Parametric uncertainty with perturbations restricted to be real on 12 CD mechanisms

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Abstract

12 actual CD mechanisms that differ by having worst-case behaviors with respect to various properties are considered. An uncertainty model is based on a parametric description of plants where the perturbations are restricted to be real and μ-theory is applied to design a controller meeting the worst-case focus deviations. Satisfactory results were achieved in the 12 CD mechanisms with the synthesized controller after a controller order reduction.

1 Introduction

Optical Disc Drives (ODD) are mainly characterized by the absence of the physical contact between the pick-up and the disc. Feedback control is necessary to control the position of the focus point of the laser in order to read the data. Two main control loops can be identified: the focus loop which maintains the focus point of the laser on the signal layer, and the radial loop which follows the track. Since the compact disc player was introduced in the market two decades ago, more products based in the same technology have been developed, as CD-ROM and DVD players having a track pitch of 1.6 μm and 0.78 μm respectively. Performance requirements to the position controllers have therefore increased at the same time. In parallel to the development of ODD's, much effort has been spent to solve the multivariable robustness analysis and synthesis where different classes of uncertainties have been considered. Unstructured uncertainties (full-block complex perturbation uncertainties) can be used in the H_∞ framework, see [DGKF89]. In general, a less conservative controller is achieved if the control problem is formulated in the μ framework which considers structured uncertainties. Common to both approaches is the description of model uncertainties as transfer functions, which are norm-bounded but otherwise unknown. Parametric uncertainty representation in the H_∞ framework is an even less conservative approach. However, parametric uncertainty is often avoided among other things because it usually requires a large effort to model parametric uncertainty, the exact model structure is required and real perturbations are required, which are more difficult to deal with mathematically and numerically, especially when it comes to controller synthesis. Therefore, parametric uncertainty is often represented by complex perturbations, see two applications on CD-players [SWvGB94] and [SSB92]. This is of course conservative as it introduces possible plants that are not present in the original set. In order to avoid conservatism (and increase performance) a parametric uncertainty representation is chosen in this paper where the perturbations, Δ, are restricted to be real. The synthesized controller was reduced from a 94th order to a 3th order controller without loosing robust performance and was successfully implemented in the 12 CD mechanisms. The reminder of this paper is organized as follows. In Section 2 a 4th order model of the optical pick-up is presented. The nominal and uncertainty models are described in Section 3. In Section 4 the performance requirements are specified and in Section 5 the controller is synthesized. The results are presented in Section 6 and discussed in Section 7. Finally, the findings of this paper are summarized in Section 8.

2 Model of the optical pick-up

The optical pick-up is a 2-axis device, enabling a movement of the lens in two axes: vertically for focus correction and horizontally for track following. Two coils, which are perpendicular to each other are suspended between permanent magnets. A current through a coil creates a magnetic field which repeals with the magnetic field from the permanent magnet and the coil and consequently the lens will move in the corresponding direction. The pick-up can be considered as two moving masses, see Figure 1. m_1[Kg] and m_2[Kg] represent respectively the lens and the arms of the levers on which the lens is suspended. x[m] and x'[m] are their corresponding displacements from a fixed point of the optical pick-up. K_1[N/m] and K_2[N/m] are the spring moduli and C_1[N·s/m] and C_2[N·s/m] are the viscosity coefficients. f_{el}[N] is the force applied to m_1[Kg], given by eq. 1,

$$f_{el}(s) = \frac{-(Bl^2 \cdot x)s + Bl \cdot u}{R} \tag{1}$$

where Bl[N/A] is the AC-sensitivity, u[V] the voltage applied to the coil and $R[\Omega]$ the resistance of the coil. The transfer function from u[V] to x[m] is then given by,

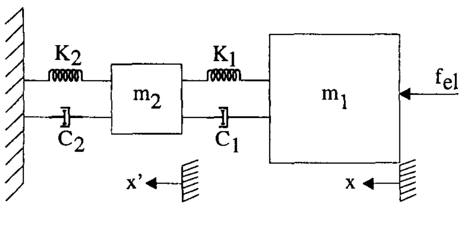

Figure 1: Translational diagram of the optical pick-up.

$$F(s) = \frac{X(s)}{U(s)} = G \cdot \frac{b_0 s^2 + b_1 s + b_2}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \qquad (2)$$

where G the gain composed by the electronics and the optical gain. Table 1 shows the coefficients of the polynomials described in eq. 2

b_0	$m_2 Bl$
b_1	$Bl(C_1 + C_2)$
b_2	$Bl(K_1 + K_2)$
a_0	$Rm_1 m_2$
a_1	$Bl^2 m_2 + Rm_1 C_1 + Rm_1 C_2 + Rm_2 C_1$
a_2	$Bl^2(C_1+C_2) + Rm_1(K_1+K_2) + Rm_2 K_1 + RC_1 C_2$
a_3	$Bl^2(K_1+K_2) + Rm_1 C_2 + RK_2 C_1$
a_4	$RK_1 K_2$

Table 1: Coefficients of the optical pick-up.

