International Journal of Control
Vol. 78, No. 14, 20 September 2005, 1091–1110

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An architecture for fault tolerant controllers

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(*Received 20 August 2003; in final form 1 April 2005*)

A general architecture for fault tolerant control is proposed. The architecture is based on the (primary) YJBK parameterization of all stabilizing compensators and uses the dual YJBK parameterization to quantify the performance of the fault tolerant system. The approach suggested can be applied for additive faults, parametric faults and for system structural changes. The modelling for each of these fault classes is described. The method allows for design of passive as well as for active fault handling. Also, the related design method can be fitted either to guarantee stability or to achieve graceful degradation in the sense of guaranteed degraded performance. A number of fault diagnosis problems, fault tolerant control problems, and feedback control with fault rejection problems are formulated/considered, mainly from a fault modelling point of view. The method is illustrated on a servo example including an additive fault and a parametric fault.

1. Introduction

There are many trends in the development of man-made systems, but one seems to be common widely across industrial areas: the systems become increasingly more complex. Elementary reliability theory tells us at least one challenge in this connection. As the complexity grows, so does the probability of critical faults occurring in the system.

This is one of the motivations for the increasing interest in the design of fault tolerant control systems, where the objective is to disallow one or several faults to develop into an overall system failure.

In the search for systematic design methods for fault tolerant control, recent research efforts have focused on deriving control laws based on a specific fault model. The best choice of fault model will depend on the purpose of the model. A number of faults can naturally be considered both as additive faults or as parametric faults. However, a random choice might not be optimal. The fault model needs to be selected with respect to the design objectives, i.e. whether only fault diagnosis is required, the objective is to preserve stability in faulty situations, or even to recover performance during faults.

In the past, additive fault models have been the most popular models, especially in connection with fault diagnosis. Modelling e.g. an actuator fault as an additive fault will generally be very useful in connection with fault detection and/or fault isolation. In connection with closed-loop systems, an actuator fault might result in instability. Using an additive fault model description in this case, the fault will be considered as an external signal entering the system. The fault signal of the model will therefore not affect the stability of the system, at least not for bounded fault signals. This small example clearly indicates that the description of possible faults in a dynamic system needs to be selected in very close relation with the application of the fault description/fault model.

In this paper, three types of faults/fault models will be considered. The three types are as follows:

- additive faults;
- parametric faults;
- system structural changes.

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International Journal of Control
ISSN 0020–7179 print/ISSN 1366–5820 online © 2005 Taylor & Francis Group Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/00207170500202249

These fault models can be considered in connection with the following applications:

- fault detection, fault isolation and fault estimation;
- fault tolerant control with stability recovery, i.e. the control system can handle faults in the system without resulting in an unstable closed loop system. Note, that additive faults cannot challenge the stability of a linear system;
- feedback control with performance recovery, i.e. the effect from the fault is minimized in the closed loop by a feedback controller.

In this paper only linear systems will be considered. However, a number of the presented results can be generalized to non-linear systems without further assumptions.

The results of this paper relate to the areas of fault tolerant control and robust control. These areas are very well described in a large number of papers and books. Without going into details, let us mention the books by Basseville and Nikiforov (1993), Chen and Patton (1998) and Gertler (1998) for a good introduction to the area of fault diagnosis. In Patton (1997a, b) and Blanke *et al.* (2000, 2001) and references therein, good introductions to the area of fault tolerant control can be found. Most of these papers describe various concepts for FTC. However, in the past years, also a number of theoretical results has been presented in this area, see e.g. Wang and Wu (1993), Wu (1993), Wu and Chen (1996), Staroswiecki *et al.* (1999), Wu *et al.* (2000), Stoustrup and Niemann (2001), Zhou and Ren (2001), Staroswiecki *et al.* (2002). The area of robust control has been investigated in a large number of books and papers. Let us only mention the books written by Zhou *et al.* (1995) and Skogestad and Postlethwaite (1996).

A significant application area of fault tolerant control deserving specific mention is the area of reconfigurable flight systems which has been a pioneering area for several of the methodologies. To mention a few references in this area, we point to Boskovic and Mehra (1998), Boskovic *et al.* (1998), Ganguli *et al.* (2002) in which further references can be found.

Two main classes of approaches can be distinguished in the literature on fault tolerant control: active FTC and passive FTC. In active FTC, the controller is reconfigured whenever a fault is detected. In passive FTC, the controller is fixed; its fault tolerance is obtained by an *a priori* design based on the fault models, such that this fixed controller is able to handle all possible faults. Recently, an existence result has been shown for the fault tolerant stabilization problem in the paper by Stoustrup and Blondel (2004) where further references to passive FTC can be found. Active FTC relies on fault detection, fault isolation, and fault estimation. An approach to fault estimation which can be integrated with the FTC approach presented in this paper, can be found in Stoustrup and Niemann (2002).

The focus in this paper will be on using various fault models in connection with FTC. The paper will give an overview of the various design problems for controller reconfiguration in a general FTC architecture, depending on the type of faults.

A general architecture based on the YJBK parameterization will be proposed which allows us to handle all fault model types and to implement solutions for all the design problems described. The architecture is based on the results presented in Niemann and Stoustrup (2002, 2005).

This paper is organized as follows. In section 2, the system setup is given for three different fault types together with a number of definitions. The YJBK parameterization is first introduced in section 3. A new controller architecture for FTC is introduced in section 4 followed by a study of controller reconfiguration in the FTC architecture for the three types of fault models in section 5. An example is considered in section 6. Finally, we arrive at a conclusion in section 7.

1.1. *Nomenclature*

Capital letters will denote matrices or matrix valued functions. A^T is the transposed of A. A nominal system is described by Σ and a stabilizing feedback controller for Σ is given by Σ_C. Further, let an uncertain system or faulty system be given by Σ_Θ, where $\Theta \in \underline{\Theta}$ represents the model uncertainty, finite sets of the fault parameters or the input fault signals. A more detailed description of Σ_Θ is given below. The interconnection of the nominal system and the feedback controller is given by $\Sigma \times \Sigma_C$.

$\mathcal{F}_l(X, Y) = X_{11} + X_{12}Y(I - X_{22}Y)^{-1}X_{21}$ is the lower Linear Fractional Transformation (LFT) of (X, Y). The upper LFT of (X, Y) is given by $\mathcal{F}_u(X, Y) = X_{22} + X_{21}Y(I - X_{11}Y)^{-1}X_{12}$.

For simplicity, transfer functions are not equipped with an explicit dependency of a complex variable 's', as it should not be possible to confuse matrices and transfer functions when considering the context. In a few cases, the word 'dynamic' has been added to explicitly refer to a transfer function rather than a matrix.

