

# PERIODIC $H_2$ SYNTHESIS FOR SPACECRAFT ATTITUDE DETERMINATION AND CONTROL WITH A VECTOR MAGNETOMETER AND MAGNETORQUERS

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**Abstract:** A control synthesis for a spacecraft equipped with a set of mutually perpendicular coils and a vector magnetometer is addressed in this paper. The interaction between the Earth's magnetic field and an artificial magnetic field generated by the coils produces a control torque. Comparison between the expected magnetic field vector and the true magnetometer data is used for the attitude determination. The magnetic attitude control and determination is intrinsically periodic due to periodic nature of the geomagnetic field variation in orbit. The control performance is specified by the generalized  $H_2$  operator norm. The paper proposes an LMI solution to this problem.

**Keywords:** Attitude control, optimal control, periodic systems, linear matrix inequality

## 1. INTRODUCTION

A tremendous progress in micro-electronics observed in the last two decade made small, inexpensive spacecraft missions very attractive, and technologically viable. However, due to reduced allocated mission cost, the hardware including the sensors and the actuators is often very simple, furthermore the redundancy is limited or completely avoided. The attitude control system from this perspective has to be sophisticated enough to fully utilize the existent hardware, but at the same time computationally as simple as possible to increase the reliability.

Probably the most typical actuator/sensor configuration currently in use is a combination of magnetorquer coils and a three axis magnetometer. This or similar configuration was used

on the British UoSat satellites, Danish Ørsted, South African SunSat, German Champ, Portuguese PoSat. A vital common feature of the magnetometer and the magnetorquer is that they rely on the magnetic field of the Earth. The interaction of the geomagnetic field and artificially generated field in the coils produces a control torque, whereas the comparison of the modeled and the measured magnetic field of the Earth provides attitude information. The attitude estimation and control schemes developed in this paper use an observation that the magnetic field is periodic<sup>2</sup>.

The idea in this paper is to consider the spacecraft as a linear periodic system, and to solve the  $H_2$  control synthesis problem. The optimization problem is formulated in this article by certain

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<sup>2</sup> The time propagation of the geomagnetic field vector observed from an Earth stabilized spacecraft is a superposition of two periodic motions: orbital and the Earth spin. If a ratio of the two periods is a rational number the geomagnetic field observation is periodic

linear matrix inequalities. There is a great number of publications treating the control synthesis expressed by LMIs, however only very recently periodic systems have been addressed. (Souza and Trofino, 2000) and (Bittanti and Colaneri, 1999) have treated the robust stability problem of a periodic system with the LMI technique. (Dullerud and Lall, 1999) have extended the LMI approach to  $H_\infty$  control synthesis for the LTI systems, (Gahinet and Apkarian, 1991), to periodic ones. The attitude control approach presented in this paper uses the method developed in (Wisniewski and Stoustrup, 2001).

The literature on attitude control treats the magnetic estimation and control separately. There is a great number of publications solving the magnetic estimation problem using the Kalman filter with a time dependent Riccati equation, e.g. (Lefferts *et al.*, 1982), (Psiaki *et al.*, 1990), (Natanson, 1993), (Challa *et al.*, 1997), (Psiaki, 1999), (Bak, 1999). A concept for attitude control based on electromagnetic actuation has gained a comparable attention lately. The early work was based on an idea of designing magnetic controller for the system with averaged parameters, rather than time varying. This design strategy was used both for bias momentum satellites (Camillo and Markley, 1980), (Hablani, 1995), (Hablani, 1997) and three axis control (Martel *et al.*, 1988). In the recent papers more sophisticated control schemes were proposed, where not only the linear, (Cavallo *et al.*, 1993), (Arduini and Baiocco, 1997), (Wisniewski and Markley, 1999), (Wisniewski, 2000) but also nonlinear control methods, (Steyn, 1994), (Wisniewski and Blanke, 1996), (Tabuada *et al.*, 1999) were in focus.

