Proceedings of the 42nd IEEE
Conference on Decision and Control
Maui, Hawaii USA, December 2003

Multi-Dimensional Gain Scheduling with Application to Power Plant Control

Jan Dimon Bendtsen Jakob Stoustrup Klaus Trangbæk
Department of Control Engineering, Aalborg University
Fredrik Bajersvej 7C, 9220 Aalborg East, Denmark.
Email: {dimon,jakob,ktr}@control.auc.dk

Abstract

This paper deals with gain scheduling control of a power plant model, which is an example of a multi-dimensional nonlinear system. Linear observer-based controllers are designed for a number of linear approximations of the system model in a set of operating points, and gain-scheduling control can subsequently be achieved by interpolating between each controller. We use the Youla-Jabr-Bongiorno-Kucera parameterization to achieve a smooth scheduling between the controllers. However, for multi-dimensional systems it is often not straightforward to obtain appropriate scheduling parameters. To address this issue, we propose a systematic approach to scheduling between controllers using multiple scheduling parameters. The approach is tested on a simple, but highly nonlinear model of a coal-fired power plant.

1 Introduction

Gain scheduling control is a celebrated approach to tracking control of "well-behaved" nonlinear systems, which has been employed in numerous control applications. See e.g. [5] for a general survey of gain scheduling control techniques. These schemes involve linearization of the system model in an appropriate set of operating points, followed by synthesis of one or more linear controllers for the system in these points, for instance using robust or optimal design methods (see e.g., [6]). However, it is important to note that, even if two stabilizing controllers K_1 and K_2 are designed for the same system, there is no guarantee that a simple linear combination of the two controllers $K = \alpha K_1 + (1-\alpha)K_2$, where $\alpha \in [0;1]$ is a scheduling variable, stabilizes the system for $0 < \alpha < 1$.

[3] provided a framework for gain scheduling control based on the Youla-Jabr-Bongiorno-Kucera (YJBK) parameterization of all stabilizing controllers. By using the YJBK parameterization of all stabilizing controllers for the interpolation it is possible to switch between individual stabilizing controllers in a stable manner. [1] elaborated upon this idea by proposing a scheme that, based on several linearized models extracted from an artificial neural network, provided the basis for the design of a number of controllers in different operating points and subsequent gain scheduling using the YJBK parameterization.

However, the results above considered the case where the scheduling parameter at any given time could be considered a scalar. For multi-dimensional nonlinear systems it is often not straightforward to obtain appropriate, simple scheduling parameters. To address this issue, we intend to expand further upon the aforementioned ideas and propose a systematic approach to scheduling between multiple controllers designed in different *combinations* of operating points, for instance in a p-dimensional grid. When the number of controllers is greater than p, the number of scheduling parameters may become larger than the number of measurements specifying the operating point. In this paper we propose a performance functional that yields a unique, continuous solution to the scheduling problem in any point in the operating space. When the current state of the plant is exactly at an operating point, the performance functional is minimized by choosing the controller in that operating point and all other scheduling parameters are set to 0. Elsewhere in the operating space the controllers are weighted such that the greatest emphasis is placed on the controller 'closest' to the current plant state. We also discuss how to handle integrators in this framework. The gain scheduling approach proposed in this paper is tested on a very simple, but highly nonlinear model of a coal-fired power plant.

2 Gain Scheduling Control

In this section, we will provide a brief review of the framework established in [3], on which we base the controller synthesis. We will provide all results in this section in discrete time, although they are equally valid in continuous time.

2.1 Basic Controller Parameterization

Consider the system G with the state space realization

$$G(z) = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_v & D_{vw} & D_{vu} \\ C_y & D_{yw} & D_{yu} \end{array}\right] \quad (1)$$

where $y \in \mathbb{R}^{p_y}$ is the measurement vector, $u \in \mathbb{R}^{m_u}$ is the control vector, $v \in \mathbb{R}^{p_v}$ is the signal to be controlled (which

0-7803-7924-1/03/$17.00 ©2003 IEEE

may coincide with y) and $w \in \mathbb{R}^{p_w}$ is a disturbance vector containing noise and command signals. If the subsystem G_{yu} given by the matrices (A, B_u, C_y, D_{yu}) is stabilizable and detectable, G can be stabilized by an observer-based feedback controller (see e.g. [6]). This setup is illustrated in the left part of Figure 1.