2. Definitions and system setup

2.1. *System setup*

The general systems will now be described in detail by using transfer functions. Consider the following generalized nominal $(r+m)\times(q+p)$ system,

$$\Sigma\colon \begin{cases} e = G_{ed}d + G_{eu}u \\ y = G_{yd}d + G_{yu}u, \end{cases} \qquad (1)$$

where $d \in \mathcal{R}^r$ is a disturbance signal vector, $u \in \mathcal{R}^m$ the control input signal vector, $e \in \mathcal{R}^q$ is the external output signal vector to be controlled, and $y \in \mathcal{R}^p$ is the measurement vector. $G_{\xi\zeta}$ is the transfer function between input ζ and output ξ.

Further, let the system be controlled by a stabilizing dynamic feedback controller given by

$$\Sigma_C\colon \left\{ u = Ky \right. \qquad (2)$$

resulting in the closed-loop system

$$\Sigma_{cl} = \Sigma \times \Sigma_C.$$

Let Σ_Θ the generalized system in (1) include faults. Three different types of faults will now be introduced. First, let us consider systems with additive faults, Σ_Θ is then given by Σ_A

$$\Sigma_A\colon \begin{cases} e = G_{ed}d + \sum_{i=1}^{k} G_{ef,i}f_i + G_{eu}u \\ \quad = G_{ed}d + G_{ef}f + G_{eu}u \\ y = G_{yd}d + \sum_{i=1}^{k} G_{yf,i}f_i + G_{yu}u \\ \quad = G_{yd}d + G_{yf}f + G_{yu}u, \end{cases} \qquad (3)$$

where f_i signifies the i-th fault for each $i = 1, 2, \ldots, k$. It is further assumed that the f_i is bounded and not correlated with the system state. The fault signal vector $f \in \mathcal{R}^k$ is a collection of fault signals f_i, $i = 1, 2, \ldots, k$, into a vector. Also, it is common in the fault detection and isolation setting for model uncertainties to be described as external input signals in the same manner as disturbance signals w. In other words, w here can be thought of representing both external disturbance signals and signals that might arise due to model uncertainties, see e.g. Frank and Ding (1994).

However, in the cases where we would like to detect, isolate and/or estimate parameter changes or uncertainty variations in the system, the fault model described by (3) cannot in general be applied. In the case where the system includes parametric faults, Σ_Θ can be described by Σ_P

$$\Sigma_P\colon \begin{cases} z = G_{zw}w + G_{zd}d + G_{zu}u \\ e = G_{ew}w + G_{ed}d + G_{eu}u \\ y = G_{yw}w + G_{yd}d + G_{yu}u, \end{cases} \qquad (4)$$

where $w \in \mathcal{R}^{k_w}$ and $z \in \mathcal{R}^{k_z}$ are the external input and output vectors. The connection between the external output and the external input is given by

$$w = \theta z,$$

where θ represents the parametric faults in the system. Note that the above description is also applied in connection with the description of systems including model uncertainties, see e.g. Zhou *et al*. (1995). In this case, the connection between the external output and the external input is given by

$$w = \Delta z$$

where $\Delta \in \underline{\Delta}$ represent the model uncertainties. The system is the described by Σ_Δ. Closing the loop from w to z in Σ_P by using θ, we get

$$\Sigma_{P,\theta} = \mathcal{F}_u(\Sigma_P, \theta).$$

Faults might change the structure of the system. Based on a structural change of the nominal system in (1) due to faults, the general system Σ_Θ then takes the following form:

$$\Sigma_{S_i}\colon \begin{cases} e = \tilde{G}_{ed,i}d + \tilde{G}_{eu,i}u \\ y = \tilde{G}_{yd,i}d + \tilde{G}_{yu,i}u \end{cases}, \quad i = 0, \ldots, k, \qquad (5)$$

where $\tilde{\cdot}$ indicates a change in transfer matrix, a change in the number of system states and number of inputs and outputs. Note that $i = 0$ is defined as the nominal model, $\Sigma_{S_0} = \Sigma$. Below, it will be argued in further detail, that it is unnecessary to define new input, output, or disturbance signals, as 'missing' signals can be modeled by transfer matrices with appropriate zero entries.

It should be pointed out in connection with the fault model given by (5), it is the most direct way to describe a change in the system as a consequence of faults in the system. However, in many cases a more detailed model description can be obtained by using a parametric fault model.

Throughout the paper, it will be assumed that faults only occur one at a time.

In some of the stability results of this paper, it is an implicit assumption, that the system remains detectable and stabilizable after a fault has occured, such that set of stabilizing controllers remains non-empty.

Finally, the methods proposed assume that knowledge of post-fault scenarios is available *a priori*. As an alternative, they can be identified. Such an approach, however, lies outside the scope of this paper.

2.2. *Example*

An example is introduced here in connection with a description of the three types of faults introduced above. Let's consider the system described by the block diagram shown in figure 1.

The system is described by the following equations:

$$\left.\begin{aligned} z_1 &= x_1 = G_1 u \\ z_2 &= x_2 = G_2 x_1 \\ y_1 &= G_{\text{sen},1} x_1 + f_1 \\ y_2 &= G_{\text{sen},2} x_2 + f_2, \end{aligned}\right\} \tag{6}$$

where z_1 and z_2 are the signals to be controlled, u is the control input and y_1, y_2 are two measurement signals. f_1 and f_2 represent additive sensor faults as e.g. bias in the measurement signals or an increase in the measurement noise due to a loss of filtering in the sensors. k_1 and k_2 represent the gains of the two sensors. A change in k_1 and k_2 is directly modelled as parametric faults given by

$$\begin{aligned} k_1 &= k_{10}(1+\theta_1), \quad \theta_1 \in [-1, 0] \\ k_2 &= k_{20}(1+\theta_1), \quad \theta_2 \in [-1, 0], \end{aligned}$$

where k_{10} and k_{20} are the nominal gains and θ_1, θ_2 are the parametric faults.

The structure of the system changes if e.g. a sensor falls out. Let's assume that we only want to control z_1 and the second measurement signal y_2 is not available. In this case, the new system is given by

$$\left.\begin{aligned} z_1 &= x_1 = G_1 u \\ y_1 &= G_{\text{sen},1} x_1 + f_1 \end{aligned}\right\} \tag{7}$$

i.e. a reduction of the nominal system given by (6).

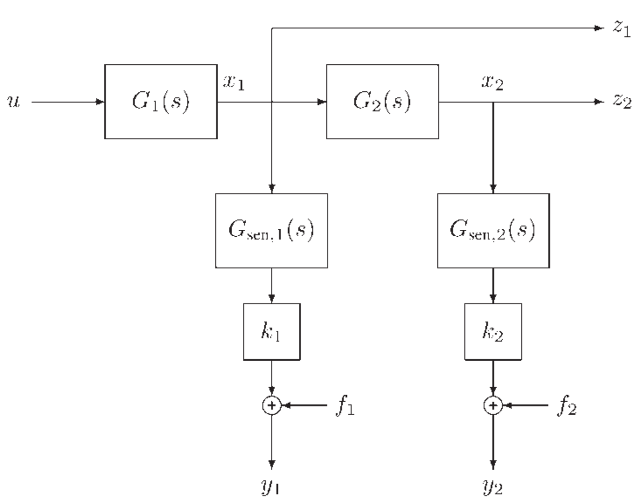

Figure 1. Block diagram of a dynamic system including two sub-systems and two sensors.

