

Applying Parametric Fault Detection to a Mechanical System

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Abstract

A way of doing parametric fault detection is described. It is based on the representation of parameter changes as linear fractional transformations (lfts). We describe a model with parametric uncertainty. Then a stabilizing controller is chosen and its robustness properties are studied via μ . The parameter changes (faults) are estimated based on estimates of the fictitious signals that enter the delta block in the lft. These signal estimators are designed by H_∞ techniques. The chosen example is an inverted pendulum.

1 Objectives and Motivation

The problem of fault detection and identification (FDI) may be seen as a part of a more general and interdisciplinary problem known as fault tolerant control [8]. A fault tolerant control system can be based on the idea that an adequate controller should be robust to small faults and that more important faults should be detected and identified by a compatible FDI system. By compatibility here we understand that the controller should cope with faults that the FDI system can not detect with the sufficient degree of certainty and that the FDI system should detect faults to which the controller is not robust. We will treat a subset of this fault tolerant control problem, the design of the FDI system.

The problem of model based FDI has been receiving increasing attention from the research community since the beginning of the seventies [11]. The majority of the works and methodologies have modeled faults as

additive, that is, faults are modeled as exogenous perturbations affecting the system [3], [5], [4]. On of the few exceptions to this is the use of parameter identification methodologies in fault detection and identification [6]. We can argue that many faults are better modeled as parametric. Another problem with the modeling of faults as exogenous inputs is that an exogenous input can not destabilize a system and a parameter change may do that.

As we believe that the real nature of many faults is in fact parametric we consider parametric faults described as such. We are interested in designing an FDI system that is capable of detecting parametric faults and is compatible with the robustness margins of a given controller. Here we develop and apply an approach first suggested in [10]. For simplicity and to keep a better track of what is happening an inverted pendulum model was chosen as example.

2 Nominal Model and a Controller

Consider the inverted pendulum shown in figure 1. While θ and $\dot{\theta}$ are small the dynamic equations of the pendulum can be linearized and its state equation as shown in 1. The state vector was chosen to be $\alpha = \begin{bmatrix} \theta & \dot{\theta} & x & \dot{x} \end{bmatrix}^T$ and the state equation is written in standard matrix form as $\dot{\alpha} = A\alpha + Bu$, the output equation is $y = Cx + Du$ and as a zero D matrix and C will be the identity matrix if we assume all states are measured.

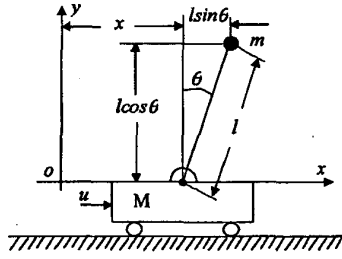


Figure 1: The inverted pendulum.

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+m}{Ml}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u \quad (1)$$

We consider the nominal parameter values $M = 2Kg$, $m = 0.1Kg$, $l = 0,5m$, $g = 9,8m.s^{-1}$. Obvious the inverted pendulum is an unstable system. The nominal model poles are $p_{1,2,3,4} = 0, 0, \pm 4.5365$. In what follows a stable system is convenient. A controller is needed.

For the moment, the controller and the FDI system will be designed in two separate processes. To proceed with the FDI design, with need the monitored process to belong to RH_{∞} . So a feedback controller was designed for the nominal model by pole placement.

Assume all states are available for feedback. The closed loop poles were placed at

$$p_{1,2} = -1 \pm 1.732i, \quad p_3 = p_4 = -5 \quad (2)$$

The controller was designed as a regulator and its gain matrix, K , is shown in equation 3. The chosen dominant poles make the answer of the closed loop similar to the one of a second order system with $\zeta = 0.5$ and natural non-damped frequency $\omega_n = 2rad \cdot s^{-1}$.

$$K = \begin{bmatrix} -74.698 & -16.587 & -10.194 & -9.174 \end{bmatrix} \quad (3)$$

Besides stabilizing the system, the objective of the controller is to allow the following of a reference for the position of the cart maintaining the pendulum as close as possible to the vertical position. The closed loop is shown in figure 2 where $K_1 = [0 \ 0 \ 1 \ 0]^T$ means a conversion of a reference of position to a reference of state.

