *Robust and optimal control.* Prentice Hall, 1995.