

PASSIVE FAULT TOLERANT CONTROL OF A DOUBLE INVERTED PENDULUM - A CASE STUDY EXAMPLE

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Abstract: A passive fault tolerant control scheme is suggested, in which a nominal controller is augmented with an additional block, which guarantees stability and performance after the occurrence of a fault. The method is based on the Youla parameterization, which requires the nominal controller to be implemented in observer based form. The proposed method is applied to a double inverted pendulum system, for which an \mathcal{H}_∞ controller has been designed and verified in a lab setup. In this case study, the fault is a degradation of the tachometer loop.

Keywords: \mathcal{H}_∞ controller design, Youla parameterization, Fault tolerant control, observer based controller implementation, simulation.

1. INTRODUCTION

The pendulum system is one of the classical examples used in connection with feedback control. The single inverted single pendulum is a standard example in many text books dealing with classical as well as modern control. The reason is that the system is quite simple, non-linear and unstable. In connection with the classical control, the single inverted pendulum system has among other things been used to show that the system cannot be stabilized by using just a P controller. In spite of the fact that the system is unstable, the design of stabilizing controllers for the system can be done reasonably easy. However, this is not the case when considering the quite more complicated double inverted pendulum system. It is much more challenging to design/tune stabilizing controllers for this system. Therefore, more advanced controller architectures and advanced design methods should be applied. Previous work has involved different types of model based controllers designed by using e.g. \mathcal{H}_2 based

methods, \mathcal{H}_∞ based methods and μ based methods. An investigation of different robust controllers for the double inverted pendulum system has been described in (Niemann and Poulsen, 2003; Poulsen, 2001).

In this paper, the double inverted pendulum system will be applied in connection with design of a fault tolerant controllers. The area of FTC has been an increasing research area the past 5 - 10 years, see (Blanke *et al.*, 2000; Blanke *et al.*, 2001; Wu and Chen, 1996; Wu *et al.*, 2000) and the references in these. The reason is the increasing use of more and more complicated control systems. This will in general require a supervision on top of the control level to handle faulty situations in a systematic way. One part of this supervision is to use fault tolerant controllers (FTC). The key idea of using FTC is to keep the closed loop system stable and accept a reduced performance when (critical) faults occur in the system. This can be done either by a reconfiguration of the feedback controller, (Blanke *et al.*, 2000), or by using a passive

approach, where the fault tolerance is included in the controller architecture, see e.g. (Niemann and Stoustrup, 2002b) or (Niemann and Stoustrup, 2002b). A preliminary version of this passive FTC architecture can be found in (Stoustrup and Niemann, 2001; Zhou and Ren, 2001). The passive FTC concept will be applied in this paper. The advantages with the concept is that no time delay will be included in the controller due to detection of faults and a following reconfiguration of the controller. The disadvantage with the passive concept is that it can only handle a single fault or a few faults.

The passive FTC architecture is based on the Youla parameterization of all stabilizing controllers, (Tay *et al.*, 1997). The nominal feedback controller for the fault free system is applied as the basis for the Youla parameterization. The Youla parameter is applied both in connection with the feedback controller for the fault free system to optimize the closed loop performance and in the faulty case for stabilizing the closed-loop system. This approach results in a multi objective design of the Youla parameter.

In this example, the FTC controllers are designed with respect to a single fault in the tacho loop in the motor, i.e. a broken tacho loop or a major reduction of the tacho gain. A broken tacho loop or a major reduction of the tacho gain will result in an unstable closed loop system if no corrective measures are taken. Due to the limitations in the system, the fault tolerant part of the controller need to be active immediately after the tacho fault appear. Even a minor time delay between the fault appears and the FTC controller becomes active cannot be accepted in this case.

The rest of this paper is organized as follows. In Section 2, the double inverted pendulum system is shortly described, including a design of controllers for the fault free system. The fault tolerant controller architecture is shortly introduced and the FTC design with respect to a fault at the tacho loop is described in Section 3. Section 4 include a simulation of the fault tolerant controller on the double inverted pendulum system followed by a conclusion in Section 5.