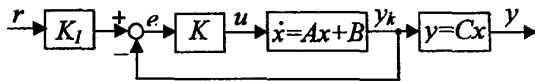


Figure 2: The closed loop.

It can be shown that the steady state tracking error to a step reference input is zero. Simulation results of the response of the closed loop system to a step input and observation of the closed loop frequency response show a reasonably good behaviour and also show that the con-

troller does not ask too much from the actuator. From these points of view it seems a reasonable controller.

The robustness of the closed loop system to parameter changes will be studied after an adequate description of the parametric uncertainty. (Another way to proceed could be to design an H_{∞} controller by (μ synthesis), with previously defined robustness properties).

3 Parametric Uncertainty

Consider that each of the three parameters in the nominal linearized model of the inverted pendulum is known with uncertainty or may change from its nominal value. The perturbed values of the parameter will be represented as in equations 4 to 6 where m_0 , M_0 and l_0 represent the nominal values of the parameters.

$$m = m_0(1 + \delta_m) \quad (4)$$

$$M = M_0(1 + \delta_M) \quad (5)$$

$$l = l_0(1 + \delta_l) \quad (6)$$

Consider again equation 1. We want to represent all possible parameter variations as an upper linear fractional transformation. There are several ways to do this, it can be done analytically [7], or using block diagrams [12]. We will use this last way because it is shorter and easier.

Consider equation 1 written as in equation 7 where $a = \frac{M+m}{Ml}g$, $b = -\frac{m}{M}g$, $c = \frac{1}{Ml}$, $d = \frac{1}{M}$. Equation 7 can be represented by a block diagram as in figure 3. In that figure we consider the outputs $[y_1 \ y_2]^T = [\dot{\theta} \ x]^T$. Other output equations would be treated in the same way.

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ b & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ c \\ 0 \\ d \end{bmatrix} u \quad (7)$$

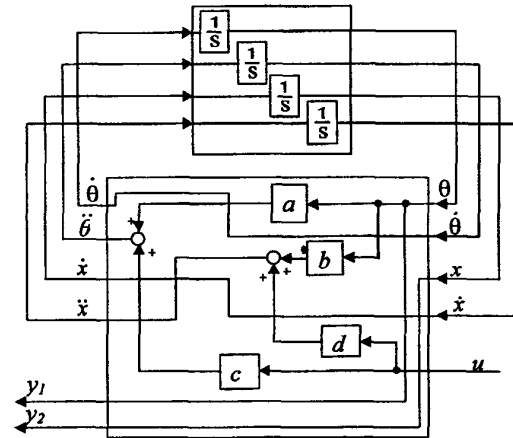


Figure 3: Block diagram of the nominal model. To represent the parameter uncertainties, or perturba-

tions, we treat each of the a , b , c and d blocks separately. Each one of the blocks is represented with the uncertainties as Δ -blocks. After, the results can be reunited into the overall diagram. After simplification this diagram is the desired linear fractional representation and can be transformed into a set of equations describing the perturbed model. The perturbed model final block diagram and the corresponding model matrices are represented in figure 4 and equations 8. Note that the model is in open loop.

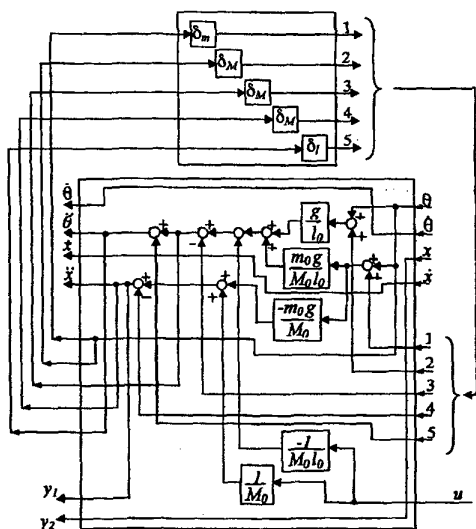


Figure 4: Block diagram of the perturbed model.

