Fault Detection and Isolation in Systems with Parametric Faults

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Abstract

The problem of fault detection and isolation of parametric faults is considered in this paper. A fault detection problem based on parametric faults are associated with internal parameter variations in the dynamical system. A fault detection and isolation method for parametric faults is formulated in a standard setup and a synthesis method for the fault detector is given. Further, fault detection problems with both parametric faults and faults described by external input signals are also shortly considered.

1 Introduction

In the control community, increasing attention has been paid to the fact that availabity and security are equivally important to most industries as optimality. If a control system fails for just a short period of time, all that is gained by optimality might be lost due to costs of inavailability and/or damages. Thus, a substantial amount of research has been dedicated recently to the area of fault detection and isolation (FDI).

A useful survey on early work on FDI can be found in [Wil76, Fra90] and in [PFC89]. Many of these techniques are observer based, such as [DG96, MM91]. These methods have since been refined and extended.

In the classical approach to FDI, faults are most frequently modeled as additive exogenous signals.

This perspective is highly relevant to FDI problems, but leaves unanswered the following problem: how are faults detected that are not associated with sensors and actuators, but rather with internal parameter variations? How are the situation detected early, when oil is leaking in a hydraulic system, or when the rotor in an induction motor is overheated? Such fault detection problems can not directly be described by using the standard FDI description by an additive description, [BN93, NS97, PFC89]. Instead, a parametric description of the system variation needs to be applied in connection.

This problem is the subject of this paper. The approach taken fits very well with the uncertainty descriptions given in the papers mentioned above. However, dynamic model uncertainty descriptions are not explicitly integrated in the models described below for reasons of clarity.

The rest of this paper is organized as follows. The problem formulation is given in Section 2. Section 3 includes the main results follows by Section 4, where systems including both parametric faults and additive faults are considered. A conclusion is given in Section 5.

2 Problem Formulation

The approach taken is to model a potentially faulty component as a nominal component in parallel with a (fictitious) error component. The optimization procedure suggested in the paper then tries to detect the outgoing signal from the error component. This works of course only well in cases where the component is reasonably well excited, but on the other hand, if the component is not excited at all, there is absolutely no way to detect whether it is faulty, in theory or practice!

In the following we shall describe the steps needed in order to model a faulty system in the form needed for modern optimization tools, embarking from a set of physical equations.

We shall consider a plant described by a model of the following form:

$$\begin{array}{rcccc} \dot{x} & = & A_\Delta x & + & Bd \\ y & = & C_2 x & + & Dd \end{array} \tag{1}$$

where A_Δ is a matrix that may deviate from a nominal value A_0, by a (possibly nonlinear) dependency of a fault. d is a vector valued signal that comprises all exogenous signals, such as disturbances, noise, and command signals.

Hence, in this setting we do not allow directly for faults manifesting themselves in the input and/or output matrices (B/C) matrices which might be relevant in practice, e.g. in connection with gain variations. However, it is quite easy to model such faults as well in the setup given by (1). The trick is to introduce an input filter, for instance of the form $\frac{1}{\tau s+1}$ with τ sufficiently small, and associate the fault with the fictitious state introduced in this way.

The next step in the modeling procedure is to approximate the possibly nonlinear parameter dependencies of A_Δ with polynomial (in full generality: multinomial) or rational ones. Here, the following considerations must be taken:

- rational approximations of a specified order are usually better than polynomial approximations of the same order
- polynomial approximations of a specified order give better numerical results than rational approximations of the same order in the algorithm given in this paper

In conclusion, at least for small or medium variations, polynomial approximations will give better results than rational ones, but either can be considered for any application. To obtain a polynomial approximation, the obvious approach is to compute a multivariate Taylor series. For rational approximation the number of methods are legio. (For example, the function $\sin(\delta), -1 < \delta < 1$ is approximated very well by the rational function $f_2(\delta) = \frac{\delta}{1+0.185\delta^2}$ but equally well by the polynomial function $f_1(\delta) = \delta - \frac{1}{6}\delta^3 + \frac{1}{120}\delta^5$.)

We are now faced with a model of the form (1) where A_Δ takes the form:

$$A_\Delta = A_0 + \sum_i f_i(\delta_1, \ldots, \delta_p) A_i \tag{2}$$

where each f_i are polynomial or rational functions of the parameters $\delta_1, \ldots, \delta_p$, satisfying $f_i(0, \ldots, 0) = 0$ (the non-faulty operation mode). Typically, each A_i will have only entries with values 0 and 1.