2. MODEL OF A DOUBLE INVERTED PENDULUM

In the following, a short description of the double inverted pendulum system is given. Both the nominal as well as the laboratory model are considered. A more detailed description can be found in (Niemann and Poulsen, 2003; Poulsen, 2001).

The double inverted pendulum consist of a cart placed on a track, and two aluminium arms connected to each other. These are constrained to rotate within a single plane. The axis of the rotation is perpendicular to the direction of the motion of the cart. The cart is attached to the bottom of the pendulum, and moving along

a linear low friction track. The cart is moved by an exerting force by a servo motor system.

Some data for the complete system are as follows:

mass of cart	m	0.81kg
length of track	l_t	1.34m
mass of lower arm	m_1	0.548kg
length of lower arm	l_1	0.535m
inertia of lower arm	I_1	$2.678 \cdot 10^{-2} \text{kgm}^2$
mass of upper arm	m_2	0.21kg
length of upper arm	l_2	0.512m
inertia of upper arm	I_2	$5.217 \cdot 10^{-3} \text{kgm}^2$

A nonlinear model for the complete system can be derived by using Newton's second and third laws on every part of the system. Based on this nonlinear model, a linear model can be derived by a linearization of the nonlinear model around the working point. The linear model Σ_G for the complete system can be described by the following state space description

$$\Sigma_G : \begin{cases} \dot{x} = Ax + B_w w + B_u u \\ z = C_z x + D_{zw} w + D_{zu} u \\ y = C_y x + D_{yw} w + D_{yu} u \end{cases} \quad (2.1)$$

where x is the state, w is the exogenous inputs, u is the control input and (the output voltage U), y is the measurement output. z is an external output vector, see below. The linear model is of order 7 with the following states:

$$x = [\theta_1 \ \dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2 \ x_c \ \dot{x}_c \ i]^T \quad (2.2)$$

where θ_1 is the angle between vertical and the lower arm, $\dot{\theta}_1$ is the angular velocity related to θ_1 , θ_2 is the angle between vertical and the upper arm, $\dot{\theta}_2$ is the angular velocity related to θ_2 , x_c is the cart position, \dot{x}_c is the velocity of the cart and i is the motor current.

The exogenous input vector are given by

$$w = [r_c \ M_{d1} \ M_{d2} \ n_1 \ n_3 \ M_{dm} \ n_x]^T \quad (2.3)$$

where r_c is the cart position reference, M_{d1} and M_{d2} are the torque disturbance on the joint on the lower arm and on the upper arm, respectively, n_1 and n_3 are noise signal in measuring θ_1 and $\theta_3 = \theta_1 - \theta_2$, respectively, M_{dm} is the torque disturbance on the motor, and n_x is the noise signal in the measuring of the cart position x_c .

The measurement vector y is given by

$$y = [e_c \ \theta_1 \ \theta_3]^T \quad (2.4)$$

where e_c is the cart position error $r_c - x_c$.

2.1 Nominal Controller Design

A nominal feedback controller is designed by using the standard \mathcal{H}_∞ design approach, (Skogestad and Postlethwaite, 1996; Zhou *et al.*, 1995). The system setup given by (2.1) is extended by a multiplicative output uncertainty described by

$$G_p = (I + W_o \Delta_o) G \quad (2.5)$$

where the perturbation matrix Δ_o satisfies $\|\Delta_o\|_\infty \leq 1$ as in (Skogestad and Postlethwaite, 1996) and W_o is a weight that indicates a percentage of error as function of frequency. The performance for the system is described by including an external output vector z given by

$$z = [e_c \ \theta_1 \ \theta_2 \ u \ i]^T \quad (2.6)$$

where u is the control signal and i is the current in the motor. The complete design setup is shown in 1. W_p is a weighting matrix for the performance specification.