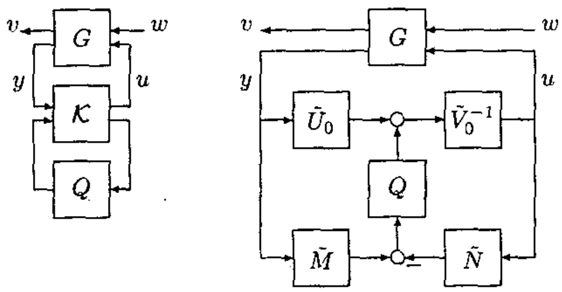

Figure 1: *Left: The interconnection of the system G and the observer-based controller $K(Q) = \mathcal{K} \star Q$, where $\star$ denotes the star product [6]. Right: The controller is implemented using coprime factorizations of the controller and system.*

Let $G_{yu}(z) = C_y(zI - A)^{-1}B_u + D_{yu}$ be written using coprime factorization as

$$G_{yu}(z) = NM^{-1} = \tilde{M}^{-1}\tilde{N} \quad (2)$$

with $N, M, \tilde{M}, \tilde{N} \in \mathcal{RH}_\infty$. Further, let a number of controllers for G_{yu} be given by

$$K_i(z) = U_iV_i^{-1} = \tilde{V}_i^{-1}\tilde{U}_i,\ i = 0, \dots, \nu \quad (3)$$

where $U_i, V_i, \tilde{U}_i, \tilde{V}_i \in \mathcal{RH}_\infty$. These coprime factorizations can be chosen to satisfy the double Bezout equation

$$\begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_i \\ N & V_i \end{bmatrix} = \begin{bmatrix} M & U_i \\ N & V_i \end{bmatrix} \begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

for $i = 0, \dots, \nu$. All stabilizing controllers for G_{yu} based on any stabilizing K_0 can now be written according to the YJBK parameterization

$$K(Q) = \mathcal{K} \star Q = K_0 + \tilde{V}_0^{-1}Q(I + V_0^{-1}NQ)^{-1}V_0^{-1} \quad (4)$$

i.e., the linear fractional transformation setup depicted in the left part of Figure 1. We now have the following result, adapted from [6].

Theorem 1 *Let a number of stabilizing controllers (3) be given for a system (2). Then $K_i, i = 0, \dots, \nu$ can be implemented as $K(Q_i) = \mathcal{K} \star Q_i$, with $Q_i \in \mathcal{RH}_\infty$ given by $Q_i = \tilde{U}_iV_0 - \tilde{V}_iU_0 = \tilde{V}_i(K_i - K_0)V_0$.*

Proof: Follows by inserting $Q_i = \tilde{V}_i(K_i - K_0)V_0$ in (4), rewriting the expression as

$$K(Q_i) = K_0 + \tilde{V}_0^{-1}\tilde{V}_i(I + (K_i - K_0)N\tilde{V}_i)^{-1}(K_i - K_0)$$

and using the Bezout identity to show that $I + (K_i - K_0)N\tilde{V}_i = \tilde{V}_0^{-1}\tilde{V}_i$. ∎

Theorem 1 states that it is possible to implement any stabilizing controller as a function of a stable parameter system Q based on another stabilizing controller, as depicted in the right part of Figure 1. This implies that it is possible to change between two controllers online, say, from K_0 to K_i, in a smooth fashion by scaling the Q_i parameter by a factor $\alpha \in [0; 1]$.

In this paper we employ the YJBK theory to change from a controller designed in one operating point to another controller designed in a different operating point of a nonlinear system. Thus we implicitly assume that the nonlinear system is sufficiently well-behaved for the resulting gain scheduled controller to stabilize it in between the operating points. This is not explicitly guaranteed by the YJBK-parameterization, which only ensures stability while changing controllers in one operating point.