In connection with fault diagniosis, it is possible to model all three types of faults as additive faults. The parametric faults in k_1 and k_2 can be modelled by

$$\begin{aligned} f_1 &= -\theta_1 G_{\text{sen},1} x_1 \\ f_2 &= -\theta_2 G_{\text{sen},2} x_2. \end{aligned}$$

A loss of the second sensor can be described by

$$f_2 = -G_{\text{sen},2} x_2 = -G_{\text{sen},2} G_2 x_1$$

i.e. $y_2 = 0$.

The most direct way to handle the controller reconfiguration problem is to use the different fault types directly, instead of transforming them to additive faults.

2.3. *Definitions*

Based on the system setup given above, let us give a number of definitions in connection with feedback control and fault tolerant control. The definition of stable feedback control and robust feedback control are given in Zhou *et al.* (1995). The definitions of fault tolerant control are equivalent with the definitions given in e.g. Blanke *et al.* (2000).

From Boyd and Barratt (1991), we have the following definition of a design specification:

Definition 1: A design specification $\mathcal{D}$ is a boolean function or test on the closed loop transfer matrix $\Sigma_{cl} = \Sigma \times \Sigma_C$.

An equivalent definition of design specification can be found in Staroswiecki and Gehin (2001) as the standard control problem (SCP) and later used in Blanke *et al.* (2003).

The design specification given by $\mathcal{D}$ is tests on the closed loop transfer matrix Σ_{cl} that is either satisfied or not satisfied, i.e. the result of the tests has a pass or fail as the outcome. The design specifications can be given in both the time domain and/or the frequency domain.

It is possible to combine a number of design specifications as e.g.

$$\mathcal{D}_{\text{stable}} \wedge \mathcal{D}_{\text{performance}} \colon \Sigma_{cl}$$

meaning that the closed loop transfer matrix Σ_{cl} must satisfy both the stability condition given by $\mathcal{D}_{\text{stable}}$ and the performance condition given by $\mathcal{D}_{\text{performance}}$.

It is also possible to order the design specifications in some cases. Let $\mathcal{D}_1$ and $\mathcal{D}_2$ be two sets of design specifications that can be compared. We will say that $\mathcal{D}_1$ is tighter or stronger than $\mathcal{D}_2$ if all transfer matrices satisfying $\mathcal{D}_1$ also satisfy $\mathcal{D}_2$. If at least one transfer function exists that satisfies $\mathcal{D}_2$ but not $\mathcal{D}_1$, we will say that $\mathcal{D}_1$ is strictly tighter or strictly stronger than $\mathcal{D}_2$.

Based on Definition 1 and the system setup given in section 2.1, we can now define the following design problems.

Problem 1: The design problem for closed loop stability of the nominal system is given by

$$\mathcal{D}_{\text{stable}}\colon \Sigma_{cl}$$

where $\mathcal{D}_{\text{stable}}$ is the stability specification and Σ_{cl} is the nominal closed loop transfer matrix.

Problem 2: The open loop design problem for performance of the nominal system is given by

$$\mathcal{D}_{\text{performance},1}\colon \Sigma_{\text{open}}$$

where $\mathcal{D}_{\text{performance},1}$ is the performance specification and Σ_{open} is the nominal closed loop transfer matrix.

Problem 3: The design problem for performance of the nominal system is given by

$$\mathcal{D}_{\text{stable}} \wedge \mathcal{D}_{\text{performance},1}\colon \Sigma_{cl}$$

where $\mathcal{D}_{\text{stable}}$ is the stability specification, $\mathcal{D}_{\text{performance},1}$ is the performance specification and Σ_{cl} is the nominal closed loop transfer matrix.

Problem 4: The design problem for robust stability is given by

$$\mathcal{D}_{\text{stable}}\colon \Sigma_{cl,\Delta}$$

where $\mathcal{D}_{\text{stable}}$ is the stability specification and $\Sigma_{cl,\Delta}$ is the closed loop transfer matrix as function of the model uncertainty given by $\Delta \in \underline{\Delta}$.

Problem 5: The design problem for robust performance is given by

$$\mathcal{D}_{\text{stable}} \wedge \mathcal{D}_{\text{performance},1}\colon \Sigma_{cl,\Delta}$$

where $\mathcal{D}_{\text{stable}}$ is the stability specification, $\mathcal{D}_{\text{performance},1}$ is the performance specification and $\Sigma_{cl,\Delta}$ is the closed loop transfer matrix as function of the model uncertainty given by $\Delta \in \underline{\Delta}$.

Problem 6: The fault tolerant control problem for stability is given by

$$\mathcal{D}_{\text{stable}}\colon \Sigma_{cl,\Theta}$$

where $\mathcal{D}_{\text{stable}}$ is the stability specification and $\Sigma_{cl,\Theta}$ is the closed loop transfer matrix as function of the fault vector/parameter given by $\Theta \in \underline{\Theta}$.

Problem 7: The fault tolerant control problem for stability is given by

$$\mathcal{D}_{\text{stable}} \wedge \mathcal{D}_{\text{performance},2}\colon \Sigma_{cl,\Theta}$$

where $\mathcal{D}_{\text{stable}}$ is the stability specification, $\mathcal{D}_{\text{performance},2}$ is the performance specification and $\Sigma_{cl,\Theta}$ is the closed loop transfer matrix as function of the fault vector/parameter given by $\Theta \in \underline{\Theta}$.

In Problem 7, it is assumed that $\mathcal{D}_{\text{performance},1}$ is stronger than $\mathcal{D}_{\text{performance},2}$.

In connection with Problems 6 and 7, it should be pointed out that the first objective in connection with FTC is to stabilize the feedback system, i.e. Problem 6. However, in some cases, it might also be possible to design the FTC part of the controller such that the closed loop performance reduction, due to fault in the system, is minimized. This is the case in situations where there are sufficient sensor and actuator redundancy, such that a realistic alternative to a 'limb home' strategy exists.

It is now possible to give the following definitions of the solutions to Problems 1–7.

Definition 2: Given a nominal dynamic system Σ and a feedback controller Σ_C. The feedback controller Σ_C is said to be a stabilizing feedback controller if and only if the closed loop transfer matrix of the interconnection $\Sigma \times \Sigma_C$ satisfies the stability specification of Problem 1.

Definition 3: Given a nominal dynamic system Σ and a feedforward controller Σ_C. The feedforward controller Σ_C is said to satisfy nominal open loop performance if and only if the open loop transfer matrix of Σ and Σ_C satisfies the performance specification of Problem 2.

Definition 4: Given a nominal dynamic system Σ and a feedback controller Σ_C. The feedback controller Σ_C is said to satisfy nominal performance if and only if the closed loop transfer matrix of the interconnection $\Sigma \times \Sigma_C$ satisfies the stability and performance specification of Problem 3.