The presented transfer function in eq. 2 is valid for focus and radial actuators. It has been chosen to study the focus loop only. Similar results however transfer immediately to the radial loop.

3 Model parameterization of 12 CD mechanisms

12 CD mechanisms that differ by having worst-case behaviors with respect to various properties were analyzed. An open and closed loop parametric system identification method described in [VSA+01] was performed yielding 12 Nyquist/bode plots which describe the dynamics of the focus actuators. A relatively simple way to model uncertainty is to represent it in the frequency domain using complex norm-bounded perturbations. This method is usually conservative as it introduces possible plants that are not present in the original set. In this paper only parametric uncertainties are used in order to allow a more accurate, and consequently less conservative, representation of the CD mechanism's dynamics.

Nominal model

With parametric uncertainty the most straightforward option is probably to make a nominal model of mean parameter values. Obviously the mean parameter values must be available. If this is not the case, another option would be to consider the Nyquist plots of the plants and encapsulate all the plants at each frequency point with a convex geometric figure. Usually the geometric figure is a disc and the center of each disc at each frequency point yield the nominal model. In this work the second option is chosen but instead of discs, another geometric figure is employed, composed by a rectangle and two semicircles, one at each end of the smallest side of the rectangle. The method is described in [VSA+02]. The resulting nominal model is a fifth order model composed by the fourth order transfer function with the structure given by eq. 2 plus an extra pole at 5[kHz] which describes the dynamics of the first order antialiasing filter. The nominal parameters are the following: $\overline{m_1}$=5.3·10^{-4}[Kg], $\overline{m_2}$=2.97·10^{-5}[Kg], $\overline{R}$=20[Ω], $\overline{Bl}$=0.2375[N/A], $\overline{C_1}$=0.0013[N·s/m], $\overline{C_2}$=0.0117[N·s/m], $\overline{K_1}$=266.99[N/m], $\overline{K_2}$=21.44[N/m], $\overline{G}$=9350.

Uncertainty model

Assuming that the total moving mass of the optical pick up is known, there are 8 parameters which can vary, namely the spring moduli K_1[N/m] and K_2[N/m], the viscosity coefficients C_1[N·s/m] and C_2[N·s/m], the coil resistance R[Ω], the AC-sensitivity Bl[N/A], the mass distribution factor between m_1[Kg] and m_2[Kg] and finally the gain G. Apart of identifying the location of zeros and poles with the mentioned method in [VSA+02], gain variations are also identified. The controller should be robust against these gain variations. In practice, a large number of CD-players have a PID-like controller structure with an adaptivity scheme in charge of reducing the gain uncertainty near the bandwidth of the controller. It would be conservative to model such gain uncertainty and the Nyquist plots were therefore adjusted above 200[Hz] such that the gain variation was approx. below 10%. At the time of synthesizing the controller it is normally preferred to have a low number of parametric uncertainties in order to reduce the computation time and order of the controller. Not all the parameters have the same impact on deviations with respect to the nominal model. Each parameter was analyzed and it was observed that three parameters were sufficient to cover approximately the same Nyquist area, not adding therefore more conservatism to the uncertainty description. The parameters together with theirs variations with respect to the nominal parameters are: the resistance R[Ω] (7%) and the spring moduli K_1[N/m] (15%) and K_2[N/m] (15%).

4 Performance specification

The aim of the focus controller is to maintain the focus point on the signal layer in order to retrieve data from the compact disc. Disturbances in the focus loop of compact disc players can be classified in two groups, disturbances which the controller should suppress: like shocks and acoustic feedback from speakers. The second group is constituted by disturbances which the controller should not react against like scratches, finger prints and dust. The first group requires a higher closed loop bandwidth than the second group. If the closed loop has a high bandwidth the controller can have a good performance in suppressing shocks but robustness is deteriorated and as another side effect the controller will be more prone to react wrongly against scratches resulting in data loose. This imposes conflictive requirements to the closed loop bandwidth of the system. In the sequel only a part of the first group of disturbances is considered, namely the vertical deviations of the compact disc. Any CD-player should at least be able to cope with this kind of disturbances. However portable CD-players should have a higher bandwidth to better cope with shocks at the expenses of a more elevated battery consumption. The worst-case vertical deviations, according to the specifications for Compact-Disc Digital Audio [Sta98], are shown in table 2.