2.2 Multi-dimensional Gain Scheduling

As long as the controller can be scheduled using a single parameter, e.g., a single output or reference signal, the implementation of the controller is fairly straightforward. The situation becomes more complicated when the scheduling has to take place using several scheduling parameters that are independent of each other, for instance if it is desired to control several outputs of a MIMO system independently of each other. The situation can be illustrated as shown in Figure 2, where y denotes a measurement and $\bar{(\cdot)}$ denotes an operating point. The left part of the figure shows a situation

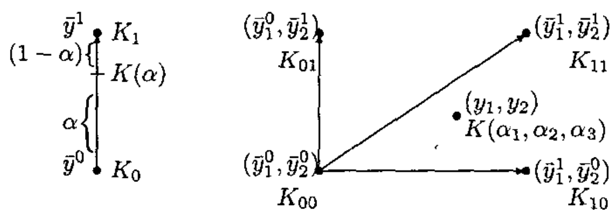

Figure 2: *Single (left) and multi-dimensional (right) gain scheduling parameters.*

where the controllers K_0 and K_1 have been designed in the operating points $\bar{y}^0$ and $\bar{y}^1$, respectively. The scheduled controller is found according to (4) as $K = K(\alpha Q)$, such that $K = K_0$ when $\alpha = 0$ and $K = K(Q) = K_1$ when $\alpha = 1$. The scheduling parameter α itself is typically found as a simple linear scaling between $\bar{y}^0$ and $\bar{y}^1$.

The right part of the figure, on the other hand, shows a situation with two measurements. The controllers $K_{ij}, i, j \in \{0, 1\}$ have been designed in each of the four operating points $(\bar{y}_1^i, \bar{y}_2^j), i, j \in \{0, 1\}$. Assume that the gain scheduling is based on K_{00}. Since K_{11} is not a linear combination of K_{10} and K_{01} it is not sufficient simply to find $\alpha_1, \alpha_2 \in [0, 1]$ as linear scalings between $\bar{y}_i^0$ and $\bar{y}_i^1, i \in \{0, 1\}$. It is necessary to include the scheduling

indicated by the 'vector' between the starting controller K_{00} and the controller K_{11} shown in the figure as well, yielding three scheduling variables. The scheduling variables must be designed such that in each operating point, the other three controllers should be completely phased out, i.e., $K = K_{00}$ for $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $K = K_{01}$ for $\alpha_1 = 0, \alpha_2 = 1, \alpha_3 = 0$, etc. Furthermore, they should be continuous everywhere in the scheduling range in order to avoid introducing unnecessary disturbances in the control loop due to abrupt changes in the controller.
To address this issue, a method of calculating the scheduling parameters yielding a continuous transition between each operating point, and which ensures the aforementioned phasing out of the other controllers in each operating point, is presented in the following.

Theorem 2 *Consider a set of $n+k$ distinct, non-zero vectors $v_1, \ldots, v_{n+k} \in \mathbb{R}^n$, where $span\{v_1, \ldots, v_n\} = \mathbb{R}^n$. For any point $x \in \mathbb{R}^n$ there exists a unique choice of coefficients $\alpha_1, \ldots, \alpha_{n+k}$, which minimizes the functional*

$$P(x, \alpha_1, \ldots, \alpha_{n+k}) = \sum_{i=1}^{n+k} \alpha_i^2 \|x - v_i\|^2 \tag{5}$$

subject to

$$x = \sum_{i=1}^{n+k} \alpha_i v_i. \tag{6}$$

Moreover, this choice of $\alpha_1, \ldots, \alpha_{n+k}$ is a continuous function of x for all $x \in \mathbb{R}^n$.

Proof: Consider an arbitrary $x \in \mathbb{R}^n$, let $a = [\alpha_1, \ldots, \alpha_n]^T \in \mathbb{R}^n$ and $b = [\alpha_{n+1}, \ldots, \alpha_{n+k}]^T \in \mathbb{R}^k$ denote column vectors containing the first n and last k coefficients, respectively, and define the matrices $\Lambda = \mathrm{col}\{v_1, \ldots, v_n\} \in \mathbb{R}^{n \times n}$ and $V = \mathrm{col}\{v_{n+1}, \ldots, v_{n+k}\} \in \mathbb{R}^{n \times k}$. Λ is invertible since $v_1, \ldots, v_n$ span $\mathbb{R}^n$, and we can thus rewrite the condition (6) as $x = \Lambda a + Vb$ or