$$\begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}; A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M_0 + m_0}{M_0} g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m_0 g}{M_0} & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{m_0 g}{M_0 l_0} & \frac{g}{l_0} & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{m_0 g}{M_0} & 0 & 0 & -1 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ -\frac{1}{M_0 l_0} \\ 0 \\ \frac{1}{M_0} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{M_0 + m_0}{M_0 l_0} g & 0 & 0 & 0 \\ -\frac{m_0 g}{M_0} & 0 & 0 & 0 \\ \frac{M_0 + m_0}{M_0 l_0} g & 0 & 0 & 0 \end{bmatrix}; C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{m_0 g}{M_0 l_0} & \frac{g}{l_0} & -1 & 0 \\ -\frac{m_0 g}{M_0} & 0 & 0 & -1 \end{bmatrix}; D_{12} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_0 l_0} \\ \frac{1}{M_0} \end{bmatrix}$$

$$D_{21} = [0], \quad D_{22} = [0]$$

4 Stability robustness

Consider the inverted pendulum in closed loop represented as a linear fractional transformation as in figure

5. The input-output transfer relation will be as in equation 9 where N is considered partitioned as in equation 10 and $e = Fr$.

$$F = F_u(N, \Delta) \triangleq \triangleq N_{22} + N_{21} \Delta (I - N_{11} \Delta)^{-1} N_{12} \quad (9)$$

$$N = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \quad (10)$$

Since the nominal system, N , and the perturbation Δ are stable, then the perturbed system will be stable if and only if $(I - N_{11} \Delta)$ has a stable inverse, that is, the determinant $\det(I - N_{11} \Delta)$ should never pass through zero. We will use $M = N_{11}$.

The stability analysis can be carried out with the help of the structured singular value, defined as in equation 11. We will compute the smallest Δ , measured in terms of $\bar{\sigma}(\Delta)$, with the appropriate structure that will destabilize the loop. The loop will be stable to all perturbations, Δ_p , with the adequate structure and having $\bar{\sigma}(\Delta_p) < (\mu_\Delta(M))^{-1}$.

$$(\mu_\Delta(M))^{-1} = \min \{ \bar{\sigma}(\Delta) : \Delta \in \Delta, \det(I - M\Delta) = 0 \} \quad (11)$$

In figure 5 the K_2 matrix selects the state error component that corresponds to the cart position.

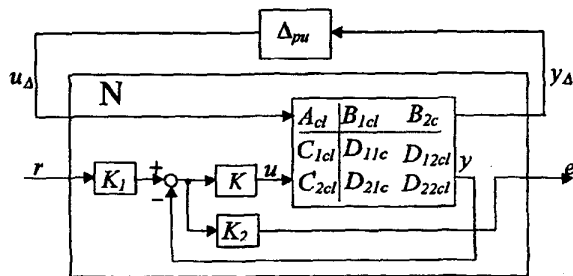


Figure 5: The closed loop in the N block.

We can compute the state space representation of the system N and extract the M system as $M(s) = N_{11}(s)$. We then use $\mu_\Delta(M)$ to determine the robustness margin of the closed loop.

In our present case, all the perturbations in the Δ block are real. The μ function is not necessarily a continuous function when all the perturbation blocks are real [2], [1]. That may cause problems in the convergence of the algorithms used to compute the μ lower bound. In fact in our case there are problems in computing the lower bound considering only real perturbations. To overcome this problems we use a trick suggested in [1], and add small complex uncertainties to the problem to give the computation better numerical properties.

The upper bound of $(\mu_\Delta(M))$ computed considering only real perturbations was $(\mu_\Delta(M)) < 1.8$. This means that no real Δ perturbation with the required structure that satisfies $\bar{\sigma}(\Delta) < 1/1.8 \approx 0.55$ will destabilize

the system. This can be a little conservative, meaning that maybe larger perturbations are allowed. To gain a better knowledge of the μ bounds some small complex perturbations were added and new μ upper and lower bounds were computed again. We also observed the destabilizing perturbations for several frequencies and for different values of the small complex perturbations. The conclusions are that the upper bound computed with only real perturbations is not too conservative. This means that each parameter may change as much as 55% without causing instability.