The third step in the problem setup is to rewrite the model (2) as a linear fractional tranformation. A general procedure to achieve this is described in [ZDG96, Section 10.2]. As a result we get a system of the form:

$$\begin{array}{rclcl} \dot{x} & = & Ax & + & B_1 w \\ f_p & = & C_f x & & \\ y & = & C_2 x & + & D_{21} w \end{array} \tag{3}$$

where

$$B_1 = \begin{pmatrix} B_f & B \end{pmatrix}, \quad D_{21} = \begin{pmatrix} 0 & D \end{pmatrix},$$
$$w = \begin{pmatrix} w_f \\ d \end{pmatrix}, \quad w_f = \Delta_{\text{par}} f_p$$

and

$$\Delta_{\text{par}} = \begin{pmatrix} \delta_1 I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \delta_p I_p \end{pmatrix}$$

where the I_i's are identity matrices. The dimension of each identity matrix depends on the order of the corresponding parameter δ_i in the polynomial or rational approximation. The matrix A will in general differ from A_0, but will be of the same dimension. Without loss of generality, the model (2) can be assumed to be normalized such that each parameter δ_i varies between -1 and 1.

This general representation of a system with parametric faults is depicted in Figure 1.

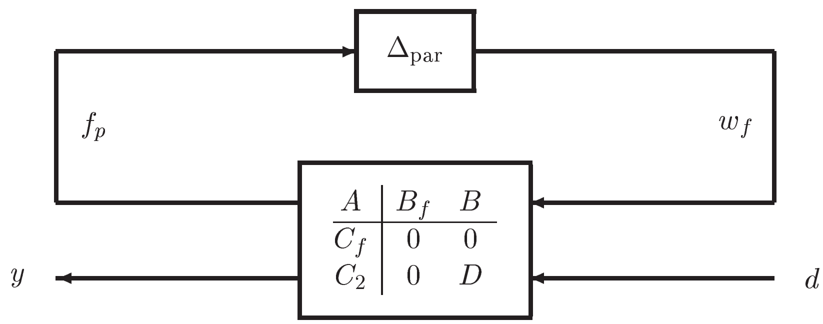

Figure 1: Formulation of a system with parametric faults as a linear fractioncal transformation in the fault parameters Δ_{par}

The next step in setting up the fault detection and isolation problem as a standard optimization problem is to introduce the isolation error e_p as:

$$e_p = f_p - \hat{f}_p$$

where $\hat{f}_p$ is the estimation of f_p to be generated.

The final model becomes:

$$\begin{array}{rclclcl} \dot{x} & = & Ax & + & B_1 w & + & B_2 u \\ z & = & C_1 x & & & + & D_{12} u \\ y & = & C_2 x & + & D_{21} w & & \end{array} \tag{4}$$

where

$$B_1 = \begin{pmatrix} B_f & B \end{pmatrix}, \quad w = \begin{pmatrix} w_f \\ d \end{pmatrix}, \quad B_2 = 0, \quad u = \hat{f}_p,$$
$$z = \begin{pmatrix} f_p \\ e_p \end{pmatrix}, \quad C_1 = \begin{pmatrix} C_f \\ C_f \end{pmatrix}, \quad D_{12} = \begin{pmatrix} 0 \\ -I \end{pmatrix},$$
$$D_{21} = \begin{pmatrix} 0 & D \end{pmatrix}$$

The null matrix B_2 and the notation $u = \hat{f}_p$ is introduced simply to be consistent with the so-called standard problem notation as e.g. in [ZDG96].

The signals and interconnection structure defined in this way is depicted in Figure 2. Note, that a fictitious performance block Δ_{perf} has been introduced. The significance of this block is the following. According to the small gain theorem, the $\mathcal{H}_\infty$ norm of the transfer function from d to e is bounded by γ **if and only if** the system in Figure 2 is stable for all Δ_{perf}, $\|\Delta_{\text{perf}}\|_\infty < \gamma$.

Hence, the problem of making the norm of the fault estimation error bounded by some quantity has been transformed to a stability problem. We shall give more details on this issue in the following section.