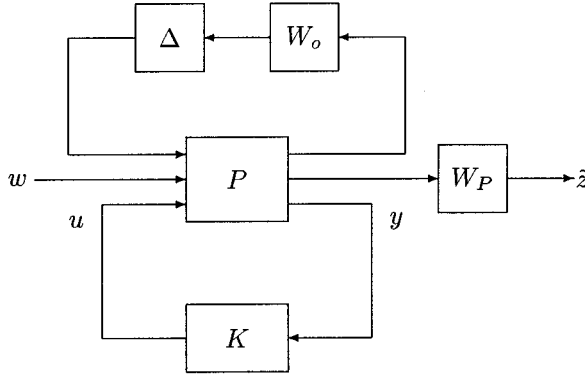


Fig. 1. The complete system setup for design of robust feedback controllers

Four different controller designs have been described in (Niemann and Poulsen, 2003), three controllers designed by using the standard \mathcal{H}_∞ optimization and one controller by using μ synthesis. In this paper, we will apply an \mathcal{H}_∞ optimized controller. The controller has only been designed for the nominal system, i.e. $W_o = 0$. The final controller is of order 11, but has been reduced to order 7, the same order as the nominal plant, see (Niemann and Poulsen, 2003). A simulation of the applied nominal controller is shown in Figure 2, and in Figure 3, the \mathcal{H}_∞ controller is applied to the laboratory system.

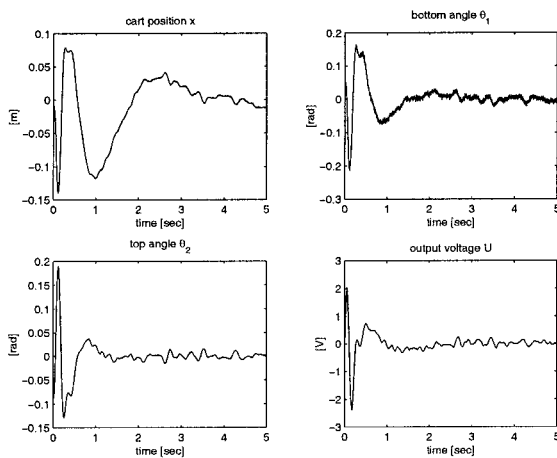


Fig. 2. Simulation of the nonlinear system with an \mathcal{H}_∞ controller. The initial conditions are: $\theta_1 = 0.05\text{rad}$ and $\theta_2 = -0.04\text{rad}$, similar to what would happen for the lab. model.

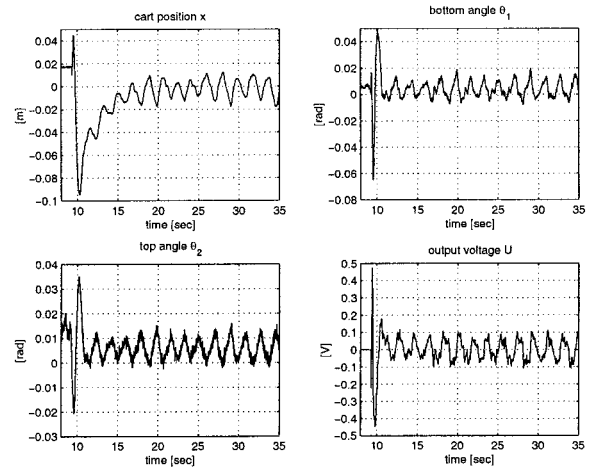


Fig. 3. Sampled data from laboratory model, running with an \mathcal{H}_∞ controller.

3. DESIGN OF FAULT TOLERANT CONTROLLER

The design of a passive fault tolerant controller for the inverted double pendulum system is based on the results described in (Niemann and Stoustrup, 2002b; Niemann and Stoustrup, 2002a).

