Optimization of signature sequences and receiver filters for downlink DS-CDMA systems

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***Abstract*— Joint optimization of both signature sequences and linear receiver filters for a downlink DS-CDMA system with multipath propagation channels is considered. The signature sequences and the receiver filters are optimized in order to minimize the sum of the mean-squared-errors at the output all the receivers while maintaining a fixed total transmitting power. The signature sequences and receiver filters are derived using a filtering approach to accommodate for the multipath propagation effects.**

I. Introduction

In a downlink DS-CDMA system with orthogonal signature sequences the performance is limited by multipath propagation which introduces both multiaccess interference (MAI) and intersymbol interference (ISI). Multiuser detection (MUD) techniques which to a large extent remedies these effects are well-known [1]. They combine interference rejection and equalization by designing appropriate receivers only.

Instead we consider joint optimization of both signature sequences and receiver filters in order to improve the overall performance. The optimized signature sequences are found using information about all the downlink channels and the receiver filters are optimized using the information of their corresponding channel only (coordinated transmission and uncoordinated reception). The downlink channel information is normally not accessible at the transmitter and thus this scheme has clear practical limitations. However, for a time-division-duplex (TDD) system the channel-impulse-responses can be estimated from uplink transmission using the channel-reciprocity. Alternatively, the estimated channel impulse responses can be fed back from the receivers to the transmitter, which requires that the channels are sufficiently slowly time-varying so that the amount of feedback information is low compared to the data-rate.

The joint optimization is performed to minimize the mean-squared-error (MSE) from the input to the output while keeping the total transmitting power fixed.

Related investigations have been reported in [2], [3], [4], [5], [6]. Fully coordinated transmitters and receivers for MIMO systems are investigated in [3]. In this work the problem formulation differs from our since the receivers are fully coordinated, i.e. all the received signals are available to each receiver.

In [4] the optimization is performed on symbol-level rather than on chip-level with a linear transformation of the information symbols. The multipath propagation is accommodated using a block-based signal model. With this approach a complete data-burst must be processed together, which can be impractical.

In [5] the signature sequences are optimized together with the receiver filters in order to minimize the total transmitting power while keeping the sum of the MSEs below a predescribed threshold. It is assumed that the effect of inter-symbol-interference can be ignored, and thus the results might not apply for systems with short spreading sequences compared to the propagation channel delay-spread.

Related investigations are reported in [6] where the authors find optimal transmitter and receiver filters. The problem considered therein is significantly different from ours since the authors consider peer-to-peer communications, e.g. a base-station transmitting to one receiver and not a multiuser system with a base-station transmitting to several receivers.

In our formulation of the problem we design optimal transmitter and receiver filters using a filtering approach to accommodate for the ISI and to avoid processing complete data-bursts together. We minimize the sum of the MSEs at the outputs of the different receivers while keeping the transmitting power fixed. Our main contribution is to solve this problem taking frequency-selective channels into account without processing an entire data-burst at a time or ignoring the effect of ISI. By means of simulations we demonstrate a large performance gain over a traditional CDMA system where only the receivers are optimized.

The following notation is used in the sequel: $(\cdot)^T$ denotes the transpose operator, $(\cdot)^*$ denotes complex conjugation, $(\cdot)^H$ denotes complex conjugation followed by the transpose operator, $\otimes$ is the Kronecker product, $\mathrm{col}(\cdot)$ is the column operator which puts all the arguments into a column vector, scalars and time-series are written using math-type font, vectors are written as lower-case bold symbols and matrices are written as upper-case bold symbols.