We would like to detect changes greater than a certain initial uncertainty, say 20% in each parameter. In view of that some experiences were made to see how much each parameter may change before generating instability if the others stay inside the initial uncertainty interval. The results are as follows:

- Parameter M can augment 88% ($\delta_M < 0.88$) before instability if $|\delta_m| \leq 0.2$ and $|\delta_l| \leq 0.2$;
- Parameter l can augment 100% ($\delta_l < 1.07$) before instability if $|\delta_M| \leq 0.2$ and $|\delta_m| \leq 0.2$;
- Parameter m can augment as much as 20 times its nominal value before instability if $|\delta_l| \leq 0.2$ and $|\delta_m| \leq 0.2$;
- All parameters may decrease as far as they do not become zero.

In view of this and the above results it seems feasible to design a filter that enable us to detect parameter changes before they cause stability problems. We may even be able to identify the parameter that changed. We could consider also the problem of keeping performance of the closed loop but for the moment we will care only about stability.

5 Fault Detection and Identification

What we want to do is to detect and if possible identify parametric faults. These are changes in the parameters that go beyond an initial uncertainty ball boundary. We want to detect parameter changes before they become too big, in our case, before they compromise the stability of the closed loop system.

Consider a process with uncertain parameters represented as an upper linear fractional transformation. Consider also that we design a filter to estimate the fictitious signals, f_p , that are outputs of the generalized system and inputs of the perturbation block. The setup used to design the filter $F(s)$ is represented in figure 6 and the filter can be designed to be robust to the initial uncertainty set by μ -synthesis [10]. With μ -synthesis we perform a search for a filter that minimizes the H_∞ norm from d to e , considering all the allowable perturbations. In this context "allowable perturbations" are the initial parameter uncertainty, Δ_{par} .

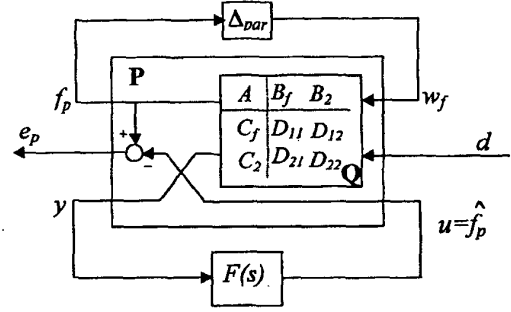


Figure 6: The residual generation setup.

If we have a filter that is able to estimate the signal f_p well we can try to do fault detection and identification. One way to do it is to estimate the elements in Δ_{par} using the estimate of \hat{f}_p and the structure of the uncertainty block.

5.1 Estimating the Δ elements

In our case study the Δ matrix has a very special structure and that structure may be used to try to estimate the elements of the block based on good estimates of \hat{f}_p . The components of f_p can be related to the inputs w_f and d of the generalized system in figure 6 by the matrices described in equations 8. The relations obtained are in equations 12 to 15.

$$f_{p1} = f_{p2} = x_1 \quad (12)$$

$$f_{p3} = \frac{M_0 + m_0}{M_0 l_0} g x_1 + \frac{m_0 g}{M_0 l_0} w_{f1} + \frac{g}{l_0} w_{f2} + w_{f3} - \frac{1}{M_0 l_0} u \quad (13)$$

$$f_{p4} = \frac{-m_0 g}{M_0} x_1 - \frac{m_0 g}{M_0} w_{f1} - w_{f4} + \frac{1}{M_0} u \quad (14)$$

$$f_{p5} = \frac{M_0 + m_0}{M_0 l_0} g x_1 + \frac{m_0 g}{M_0 l_0} w_{f1} + \frac{g}{l_0} w_{f2} + w_{f3} - w_{f5} - \frac{1}{M_0 l_0} u \quad (15)$$

Assume we know the f_{pi} (we know its estimates). We measure x_1 and we also know the input u . Then we can write equations 13 to 15 as a system of equations with unknowns w_{fi} . That system is in equations 16 where $b_1 = f_{p3} - \frac{M_0 + m_0}{M_0 l_0} g x_1 + \frac{1}{M_0 l_0} u$ and the other b_i and a_{ij} are determined in a similar way from equations 13 to 15 using known constants and measured or estimated signals.

$$\begin{aligned} b_1 &= a_{11} w_{f1} + a_{12} w_{f2} + a_{13} w_{f3} \\ b_2 &= a_{21} w_{f1} + a_{24} w_{f4} \\ b_3 &= a_{31} w_{f1} + a_{32} w_{f2} + a_{33} w_{f3} + a_{35} w_{f5} \end{aligned} \quad (16)$$