$$a = \Lambda^{-1}(x - Vb). \tag{7}$$

Let $\lambda_i, i = 1, \ldots, n$ denote the i'th row vector of Λ^{-1}. Eqn. (7) is inserted in eqn. (5), yielding

$$\begin{aligned} P(\cdot) &= \sum_{i=1}^{n} \alpha_i^2 \|x - v_i\|^2 + \sum_{i=n+1}^{n+k} \alpha_i^2 \|x - v_i\|^2 \\ &= \sum_{i=1}^{n} (\lambda_i(x - Vb))^2 \|x - v_i\|^2 + \sum_{i=n+1}^{n+k} \alpha_i^2 \|x - v_i\|^2 \\ &= (\zeta - Mb)^T (\zeta - Mb) \end{aligned}$$

where we have defined $\zeta \in \mathbb{R}^{n+k}$ and $M \in \mathbb{R}^{n+k \times k}$ as

$$\zeta = \begin{bmatrix} \|x - v_1\| \lambda_1 x \\ \vdots \\ \|x - v_n\| \lambda_n x \\ 0_{k \times 1} \end{bmatrix}$$

and

$$M = \begin{bmatrix} \|x - v_1\| \lambda_1 V \\ \vdots \\ \|x - v_n\| \lambda_n V \\ \mathrm{diag}\{\|x - v_{n+1}\|, \ldots, \|x - v_{n+k}\|\} \end{bmatrix}$$

Since the bottom k rows of M are diagonal, M can only lose column rank if one of the diagonal elements is zero, which may only happen if x is exactly equal to one of the vectors $v_{n+1}, \ldots, v_{n+k}$. Assume $x = v_{n+\tau}$ for some $\tau \in \{1, \ldots, k\}$ and observe that the τ'th column in the upper block in M is given as

$$m_\tau = \mathrm{diag}\{\|x - v_1\|, \ldots, \|x - v_n\|\} \Lambda^{-1} v_{n+\tau}.$$

Obviously, as $v_1, \ldots, v_{n+k}$ are distinct, x must be different from $v_1, \ldots, v_n$. This implies that the product $\mathrm{diag}\{\|x - v_1\|, \ldots, \|x - v_n\|\} \Lambda^{-1}$ has rank n, and since $v_1, \ldots, v_{n+k}$ are assumed to be non-zero, m_τ has rank 1 and M can be seen to have full column rank.
The optimal least-squares solution (a^*, b^*) to the quadratic optimization problem (5) thus exists, is unique and given as

$$b^* = (M^T M)^{-1} M^T \zeta, \quad a^* = \Lambda^{-1}(x - Vb^*). \tag{8}$$

Finally, to complete the proof we only need to note that since (a^*, b^*) are given as products and sums of matrices that depend continuously on x, the solution must also depend continuously on x. ∎

At any point in the space in which the operating points exist, the choice of scheduling parameters given in eqn. (8) weights the 'closest' controller highest. Obviously, if the current state of the plant is exactly at an operating point, the performance functional is minimized by choosing the controller in that operating point and setting all other scheduling parameters to 0. In the following we will demonstrate the practical usefulness of the proposed scheduling method on a simulation model of a power plant, after addressing some implementation issues.

2.3 Application

Here, we outline the construction of Q and how to include integral action in the controller in order to remove any steady state errors that might arise from unmodelled dynamics, etc.
The integrator is included in the controller by augmenting the controller by an extra state defined as the integral of the control error $e = y - y_{ref}$, which corresponds to placing a controller pole in $z = 1$. However, we observe that both of the coprime factors $\tilde{U}_i$ and $\tilde{V}_i$ in eqn. (3) must be stable. This means that including an integrator on either side of the summing point in the middle of Figure 1 will add a pole in $z = 1$, violating the conditions for Theorem 1 to hold. However, it is possible to circumvent this difficulty by factorizing the integrator into the following coprime factor-

ization:

$$\frac{z^{-1}}{1-z^{-1}} = \left(\frac{1-z^{-1}}{1-rz^{-1}}\right)^{-1} \left(\frac{z^{-1}}{1-rz^{-1}}\right) = \tilde{V}_I^{-1}\tilde{U}_I \quad (9)$$

where $0 < r < 1$, yielding $\tilde{V}_I, \tilde{U}_I \in \mathcal{RH}_\infty$. Next, we