The  $H_2$  attitude control synthesis addressed in this paper is not completely new in (Wisniewski and Markley, 1999) and (Wisniewski, 2000) the equivalent  $L_2$  magnetic control problem was addressed. The solution proposed involved a periodic Riccati equation. This paper tackles the problem using the linear matrix inequalities. General schemes for the control synthesis and its dual problem, estimation are proposed. From the implementation point of view the advantage of the LMI approach is that the computer burden of the control synthesis is in the off-line calculation, whereas the on-board algorithm is simple. This makes the fix-point implementation possible.

## 2. LMI

The argument for using this paradigm is that the separation principle is valid for a periodic system (Prato and Ichikawa, 1988).

Consider a system of specifications used for the standard  $H_2$  synthesis

$$\begin{aligned} s_1 : \begin{bmatrix} \mathbf{w} \\ \mathbf{u} \end{bmatrix} &\mapsto \begin{bmatrix} \mathbf{z} \\ \mathbf{y} \end{bmatrix}, \\ \mathbf{x}(t+1) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}_1(t)\mathbf{w}(t) + \mathbf{B}_2(t)\mathbf{u}(t) \\ \mathbf{z}(t) &= \mathbf{C}_1(t)\mathbf{x}(t) + \mathbf{D}_{12}(t)\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}_2(t)\mathbf{x}(t) + \mathbf{D}_{21}(t)\mathbf{w}(t), \end{aligned} \quad (1)$$

where the system matrices are periodic  $\mathbf{B}_1(t+N) = \mathbf{B}_1(t) \in \mathbb{R}^{s \times n}$ ,  $\mathbf{B}_2(t+N) = \mathbf{B}_2(t) \in \mathbb{R}^{m \times n}$ ,  $\mathbf{C}_1(t+N) = \mathbf{C}_1(t) \in \mathbb{R}^{n \times r}$ ,  $\mathbf{C}_2(t+N) = \mathbf{C}_2(t) \in \mathbb{R}^{n \times p}$ ,  $\mathbf{D}_{12}(t+N) = \mathbf{D}_{12}(t) \in \mathbb{R}^{m \times r}$ , and  $\mathbf{D}_{21}(t+N) = \mathbf{D}_{21}(t) \in \mathbb{R}^{s \times p}$ .

We shall first assume full state space information, i.e.  $\mathbf{C}_2 = \mathbf{I}$ , and  $\mathbf{D}_{21} = \mathbf{0}$ , and periodic state feedback  $\mathbf{u}(t) = \mathbf{K}(t)\mathbf{x}(t)$ ,  $\mathbf{K}(t+N) = \mathbf{K}(t)$ . The objectives of the control design is to compute a gain  $\mathbf{K}(t)$  for which the transfer function

$$\begin{aligned} s_c : \mathbf{w} &\mapsto \mathbf{z}, \\ \mathbf{x}(t+1) &= \mathbf{A}_c(t)\mathbf{x}(t) + \mathbf{B}_1(t)\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}_c(t)\mathbf{x}(t), \end{aligned} \quad (2)$$

where  $\mathbf{A}_c(t) = \mathbf{A}(t) + \mathbf{B}_2(t)\mathbf{K}(t)$ ,  $\mathbf{C}_c(t) = \mathbf{C}_1(t) + \mathbf{D}_{12}(t)\mathbf{K}(t)$  satisfies

$$\|s_c\|_2 < \gamma \quad (3)$$

The main results are summarized in the following theorem

*Theorem 1.* (Wisniewski and Stoustrup, 2001) Consider a periodic discrete time system  $s_c$ ,  $(\mathbf{A}(t), \mathbf{B}_2(t))$  stabilizable. The suboptimal  $H_2$  problem Eq. (3) is solvable if and only if there exists a symmetric periodic matrix  $\mathbf{Q}(t)$  and a periodic  $\mathbf{Z}(t)$  such that