3.1 Observer Based Controllers

The fault tolerant controller is implemented as a Youla parameterized controller. The implementation of the Youla parameterized controller is based on an observer based feedback controller, (Niemann and Stoustrup, 2002a). The Youla parameterized controller is given by, (Niemann and Stoustrup, 2002a; Tay *et al.*, 1997)

$$K(Q) = \mathcal{F}_l(J_K, Q) \quad (3.1)$$

or

$$\begin{pmatrix} u \\ r \end{pmatrix} = J_K \begin{pmatrix} y \\ s \end{pmatrix}$$

with $s = Qr$ and where J_K is given by

$$J_K = \left(\begin{array}{c|c} \frac{A + B_u F + L C_y}{F} & \begin{matrix} -L & B_u \\ 0 & I \end{matrix} \\ \hline -C_y & \begin{matrix} I & 0 \end{matrix} \end{array} \right) \quad (3.2)$$

where F is a stabilizing state feedback gain such that $A + B_u F$ is stable and L is a stabilizing observer gain such that $A + L C_y$ is stable. The FTC controller architecture is shown in Figure 4.

The final 7th order controller (designed by using the μ design method followed by a model reduction) is transformed into an observer based controller by using the method described in (Alazard and Apkarian, 1999; Alazard and Apkarian, 2002).

3.2 FTC Design Setup

The FTC controller is designed with respect to a broken or damaged tachometer loop. The tachometer fault is

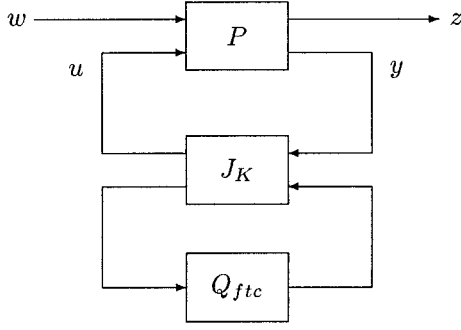


Fig. 4. The architecture setup for the passive fault tolerant controller.

described by using a multiplicative (parameter) fault model, (Niemann and Stoustrup, 2002a), given by

$$\Sigma_M : \begin{cases} z_\theta = G_{z_\theta w_\theta} w_\theta + G_{z_\theta w} w + G_{z_\theta u} u \\ z = G_{zw_\theta} w_\theta + G_{zw} w + G_{zu} u \\ y = G_{yw_\theta} w_\theta + G_{yw} w + G_{yu} u \end{cases} \quad (3.3)$$

where $w_\theta \in \mathbb{R}^{k_w}$ and $z_\theta \in \mathbb{R}^{k_z}$ are the external input and output vectors. The connection between the external output and the external input is given by

$$w_\theta = \theta z_\theta$$

where θ represent the multiplicative (parameter) faults in the system. Note that the above description is also applied in connection with description of systems including model uncertainties, see e.g. (Zhou *et al.*, 1995). In this case, θ is a scalar parameter, where $\theta = 0$ describe the fault free system and $\theta = 1$ describe the system with a broken tachometer.

The design of Q_{ftc} , the passive fault tolerant controller, will be based on an optimization of the performance of the fault free closed loop system and at the same time stabilize the faulty system. This result in a multi objective design of Q_{ftc} . Based on the system given in (3.3), the performance design problem for the fault free system is then given by, (Niemann and Stoustrup, 2002a; Tay *et al.*, 1997):

$$\|T_{zw}\|_\infty = \|T_1 + T_2 Q_{ftc} T_3\|_\infty < \gamma, \quad Q_{ftc} \in \mathcal{RH}_\infty \quad (3.4)$$

for a specified $\gamma > 0$, where T_1, T_2 and T_3 are functions of the open loop transfer functions in (3.3) for $\theta = 0$. When a fault appear in the system, in this case a broken tachometer loop, the closed loop transfer function will no longer be an affine function of the Q_{ftc} controller as in (3.4). A broken tachometer loop or a reduction of the tachometer gain result in an unstable closed loop system. The design problem for the faulty system is then a stabilization problem. Let \tilde{T}_4 be the transfer function from y_q to u_q for $\theta \in]0, 1]$. The FTC design problem is then as follows, (Niemann and Stoustrup, 2002a):

$$(I - \tilde{T}_4 Q_{ftc})^{-1} \in \mathcal{RH}_\infty \quad (3.5)$$

Note that \tilde{T}_4 is also known as the dual Youla parameter S , (Niemann and Stoustrup, 2002a). However, it is

also possible to include performance in the design of Q_{ftc} controller. This can be done by optimizing Q_{ftc} with respect to the external inputs/outputs, i.e.