II. Signal model

We consider the discrete-time equivalent lowpass representation of a downlink system, where the transmitted signal is the superposition of K synchronous DS-CDMA signals, see

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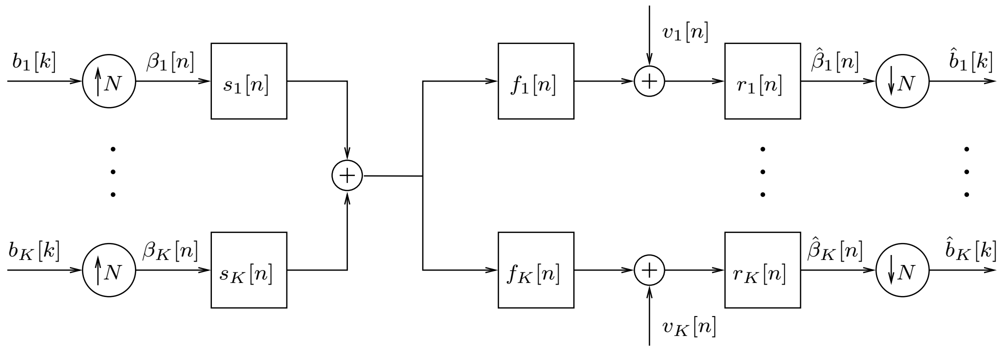

Fig. 1. Filter interpretation of downlink DS-CDMA system.

Fig. 1. The transmitted chip-rate signal can be written as

$$x[n] = \sum_{k=1}^{K} \sum_{i=-\infty}^{\infty} b_k[i] s_k[n - iN] \tag{1}$$

where $b_k[i]$ denotes the ith information symbol of user k and $s_k[n]$ is the corresponding spreading sequence of length N chips with support $\{0, 1, \ldots, N-1\}$. Let $\beta_k[n]$ is the N-fold upsampled version of $b_k[n]$. Then (1) can be recast as

$$x[n] = \sum_{k=1}^{K} s_k[n] * \beta_k[n]. \tag{2}$$

In [7] a similar approach is used to design equalizers for CDMA systems.

For simplicity we only consider BPSK modulation in the following. The signal $x[n]$ propagates to the different receivers through different channels, which can be represented as transversal filters. Thus, after noise is added, the signal received by user k can be written as

$$y_k[n] = f_k[n] * x[n] + v_k[n]. \tag{3}$$

For convenience we consider the different channel impulse responses $f_k[n]$ to have the same length L. The noise is assumed to be circularly symmetric additive white Gaussian noise with a variance σ_k^2. In this correspondence we only consider linear receivers.

Normally a linear receiver or equalizer is represented as a non-causal transversal filter [8] followed by downsampling in the case of fractionally spaced equalizers. In practice the receiver filter is made causal by a time-shift, which introduces a delay in the symbol estimates. Let such a non-causal transversal filter be denoted by $r_k[n]$, then the equalized output at the kth user is

$$z_k[n] = r_k[n] * y_k[n]. \tag{4}$$

The equalization is performed to obtain a close replica of the input

$$\hat{\beta}_k[n] \approx \beta_k[n], \quad n = \cdots - N, 0, N, \ldots \tag{5}$$

and the symbol estimates are obtained as

$$\hat{b}_k[j] = \operatorname{sign}(\hat{\beta}_k[jN]), \quad j = \ldots, -1, 0, 1, \ldots \tag{6}$$

III. Algorithm

We wish to design signature sequences and receiver filters that minimize the total mean-squared-error

$$\sum_{k=1}^{K} E[|\beta_k[jN] - \hat{\beta}_k[jN]|^2]$$

while retaining the same transmitting power K. This is a non-trivial biconvex optimization problem. In this paper we use a suboptimal approach proposed by numerous authors, see e.g. [2], [5], [3], [4].

We start by optimizing the signature sequences for fixed receiver filters and then subsequently optimize the receiver filters for fixed signature sequences. The two optimization steps are iterated until convergence. In general this will not give a global optimal solution for signatures and receivers filters.

A. Optimizing signature sequences for fixed receivers

We start by fixing the receiver filters. They can be initialized to e.g. random vectors. We combine the channel impulse responses $f_k[n]$ and the receiver filters $r_k[n]$ and denote the combined (non-causal) filter $g_k[n]$ to get the equivalent signal model in Fig. 2.