Definition 5: Given a dynamic system Σ_Δ and a feedback controller Σ_C. It is assumed that $\Delta \in \underline{\Delta}$ represents the model uncertainty. The feedback controller Σ_C is said to be a robustly stabilizing feedback controller if and only if the closed loop transfer matrix of the interconnection $\Sigma_\Delta \times \Sigma_C$ satisfies the stability specification of Problem 4.

Definition 6: Given a dynamic system Σ_Δ and a feedback controller Σ_C. It is assumed that $\Delta \in \underline{\Delta}$ represents the model uncertainty. The feedback controller Σ_C is said to be a robust performance feedback controller if and only if the closed loop transfer matrix of the interconnection $\Sigma_\Delta \times \Sigma_C$ satisfies the stability and performance specification of Problem 5.

Definition 7: Given a set of dynamic systems Σ_Θ and a feedback controller Σ_C. It is assumed that $\Theta \in \underline{\Theta}$ represents a finite number of faulty parameter sets. The feedback controller Σ_C is said to be a fault tolerant feedback controller if and only if the closed loop transfer matrix of the interconnection $\Sigma_\Theta \times \Sigma_C$ satisfies the stability specification of Problem 6.

Definition 8: Given a set of dynamic systems Σ_Θ and a feedback controller Σ_C. It is assumed that $\Theta \in \underline{\Theta}$ represents a finite number of faulty parameter sets or the fault input signals. The feedback controller Σ_C is said to be a fault tolerant feedback controller with performance specifications if and only if the closed loop transfer matrix of the interconnection $\Sigma_\Theta \times \Sigma_C$ satisfies the stability and performance specification of Problem 7.

In the rest of this paper, the performance specifications given in Problems 3, 5 and 7 are limited to disturbance rejection specifications. The disturbance rejection specifications are given as a specification of the $\mathcal{H}_2$ or the $\mathcal{H}_\infty$ norm of the closed loop transfer matrix from external input d to external output e. Using the $\mathcal{H}_2$ norm as the specification, $\mathcal{D}_{\text{performance}}$ is given by

$$\mathcal{D}_{\text{performance}} = \mathcal{D}_{\mathcal{H}_2} = \min \|\Sigma \times \Sigma_C\|_2. \tag{8}$$

Using the $\mathcal{H}_\infty$ norm as the specification, $\mathcal{D}_{\text{performance}}$ is given by

$$\left.\begin{array}{l} \mathcal{D}_{\text{performance},1} = \mathcal{D}^{\gamma_1}_{\mathcal{H}_\infty} = \|\Sigma \times \Sigma_C\|_\infty < \gamma_1 \\ \mathcal{D}_{\text{performance},2} = \mathcal{D}^{\gamma_2}_{\mathcal{H}_\infty} = \|\Sigma \times \Sigma_C\|_\infty < \gamma_2, \end{array}\right\} \tag{9}$$

where $\gamma_1 \le \gamma_2$.

3. The YJBK parameterization

Before considering the three different FTC design cases, the (primary) YJBK parameterization and the dual YJBK parameterization are shortly introduced. The controller architecture applied for the FTC in the following will be based on the YJBK parameterization. The YJBK parameterization has also been applied in connection with FTC in Stoustrup and Niemann (2001), Zhou and Ren (2001).

The YJBK parameterization was first derived by Youla *et al.* and independently by Kucera. It has been described in Youla *et al.* (1976a, b), Kucera (1979) and later used in many cases in connection with feedback control, see e.g. Boyd *et al.* (1988), Boyd and Barratt (1991), Grimble (1994), Dahleh and Diaz-Bobillo (1995), Zhou *et al.* (1995), Tay *et al.* (1997), Anderson (1998).

3.1. *The Primary YJBK parameterization*

Let a coprime factorization of the system $G_{yu}(s)$ from (1) and a stabilizing controller $K(s)$ from (2) be given by

$$\left.\begin{array}{ll} G_{yu} = NM^{-1} = \tilde{M}^{-1}\tilde{N}, & N, M, \tilde{N}, \tilde{M} \in \mathcal{RH}_\infty \\ K = UV^{-1} = \tilde{V}^{-1}\tilde{U}, & U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty, \end{array}\right\} \tag{10}$$

where the eight matrices in (10) must satisfy the double Bezout equation given by, see Zhou *et al.* (1995)

$$\begin{aligned} \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} &= \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix}\begin{pmatrix} M & U \\ N & V \end{pmatrix} \\ &= \begin{pmatrix} M & U \\ N & V \end{pmatrix}\begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix}. \end{aligned} \tag{11}$$

Based on the above coprime factorization of the system $G_{yu}(s)$ and the controller $K(s)$, we can give a parameterization of all controllers that stabilizing the system in terms of a stable parameter $Q(s)$, i.e. all stabilizing controllers are given by Tay *et al.* (1997)

$$K(Q) = U(Q)V(Q)^{-1}, \tag{12}$$

where

$$U(Q) = U + MQ,\ V(Q) = V + NQ, \quad Q \in \mathcal{RH}_\infty$$

or by using a left factored form

$$K(Q) = \tilde{V}(Q)^{-1}\tilde{U}(Q), \tag{13}$$

where

$$\tilde{U}(Q) = \tilde{U} + Q\tilde{M}, \tilde{V}(Q) = \tilde{V} + Q\tilde{N}, \quad Q \in \mathcal{RH}_\infty.$$

Using the Bezout equation, the controller given either by (12) or by (13) can be realized as an LFT in the parameter Q,

$$K(Q) = \mathcal{F}_l(J_K, Q), \tag{14}$$

where J_K is given by

$$J_K = \begin{pmatrix} UV^{-1} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix} = \begin{pmatrix} \tilde{V}^{-1}\tilde{U} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix}. \tag{15}$$

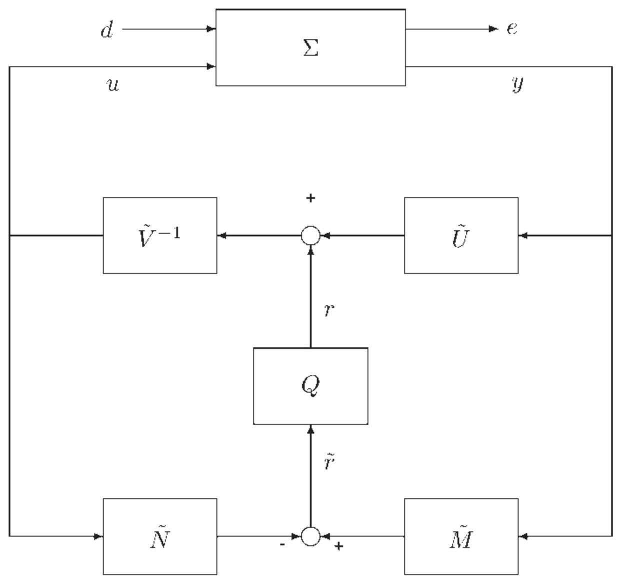

Figure 2. Controller structure with parameterization.