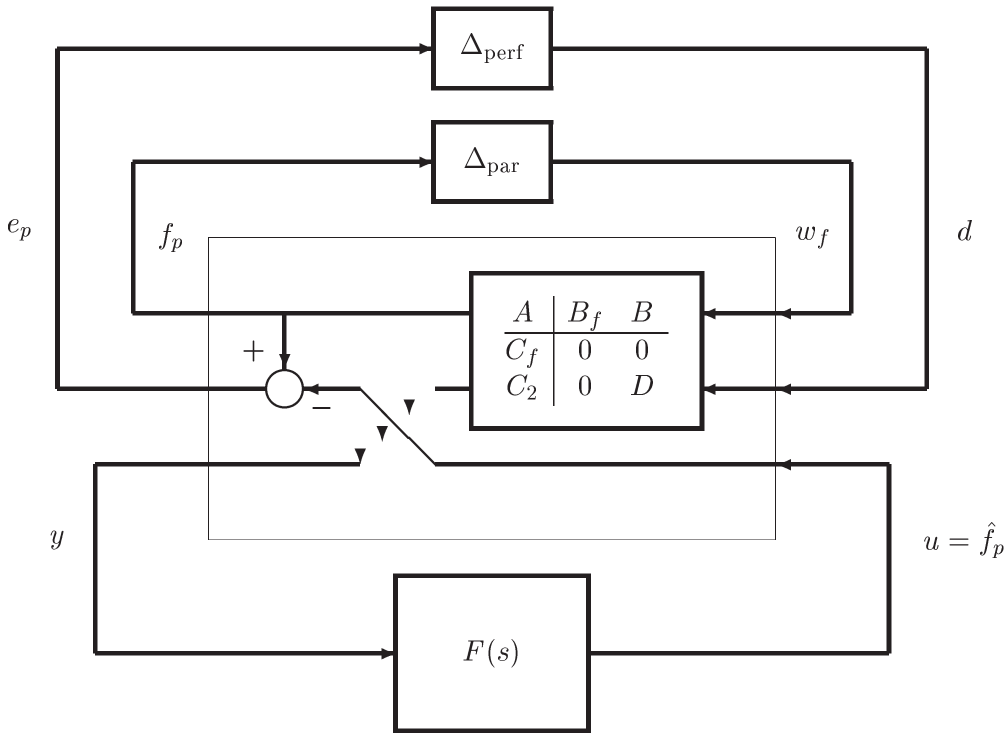

Figure 2: Interconnection structure with parametric faults Δ_{par} in linear fractional representation and a fictitious performance block Δ_{perf}

Finally, we introduce

$$\Delta = \begin{pmatrix} \Delta_{\text{par}} & 0 \\ 0 & \Delta_{\text{perf}} \end{pmatrix}$$

Extracting this block from the diagram in Figure 2, gives Figure 3 which shows the final standard problem formulation.

3 Main Results

The main result is:

Theorem 1 *Let $F(s)$ be a linear filter applied to the system (4) as $u = F(s)y$, and assume that $F(s)$ satisfies:*