$$\begin{aligned} &(\mathbf{W}_1(t)^T \mathbf{A}(t) + \mathbf{W}_2(t)^T \mathbf{C}_1(t)) \mathbf{Q}(t-1) \\ &\times (\mathbf{A}(t)^T \mathbf{W}_1(t) + \mathbf{C}_1(t)^T \mathbf{W}_2(t)) \\ &- \mathbf{W}_1(t)^T \mathbf{Q}(t) \mathbf{W}_1(t) - \mathbf{W}_2(t)^T \mathbf{Z}(t) \mathbf{W}_2(t) < \mathbf{0}, \end{aligned} \quad (4)$$

$$\begin{bmatrix} \mathbf{Q}(t) & \mathbf{B}_1(t) \\ \mathbf{B}_1(t)^T & \mathbf{Z}(t) \end{bmatrix} > \mathbf{0}, \quad (5)$$

$$\text{tr} \left( \sum_{t=0}^{N-1} \mathbf{Z}(t) \right) < N\gamma^2, \quad (6)$$

where  $\text{im} \begin{bmatrix} \mathbf{W}_1(t) \\ \mathbf{W}_2(t) \end{bmatrix} = \ker [\mathbf{B}_2(t)^T \mathbf{D}_{12}(t)^T]$ .

The  $H_2$  control synthesis is decomposed into a feasibility problem of finding symmetric periodic matrices  $\mathbf{Q}(t)$  and  $\mathbf{Z}(t)$  meeting the inequalities (4) to (6) and a problem of finding a periodic control gain  $\mathbf{K}(t)$  satisfying the following LMI (Wisniewski and Stoustrup, 2001)

$$\begin{aligned}
& \begin{bmatrix} -\mathbf{Q}(t) & \mathbf{A}(t) & \mathbf{0} \\ \mathbf{A}(t)^T & -\mathbf{Q}^{-1}(t-1) & \mathbf{C}_1(t)^T \\ \mathbf{0} & \mathbf{C}_1(t) & -\mathbf{I} \end{bmatrix} \\
& + [\mathbf{B}_2(t)^T \mathbf{0} \mathbf{D}_{12}(t)^T]^T \mathbf{K}(t) [\mathbf{0} \mathbf{I} \mathbf{0}] \\
& + [\mathbf{0} \mathbf{I} \mathbf{0}]^T \mathbf{K}(t)^T [\mathbf{B}_2(t)^T \mathbf{0} \mathbf{D}_{12}(t)^T]^T \\
& < \mathbf{0}.
\end{aligned} \tag{7}$$

The following algorithm will be used in Section 3 for the periodic state feedback synthesis

*Algorithm 1.*

- (1) Find using a symmetric matrix  $\mathbf{Q}(t)$  and a matrix  $\mathbf{Z}(t)$  for  $t = 0 \dots N-1$  satisfying LMIs (4) to (6).
- (2) For each  $t = 0 \dots N-1$  find a matrix  $\mathbf{K}(t)$ , which satisfies LMI (7).

The observer synthesis reduces to an application of the duality argument. The following system is considered

$$\begin{aligned}
s_o : \mathbf{w} &\mapsto \mathbf{z}, \\
\mathbf{x}(t+1) &= \mathbf{A}_o(t)\mathbf{x}(t) + \mathbf{B}_o(t)\mathbf{w}(t) \\
\mathbf{z}(t) &= \mathbf{C}_1(t)\mathbf{x}(t),
\end{aligned} \tag{8}$$

where  $\mathbf{A}_o(t) = \mathbf{A}(t) + \mathbf{L}(t)\mathbf{C}_2(t)$ ,  $\mathbf{B}_o(t) = \mathbf{B}_1(t) + \mathbf{L}(t)\mathbf{D}_{21}(t)$ . The observer synthesis is such that the gain  $\mathbf{L}(t)$  fulfills

$$\|s_o\|_2 < \gamma \tag{9}$$

The problem Eq. (9) is solvable if and only if there exists a symmetric periodic matrix  $\mathbf{Q}(t)$  and a periodic  $\mathbf{Z}(t)$  such that