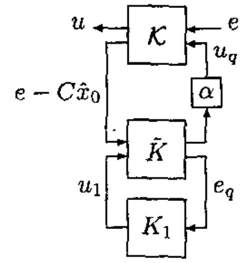

Figure 3: *The interconnection of the controller* $K(Q) = \mathcal{K} \star \tilde{K} \star K_1$.

present how to find Q once a number of controllers have been found in individual operating points. For reasons of clarity we present the method for two controllers designed for two operating points with linearizations (A_0, B_0, C_0) and (A_1, B_1, C_1), respectively, and one scheduling parameter, but the procedure is easily generalized to more controllers. Refer to Figure 3. Let $\mathcal{K}$, the augmented controller on which we base the YJBK scheduling, be given as (see e.g. [6])

$$\mathcal{K} = \left[\begin{array}{cc|cc} A_0 + B_0F_0 + L_0C_0 & B_0 & -L_0 & B_0 \\ 0 & I & F_{I0} & rI \\ \hline F_0 & I & 0 & I \\ C_0 & 0 & -I & 0 \end{array}\right] \quad (10)$$

where $A_0 + B_0F_0$ and $A_0 + L_0C_0$ are stable matrices (i.e., the norms of all eigenvalues are less than one), and $rI \in \mathbb{R}^{p \times p}$ represents the integrators included for each measurement output channel, factorized as described above. $\mathcal{K}$ takes the signals e and u_q as inputs and yields the outputs u, which is applied to the plant, and e_q, which is fed to Q. As depicted in Figure 1, $K(Q)$ is formed as a linear fractional transformation of $\mathcal{K}$ and Q scaled by α, i.e., $K(\alpha Q) = \mathcal{K} \star (\alpha Q)$. In particular, for $\alpha = 0$, the resulting controller becomes

$$K_0 = \left[\begin{array}{cc|c} A_0 + B_0F_0 + L_0C_0 & B_0 & -L_0 \\ 0 & I & F_{I0} \\ \hline F_0 & I & 0 \end{array}\right] \quad (11)$$

which can be recognized as a standard observer-based controller. Correspondingly, when $\alpha = 1$ we must have $K(Q) = K_1$ where

$$K_1 = \left[\begin{array}{cc|c} A_1 + B_1F_1 + L_1C_1 & B_1 & -L_1 \\ 0 & I & F_{I1} \\ \hline F_1 & I & 0 \end{array}\right] \quad (12)$$

Note that, in the LFT setup, K_1 takes e_q as input and yields u_1 as output. Hence we may find Q as $Q = \tilde{K} \star K_1$, where

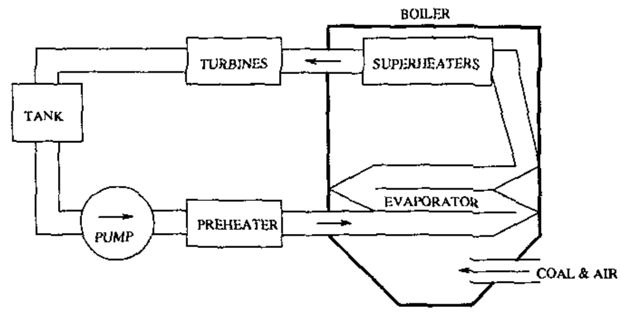

Figure 4: *Coal-fired power plant.*

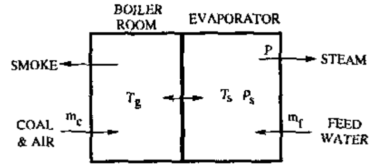

Figure 5: *Simplified model of boiler.*

$\tilde{K}$ is chosen such that $K_0 \star \tilde{K}$ is an identity system. Fairly straightforward calculations yield

$$\tilde{K} = \left[\begin{array}{cc|cc} A_0 & 0 & L_0 & B_0 \\ F_{I0}C_0 - rF_0 & (1-r)I & -F_{I0} & rI \\ \hline -F_0 & -I & 0 & I \\ C_0 & 0 & -I & 0 \end{array}\right] \quad (13)$$

where $\tilde{K}$ takes u_1 and the prediction error from K_0 as inputs and yields u_q and e_q as outputs.