Conditions	Parameter	Specification
below 500 [Hz]	Max. deviation	± 500 [μm]
	Max. vertical accel.	10 [m/s^2]
above 500 [Hz]	Max. deviation	± 1.0 [μm]

Table 2: Standardized vertical deviations from nominal position of the information layer specified at the disc scanning velocity $v_a = 1.2 \ldots 1.4$[m/s]

The vertical deviation $w(t)$ is modeled as a harmonic and by differentiating it twice, an expression for the acceleration $\ddot{w}_(t)$ is obtained.

$$\begin{aligned} w(t) &= A_w \sin(2\pi f t) \\ \dot{w}(t) &= A_w 2\pi f \cos(2\pi f t) \\ \ddot{w}(t) &= -A_w (2\pi f)^2 \sin(2\pi f t) \end{aligned} \tag{3}$$

where A_w is the maximum amplitude of the vertical disturbance, which is limited by the maximal acceleration $\ddot{w} = 10[m/s^2]$. That is, the highest frequency below 500[Hz] where $A_w = 500[\mu m]$ is given by:

$$\frac{1}{2\pi}\sqrt{\frac{max(|\ddot{w}|)}{max(|A_w|)}} = \frac{1}{2\pi}\sqrt{\frac{10}{500 \cdot 10^{-6}}} \simeq 22.5[Hz] \tag{4}$$

Above 22.5[Hz] the amplitude is limited by the specified maximum acceleration up to the frequency where the maximum vertical disturbance is $A_{xd} = 1[\mu m]$.

$$\frac{1}{2\pi}\sqrt{\frac{max(|\ddot{w}|)}{max(|A_w|)}} = \frac{1}{2\pi}\sqrt{\frac{10}{1 \cdot 10^{-6}}} \simeq 503.3[Hz] \tag{5}$$

Above 503.3[Hz] the maximum vertical disturbance is $A_w = 1[\mu m]$. For clarity, the restrictions on the vertical deviations are visualized in Figure 2.

The specifications given in table 2 can be formulated in a natural way as the requirements for the output sensitivity function of the focus loop $S = (I + GK)^{-1}$. The focus error, z, should not be higher than 2[μm] [BBHP85], that is $z = Sw < 2[\mu m]$, which imposes the following requirement

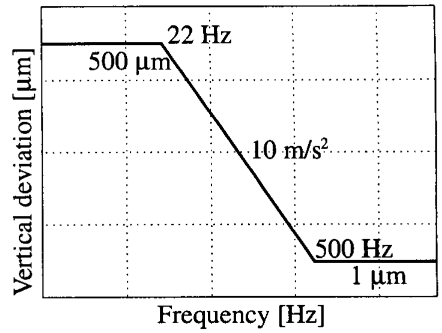

Figure 2: Graphical representation of the vertical deviation of the disc.

to the sensitivity function: $S \leq w^{-1}2 = W_P$. W_P is the performance weight given by the "inverse" of the vertical deviation of the disc.

$$W_P(s) = \frac{(s/M^{1/3} + w_B)^3}{(s + w_B A^{1/3})^3} \cdot \frac{(s + w_I)}{(s + 0.001)} \tag{6}$$

where M=2, A=0.004 and w_B=$2\pi \cdot 230$ [rad/sec]. A has been chosen a factor two lower than what it is required in order to better attenuate lower frequency disturbances. As it is desired to eliminate constant steady-state errors, an integrator is added. The integrator is slightly moved to the left half plan otherwise it becomes an uncontrollable pole of the feedback system and the H_∞ assumptions are violated [aJCDG96]. In the same reference a reformulation of the control problem is given if it desired to have a pole in the imaginary axis. w_I=$2\pi \cdot 10$[rad/sec] is in charge of removing the integral action. The specifications on the sensitivity function put a lower bound on the bandwidth but not an upper one, and nor does it allow to specify the roll-off of $L(s)$ above the bandwidth. Usually another performance weight is specified, for example on the complementary sensitivity or on the control signal. For simplicity, only the sensitivity weight is specified in this paper and as the results show later, a satisfactory controller is obtained.