Reorganizing the controller $K(Q)$ given by (14) results in the closed loop system depicted in figure 2, see also Tay *et al*. (1997).

The main observation to be exploited in the solution to the fault tolerant control problem, is the following relatively simple expression for the transfer function from the external input d to the external output e terms of the parameter Q

$$\begin{aligned} e &= (G_{ed} + G_{eu}K(Q)(I - G_{yu}K(Q))^{-1}G_{yd})d \\ &= (G_{ed} + G_{eu}U\tilde{M}G_{yd} + G_{eu}MQ\tilde{M}G_{yd})d, \end{aligned}$$

where (11) has been exploited. Note, that the transfer function relating d and e is affine in Q.

3.2. *The dual YJBK parameterization*

The dual YJBK parameterization gives a parameterization in term of a stable parameter S of all systems stabilized by a given controller. The parameterization is given by Tay *et al*. (1997)

$$G(S) = N(S)M(S)^{-1}, \tag{16}$$

where

$$N(S) = N + VS, M(S) = M + US, \quad S \in \mathcal{RH}_\infty$$

or by using a left factored form:

$$G(S) = \tilde{M}(S)^{-1}\tilde{N}(S), \tag{17}$$

where

$$\tilde{N}(S) = \tilde{N} + S\tilde{V},\ \tilde{M}(S) = \tilde{M} + S\tilde{U}, \quad S \in \mathcal{RH}_\infty.$$

An LFT representation of (16) or (17) is given by

$$G(S) = \mathcal{F}_l(J_G, S), \tag{18}$$

where J_G is given by

$$J_G = \begin{pmatrix} NM^{-1} & \tilde{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix}. \tag{19}$$

The interpretation of the dual YJBK parameter S can be investigated from the primal YJBK parameterization shown in figure 2. It turns out that the dual YJBK parameter S is the open loop transfer function from r to $\tilde{r}$ in figure 2 (Tay *et al*. 1997), i.e.

$$S = \mathcal{F}_u(J_K, G_{yu}(S)).$$

This fact can be used in connection with the estimation of system changes.

In table 1, S has been calculated for a number of different types of model uncertainties, These equations for S will be applied in the following in connection with parametric faults. The calculation of S as function of Δ is given in Appendix A for the general case.

4. Fault tolerant controller architecture

In the sequel, an architecture for fault tolerant controllers will be proposed, based on the YJBK parameterization shown in the block diagram in figure 2. There is a number of reasons for using the architecture from the YJBK parameterization in connection with FTC.

Before going into details with the FTC architecture, it is important to point out that a FTC controller consists of two parts: a fault diagnosis (FDI-FTC) part and a controller reconfiguration (CR-FTC) part. The first part is used for detection and/or isolation of faults in the system. The second part is a reconfiguration of the feedback controller based on information from the FDI-FTC block.

Using an FTC architecture based on the YJBK parameterization, the Q parameter will be the CR-FTC part of the FTC controller. This means that the CR-FTC part of the feedback controller is a modification of the existing controller. Thus, a controller change when a fault appears in the system is not a complete shift to another controller, but only a modification of the existing controller by adding a correction signal in the

Table 1. The connection between different system uncertainty descriptions in terms of Δ and the dual YJBK parameter S.

System description, $G_{yu}(\theta)$	The dual YJBK parameter, $S(\Delta)$
$G_{yu}(\Delta) = (I+\Delta)G_{yu}$	$S(\Delta) = \tilde{M}\Delta(I - N\tilde{U}\Delta)^{-1}N$
$G_{yu}(\Delta) = G_{yu}(I+\Delta)$	$S(\Delta) = \tilde{N}\Delta(I - U\tilde{N}\Delta)^{-1}M$
$G_{yu}(\Delta) = G_{yu} + \Delta$	$S(\Delta) = \tilde{M}\Delta(I - U\tilde{M}\Delta)^{-1}M$
$G_{yu}(\Delta) = G_{yu}(I-\Delta)^{-1}$	$S(\Delta) = \tilde{N}\Delta(I - M\tilde{V}\Delta)^{-1}M$
$G_{yu}(\Delta) = (I-\Delta)^{-1}G_{yu}$	$S(\Delta) = \tilde{M}\Delta(I - V\tilde{M}\Delta)^{-1}N$
$G_{yu}(\Delta) = G_{yu}(I-\Delta G_{yu})^{-1}$	$S(\Delta) = \tilde{N}\Delta(I - N\tilde{V}\Delta)^{-1}N$
$G_{yu}(\Delta) = (N+\Delta_N)(M+\Delta_M)^{-1}$	$S(\Delta) = \begin{pmatrix} -\tilde{N} & \tilde{M} \end{pmatrix}\begin{pmatrix} \Delta_M \\ \Delta_N \end{pmatrix}\left(I + \begin{pmatrix} \tilde{V} & -\tilde{U} \end{pmatrix}\begin{pmatrix} \Delta_M \\ \Delta_N \end{pmatrix}\right)^{-1}$
$G_{yu}(\Delta) = (\tilde{M}+\Delta_{\tilde{M}})^{-1}(\tilde{N}+\Delta_{\tilde{N}})$	$S(\Delta) = \left(I + \begin{pmatrix} \Delta_{\tilde{M}} & \Delta_{\tilde{N}} \end{pmatrix}\begin{pmatrix} -U \\ V \end{pmatrix}\right)^{-1}\begin{pmatrix} \Delta_{\tilde{M}} & \Delta_{\tilde{N}} \end{pmatrix}\begin{pmatrix} M \\ -N \end{pmatrix}$

nominal controller, the r signal in figure 2. However, it should be pointed out that it is possible to modify the controller arbitrarily by designing the YJBK parameter Q, see e.g. Tay *et al*. (1997), Niemann *et al*. (2002).

Another important thing is that the architecture also includes a parameterization of all residual generators. All residual signals can be described by, Frank and Ding (1994), Gertler (1998)

$$r = Q_{FDI}\tilde{r} = Q_{FDI}(\tilde{M}y - \tilde{N}_u u). \tag{20}$$

This means that it is possible to combine both fault diagnosis and fault tolerant control in the same architecture without any problems. A block diagram for this FTC architecture based on the YJBK parameterization is shown in figure 3 for three potential parametric faults – the generalization to any number of faults should be obvious.

The above controller architecture applied for FTC shown in figures 2 and 3 has a fixed structure with respect to the number of measurement signals and control signals. This will not in general be the case in real applications. Here, faults in e.g. sensors can be handled by applying other sensors in the system, i.e. the measurement output from the system is changed. Equivalent to faults in connection with the actuators in the system. This type of system change has not directly been included in the system description given by (4) or (5). However, it is possible to include a change of sensors and/or actuators in the FTC architecture given above.