3 Power Plant Control Simulation

Figure 4 illustrates how the considered power plant works. Water is pumped from a feed water tank through a preheater and into the boiler. In the boiler, the water evaporates in the evaporator and the temperature is further increased in the superheaters. The superheated steam is then expanded through the turbines, which drive a number of generators producing electricity. After the turbines the water is led back to the feed water tank. Figure 5 shows the simplified model of the boiler used here. The gas in the boiler room and the steam in the evaporator are lumped together into two average states. Assuming that the mass flow of the smoke (and ashes) equals the mass flow of coal and air, just three state variables are left: the temperature and density of the steam, T_s and ρ_v, along with the temperature of the smoke, T_g. The controlled inputs are the mass flow of coal, $\dot{m}_c$, and the mass flow of the feed water, $\dot{m}_f$.
The heat flux from the coal and air is modelled as

$$Q_c = \dot{m}_c h_c + \dot{m}_a h_a,$$

where h_c and h_a are the specific enthalpies of the coal and air, and $\dot{m}_a$ is the mass flow of air. The heat flux of the smoke is modelled as

$$Q_g = (\dot{m}_c + \dot{m}_a)c_g T_g,$$

where c_g is the specific heat capacity of the smoke. This gives the following time derivative of T_g:

$$\frac{dT_g}{dt} = \frac{1}{c_g m_g}(Q_c - Q_g - Q_w), \tag{14}$$

where m_g is the mass of the smoke (and ashes) and Q_w is the heat flux through the evaporator wall modelled as

$$Q_w = \alpha_w(T_g - T_s) + \epsilon_w(T_g^4 - T_s^4),$$

where α_w and ϵ_w are heat transfer coefficients of the wall. The time derivative of T_s is modelled as

$$\frac{dT_s}{dt} = \frac{\rho_s \frac{\partial h_s}{\partial \rho_s}(\dot{m}_s - \dot{m}_f) + \dot{m}_f(h_f - h_s) + Q_w}{C_w + V\rho_s \frac{\partial h_s}{\partial T_s}}, \tag{15}$$

where $h_s(T_s, \rho_s)$ is the enthalpy of the steam, h_f is the enthalpy of the feed water, C_w is the heat capacity of the wall, V is the volume of the evaporator, and $\dot{m}_s$ the mass flow of steam out of the evaporator modelled as (ref. [2])

$$\dot{m}_s = \beta_v\sqrt{(P^2 - P_0^2)/T_s},$$

where $P(T_s, \rho_s)$ is the pressure of the steam, P_0 is the pressure in the tank, and β_v is a flow coefficient. The final time derivative needed is that of ρ_s which is simply given by

$$\frac{d\rho_s}{dt} = \frac{\dot{m}_f - \dot{m}_s}{V}. \tag{16}$$

By assuming

$$\begin{array}{lllllll} h_c & = & 25\,MJ/kg, & h_a & = & 570\,kJ/kg, \\ c_g & = & 1280\,J/(kgK), & m_g & = & 1677\,kg, \\ \alpha_w & = & 12\,kW/K, & \epsilon_w & = & 0.00068\,W/K^4, \\ C_w & = & 103\,MJ/K, & V & = & 28.3\,m^3, \\ h_f & = & 1400\,kJ/kg, & P_0 & = & 6.2\,MPa \end{array}$$

and $\beta_v = 0.00031\ kgK^{1/2}/(sPa)$ to be constants we have a third order dynamical model given by Equations (14), (15), and (16). With $\dot{m}_a$ calculated as a function of $\dot{m}_c$ the model has two control inputs $\dot{m}_c$ and $\dot{m}_f$.

The values of the constants were found by fitting the model to measurement data from an actual 400 MW power plant. The fitted model showed good agreement with the actual data, considering how simple it is.