$$\begin{aligned}
& (\mathbf{W}_1(t)^T \mathbf{A}(t)^T + \mathbf{W}_2(t)^T \mathbf{B}_1(t)^T) \mathbf{Q}(t-1) \\
& \times (\mathbf{A}(t) \mathbf{W}_1(t) + \mathbf{B}_1(t) \mathbf{W}_2(t)) \\
& - \mathbf{W}_1(t)^T \mathbf{Q}(t) \mathbf{W}_1(t) - \mathbf{W}_2(t)^T \mathbf{W}_2(t) < \mathbf{0},
\end{aligned} \tag{10}$$

$$\begin{bmatrix} \mathbf{Q}(t) & \mathbf{C}_2^T(t) \\ \mathbf{C}_2(t) & \mathbf{Z}(t) \end{bmatrix} > \mathbf{0}, \tag{11}$$

$$\text{tr} \left( \sum_{t=0}^{N-1} \mathbf{Z}(t) \right) < N\gamma^2, \tag{12}$$

where  $\text{im} \begin{bmatrix} \mathbf{W}_1(t) \\ \mathbf{W}_2(t) \end{bmatrix} = \ker [\mathbf{C}_2(t) \mathbf{D}_{21}(t)]$ . The observer gain is a solution of the following LMI

$$\begin{aligned}
& \begin{bmatrix} -\mathbf{Q}(t) & \mathbf{A}(t)^T & \mathbf{0} \\ \mathbf{A}(t) & -\mathbf{Q}^{-1}(t-1) & \mathbf{B}_1(t) \\ \mathbf{0} & \mathbf{B}_1(t)^T & -\mathbf{I} \end{bmatrix} \\
& + [\mathbf{C}_2(t) \mathbf{0} \mathbf{D}_{21}(t)]^T \mathbf{L}(t)^T [\mathbf{0} \mathbf{I} \mathbf{0}] \\
& + [\mathbf{0} \mathbf{I} \mathbf{0}]^T \mathbf{L}(t) [\mathbf{C}_2(t) \mathbf{0} \mathbf{D}_{21}(t)] < \mathbf{0}.
\end{aligned} \tag{13}$$

The algorithm for periodic observer design is

*Algorithm 2.*

- (1) Find a symmetric matrix  $\mathbf{Q}(t)$  and a matrix  $\mathbf{Z}(t)$  for  $t = 0 \dots N-1$  satisfying LMIs (10) to (12).
- (2) For each  $t = 0 \dots N-1$  find  $\mathbf{L}(t)$  which satisfies (13).

In spacecraft applications a time invariant control/observer gains are often desirable in simple on-board implementations. In this case the step 1 in the Algorithm 1 and 2 remains unchanged, whereas the periodic  $\mathbf{K}(t)$  and  $\mathbf{L}(t)$  may be substituted by the time invariant matrix  $\mathbf{K}$  and  $\mathbf{L}$  in the step 2. The drawback of this approach is that the  $\mathbf{K}$  and  $\mathbf{L}$  matrices do not correspond to the optimal constant observer and controller gains.

### 3. MAGNETICALLY ACTUATED SPACECRAFT

The objectives of this section is to apply Algorithms 1 and 2 for the three-axis attitude control of a spacecraft in a low, highly inclined Earth orbit. The spacecraft is actuated by three mutually perpendicular electromagnetic coils. The interaction between the geomagnetic field and the magnetic field in the coil produces the control torque. The comparison between the expected magnetic field vector and the true magnetometer data is used for the attitude determination.

#### 3.1 Spacecraft Model

The satellite considered in this study is modeled as a rigid body in the Earth gravitational field influenced by the aerodynamic drag torque and the control torque generated by the magnetorquers. The attitude is parameterized by the unit quaternion providing a singularity free representation of the kinematics (Goldstein, 1980), (Wertz, 1990).