$$\|\tilde{T}_{zw}(Q_{ftc})\|_\infty < \gamma, \quad Q_{ftc} \in \mathcal{RH}_\infty \quad (3.6)$$

where

$$\tilde{T}_{zw}(Q_{ftc}) = \tilde{T}_1 + \tilde{T}_2 Q_{ftc} (I - \tilde{T}_4 Q_{ftc})^{-1} \tilde{T}_3$$

Based on this, it is possible to formulate a number of passive FTC design problems. In the first design problem, the main passive fault tolerant control design problem is given. Here, the stability conditions for the nominal and the faulty system is the main design condition.

Problem 1. The passive fault tolerant controller design problem is to design Q_{ftc} , $Q_{ftc} \in \mathcal{RH}_\infty$ such that

$$(I - \tilde{T}_4 Q_{ftc})^{-1} \in \mathcal{RH}_\infty$$

In the next design problem, the FTC part of the controller is optimized with respect to the performance of the nominal closed loop system together with the stability condition.

Problem 2. Let $\gamma > 0$ be given. The passive fault tolerant controller design problem with respect to an \mathcal{H}_∞ optimization of the nominal performance is to design Q_{ftc} , $Q_{ftc} \in \mathcal{RH}_\infty$ such that

$$\begin{aligned} \|T_1 + T_2 Q_{ftc} T_3\|_\infty &< \gamma \\ (I - \tilde{T}_4 Q_{ftc})^{-1} &\in \mathcal{RH}_\infty \end{aligned}$$

In the last FTC design problem, the FTC part of the controller is designed with respect to both the stability of the faulty system and with respect to optimize the performance of both the nominal system and the faulty system.

Problem 3. Let $\gamma_2 \geq \gamma_1 > 0$ be given. The passive fault tolerant controller design problem with respect to an \mathcal{H}_∞ optimization of the nominal performance and the performance in the faulty system is to design Q_{ftc} , $Q_{ftc} \in \mathcal{RH}_\infty$ such that

$$\begin{aligned} \|T_1 + T_2 Q_{ftc} T_3\|_\infty &< \gamma_1 \\ \|\tilde{T}_1 + \tilde{T}_2 Q_{ftc} (I - \tilde{T}_4 Q_{ftc})^{-1} \tilde{T}_3\|_\infty &< \gamma_2 \end{aligned}$$

The design of Q_{ftc} for the double inverted pendulum system has been derived by using a slightly modified version of Problem 3. Problem 3 is a multi objective design problem. Instead, the design of Q_{ftc} has been done with respect to optimizing the performance of the faulty system followed by a validation of the performance for the nominal closed loop system. An \mathcal{H}_∞ design method has used for the design of Q_{ftc} . Using this method for the design of Q_{ftc} given that it is not possible to design a stable Q_{ftc} with a complete

broken tacho loop. Instead, a reduction of the tacho gain with 70% is considered for the FTC design.

The final controller Q_{ftc} is of order 18. The controller order has not been reduced in the simulation. However, it is possible to reduce it to a much lower order without any problems. The magnitude of the nominal \mathcal{H}_∞ controller K_{nom} as well as for the FTC controller $K(Q_{ftc})$ is shown in Figure 5. It is clear that the gain of the FTC controller has been reduced compared with the nominal controller. As a consequence of this, a reduction in the performance of the nominal closed loop system is expected.