$$\|\mathcal{F}_\ell(G_{zw}, F)\|_\mu < \gamma$$

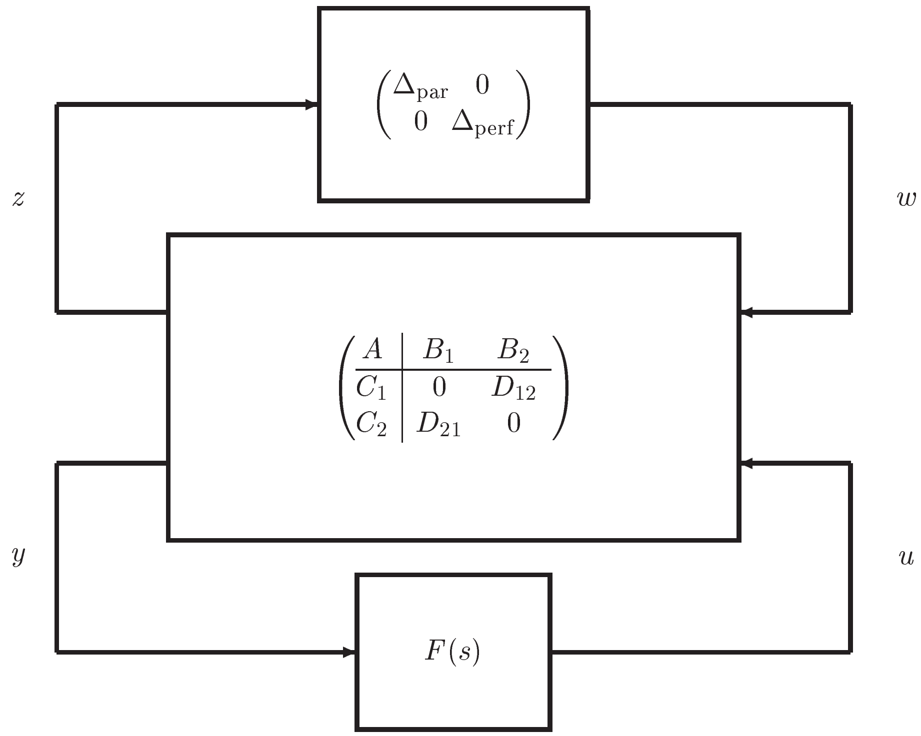

Figure 3: Standard problem formulation. The middle block is the same as the one indicated by a thin line in Figure 2

then the resulting isolation error is bounded by:

$$\|e_p\| < \gamma N$$

where N is the excitation level of the system, i.e. $\|w\| = N$.

In the following we shall present a synthesis procedure for $F(s)$.

A number of different more or less complicated synthesis methods can be applied on the above design problem given in Theorem 1. The main problem with the above design problem is that the perturbation block Δ consists of both real and complex perturbations. The standard μ synthesis method, [ZDG96], can not in general be applied without introducing conservatism in the design. The reason is that the standard μ synthesis method can only handle complex perturbations. A number of alternative synthesis methods for mixed perturbations has been considered in [FFDM96] in connection with design of a missile autopilot.

Indeed, it is possible to apply the standard μ synthesis, if an additional scaling matrix is introduced in the method. This extra scaling matrix takes into account the difference between the mixed and the complex μ. In the following, the complex μ and the modified μ synthesis methods are shortly described.

3.1 μ Synthesis

We may now formulate an optimal robust performance problem in terms of μ:

$$F(s) = \arg \min_{F(s) \in \mathbf{F}} \left\| \mu_{\Delta} \left(\mathcal{F}_l(\tilde{G}(s), F(s)) \right) \right\|_{\infty} \tag{5}$$

where $\mathbf{F}$ denotes the set of all nominally stabilizing controllers (there might not exist an admissible controller achieving the minimum, but we make this abuse of notation for convenience). $\tilde{G}$ is the system, see Figure 3. Unfortunately (5) is not tractable since μ cannot be directly computed. Rather the upper bound $\beta_{\min}$ is used to formulate the control problem:

$$F(s) = \arg\min_{F\in\mathbf{F}} \sup_{\omega} \inf_{D(\omega)\in\mathbf{D},G(\omega)\in\mathbf{G}} \inf_{\beta(\omega)\in\mathcal{R}_+} \{\beta(\omega)\,|\bar{\sigma}(\Sigma(\omega)) \le 1\} \tag{6}$$

where

$$\Sigma(\omega) = \left(\frac{D(\omega)\mathcal{F}_\ell(\tilde{G}(j\omega),F(j\omega))D^{-1}(\omega)}{\beta(\omega)} - jG(\omega)\right)\left(I + G^2(\omega)\right)^{-\frac{1}{2}} .$$

where

$$\mathbf{D} = \{diag\,(D_1,\cdots,D_p,dI_{perf}) \,\Big|\, D_i \in \mathcal{C}^{k_i\times k_i}, D_i^* = D_i > 0, d\in\mathcal{R}, d>0\Big\}$$

$$\mathbf{G} = \{diag\,(G_1,\cdots,G_p,O) \,\Big|\, G_i \in \mathcal{C}^{k_i\times k_i}, G_i = G_i^*\Big\} .$$

The structure of $\mathbf{D}$ and $\mathbf{G}$ depend on the structure of the perturbation block Δ.

For purely complex perturbations, the control problem reduce to

$$F(s) = \arg\min_{F\in\mathbf{F}} \sup_{\omega} \inf_{D\in\mathbf{D}} \{\bar{\sigma}\,(D(\omega)\,\mathcal{F}_\ell\left(\tilde{G}(j\omega),F(j\omega)\right)D^{-1}(\omega))\Big\} . \tag{7}$$

The control problems (6) and (7) are both *scaled $\mathcal{H}_\infty$ optimization problems.* Scaled $\mathcal{H}_\infty$ optimizations have recently been an area of intensive research within the automatic control community. However, no solution to (6) or (7) has yet been found. Rather iterative approximate solution procedures have been developed for both purely complex and mixed perturbation sets.