The control torque,  $\mathbf{N}_{ctrl}$ , of the magnetically actuated satellite always lies perpendicular to the geomagnetic field vector,  $\mathbf{b}$ . Therefore a magnetic moment,  $\mathbf{m}$ , generated in the direction parallel to the local geomagnetic field has no influence on the satellite motion. This can be explained by the following equality

$$\mathbf{N}_{ctrl} = (\mathbf{m}_{\parallel} + \mathbf{m}_{\perp}) \times \mathbf{b} = \mathbf{m}_{\perp} \times \mathbf{b}, \tag{14}$$

where  $\mathbf{m}_{\parallel}$  is the component of the magnetic moment parallel to  $\mathbf{b}$ , whereas  $\mathbf{m}_{\perp}$  is perpendicular to the local geomagnetic field.

Concluding, the necessary condition for power optimality of a control law is that the magnetic moment lies on a plane perpendicular to the geomagnetic field vector.

Consider the following mapping

$$\tilde{\mathbf{m}} \mapsto \mathbf{m} : \mathbf{m} = \tilde{\mathbf{m}} \times \mathbf{b}/|\mathbf{b}|^2 \quad (15)$$

where  $|\cdot|$  denotes the standard Euclidean norm, and  $\tilde{\mathbf{m}}$  represents a new control signal for the satellite. Now, the magnetic moment,  $\mathbf{m}$ , is exactly perpendicular to the local geomagnetic field vector and the control theory for a system with unconstrained input  $\tilde{\mathbf{m}}$  can be applied. The direction of the signal vector  $\tilde{\mathbf{m}}$  (contrary to  $\mathbf{m}$ ) can be chosen arbitrarily by the controller.

The model of the sensor, a three axis magnetometer, will be developed in the following. For simplicity of this exposition it is assumed that the magnetometer measurements are provided in a coordinate system spanned on the principal axes, denoted a body coordinate system. The model of the vector magnetometer is then

$$\mathbf{y}(t) = \mathbf{R}(\mathbf{q})\mathbf{b}(t), \quad (16)$$

where  $\mathbf{R}(\mathbf{q})$  is the rotation matrix corresponding to the attitude quaternion  $\mathbf{q}$  and describing a rotation from an orbit fixed coordinate system e.g. Local-Vertical-Local-Horizontal Coordinate System (LVLH) to the body coordinate system; for complete definition of the involved coordinate systems the reader is referred to (Wisniewski *et al.*, 2000). The vector  $\mathbf{b}(t)$  is the local magnetic field seen from the orbit (LVLH) and reproduced in this paper by the 8th-degree IGRF model (Wertz, 1990).

Locally the attitude can be represented by three coordinates. In this work three components of the vector part of the attitude quaternion are used. The continuous time linear model of the satellite motion is given in terms of the angular velocity and the vector part of the attitude quaternion (Wisniewski, 2000)

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} \delta\Omega \\ \delta\mathbf{q} \end{bmatrix} &= \mathbf{A}_s \begin{bmatrix} \delta\Omega \\ \delta\mathbf{q} \end{bmatrix} + \mathbf{B}_s(t)\tilde{\mathbf{m}} \\ \mathbf{y}(t) &= \mathbf{C}_s(t) \begin{bmatrix} \delta\Omega \\ \delta\mathbf{q} \end{bmatrix}, \end{aligned} \quad (17)$$

where

$$\mathbf{A}_s = \begin{bmatrix} 0 & 0 & -\sigma_x\omega_o & -6\omega_o^2\sigma_x & 0 & 0 \\ 0 & 0 & 0 & 0 & 6\omega_o^2\sigma_y & 0 \\ -\omega_o\sigma_z & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \omega_o \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\omega_o & 0 & 0 \end{bmatrix},$$

$$\sigma_x = \frac{I_y - I_z}{I_x}, \sigma_y = \frac{I_z - I_x}{I_y}, \sigma_z = \frac{I_x - I_y}{I_z},$$

$$\mathbf{B}_s(t) = \begin{bmatrix} \mathbf{I}^{-1} \begin{bmatrix} -b_y^2 - b_z^2 & b_x b_y & b_x b_z \\ b_x b_y & -b_x^2 - b_z^2 & b_y b_z \\ b_x b_z & b_y b_z & -b_x^2 - b_y^2 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{bmatrix},$$