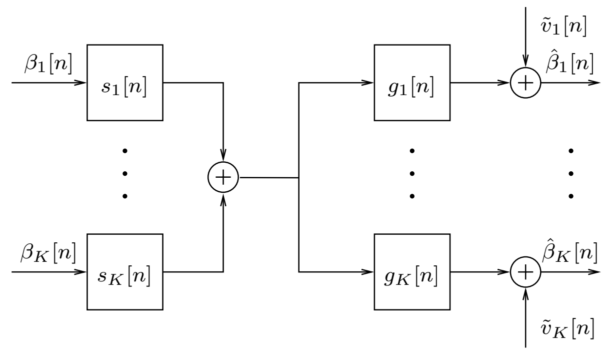

Fig. 2. Signal model for fixed receivers.

Then, the sequence at the output of the ith receiver filter is

$$\hat{\beta}_i[n] \quad = \quad \sum_{j=1}^{K} s_j[n] * g_i[n] * \beta_j[n] + \tilde{v}_i[n] \tag{7}$$

with implicit definition of the filtered noise $\tilde{v}_i[n] = v_i[n] * r_i[n]$. Define $\psi_{ij}[n] = g_i[n] * \beta_j[n]$. Then

$$\hat{\beta}_i[n] = \sum_{j=1}^{K} s_j[n] * \psi_{ij}[n] + \tilde{v}_i[n]. \tag{8}$$

If we further define $\mathbf{s}_j = (s_j[N-1], \ldots, s_j[0])^T$ and $\boldsymbol{\psi}_{ij}[n] = (\psi_{ij}[n-N+1], \ldots, \psi_{ij}[n])^T$, where N is the length of the signature sequences, we get

$$\hat{\beta}_i[n] = \sum_{j=1}^{K} \mathbf{s}^T \boldsymbol{\psi}_{ij}[n] + \tilde{v}_i[n]. \tag{9}$$

In order to isolate the input signal from (9) we write the convolution as shown in (10) at the top of the following page. In (10) the number of coefficients in $g_k[n]$ is $G_2 + G_1 + 1 = L_R + L - 1$ where L_R is the number of coefficients for the receiver filter $r_k[n]$ and L is the number of coefficients of the propagation channel $f_k[n]$, i.e. the length of $g_k[n]$ is given by the length of $f_k[n] * r_k[n]$. For simplicity we assume the same values of L, L_R, G_1 and G_2 for all users.

We choose G_1 such that $\beta_i[n]$ is the middle element of $\boldsymbol{\beta}_i[n]$, i.e. $G_1 = \lceil L_\beta/2 \rceil$ where $L_\beta = N + L_R + L - 2$. This implicitely gives the value of $G_2 = L_R + L - G_1 - 2$. Note that all users will have non-causal parts due to the assumption of common filter lengths. The choice of G_1 is not unique. As for traditional equalization using inverse filtering, the performance is highly dependent on this choice. For certain channel realizations other choices might give better results, but a brute-force approach trying all possible delays is clearly not feasible.

We can now cast the output of the ith receiver filter as

$$\hat{\beta}_i[n] = \sum_{j=1}^{K} \mathbf{s}_j^T \mathbf{G}_i \boldsymbol{\beta}_j[n] + \tilde{v}_i[n]. \tag{11}$$

By defining

$$\begin{aligned}
\mathbf{s} &= \operatorname{col}(\mathbf{s}_1^*, \ldots, \mathbf{s}_K^*) \\
\mathbf{G} &= [\mathbf{I}_K \otimes \mathbf{G}_1^* \;\ldots\; \mathbf{I}_K \otimes \mathbf{G}_K^*] \\
\mathbf{B}[n] &= \mathbf{I}_K \otimes \operatorname{col}(\boldsymbol{\beta}_1[n], \ldots, \boldsymbol{\beta}_K[n]) \\
\mathbf{v}[n] &= \operatorname{col}(\tilde{v}_1[n] \;\ldots\; \tilde{v}_K[n])
\end{aligned}$$

the vector of estimates can be written in a suitable form as

$$\hat{\boldsymbol{\beta}}[n]^H = \mathbf{s}^H \mathbf{G} \mathbf{B}[n] + \mathbf{v}[n]^H. \tag{12}$$