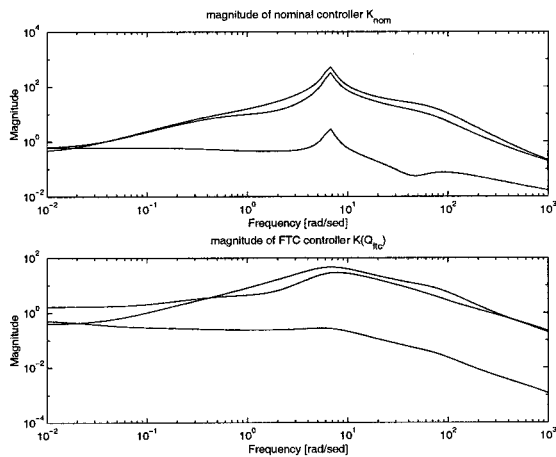


Fig. 5. The magnitude of the nominal controller K_{nom} and of the FTC controller $K(Q_{ftc})$.

Simulations of the faulty system with the fault tolerant controller are shown in the following section.

4. SIMULATION RESULTS

A number of simulations with the faulty system are shown in this section. The FTC system is simulated under the following conditions: At $t = 0.5\text{sec.}$, the Q_{ftc} is included in the closed loop system. At $t = 2.0\text{sec.}$, the gain of the tacho loop is reduced with 70%. The results of the simulations are shown in Figure 6 - 9.

In Figure 6, the faulty system is simulated with the nominal controller. It is clear that the faulty closed loop system is unstable.

The performance of the fault free system when the FTC controller $K(Q_{ftc})$ is applied can be seen from Figure 7. A reduction of the performance of the closed loop system is the result of using the FTC controller compared with the closed loop system based on K_{nom} - compare with the simulation in Figure 2. This is also in line with results shown in Figure 5. In Figure 8, the faulty system has been simulated when the FTC controller has been applied. As it can be seen, the closed loop system is now stable. It is also clear that the performance of the closed loop has been reduced compared with the fault free system, see Figure 2. In

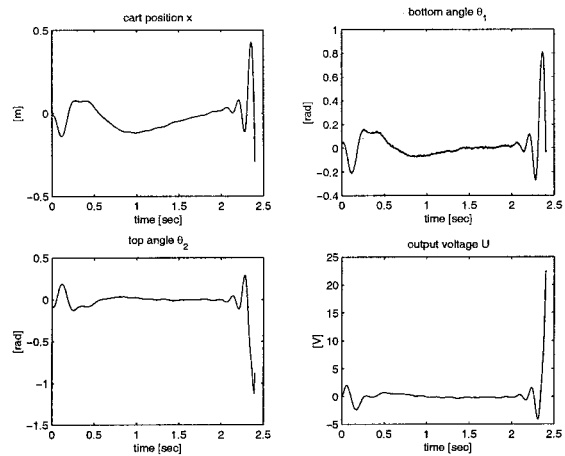


Fig. 6. Simulation of the nonlinear system with the \mathcal{H}_∞ controller. The initial conditions are: $\theta_1 = 0.05\text{rad}$ and $\theta_2 = -0.04\text{rad}$. The gain in the tacho loop is reduced with 70% at $t = 2\text{sec.}$

Figure 9, the two control signals (u and y_q) are shown. It is quite clear that the Q_{ftc} part of the controller is very active after the fault has appeared in the system. This part of the controller need to take over for the reduced tacho feedback loop.