5 mixed-μ synthesis

All the existing algorithms for μ-synthesis are based on H_∞ synthesis, see e.g. [DGKF89]. In the following only parametric uncertainties where perturbations are restricted to be real are used in order to allow a more accurate and consequently less conservative representation of the CD mechanism's dynamics. A synthesis procedure based on the parametric uncertainty model proposed in this paper relies on mixed μ-synthesis, i.e. optimization for systems with both real and complex uncertainties. Examples of algorithms for mixed μ-synthesis are [YD90, YND92] and [TCASN95]. The latter one, which involves solving a series of scaled DK-iterations is used in this work.

Figure 3 depicts the generalized plant with parametric uncertainties. The task of the positioning controllers in CD

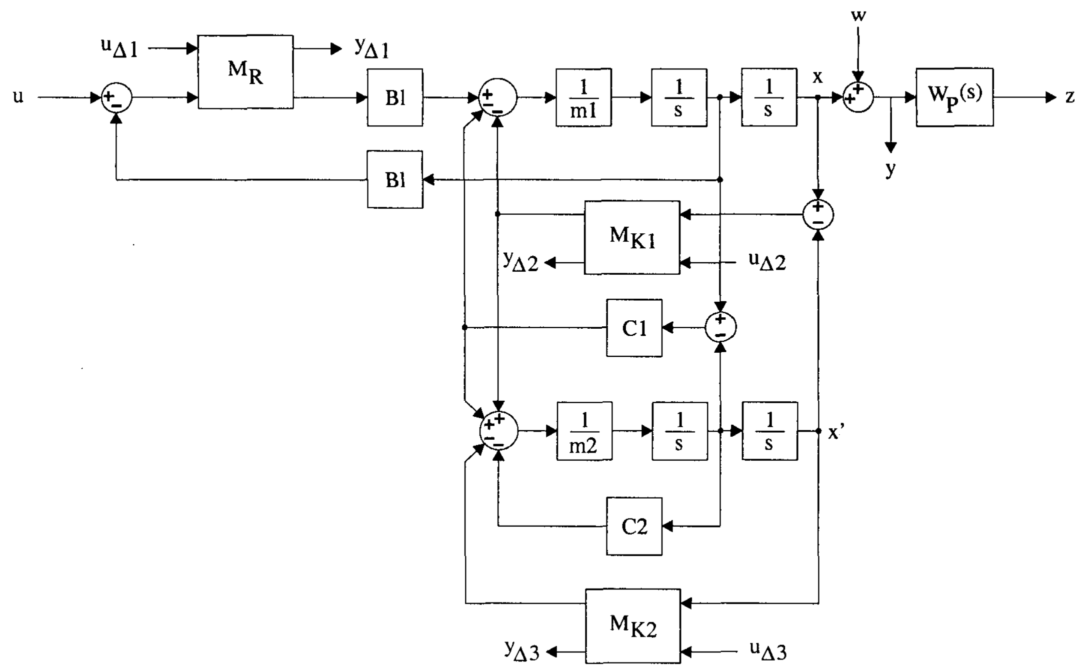

Figure 3: Generalized plant with parametric uncertainties and performance weight on the output sensitivity.

players is to minimize the measurable distance, y, between the position of the focus point x, and the position of the signal layer, w, where the information is contained. The performance weight, $W_P(s)$, which is the inverse of the desired sensitivity function is therefore placed at the output. The parameter uncertainties on the resistance $R[\Omega]$ (7%) and the spring moduli K_1[N/m] (15%) and K_2[N/m] (15%) are described by respectively one upper and two lower fractional transformations. These parameters are allowed to vary as given below with perturbations restricted to be real:

$$
\begin{aligned}
R &= \overline{R}(1+0.07\delta_R) & -1 \le \delta_R \le 1\\
K1 &= \overline{K1}(1+0.15\delta_{K1}) & -1 \le \delta_{K1} \le 1\\
K2 &= \overline{K2}(1+0.15\delta_{K2}) & -1 \le \delta_{K2} \le 1
\end{aligned}
$$

The complex uncertainties comes into picture when performance requirements are specified, in this case on the output sensitivity, see section 4. Robust performance is investigated by performing a robust stability analysis on the structure shown in Figure 4.

$\hat{\Delta}$ is composed by the diagonal parametric uncertainty matrix and a complex scalar $\delta_P \in \mathbf{C}, \quad |\delta_P| \le 1$. The task of the algorithm for mixed-μ synthesis is to find a stabilizing controller K, which satisfies following conditions, where $N(s)$ is the result of the lower fractional transformation of the generalized plant and the controller, $N(s) = F_l(P,K)$. (NS stands for nominal internal stability).