Let us consider the system G_{yu} given by (1). Assume that only a subset of the sensors and the actuators has been applied for the nominal feedback controller

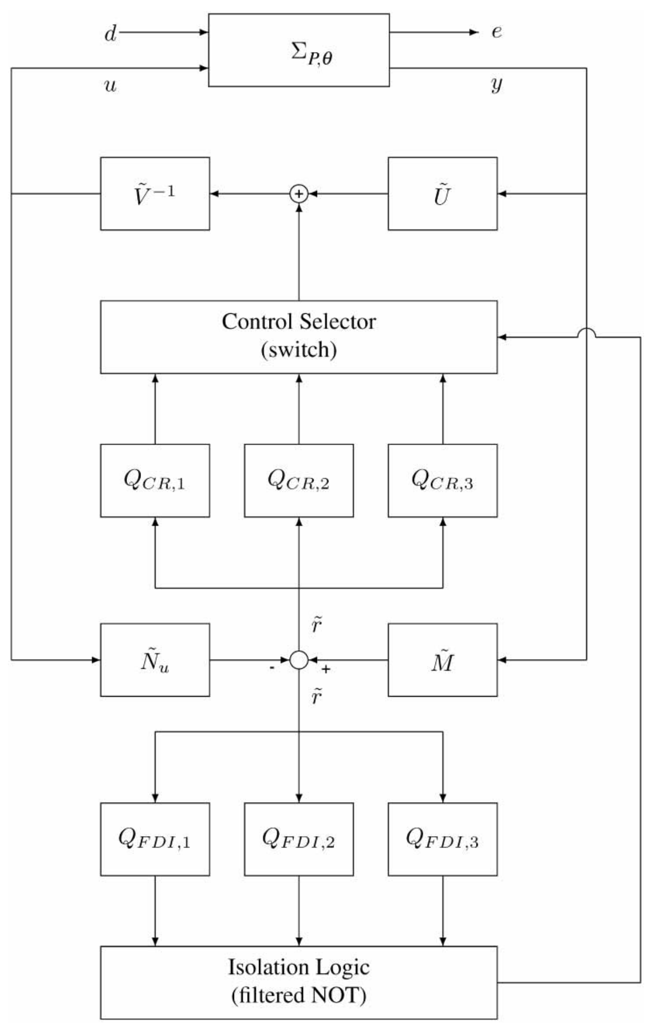

Figure 3. Fault tolerant scheme with three potential parametric faults. The residual signal is used both for isolation and for feedforward in the fault handling.

$K(s)$ given by (2). Let the system G_{yu} be partitioned as follows:

$$G_{yu} = \begin{pmatrix} G_{yu,00} & G_{yu,01} \\ G_{yu,10} & G_{yu,11} \end{pmatrix}. \tag{21}$$

Further, let us use the controller given by

$$K(s) = \begin{pmatrix} K_0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{22}$$

Based on this controller, the YJBK matrices then take the following form

$$\begin{pmatrix} M & U \\ N_u & V \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} M_0 & M_1 \\ 0 & I \end{pmatrix} & \begin{pmatrix} U_0 & 0 \\ 0 & 0 \end{pmatrix} \\ N_u & \begin{pmatrix} V_0 & 0 \\ V_1 & I \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N}_u & \tilde{M} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \tilde{V}_0 & \tilde{V}_1 \\ 0 & I \end{pmatrix} & \begin{pmatrix} -\tilde{U}_0 & 0 \\ 0 & 0 \end{pmatrix} \\ -\tilde{N}_u & \begin{pmatrix} \tilde{M}_0 & 0 \\ \tilde{M}_1 & I \end{pmatrix} \end{pmatrix}.$$

The YJBK parameterized controller $K(Q)$ given by (12) then takes the following form:

$$\begin{aligned} K(Q) &= U(Q)V(Q)^{-1} \\ &= \begin{pmatrix} K_0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \tilde{V}_0^{-1} & -\tilde{V}_0^{-1}\tilde{V}_1 \\ 0 & I \end{pmatrix} Q \\ &\quad \times \left(I + \begin{pmatrix} V_0^{-1} & 0 \\ -V_1 V_0^{-1} & I \end{pmatrix} N_u Q \right)^{-1} \begin{pmatrix} V_0^{-1} & 0 \\ -V_1 V_0^{-1} & I \end{pmatrix}. \end{aligned} \tag{23}$$

A block diagram of the $K(Q)$ given by (23) is shown in figure 4.

From figure 4, it is possible to calculate the transfer function from r to $\tilde{r}$. The open loop transfer function is the zero transfer function in the fault free case. As a direct consequence of this, the closed loop transfer function will be an affine function of the Q parameter. Note that the $\tilde{r}$ (or r) vector can still be applied in connection with fault diagnosis and/or fault isolation.

It is clear from both figure 4 and (23) that the controller architecture will allow us to use other sensors and/or actuators that are used in connection with the nominal controller K_0.

The result of this generalization of applying the YJBK parameterization in connection with fault tolerant control is that it is not a limitation of the controller structure. The structure can handle the problem of changing sensors and/or actuators without any problems, which is normal in connection with fault tolerant control. The above general controller architecture can directly be applied in connection with the results given in section 5.

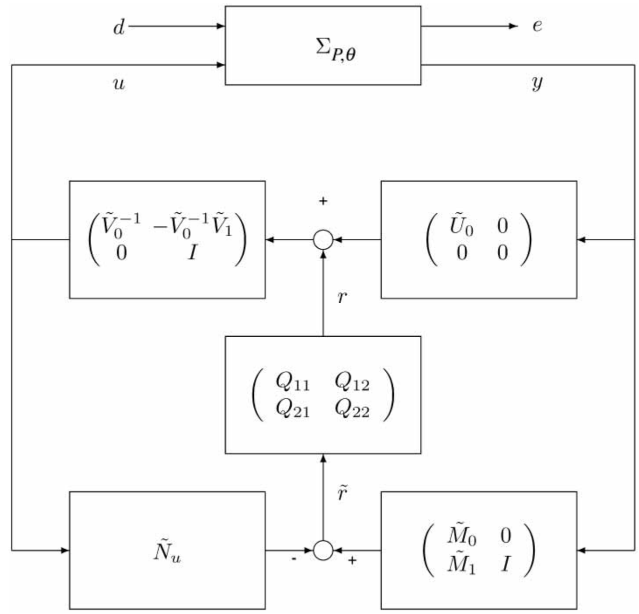

Figure 4. Block diagram for controller with a Q parameterization. Note that the nominal controller does not use the measurements employed after reconfiguration.

A simplified FTC architecture is applied in Niemann and Stoustrup (2005) in connection with passive fault tolerant control. The idea in the passive FTC is to remove the FDI part in the architecture. The single CR-FTC controller Q is designed to handle all faults in the system. Further, the CR-FTC controller will be included all the time. This means that Q will be included as an open loop transfer function in the nominal system and in a feedback loop in the faulty system. The advantage by this passive FTC architecture is that delays due to fault isolation are removed from the FTC loop.

5. Controller reconfiguration

Just as in fault diagnosis, the controller reconfiguration problem will depend strongly on the type of faults that can appear in the system. In this paper, the various controller reconfiguration design problems will be described for the three different model structures given in section 2. Especially in connection with CR-FTC for systems with structural changes, the solution (the selected controller structure, type etc.) will depend strongly on the specific case. No general method with explicit design formulae exists that can handle the general case. Much better design results can be obtained by using dedicated design methods.