The signature sequences are found as the solution to the following problem

$$\begin{aligned}
&\text{minimize} \quad E[\|\boldsymbol{\beta}[jN] - \hat{\boldsymbol{\beta}}[jN]\|^2] \\
&\text{subject to} \quad \mathbf{s}^H \mathbf{s} = 1.
\end{aligned}$$

The objective of the problem can be expanded as

$$\sigma_\beta^2 - \mathbf{s}^H \mathbf{G} \mathbf{d} - \mathbf{d}^T \mathbf{G}^H \mathbf{s} + \mathbf{s}^H \mathbf{G} \mathbf{E} \mathbf{G}^H \mathbf{s} + \sum_{k=1}^{K} \|\mathbf{r}_k\|^2 \sigma_k^2$$

where $\mathbf{d} = E[\mathbf{B}[jN]\boldsymbol{\beta}[jN]]$, $\mathbf{E} = E[\mathbf{B}[jN]\mathbf{B}[jN]^H]$ and $\|\mathbf{r}_k\|^2$ is the sum of the squared filter weights for the kth receiver. Assuming uncorrelated input signals, i.e. $E[b_i[k]b_j[k+d]] = \delta(i-j)\delta(d)$, the vector $\mathbf{d}$ will be a vector consisting of zeros and ones with K one-elements; one for each user. From the definition of $\mathbf{B}[n]$ and $\boldsymbol{\beta}[n]$ the index of the one-element corresponding to user i can be seen to be $(i-1)(K+1)L_\beta + G_1$ for $i = 1, \ldots, K$. Similarly, $\mathbf{E}$ will be a zero-matrix with one-elements on part of the diagonal. These diagonal indices can be computed as $(i-1)L_\beta + (j-1)KL_\beta + G_1 + kN$ for $i, j = 1, \ldots, K$ and $1 \le (G_1 + kN) < L_\beta$.

We thus have the following (non-convex) quadratic minimization problem with quadratic equality constraints

$$\begin{aligned}
&\text{minimize} \quad \mathbf{s}^H \mathbf{G} \mathbf{E} \mathbf{G}^H \mathbf{s} - \mathbf{s}^H \mathbf{G} \mathbf{d} - \mathbf{d}^T \mathbf{G}^H \mathbf{s} \\
&\text{subject to} \quad \mathbf{s}^H \mathbf{s} = 1.
\end{aligned}$$

This problem is the well-studied trust region problem, see e.g. [9, p.78], which has a solution for $\mathbf{s}$ as

$$\mathbf{s} = (\mathbf{G}\mathbf{E}\mathbf{G}^H + \nu \mathbf{I})^{-1} \mathbf{G} \mathbf{d} \tag{13}$$

where ν can be given the interpretation of a Lagrange multiplier to enforce the equality constraint. The value for ν can be found efficiently using e.g. the Newton-Raphson algorithm. Surprisingly, the trust region problem also has a simple closed form solution in terms of eigenvalues of a special matrix. This result is stated in the appendix without proof for brevity of space.

B. Optimizing receiver filters for fixed signature sequences

Once the signature sequences are fixed, we design optimal linear receivers. Let $h_{ij}[n] = s_i[n] * f_j[n]$. Then we get a reduced signal model for deriving optimal linear receivers, see Fig. 3.

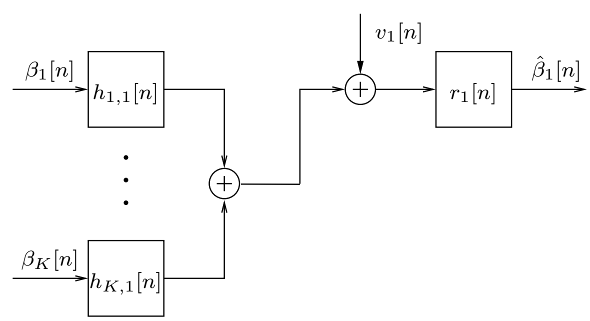

Fig. 3. Reduced signal model for user 1. Signature sequences are fixed.