The method presented in Section 2 is applied to the simulation model of the power plant. The control objective is to maintain the steam temperature, T_s, at $700K$ while keping the steam pressure at a desired reference value using the control inputs $\dot{m}_f$ and $\dot{m}_c$. The operating point is determined by the desired steam pressure, $P_{ref} \in [225; 400]bar$, and the enthalpy of the feed water, $h_f \in [350; 2100]kJ/kg$. Four operating points are chosen: $w_1 : (P_{ref} = 400bar, h_f = 2100kJ/kg)$, $w_2 : (P_{ref} = 225bar, h_f = 350kJ/kg)$, $w_3 : (P_{ref} = 400bar, h_f = 350kJ/kg)$, and $w_4 : (P_{ref} = 225bar, h_f = 2100kJ/kg)$. In each of these operating points a linearized model of the plant is obtained with a sampling period of $5s$, and a discrete time LQR/LQE controller with integral action is designed for the corresponding model with emphasis on disturbance rejection. The controllers are scheduled according to P_{ref} and h_f using the method described in Section 2.2, with the controller designed for the operating point w_4 as K_{00}. In the simulation P_{ref} and h_f follow trajectories that stay at each operating point for $2560s$ and then slowly ramp to the next. While at an operating point the system is subjected to various disturbances illustrated in Figure 6 which shows operation at w_4. The system is subjected to a 10% drop in the enthalpy of the coal, then P_{ref} is stepped up and down, and finally there is a 5% increase in the flow coefficient β_v. Figure 7 shows the entire simulation going through the four operating points in the order $w_4 \rightarrow w_2 \rightarrow w_3 \rightarrow w_1$. Halfway between the operating points, the reference is kept constant for a short period of time, and the system is subjected to the same disturbances as in the operating points. In the left plots, the gain scheduling is turned off; only K_{00} is used. In the right plots, the controllers are scheduled as described in Section 2. The top figure shows the scheduling weights found by the method described in Section 2.2. When the scheduling is turned off, the performance is seriously affected especially in w_2 and during the transition from w_2 to w_3. With the proposed scheduling method, on the other hand, the transitions run smoothly and with good disturbance rejection, including in between the operating points.

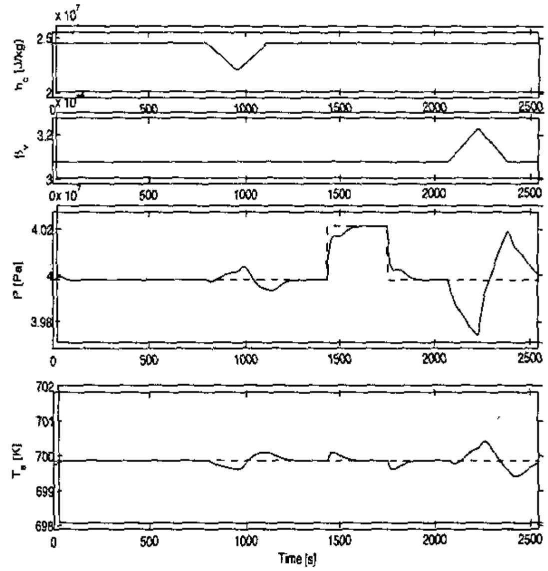

Figure 6: *First 3000 s of simulation. Top: specific enthalpy of coal. Second: flow coefficient. Third: Steam pressure. Bottom: Steam temperature. References shown with dashed lines.*