In the following, only performance specifications based on the $\mathcal{H}_\infty$ norm are considered. However, performance problems based on the $\mathcal{H}_2$ norm can be handled in an equivalent way.

5.1. *CR-FTC for systems with additive faults*

In numerous systems, faults are described as additive faults. In connection with CR-FTC, this may not be very useful. The reason is that the additive faults can be considered as external input signals to the system, at least if they are assumed to be uncorrelated with the system states. External input signals will not cause any changes in the system dynamics. Specifically, they are not able to change the stability of the closed-loop system, see e.g. Zhou *et al.* (1995). Consider for example faults on an actuator. Such faults will in general affect the stability margins of the closed-loop system. CR for systems with additive faults is therefore only relevant if the faults that can appear enter the system outside the closed loop. Consider the general 2×2 system setup with additive faults given by (3). Closing the system by a stabilizing controller $K(s)$ given by (2) gives the following closed loop transfer function:

$$\begin{aligned} e = &(G_{ed} + G_{eu}K(I - G_{yu}K)^{-1}G_{yd})d \\ &+ (G_{ef} + G_{eu}K(I - G_{yu}K)^{-1}G_{yf})f. \end{aligned} \tag{24}$$

From (24), it is clear that bounded additive faults can not affect the closed-loop stability – only the performance of the system will be affected. The main CR-FTC problem as defined in Definition 7 does not exist in this case. Instead the design of a feedback controller needs to be done with respect to minimizing the effect from additive faults on the nominal closed loop transfer function as defined in Definition 3. This problem is equivalent to a disturbance rejection problem.

The design of the controller can be done in two steps. First a nominal controller $K_0(s)$ is designed such that the nominal performance is satisfied. In the second step, the YJBK parameterized controller $K(Q)$ given by (13) or (14) is applied, based on the nominal controller $K_0(s)$. Using $K(Q)$ as the feedback controller for the system given by (3) results in the following closed-loop system:

$$\begin{aligned} e &= (G_{ed} + G_{eu}U\tilde{M}G_{yd} + G_{eu}MQ\tilde{M}G_{yd})d \\ &\quad + (G_{ef} + G_{eu}U\tilde{M}G_{yf} + G_{eu}MQ\tilde{M}G_{yf})f \\ &= (T_{1d} + T_2QT_{3d})d + (T_{1f} + T_2QT_{3f})f. \end{aligned} \tag{25}$$

From the above closed loop transfer function, it is clear that the CR-FTC problem, i.e. the design of Q, is equivalent to a disturbance rejection problem. Standard optimization methods can be applied directly for the design of a stable YJBK parameter Q. Using a standard method for the design of Q, the closed loop transfer function in (25) can be written as an LFT given by

$$e = \mathcal{F}_l(P_A, Q)\begin{pmatrix} d \\ f \end{pmatrix}, \tag{26}$$

where

$$P_A = \begin{pmatrix} \begin{pmatrix} T_{1d} & T_{1f} \end{pmatrix} & T_2 \\ \begin{pmatrix} T_{3d} & T_{3f} \end{pmatrix} & 0 \end{pmatrix}.$$

The standard setup design problem is shown in figure 5.

Based on the setup in figure 5 for the CR-FTC design of Q, Problem 2 with an $\mathcal{H}_\infty$ performance specification is then given by

Problem 8: For a given number $\gamma > 0$, the $\mathcal{H}_\infty$ sub-optimal CR-FTC performance problem for system with additive faults is defined as the problem of designing, if existent, a feedback controller Q, such that the closed loop transfer function $T_{A,cl}$ is stable and the $\mathcal{H}_\infty$ norm of $T_{A,cl}$ is less than or equal to γ, where $T_{A,cl}$ is given by

$$T_{A,cl} = \mathcal{F}_l(P_A, Q)$$

and P_A is given by

$$P_A = \begin{pmatrix} \begin{pmatrix} G_{ed} + G_{eu}U\tilde{M}G_{yd} & G_{ef} + G_{eu}U\tilde{M}G_{yf} \end{pmatrix} & G_{eu}M \\ \begin{pmatrix} \tilde{M}G_{yd} & \tilde{M}G_{yf} \end{pmatrix} & 0 \end{pmatrix}.$$

Note that the exact, the almost exact and the optimal design problems for Q have been considered in details in Saberi *et al.* (2000).

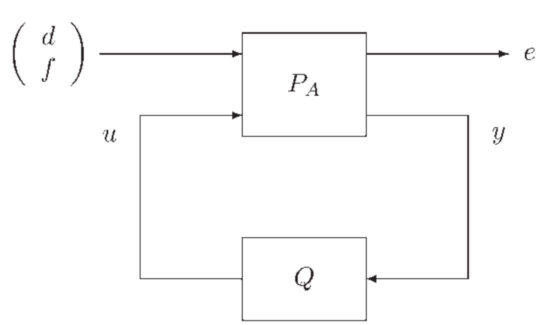

Figure 5. The standard setup for design of Q for systems with additive faults.

Combining CR-FTC with a fault isolation method gives a possibility of designing a number of Q-controllers, every single one dedicated to a single fault case. When faults appear in the system, the specified Q-controller to the given fault case can then be selected. It is also important to note that the Q controller needs to be decoupled when there are no faults in the system, otherwise the closed loop transfer function will be modified, see (25). It is clear that the FTC problem in this case is a performance problem and the closed loop stability will not be affected by the additive faults (modeled as bounded external signals).

5.2. *CR-FTC for systems with parametric faults*

In this case, the closed-loop stability can be affected by the parametric faults, if G_{yu} depends on the parametric faults. As in section 5.1, a YJBK parameterized controller $K(Q)$ is applied, where the nominal controller $K(0) = K_0$ is designed for the nominal system. The YJBK parameter is then applied for obtaining CR-FTC, i.e. Q needs to stabilize the closed-loop system when a fault has appeared in the system. The stability of the closed loop system requires stability of the nominal closed-loop system and closed-loop stability of a loop where both Q and the parametric faults θ are included (Tay *et al.* 1997). The stability of the closed-loop system is satisfied by the design of the nominal feedback controller $K(0)$. The other closed-loop system that needs to be stable is given by

$$\tilde{S}(Q) = (I - QS(\theta))^{-1}, \tag{27}$$

where $S(\theta)$ is the dual YJBK parameter, depending on the parametric faults θ.