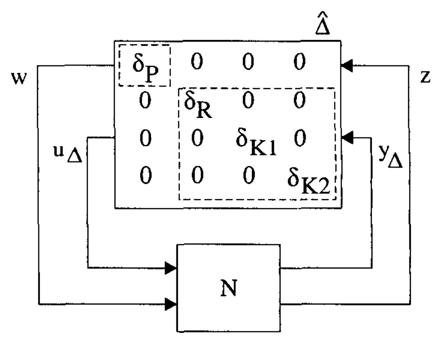

Figure 4: Robust stability analysis

1. Nominal performance:

$$NP: \quad \overline{\sigma}(N_{22}) = \mu_{\Delta_P} < 1, \quad \forall\omega, \quad and \quad NS$$

2. Robust stability:

$$RS: \mu_{\Delta}(N_{11}) < 1, \quad \forall\omega, \quad and \quad NS$$

3. Robust performance:

$$RP: \mu_{\hat{\Delta}}(N) < 1, \quad \forall\omega, \hat{\Delta} = \begin{bmatrix} \Delta_P & 0 \\ 0 & \Delta \end{bmatrix} \quad and \quad NS$$

6 Results

Figure 5 shows the obtained worst-case bode plot from the system identification method described previously mentioned (gain uncertainty has been reduced above 200[Hz]) and it also shows the worst-case bode plot which the parametric uncertainties cover. Below 100[Hz] the bode plots

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are basically not discernable from each other. Above 100[Hz] the *noisy* bode plot is the measured one and the smooth bode plot corresponds to the one described mathematically with the parameter uncertainties.

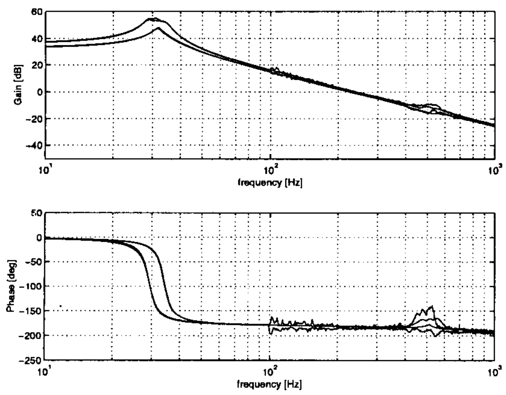

Figure 5: Measured worst-case bode plot of the 12 CD mechanisms and worst-case bode plot described by the parametric uncertainties.

Figure 6 illustrates the stabilizing 94th order (solid) controller synthesized by mixed μ-synthesis algorithm. The dashed trace is the reduction to a 3th order controller.

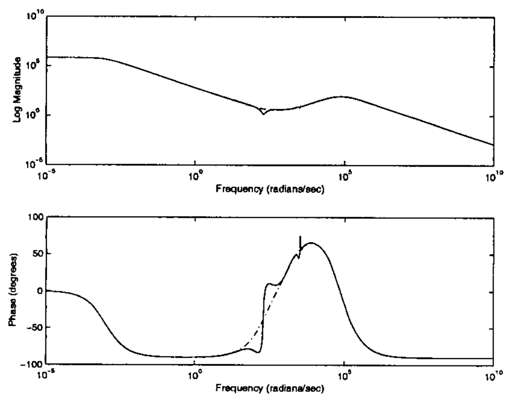

Figure 6: 94th order controller (solid) and 3th order controller (dashed).

Figure 7 shows the plot of $\mu\hat{\Delta}(N)$ along frequency for the 94th order controller (solid) and for the 3th order controller (dashed).

Figure 8 illustrates the simulated sensitivity plot of the 12-CD mechanisms (solid) and the plot of the sensitivity requirements (dashed) where the integrator effect on the weight can be observed.

Finally table 3 shows the *gain margin* of each CD player, understood in the sequel as the maximum allowable gain increase/decrease of the controller with respect to its nominal gain before instability is met. The gain of each controller

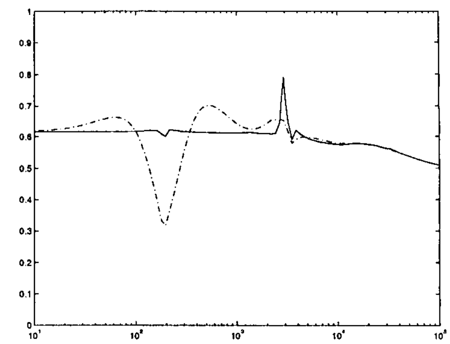

Figure 7: Robust performance of the 94th order controller (solid) and robust performance of the 3th order controller (dashed).