3.2 Complex μ Synthesis

An approximation to complex μ synthesis can be made by the following iterative scheme. For a fixed controller $F(s)$, the problem of finding $D(\omega)$ at a set of chosen frequency points ω is just the complex μ upper bound problem which is a convex problem with known solution. Having found these scalings we may fit a real rational stable minimum phase transfer function matrix $D(s)$ to $D(\omega)$ by fitting each element of $D(\omega)$ with a real rational stable minimum phase SISO transfer function. We may impose the extra constraint that the approximations $D(s)$ should be minimum phase (so that $D^{-1}(s)$ is stable too) since any phase in $D(s)$ is absorbed into the complex perturbations. For a given magnitude of $D(\omega)$, the phase corresponding to a minimum phase transfer function system may be computed using complex cepstrum techniques. Accurate transfer function estimates may then be generated using standard frequency domain least squares techniques.

For given scalings $D(s)$, the problem of finding a controller (in our case a filter) $F(s)$ which minimizes the norm $\|\mathcal{F}_\ell(D(s)\tilde{G}(s)D^{-1}(s),F(s))\|_{\mathcal{H}_\infty}$ will be reduced to a standard $\mathcal{H}_\infty$ problem. Repeating this procedure several times will yield the complex μ upper bound optimal controller provided the algorithm converges. Even though the computation of the D scalings and the optimal $\mathcal{H}_\infty$ controller are both convex problems, the iteration procedure is *not jointly convex* in $D(s)$ and $F(s)$ and counter examples of convergence has been given [Doy85]. However, the iteration seems to work quite well in practice and has been successfully applied to a large number of applications. Furthermore, with the release of the MatLab *μ-Analysis and Synthesis Toolbox*, commercially available software now exists to support complex μ synthesis using this iteration.

3.3 Mixed μ Synthesis

A detailed description of the mixed μ synthesis method described in the following can be found in [NSTCA97, TCASN95].

The main idea of the proposed mixed μ iteration scheme is to perform a scaled complex μ synthesis where the difference between mixed and complex μ is taken into account through an additional scaling matrix ?(s). Given the augmented system $\tilde{G}(s)$, a stabilizing controller $F_1(s)$ (e.g. an $\mathcal{H}_\infty$ optimal controller) we may compute upper bounds for μ across frequency given both the "true" mixed perturbation set Δ and the fully complex approximation Δ^c, i.e. δ_i are considered as a complex parameter. In order to "trick" the $\mathcal{H}_\infty$ optimization in the next iteration to concentrate more on mixed μ, we will construct an open loop system $\tilde{G}_{\mathrm{D}\Gamma_1}(s)$ which, when closed with the previous controller, has frequency response equal to the mixed μ upper bound just computed. In the mixed μ iteration, however, the structure of the approximation is different. $\tilde{G}_{\mathrm{D}\Gamma} = ?\, D\tilde{G}D^{-1}$ is constructed by applying two scalings to the original system $\tilde{G}(s)$. A D scaling such that $\bar{\sigma}(\mathcal{F}_\ell(D\tilde{G}D, F))$ approximates the complex μ upper bound and a ? scaling to shift from complex to mixed μ. In each iteration, ? can be computed as

$$?_i(s) = \begin{bmatrix} \gamma_i(s) I_{n_{ze}} & 0 \\ 0 & I_{n_y} \end{bmatrix}$$

where

$$\begin{aligned} \gamma_i(j\omega) &= (1-\alpha_i)|\gamma_{i-1}(j\omega)| \\ &\quad + \alpha_i \frac{\hat{\mu}_\Delta\big(\mathcal{F}_\ell(\tilde{G}(j\omega), F_i(j\omega))\big)}{\hat{\mu}_{\Delta^c}\big(\mathcal{F}_\ell(\tilde{G}(j\omega), F_i(j\omega))\big)} \end{aligned}$$

