FAST AND ROBUST MEASUREMENT OF OPTICAL CHANNEL GAIN

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ABSTRACT

We present a numerically stable and computational simple method for fast and robust measurement of optical channel gain. By transmitting adaptively designed signals through the channel, good accuracy is possible even in severe noise conditions.

1. INTRODUCTION

The measuring of optical channel gains is a key element in many applications. Measuring channel gains means determining the change in intensity when a signal is transmitted from an emitter to a receiver. A well-known and simple application is an automatic door, which responds whenever a person is reflecting the emitted signal, and thereby significantly increasing the channel gain. Another example is measuring the thickness of paper. A more subtle example is determination of spatial position by comparing the intensities of a multitude of reflections from a single object. A typical way of making this type of measurements is emitting a simple signal, such as a harmonic or square wave signal, since they are both easily constructed and measured with analog electronics. Such solutions have two major disadvantages: The signals are sensitive to frequency located disturbances, and it is difficult to detect and avoid/neutralize such disturbances.

We propose a measuring method which is highly accurate in moderate noise conditions, and less accurate, but very robust, in severe noise conditions. This is achieved by using two closely related digital signal design algorithms; a "best case" and a "worst case" algorithm. The former is based on the wavelet transform (WT), while the latter is based on the Rudin-Shapiro transform (RST). They are both simple, numerically stable, and post-processing friendly making them ideal for implementation e.g. in a fixed point DSP. By introducing a signal processor it becomes possible to continuously redesign the signals for improved SNR, and thereby maintaining the accuracy in changing and/or severe noise conditions.

2. DESIGNING THE DIGITAL SIGNALS

The two design algorithms are based on the wavelet packet transform scheme; it is fast, numerically stable, works well in fixed point arithmetics, and has low program complexity.

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The best case algorithm uses the classical WT to create signals which are near-orthogonal to expected noise occurrences, while the other algorithm uses the RST to create an all-spectrum signal, which by nature has low sensitivity with respect to time and frequency located noise occurrences. The difference is essentially that the WT algorithm "searches for holes" in the current noise, while the RST algorithm spreads information in time and frequency to reduce the impact of localized disturbances. The prefered method depends on the noise conditions. If there are easy-to-find holes in the noise, the former can provide very accurate measurements. If, however, the noise is difficult to define or is changing rapidly, the latter method provides lesser accurate, but more robust measurements.

A good introduction to the wavelet theory is Wickerhauser [6]. A mathematically rigorous treatment of the subject is given in Daubechies [3]. For more material on Rudin-Shapiro polynomials see Brillhart [1]

2.1. The wavelet transform

The WT based algorithm takes a simple, time localized signal (see figure 1 for an example), and inversely wavelet packet transform it, which results in a frequency localized signal. A typical maximum spread is also shown in figure 1. After transmission the signal is forwardly transformed to reproduce the original, now noisy simple signal, and by inner product with the "clean" original signal the transmission intensity (the channel gain) is determined. Since the original signal is completely known, it is also possible to obtain an estimated accuracy of the channel gain measurement. This is accomplished by taking inner product between the transmitted, transformed signal and a number of signals orthogonal to the original signal. If these quantities are small the transmission was most likely subject to only mild noise. This trick provides an easily calculated guideline to how much one can trust the current measurement. If each measurement is vital a number of signal restoration procedures (not further described here) can be applied. These also benefit from the complete knowledge of the original signal. Note that the small number of non-vanishing coefficients of the original signal in all cases significantly reduce the amount of calculations.

Because the transform is linear and has perfect reconstruction, it is also easy to make a good sample by sample estimation of the noise, which provides valuable information on any disturbances. This makes it possible to do real time adaptation of the signal. The mehtods combines the ability of the WT to produce predefined trade-offs between time and frequency information with the freedom in design of the original signal.

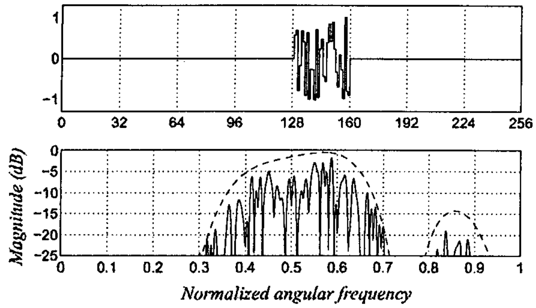

Figure 1: Uppermost a simple signal (here a sampled chirp). Below (in solid) the absolute value of the Fourier transform of the 3-scale inverse wavelet packet transform with symlets 6 [4]. The dashed curve shows the maximal frequency spreading for any (suitably normalized) signal with coefficients vanishing outside [128; 159].