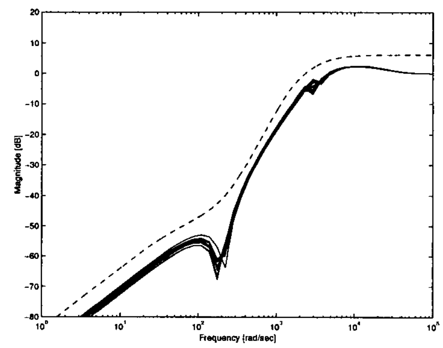

Figure 8: Sensitivity plot of the 12 CD mechanisms (solid) and plot of the sensitivity requirements (dashed).

was increased and decreased until the instability points where meet, which yield in two gain margins. In the table, the lowest gain margins are shown.

7 Discussion

A nominal model was obtained in the work as a result of fitting the center of the chosen geometric figures in Section 3 with a rational function. The obtained parameters from that fit were compared to the nominal parameters from the data sheet of the CD drives and it could be concluded that realistic values were obtained. From Figure 5 it can be seen that three mentioned parameters are sufficient to describe the uncertainties in the 12 CD mechanisms. However the phase uncertainty is not fully covered which indicates that the uncertainty description is partially optimistic around 500[Hz]. The synthesized controller had an elevated order (94th), therefore is was reduced significantly but without deteriorating noticeably the properties of the controller. This reduction gives in fact a better robust performance, see Figure. 7, specially around 30[Hz] where the first resonance peak is present. This is a consequence of the mixed μ-synthesis algorithm which tends to flatten the open loop gain $L(s)$. Intuitively, the open loop gain around 30[Hz] should have a high gain independently on the phase uncer-

CD mechanism	gain margin [dB]
CD1	15.7
CD2	15.5
CD3	16.4
CD4	15.0
CD5	15.2
CD6	13.1
CD7	15.4
CD8	15.8
CD9	15.0
CD10	15.8
CD11	14.3
CD12	15.1

Table 3: Measured gain margin in each CD mechanism.

tainty. The 3th order controller is not able to flatten this peak and therefore has a better nominal and robust performance. It is worth to mention that a large amount of CD-players in the electronic market have this 3th order controller structure in the servos of the optical pick-up. From the robust performance analysis shown in the paper it can explain why this controller structure has a wide acceptance among the CD-player producers: it is simple yet effective. The spike on the 94th order controller around 500[Hz] could lead to think that the algorithm did not run a sufficient high number of iterations. Efforts were made by the authors of the algorithm, see [TCASN95], to reduce, at the best avoid, this "pop-up" type phenomenon by giving the possibility to select a user-defined parameter, α, for each iteration. Many iterations were evaluated where the only parameter changed was α and the here presented controller was the best achieved. The gap up to 1 indicates that a tighter performance weight could have been chosen, which is also corroborated by the sensitivity plots, see fig. 8. That is, there is room to optimize the 3th order controller to cope not only with the vertical deviations of the disc (self-pollution) but also, up to some extent, with mechanical disturbances as shocks. The 3th order controller was tested in 12 different CD mechanisms and the gain margin was calculated, see table 3. Care should be taken in how to understand these values. It is very difficult to perform the test in a controlled way such that the disturbances acting on each CD mechanism are completely equal. Therefore a detailed comparison of the results presented in the table should be avoided. What it can be concluded at least is that the implemented controller had very similar gain margins on each CD mechanism in spite of the fact that they had worst-case behaviors. It does not seem that the challenges in designing the controllers for the optical pick-up in CD-players consist in the ability to cope with parameter uncertainties but in the ability to cope with the conflicting groups of disturbances: high bandwidth to better attenuate mechanical shocks and low bandwidth to be more immune against surface defects.

8 Conclusion

12 actual CD mechanisms that differ by having worst-case behaviors with respect to various properties were considered. The uncertainty model is based on a parametric description of plants where the perturbations are restricted to be real and μ-theory was applied to design a controller meeting the worst-case focus deviations. A 3th order controller was obtained as a result of an order reduction of the controller. Interestingly the structure of the reduced controller is same as the one widely used in the electronic market. From the results it seems that it is not difficult to design a controller able to cope with the parameter uncertainties. The challenges are in finding a controller which is able to cope with disturbances of opposed bandwidth requirements.

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