The output of the ith receiver filter is $\hat{\beta}_i[n] = r_i[n] * (\sum_{j=1}^{K} h_{ji}[n] * \beta_j[n] + v_i[n])$. If we define

$$\begin{aligned}
\mathbf{r}_i &= (r[L_R - G_1 - 1], \ldots, r[-G_1])^H \\
\boldsymbol{\beta}_j[n] &= (\beta_j[n - L_H + 1 + G_1], \ldots, \beta_j[n + G_1])^T \\
\mathbf{v}[n] &= (v[n - L_R + G_1 + 1], \ldots, v[n + G_1])^T \\
\mathbf{H}_{ji} &= \begin{bmatrix} h_{ji}[L_H - 1] & \ldots & h_{ji}[0] & 0 & \\ & \ddots & \ddots & \ddots & \\ 0 & & h_{ji}[L_H - 1] & \ldots & h_{ji}[0] \end{bmatrix}
\end{aligned}$$

$$\boldsymbol{\psi}_{ij}[n] = \underbrace{\begin{bmatrix} g_i[G_2] & g_i[G_2-1] & \dots & g_i[-G_1] & & & 0 \\ & g_i[G_2] & g_i[G_2-1] & \dots & g_i[-G_1] & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ 0 & & & g_i[G_2] & g_i[G_2-1] & \dots & g_i[-G_1] \end{bmatrix}}_{\mathbf{G}_i} \underbrace{\begin{bmatrix} \beta_j[n-N+1-G_2] \\ \vdots \\ \beta_j[n] \\ \vdots \\ \beta_j[n+G_1] \end{bmatrix}}_{\boldsymbol{\beta}_j[n]} \tag{10}$$

where $L_H = N + L - 1$ then the output of the (non-causal) receiver transversal filter can be written as

$$\hat{\beta}_i[n] = \mathbf{r}_i^H \left(\sum_{j=1}^{K} \mathbf{H}_{ji} \boldsymbol{\beta}_j[n] + \mathbf{v}_i[n] \right). \tag{14}$$

We further define $\mathbf{H}_i = [\mathbf{H}_{1i} \ \dots \ \mathbf{H}_{Ki}]$ and $\bar{\boldsymbol{\beta}}[n] = \mathrm{col}(\boldsymbol{\beta}_1[n], \dots, \boldsymbol{\beta}_K[n])$ so that

$$\hat{\beta}_i[n] = \mathbf{r}_i^H \left(\mathbf{H}_i \bar{\boldsymbol{\beta}}[n] + \mathbf{v}_i[n] \right). \tag{15}$$

The receiver filters are obtained as the solution to the (convex) quadratic unconstrained minimization problem

$$\text{minimize } E[|\beta_i[jN] - \hat{\beta}_i[jN]|^2]$$

which by computing the gradient and equating it to zero gives a solution for the ith receiver filter as

$$\mathbf{r}_i = (\mathbf{H}_i \mathbf{E} \mathbf{H}_i^H + \sigma^2 \mathbf{I})^{-1} \mathbf{H}_i \mathbf{d}_i \tag{16}$$

where $\mathbf{d}_i$ is a zero-vector with a one at position $L_\beta(i-1)+G_1$ and $\mathbf{E}$ is a zero-matrix with ones on the diagonal elements with index $L_\beta(k-1) + G_1 + jN$ for $k = 1, \dots, K$ and $0 \leq G_1 + jN \leq L_\beta$. This receiver is the well-known MMSE receiver [1] derived at chip-rate level and for multipath propagation channels.