$$\mathbf{C}_s(t) = 2 \begin{bmatrix} \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix} & \mathbf{0} \end{bmatrix},$$

where  $\omega_o$  is the orbital rate, and  $I_x, I_y, I_z$  are components on the diagonal of the inertia tensor  $\mathbf{I}$  (the principal moments of inertia). The matrix  $\mathbf{C}_s(t)$  corresponds to a vector product  $2\mathbf{b}(t) \times$ , the control matrix  $\mathbf{B}_s(t)$  comes from the double cross product operation  $-\mathbf{b}(t) \times (\mathbf{b}(t) \times)$  divided by  $\mathbf{I}$ . The upper left 3 by 3 submatrix of  $\mathbf{A}_s$  is due to Euler coupling, the submatrix in the upper right corner arises from the gravity gradient, and the lower part of the matrix  $\mathbf{A}$  is the linearized kinematics. Notice that the matrices  $\mathbf{B}_s(t)$  and  $\mathbf{C}_s(t)$  are the periodic parts of Eq. (17). This is due to periodicity the magnetic field vector  $\mathbf{b}(t)$ .

The model (17) is used in Algorithms 1 and 2 computing the periodic observer and controller gains  $\mathbf{K}(t)$  and  $\mathbf{L}(t)$ .

#### 4. SIMULATION RESULTS

In the numerical calculations the satellite principal moments of inertia are assigned to  $[180 \ 150 \ 1]^T$ , which characterizes a spacecraft with a long gravity gradient boom. The algorithm is implemented in the Matlab LMI toolbox. The orbit is divided into  $N = 100$  samples. The normalized Earth magnetic field vector is shown in Figure 1. The components (5, 6), (6, 6) of  $\mathbf{Q}(t)$  and (2, 2), (2, 3) of the observer gain  $\mathbf{L}(t)$  in Algorithm 2 are depicted in Figure 2. The components (5, 6), (6, 6) of  $\mathbf{Q}(t)$  and (2, 2), (2, 3) of  $\mathbf{K}(t)$  in Algorithm 1 are depicted in Figure 3. It is seen that both the observer and the controller gain are periodic as expected. Comparing Figures 1 and 3 it is seen that near the polar zones, where the z-component of the geomagnetic field vector reaches maximum and minimum values, the pitch and roll gains increase, see the gray zones.

An impulse response of the closed loop system gives a reasonable interpretation of a  $H_2$  control performance. The result of the closed-loop simulation for 8 orbits is shown in Figure 4. The initial attitude is  $\mathbf{q} = [0.82 \ 0.02 \ 0.05 \ 0.57]^T$ . The components  $q_1, q_2, q_3$  can be treated for small deviations from the identity quaternion as half values of pitch, roll, and yaw angles respectively. For the spacecraft considered in the simulation study pitch and roll are passively stabilized by the gravity gradient, whereas yaw needs active

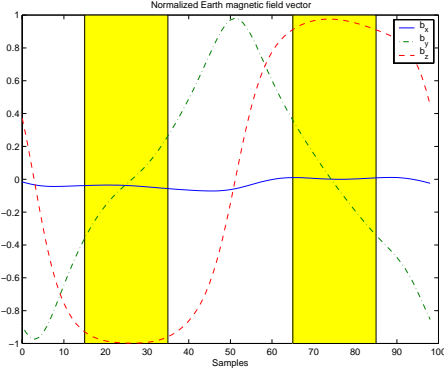


Fig. 1. The normalized magnetic field vector of the Earth computed for one orbit. The grey zones correspond to the polar regions.