Expressing each w_{fi} as $w_{fi} = \delta_i f_{pi}$ we can write equations 17 and 18. From this last equation it is possible to estimate the three different elements in the Δ matrix,

δ_m , δ_M , and δ_l . That is, if we have good estimates of the components in \hat{f}_p .

$$\begin{aligned} b_1 &= a_{11}\delta_m f_{p1} + a_{12}\delta_M f_{p2} + a_{13}\delta_l f_{p3} \\ b_2 &= a_{21}\delta_m f_{p1} + a_{24}\delta_M f_{p4} \\ b_3 &= a_{31}\delta_m f_{p1} + a_{32}\delta_M f_{p2} + a_{33}\delta_l f_{p3} + a_{35}\delta_l f_{p5} \end{aligned} \quad (17)$$

$$\begin{aligned} b_1 &= c_{11}\delta_m + c_{12}\delta_M \\ b_2 &= c_{21}\delta_m + c_{22}\delta_M \\ b_3 &= c_{31}\delta_m + c_{32}\delta_M + c_{33}\delta_l \end{aligned} \quad (18)$$

With the setup just described we can estimate the distance of each parameter to its nominal value and, hopefully, detect and identify parametric failures.

The elements c_{ij} in 18 are as defined in 19 and the solutions to the equations 18 are expressed in equations 20 to 22. Note that f_{p1} and f_{p2} are measured as they are equal to the first state of the inverted pendulum.

$$\begin{aligned} c_{11} &= a_{11}f_{p1}, \quad c_{12} = a_{12}f_{p2} + a_{31}\hat{f}_{p3}, \\ c_{21} &= a_{21}f_{p1}, \quad c_{22} = a_{24}\hat{f}_{p4}, \quad c_{31} = a_{31}f_{p1} \\ c_{32} &= a_{32}f_{p2} + a_{33}\hat{f}_{p3}, \quad c_{33} = a_{35}\hat{f}_{p5} \end{aligned} \quad (19)$$

$$\delta_m = \frac{(c_{12}b_2 - c_{21}b_1)}{(c_{12}c_{21} - c_{22}c_{11})} \quad (20)$$

$$\delta_M = \frac{b_1}{c_{12}} - \frac{c_{11}}{c_{12}}\delta_m \quad (21)$$

$$\delta_l = \frac{1}{c_{33}}(b_3 - c_{31}\delta_m - c_{32}\delta_M) \quad (22)$$

Note however that equations 20 to 22 are not defined when the c_{ij} that appear in the denominator are zero. That happens when the corresponding f_{pi} signal is zero has can be seen from equations 19. The computation of the estimates of δ_m , δ_M and δ_l must account for this restriction.

6 Filters

We want to design a filter to estimate the f_p signals as shown in figure 6. That filter should be robust to some uncertainty in the parameters, that is, it must have an acceptable performance even in the presence of uncertainty. The main idea is to minimize the H_∞ norm from d to e . One way to express the required performance is to define the amplitude of the frequency response we want from d to the estimation error, e . After choosing those weighting factors μ -synthesis procedure can be used to design the filter or we can use an H_∞ synthesis and then verify if the performance robustness is sufficient. If it is, the filter have been determined, if not, other iterations have to be done.

As the design process is not very simple and includes the choice of weighting factors that need to be determined and some other details necessary to meet the applicability conditions of the algorithms to be used we chose to proceed in smaller steps. First H_∞ filters are designed to estimate each one of the signals in f_p . This will allow an easier choice of weighting factors and a better idea of the possible performance of each filter. Next a global

H_∞ filter is designed based on the previous designs. Finally a μ -synthesis procedure could be used to design a μ filter to deal explicitly with the uncertainty.

More on μ -synthesis and H_∞ design can be found in [9] or [12].

6.1 For the H_∞ design

The design setup for the H_∞ filter is shown in figure 7. In that figure $W(s)$ is a weighting factor that helps to define the desired performance of the filter and Q is the process shown in figure 8. The design will minimize the H_∞ norm of the transfer relation from $[r \ r_m]^T$ to z .