IV. Simulations

We consider short spreading codes of length $N = 8$ chips and a fully loaded system, i.e. $K = 8$. The propagation channels $f_i[n]$ have $L = 5$ taps. The taps are independent Rayleigh distributed with exponentially decaying average power as listed in the table below:

Tap index	1	2	3	4	5
Power [dB]	0	-2.17	-4.34	-6.51	-8.69

Every channel realization is normalized to unit power, i.e. $\sum_{l=0}^{L} |f_i[l]|^2 = 1$, and for convenience all the receivers operate with identical noise powers σ^2. The receiver filters are designed with $2N = 16$ taps.

Referring to Fig. 2 we define $c_{jk}[n] = s_j[n] * g_k[n]$. As a performance measure we use the SINR at the output of the equalizers. For the kth user the SINR is given as

$$\gamma_k = \frac{|c_{kk}[0]|^2}{\sum\limits_{\substack{j \neq k \\ i \in \mathbb{Z}}} |c_{jk}[iN]|^2 + \sum\limits_{i \in \mathbb{Z} \setminus \{0\}} |c_{kk}[iN]|^2 + \|\mathbf{r}_k\|^2 \sigma_k^2}. \tag{17}$$

where the different terms in the denominator account for the multiaccess interference (MAI), the residual ISI for the user of interest and the effect of the filtered noise, respectively.

In Fig. 4 we plot the output SINR averaged both over the K users and 100 independent realizations of the downlink channels. We average over many different channel realizations since the performance of the linear receivers is highly dependent on the frequency characteristics of the channels, and generally performs badly for channel transfer functions with notches. For comparison we also plot the averaged SINR for the traditional RAKE receiver and the MMSE receiver both with orthogonal Walch codes. The MMSE receivers are implemented using a sliding window similarly to (16) also with $2N = 16$ taps for a fair comparison with the proposed algorithm.

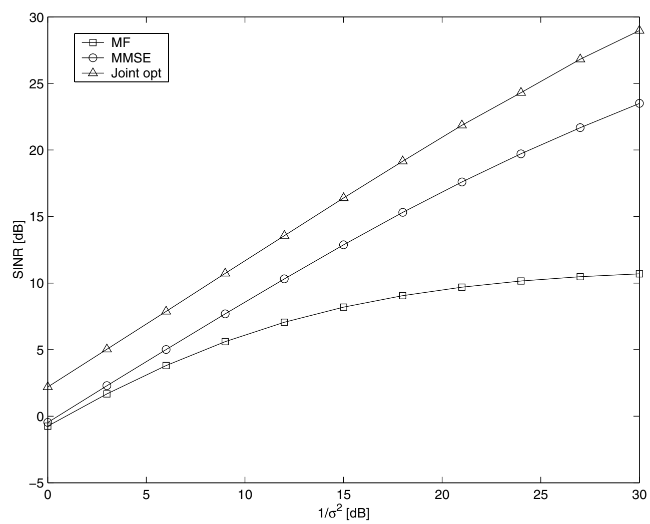

Fig. 4. Simulation results. Achieved SINR versus SNR averaged over all 8 users and 100 independent channel realizations.

In Fig. 5 we illustrate the variation in performance due to different channel realizations for the same setup. The plot shows histograms of the output SINR for the proposed algorithm.

We observe a substantial gain by jointly optimizing the signature sequences and the receiver filters, both in terms of average performance and reduced variations.