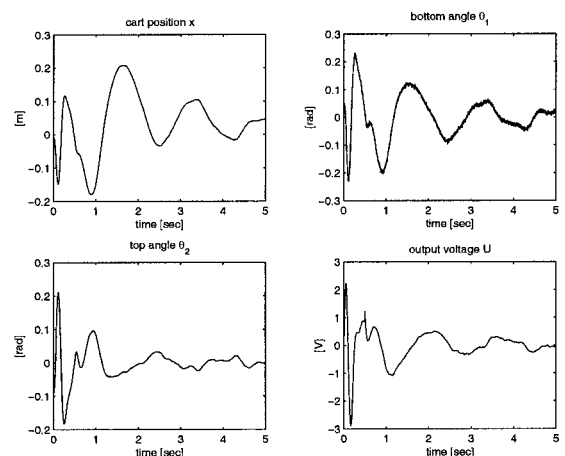


Fig. 7. Simulation of the fault free nonlinear system with the FTC controller $K(Q_{ftc})$. The initial conditions are: $\theta_1 = 0.05\text{rad}$ and $\theta_2 = -0.04\text{rad}$. The Q_{ftc} controller is included in the control loop after $t = 0.5\text{sec.}$

5. CONCLUSION

An architecture for passive fault tolerant controllers has been applied on a double inverted pendulum system. The passive FTC architecture is based on the Youla parameterization of all stabilizing controllers. Three FTC problems has been formulated for the design of Q_{ftc} for the pendulum system. The design of Q_{ftc} with respect to a fault in the tacho loop has been derived by using an \mathcal{H}_∞ optimization method. The final FTC controller has been simulated on the faulty pendulum system.

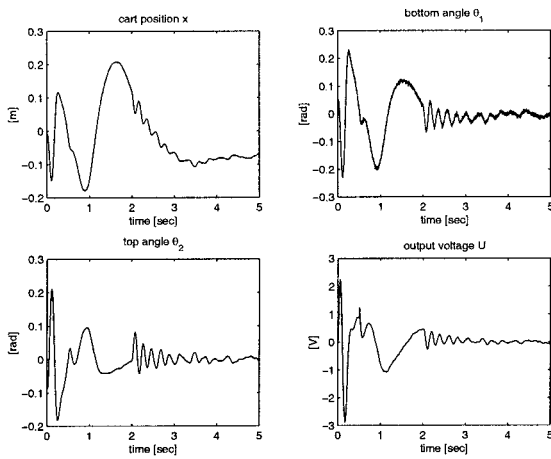


Fig. 8. Simulation of the nonlinear system with the FTC controller $K(Q_{ftc})$. The initial conditions are: $\theta_1 = 0.05rad$ and $\theta_2 = -0.04rad$. The Q_{ftc} controller is included in the control loop after $t = 0.5sec$. The gain in the tachometer loop is reduced with 70% at $t = 2sec$.

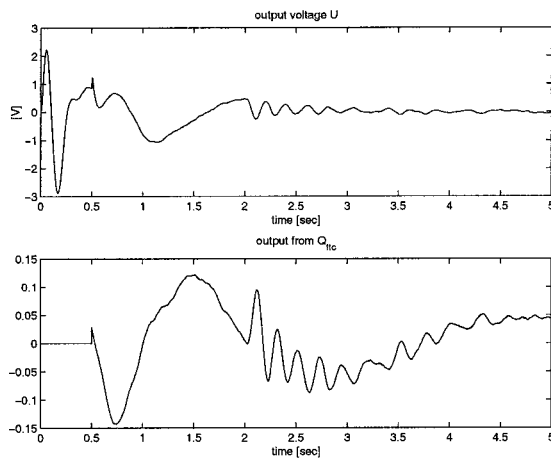


Fig. 9. The control signal from the controller $K(Q)$ and from the Q_{ftc} controller for the simulation of the nonlinear system with the H_∞ controller. The initial conditions are: $\theta_1 = 0.05rad$ and $\theta_2 = -0.04rad$. The Q_{ftc} controller is included in the control loop after $t = 0.5sec$. The gain in the tachometer loop is reduced with 70% at $t = 2sec$.

The introduction of a passive FTC controller in the loop has reduced the performance of the nominal fault free system. The design of the FTC part of the feedback controller is a trade-off between the performance of the nominal fault free pendulum system and the performance of the faulty pendulum system. In this case example, the selected FTC controller reduce the performance of the fault free system with 25 - 50% compared with the nominal controller. The performance of the faulty pendulum system is comparable with the performance of the nominal system.

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