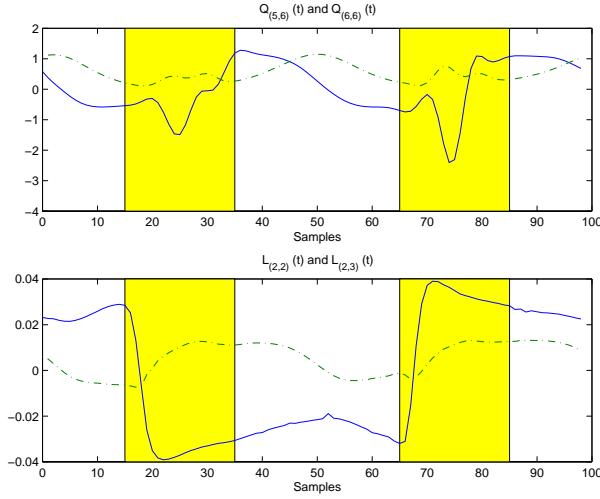


Fig. 2. The components (5,6), (6,6) of the observer matrix  $\mathbf{Q}(t)$  and (2,2), (2,3) of  $\mathbf{L}(t)$  computed for one orbit in Algorithm 2.

control action. It is seen that after four orbits, yaw is below the specified value 0.1, which corresponds to 10 deg., and the angular velocity is below  $10^{-3}$  rad/sec.

## 5. CONCLUSIONS

A periodic control scheme for  $H_2$  control synthesis was developed and implemented for the attitude control and estimation. The design algorithm presented showed the potential for on-board implementation on a small spacecraft platform equipped with a vector magnetometer and magnetorquers.

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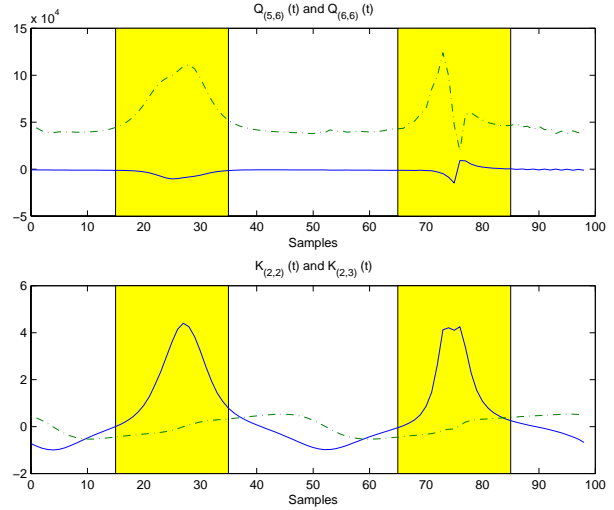


Fig. 3. The components (5,6), (6,6) of the matrix  $\mathbf{Q}(t)$  and (2,2), (2,3) of the gain  $\mathbf{K}(t)$  computed for one orbit in Algorithms 1. It is seen that the pitch and roll gains increase in the polar regions.

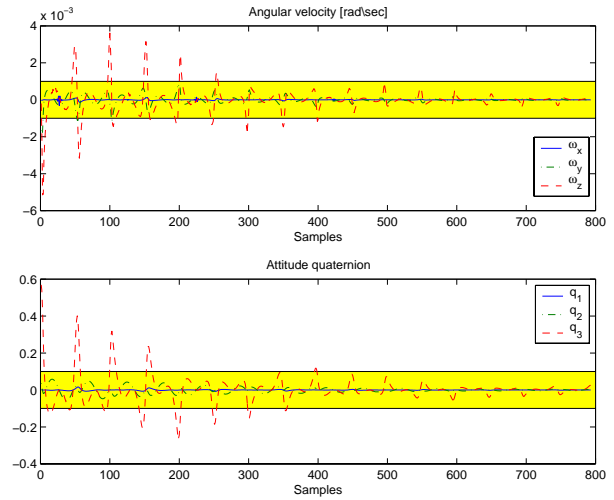


Fig. 4. The result of the closed-loop simulation: the angular velocity with respect to LVLH, and the vector part of the attitude quaternion. It is seen that after four orbits yaw is below the specified value 0.1, which corresponds to 10 deg., and the angular velocity is below  $10^{-3}$  rad/sec.

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