α_i is a certain filtering variable, see below, n_{ze} denotes the number of measurement outputs and n_{ze} denotes the number of external outputs. For perfect realizations of the scalings we will have

$$\begin{aligned} &\bar{\sigma}\left(\mathcal{F}_\ell\left(\tilde{G}_{D\Gamma_1}(j\omega), F_1(j\omega)\right)\right) = \\ &\qquad \hat{\mu}_\Delta\left(\mathcal{F}_\ell\left(\tilde{G}(j\omega), F_1(j\omega)\right)\right) \end{aligned}$$

where $\hat{\mu}_\Delta$ denoted the upper bound for μ. The controller $F_2(s)$ then will minimize the $\mathcal{H}_\infty$-norm of an augmented system which closed with the previous controller $F_1(s)$ has maximum singular value approximating mixed μ. New mixed and complex μ bounds may then be computed and the procedure may be repeated.

Applications of the mixed μ method can be found in [NSTCA97, TCASN94, TCASN95]. It is shown that the above mixed μ synthesis method are more optimal than the direct mixed μ synthesis method described in [You94].

4 A Combined FDI Setup

As mentioned above, parametric faults in actuator and sensor dynamics can easily be modeled in the approach of this paper by a simple trick. However, the additive fault description is the most used approach, see e.g. [DG96, NS97, PC96]. A system setup for parametric and additive faults will shortly be considered in the following.

Let us consider a plant described by:

$$\begin{array}{rcl} \dot{x} & = & A_\Delta x + Bd + B_{f_a} f_a \\ y & = & C_2 x + Dd + D_{f_a} f_a \end{array} \quad (8)$$

where f_a is the additive fault input vector.

For obtaining a standard optimization problem, the isolation error e_a is introduced as, in the the parametric fault case:

$$e_a = f_a - \hat{f}_a$$

where $\hat{f}_a$ is the estimate of f_a, that need to be generated by

$$\hat{f}_a = F(s)y$$

Combining the model for the parametric fault case given by (4) with the the above model in (8), gives the following complete system setup for both additive and parametric faults.

$$\begin{array}{rcl} \dot{x} & = & Ax + \tilde{B}_1 \tilde{w} + \tilde{B}_2 \tilde{u} \\ \tilde{z} & = & \tilde{C}_1 x + \tilde{D}_{12} \tilde{u} \\ y & = & C_2 x + \tilde{D}_{21} \tilde{w} \end{array} \quad (9)$$

where

$$\begin{array}{l} \tilde{B}_1 = \begin{pmatrix} B_f & B_{f_a} & B \end{pmatrix}, \quad \tilde{B}_2 = 0, \\ \tilde{u} = \begin{pmatrix} \hat{f}_p \\ \hat{f}_p \end{pmatrix}, \quad \tilde{z} = \begin{pmatrix} f_p \\ e_p \\ e_a \end{pmatrix}, \\ \tilde{C}_1 = \begin{pmatrix} C_f \\ C_f \\ 0 \end{pmatrix}, \quad \tilde{D}_{12} = \begin{pmatrix} 0 \\ -I \end{pmatrix}, \\ \tilde{D}_{21} = \begin{pmatrix} 0 & D_{f_a} & D \end{pmatrix} \end{array}$$

and

$$\tilde{w} = \begin{pmatrix} w_f \\ f_a \\ d \end{pmatrix}, \quad w_f = \begin{pmatrix} \delta_1 I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \delta_p I_p \end{pmatrix} f_p$$

The design synthesis given in Section 3 can then be applied directly to the above system. Only the Δ block need to be modified.

5 Conclusion

A systematic modeling and synthesis procedure for deriving fault detection and isolation filters for parametric faults has been presented. Further, a combined setup for fault detection and isolation in systems including both parametric as well as additive faults has been given.

The derived method includes a possibility for trading off the risk of undetected faults to the risk of false alarms.

The FDI setup considered in this paper deals only with the nominal case. However, the synthesis procedure for deriving fault detectors can quite easily be extended to handle model uncertainties, i.e. robust fault detection and isolation. This can be done by including the uncertainty blocks, $\Delta_{\text{uncertainty}}$, in the Δ block describing the the parameteric fault Δ_{par} and the performance condition Δ_{perf}. Both real as well as complex uncertainties can be handle in the setup.

The paper proposes a numerical solution based on a specific mixed μ optimization method. This is just one of many feasible solution, however, and dedending on the application and problem data, one might choose any alternative optimization method, such as other algorithms for μ optimization, or altogether different approaches, such as multiplier methods based on LMI algorithms.

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