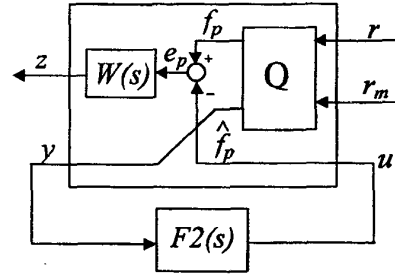


Figure 7: General design configuration.

In the Q process shown in figure 8 the signal r_m denotes measurement noise that was added to satisfy some applicability conditions of the algorithms used in the design, namely conditions over a part of the D matrix. The noise magnitude should be chosen small. In our case, for example it can be chosen to have the magnitude of 0.3% of the maximum value taken by the affected measure when the reference input is a unitary step. All the four state variables are considered measured.

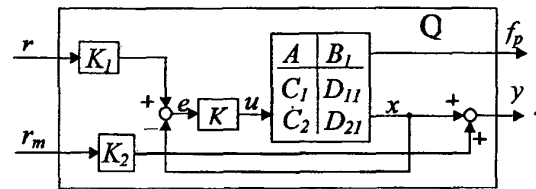


Figure 8: Base process for filter design.

6.2 A filter computed via H_∞

We want to use estimates of f_p to do parametric fault estimation. As can be seen from equations 12 to 15 we only need to estimate the three last elements of f_p . The first two are measured.

We selected one weighting factor for each of the three signals we want to estimate, f_{p3} , f_{p4} and f_{p5} . The weights are shown in equations 23 and 24. The final value achieved in the design for the γ parameter is $\gamma = 0.87$. This γ value indicates some robustness margin, meaning the filter can satisfy the performance specified by the weighting factors in the presence of some

uncertainty.

$$W_4(s) = \frac{4 \times 10^4 (s + 0.001)}{(s + 0.01)^2 (s + 10)^3} \quad (23)$$

$$W_3(s) = W_5(s) = \frac{12500}{(s + 10)^3} \quad (24)$$

The weights were chosen by means of several iterations, starting with the observation of the transfer functions from the reference input to the signal to be estimated and observing the performance. The filter obtained has 15 states, 3 outputs and 4 inputs. The performance we could obtain with the above weighting factors corresponds to a frequency response from the reference input to the estimation error of magnitude less than 10^{-3} for all frequencies below $10 \text{ rad}\cdot\text{s}^{-1}$. The problem is that the signals themselves are also small in small frequencies. A more useful measure of performance is the frequency response of a relative error. This relative error is defined as the gain from the reference to the estimation error divided by the gain from the same reference to the signal we are estimating. The relative estimation error along the frequency, for each of the 3 channels has a similar shape. In figure 9 we shown it for channel 4. This relative errors were useful in choosing the weighting factors. As we can see from the relative error plot in figure 9, only the estimates in certain frequency ranges are good enough to be used for the computations indicated in section 5.1. The estimates must be appropriately filtered to be used in those computations. Simulations will show how good can be the values of δ_m , δ_M and δ_l calculated using the equations in section 5.1 and the estimates generated by this filter. If they are good enough they can be used to detect parameter changes.

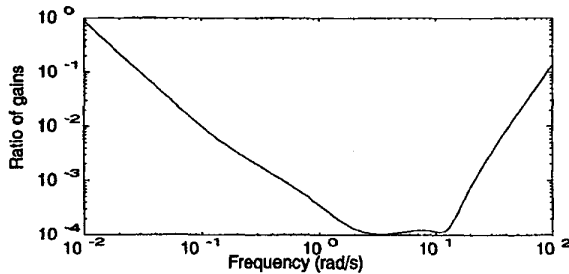


Figure 9: Relative error, channel 4.

7 Conclusions

H_∞ and μ techniques may be used to design estimation filters for parametric fault detection. They also allow verification of robustness properties of the filters. That is important because the filters should give good estimates of the signals of interest even in the presence of small parameter variations representing uncertainty. For the study case, the particular structure of the equa-

tions obtained with the upper linear fractional transformation representation of parameter variations allows the computation of estimates of two of those parameter variations from estimates of a set of fictitious signals, f_p that can be estimated from measured and/or known quantities and signals.

The same technique may be applied to other systems in a similar way. Although in this paper no external disturbances were considered they may easily be included in the formulation.

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