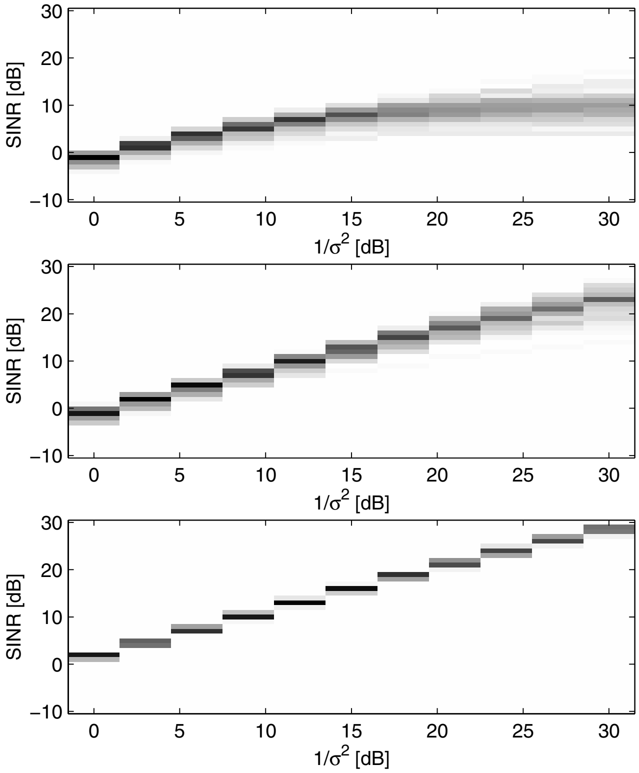

Fig. 5. Histograms of achieved SINR versus SNR. From top to bottom: RAKE, MMSE and Jointly optimized signatures and receivers.

V. Conclusions

In this correspondence we have considered joint optimization of both signature sequences and receiver filters for a downlink DS-CDMA system with uncoordinated reception using a filtering approach to accommodate for the effect of multipath propagation.

The signatures sequences have been derived together with the non-causal receiver transversal filters to minimize the sum of the mean-squared errors at output of the different receivers while keeping the total transmitting power constant.

The proposed algorithm achieves a substantial gain over a more conventional system with Walch codes and e.g. MMSE receivers both in terms of average performance, but also through much smaller variations in performance due to different channel realizations.

However, the scheme has clear practical limitations since the transmission requires knowledge of all the terminal noise powers and downlink channels, but demonstrates that a significant improvement can be obtained by also optimizing signature sequences.

From our simulations we have noticed that a fairly large number of iterations is required for convergence of the filters, especially when the noise powers are small. In general it is not expected that an alternating gradient method like ours will result in a globally optimal solution but we made the surprising observation, that the filters always converge to the same solutions (for fixed channel realizations) after a very large number of iterations. E.g. for the degenerate case where $f_i[n] = \delta[n]$ the signature sequences quickly converge to (non-binary) orthogonal sequences and the receivers converge to the corresponding time-reversed orthogonal sequences.

Appendix

In the trust-region problem we solve the problem

$$\begin{array}{ll} \text{minimize} & \mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} \\ \text{subject to} & \mathbf{x}^T\mathbf{x} = 1 \end{array} \tag{18}$$

in the variable $\mathbf{x} \in \mathbb{R}^n$ where $\mathbf{A} = \mathbf{A}^T \in \mathbb{R}^{n\times n}$ and $\mathbf{b} \in \mathbb{R}^n$ are given. In [10] we show the following result, stated here without proof.

Theorem 1: Let γ be the largest real eigenvalue satisfying

$$-\begin{bmatrix} \mathbf{A} & \mathbf{I} \\ \mathbf{b}\mathbf{b}^T & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \gamma \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} \tag{19}$$

with an associated eigenvector $(\mathbf{u}, \mathbf{v})$. Then γ is the optimal Lagrange multiplier for the problem (18). If $\mathbf{b}^T\mathbf{u} \neq 0$ the optimal solution to (18) is

$$\mathbf{x}^\star = \frac{\mathbf{v}}{\mathbf{b}^T\mathbf{u}}$$

and if $\mathbf{b}^T\mathbf{u} = 0$ an optimal solution is

$$\mathbf{x}^\star = -(\mathbf{A} + \gamma\mathbf{I})^\dagger\mathbf{b} + \alpha\mathbf{u}$$

with $\alpha = \sqrt{1 - \mathbf{b}^T(\mathbf{A}+\gamma\mathbf{I})^\dagger(\mathbf{A}+\gamma\mathbf{I})^\dagger\mathbf{b}}$.

Acknowledgement

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