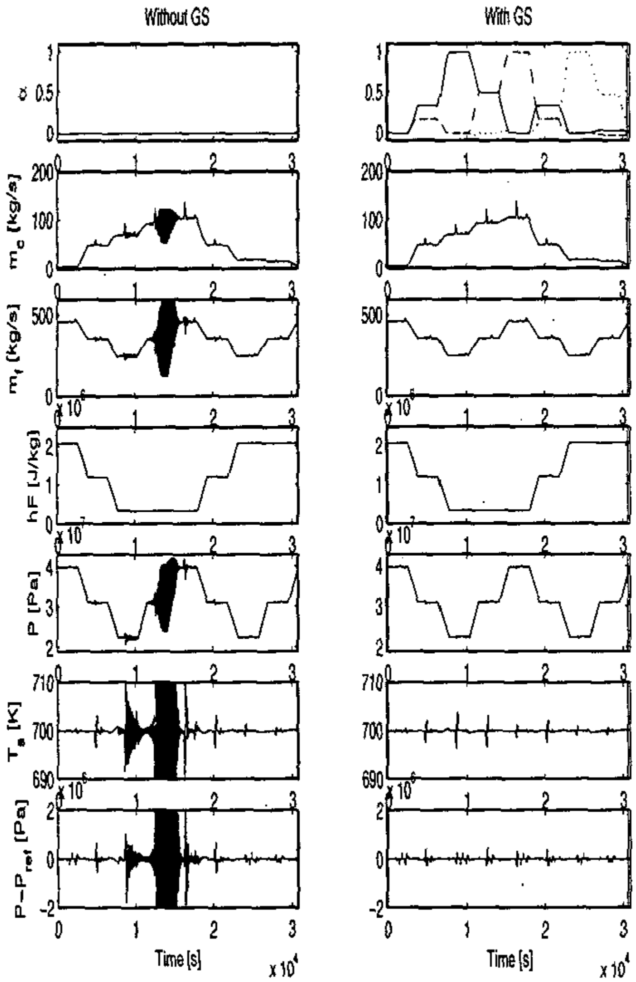

Figure 7: *Simulation in entire operating range. The fi gures show from top to bottom: Scheduling weights, mass fbw of coal, mass fbw of feed water, enthalpy of feed water, steam pressure, steam temperature, and steam pressure error. In the left plots, only the fi rst controller is used. In the right plots, the controller is gain-scheduled.*

4 Discussion

This paper presented a systematic approach to multi-dimensional gain scheduling control, with application to e.g. power plant control. It is assumed that linearized state space models of a nonlinear system are available in a number of operating points, and corresponding stabilizing controllers have been designed in each of these operating points.

It is then possible to exploit the YJBK-parameterization to achieve a scaling of the different controllers in each operating point that is guaranteed to be stable. Although this approach strictly speaking still does not guarantee stability in between operating points, at least gain scheduling design methods should guarantee stability while scheduling between controllers for a *fixed* linear model. This is satisfied for the proposed method in contrast to several of the classical methods. However, for multi-dimensional systems it is often not straightforward to obtain simple scheduling parameters. In this paper, we presented a method to calculate the scheduling factors, which weights each controller according to the distance between the current plant state and the operating point of the controller. When the plant state is exactly equal to an operating point, only the corresponding controller is active.

It was then demonstrated how to include an integration in the controller and how to derive the correct scheduling parameter system, based on linear fractional transformations between individual controllers. The feasibility of the scheme was demonstrated on a simple, but highly nonlinear simulation model of a power plant. The model was linearized in four operating points characterized by high and low pressure, and high and low enthalpy of the feed water, respectively. Discrete time LQR/LQE controllers were designed for this model with emphasis on disturbance rejection, and the proposed scheme was implemented. It was seen that the performance was seriously deteriorated when the gain scheduling was turned off.

In practice, such a gain scheduling scheme could be implemented at the medium-to-high level of the control hierarchy, where the computational demands can be met easily.

It should be pointed out that it is implicitly assumed that the transitions between the individual operating points must happen "sufficiently slowly" for the gain scheduling scheme to succeed. Further work would therefore involve considering the scheme in the gain scheduling framework presented in e.g. [4]. Other future work will investigate the nature of parameter variations for which stability guarantees can be given for gain scheduling in between operating points.

References

[1] J. Bendtsen, J. Stoustrup, "Transformation of Neural State Space Models into LFT Models for Robust Control Design," in *Proc. of the Fourth IFAC International Symposium on Robust Control Design*, June 2003

[2] S. Lu, B. W. Hogg, "Dynamic nonlinear modelling of power plant by physical principles and neural networks," *International Journal of Electrical Power and Energy Systems* Vol. 22, 1:67-78, 2000

[3] H. Niemann, J. Stoustrup, "An Architecture for Implementation of Multivariable Controllers", *Proc. of the American Control Conference*, June 1999

[4] J. S. Shamma, M. Athans, "Analysis of Gain Scheduled Control for Nonlinear Plants," *IEEE Trans. Aut. Control* Vol. 35, 898–907, 1990

[5] W. Rugh, J. S. Shamma, "Research on Gain Scheduling," *Automatica* Vol. 36, 1401–1425, 2000

[6] K. Zhou, J. Doyle, K. Glover, "Robust and Optimal Control," Prentice Hall, Inc., 1996, ISBN 0-13-456567-3