Thereby it is possible to adapt the method to virtually any type of noise, in particular disturbances with large temporal extent.

2.2. The Rudin-Shapiro transform

The RST is defined through a slightly extended version of the remarkable Rudin-Shapiro polynomials, introduced in 1951 by H. S. Shapiro in his master's thesis, and published in 1959 by Rudin [5]. Define the polynimals

$$\begin{aligned} P_{m+1}(z) &= P_m(z) + (-1)^{\delta_m} z^{2^m} Q_m(z), \quad P_0 = 1 \\ Q_{m+1}(z) &= P_m(z) - (-1)^{\delta_m} z^{2^m} Q_m(z), \quad Q_0 = 1 \end{aligned} \tag{1}$$

with $\delta_m \in \{0, 1\}$. It immediate follows that for all $|z| = 1$.

$$|P_{m+1}|^2 + |Q_{m+1}|^2 = 2|P_m|^2 + 2|Q_m|^2 = 2^{m+2}.$$

Consequently,

$$\max_{\xi} |P_m(e^{i\xi})| \leq \sqrt{2} \| P_m(e^{i\xi}) \|_2, \tag{2}$$

guaranteeing a certain flatness of the polynomials. A construction similar to (1) is found in Byrnes [2]. The coefficients of the polynomials can also be constructed with the Rudin-Shapiro transform, which is really a modified wavelet packet Haar transform. Define the unitary transform $\mathbf{H}_n : \mathbb{R}^{2^n} \mapsto \mathbb{R}^{2^n}$, $n \geq 1$, as

$$\begin{bmatrix} y_k \\ y_{k+2^{n-1}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & (-1)^k \\ 1 & -(-1)^k \end{bmatrix} \begin{bmatrix} x_{2k} \\ x_{2k+1} \end{bmatrix}$$

for $k = 0, \ldots, 2^{n-1} - 1$ when mapping $\mathbf{x}$ to $\mathbf{y}$. Then, with $\tilde{\mathbf{H}}_n$ being the inverse of $\mathbf{H}_n$,

$$\tilde{\mathbf{H}} \stackrel{\text{def}}{=} \prod_{n=1}^{N} \begin{bmatrix} \tilde{\mathbf{H}}_n & & 0 \\ & \ddots & \\ 0 & & \tilde{\mathbf{H}}_n \end{bmatrix}_{2^N \times 2^N}$$

transforms the canonical basis for $\mathbb{R}^{2^N}$ into the coefficients of the 2^N possible $P_N(z)$'s. Hence these coefficients constitutes an orthonormal basis, which not only consists of only ± 1, but

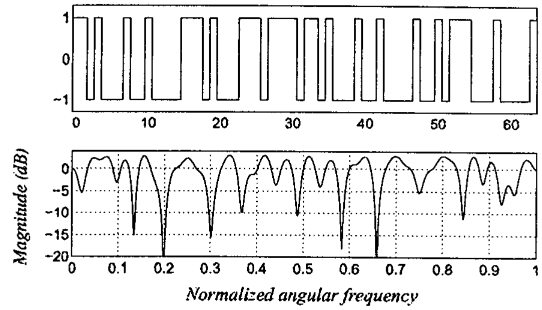

Figure 2: Uppermost is the result of applying the inverse Rudin-Shapiro transform to the 29th canonical basis vector of length 64. Below the frequency response (the absolute value of the corresponding $P_6(e^{i\xi})$ polynomial).

also, due to (2), has a remarkable frequency response. In figure 2 is an example of such a basis element and its frequency response. The RST based algorithm is applied in much the same way as the WT method. A simple signal is inversely transformed prior to transmission. This will produce an all-spectrum signal. Upon transmission the signal is forwardly transformed yielding the original, simple signal with noise. The post-processing is equivalent to that of the WT method.

3. CONCLUSION

Two computationally simple and numerically robust algorithms for measuring optical channel gains were presented. One of the algorithms provides excellent accuracy in moderate noise conditions, while the other has reduced, but very robust, accuracy even in severe noise conditions. Combining the two algorithms, either by applying the most suitable one, or jointly in two parallel systems, is easy due to their similar program and computational structure, and the result is a versatile optical channel gain method. The low complexity and numerical stability of the wavelet packet transform scheme and of the post-processing (mainly inner products) also makes this approach fast and suitable for low cost hardware implementation. For the commercial aspects of these methods, please refer to `www.beamcontrol.com`.

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