It is required that S is stable to guarantee closed-loop stability. Combining the YJBK parameterization with the dual YJBK parameterization, it is not a condition that Q and S need to be stable to guarantee closed-loop stability. Q and S just need to satisfy that the closed-loop system given by (27) is stable (Tay *et al.* 1997). Using the equation from Appendix A, $S(\theta)$ then take the following form in the general case:

$$S(\theta) = \tilde{M}G_{yw}\theta\big(I - [G_{zw} + G_{zu}U\tilde{M}G_{yw}]\theta\big)^{-1}G_{zu}M. \tag{28}$$

In connection with (28), it is important to note that the stability condition of S and/or of $\tilde{S}(Q)$ in (27) for satisfying that the faulty closed loop system is stable, is only valid if the faulty system is still detectable and stabilizable from the specified input signals u and output signals y. This is a standard condition in connection with FTC systems. If the faulty system is not detectable and/or stabilizable, additional actuators and/or sensors need to be included in the system to satisfy these two conditions.

It is important to note that if S is stable, we do not need a Q-parameter to stabilize the system. In this way, S can be used for analyzing which faults are admissible and how large they can be before the closed-loop system will become unstable.

Based on the general equation for $S(\theta)$ given by (28), we have the CR-FTC design problem in Problem 6 is given by

Problem 9: The CR-FTC problem for system with parametric faults is defined as the problem of designing, if existent, a feedback controller Q, such that $\tilde{S}(Q)$ given by

$$\tilde{S}(Q) = (I - QS(\theta))^{-1}$$

is stable, where S is given by

$$S(\theta) = \tilde{M}G_{yw}\theta\big(I - [G_{zw} + G_{zu}U\tilde{M}G_{yw}]\theta\big)^{-1}G_{zu}M$$

In the general case, the equation for $S(\theta)$ given above is quite complicated. $S(\theta)$ needs to be derived explicitly in every single case in order to reduce the complexity of $S(\theta)$. Consider two simple cases, where the parametric faults are placed at either the input to the system (actuator faults) or at the output to the system (sensor faults), i.e. the system given by (4) takes the following form:

$$\begin{pmatrix} G_{ed}(\theta) & G_{eu}(\theta) \\ G_{yd}(\theta) & G_{yu}(\theta) \end{pmatrix} = \begin{pmatrix} G_{ed} & G_{eu} + G_{eu}\theta \\ G_{yd} & G_{yu} + G_{yu}\theta \end{pmatrix} \tag{29}$$

for parametric faults at the input. The system given by (4) takes the following form for parametric faults at the output

$$\begin{pmatrix} G_{ed}(\theta) & G_{eu}(\theta) \\ G_{yd}(\theta) & G_{yu}(\theta) \end{pmatrix} = \begin{pmatrix} G_{ed} & G_{eu} \\ G_{yd} + \theta G_{yd} & G_{yu} + \theta G_{yu} \end{pmatrix}. \tag{30}$$

The dual YJBK parameter S is then given by

$$S(\theta) = \tilde{N}_u\theta(I - U\tilde{N}_u\theta)^{-1}M \tag{31}$$

for parametric faults at the input and

$$S(\theta) = \tilde{M}_u\theta(I - N_u\tilde{U}\theta)^{-1}N_u \tag{32}$$

for parametric faults at the output.

The two CR-FTC design problems for parametric faults at the input and the output are given as follows.

Problem 10: The CR-FTC problem for system with parametric input faults is defined as the problem of designing, if existent, a feedback controller Q, such that $\tilde{S}(Q)$ given by

$$\tilde{S}(Q) = (I - QS(\theta))^{-1}$$

is stable, where S is given by

$$S(\theta) = \tilde{N}_u \theta (I - U\tilde{N}_u \theta)^{-1} M.$$

Problem 11: The CR-FTC problem for system with parametric output faults is defined as the problem of designing, if existent, a feedback controller Q, such that $\tilde{S}(Q)$ given by

$$\tilde{S}(Q) = (I - QS(\theta))^{-1}$$

is stable, where S is given by

$$S(\theta) = \tilde{M}_u \theta (I - N_u \tilde{U}\theta)^{-1} N_u.$$

So far, the stability part with respect to parametric faults has been treated. This is the most important part of the CR-FTC. However, it will also in some cases be possible to design the CR-FTC controller (the Q controller) with respect to both closed-loop stability as well as closed-loop performance. Closing the loop of the system in (4) with the feedback controller $K(Q)$, we get the following closed loop transfer function, see Appendix B:

$$e = T_{P,cl}(s)d, \qquad (33)$$

where

$$T_{P,cl}(s) = G_{ed}(\theta) + G_{eu}(\theta)(U + MQ)((V - G_{yu}(\theta)U) + (N_u - G_{yu}(\theta)M)Q)^{-1} G_{yd}(\theta)$$

and

$$\begin{pmatrix} G_{ed}(\theta) & G_{eu}(\theta) \\ G_{yd}(\theta) & G_{yu}(\theta) \end{pmatrix} = \begin{pmatrix} G_{ed} + G_{ew}\theta(I - G_{zw}\theta)^{-1}G_{zd} & G_{eu} + G_{ew}\theta(I - G_{zw}\theta)^{-1}G_{zu} \\ G_{yd} + G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zd} & G_{yu} + G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu} \end{pmatrix}.$$

Again, using a standard setup, shown in figure 6, for the design of the feedback controller Q, Problem 7 is then given by

Problem 12: For a given number $\gamma > 0$, the $\mathcal{H}_\infty$ CR-FTC performance problem for system with parametric faults is defined as the problem of designing, if existent, a feedback controller Q, such that the closed loop transfer function $T_{P,cl}$ is stable and the $\mathcal{H}_\infty$ norm of $T_{P,cl}$ is less than or equal to γ, where $T_{P,cl}$ is given by

$$T_{P,cl} = \mathcal{F}_l(P_P, Q)$$

and P_P is given by

$$P_P = \begin{pmatrix} G_{ed}(\theta) + G_{eu}(\theta)U(V - G_{yu}(\theta)U)^{-1}G_{yd}(\theta) & G_{eu}(M - U(V - G_{yu}(\theta)U)^{-1}(N_u - G_{yu}(\theta)M)) \\ (V - G_{yu}(\theta)U)^{-1}G_{yd}(\theta) & -(V - G_{yu}(\theta)U)^{-1}(N_u - G_{yu}(\theta)M) \end{pmatrix}$$

At last, let us again consider the two cases with parametric input faults and output faults. The general system in (4) is then given by (29) and (30), respectively. The general closed loop transfer function in (33) is then given by

$$\begin{aligned} T_{P,cl}(\theta) &= G_{ed} + G_{eu}(I+\theta)(U + MQ) \\ &\quad \times \big((V - G_{yu}(I+\theta)U) \\ &\quad + (N_u - G_{yu}(I+\theta)M)Q\big)^{-1} G_{yd} \\ &= G_{ed} + G_{eu}(I+\theta)(U + MQ) \\ &\quad \times (I - \tilde{N}_u\theta(U + MQ))^{-1}\tilde{M}G_{yd} \end{aligned} \qquad (34)$$

for parametric faults at the input and

$$\begin{aligned} T_{P,cl}(\theta) &= G_{ed} + G_{eu}(U + MQ)((V - (I+\theta)G_{yu}U) \\ &\quad + (\tilde{N}_u - (I+\theta)G_{yu}M)Q)^{-1}(I+\theta)G_{yd} \\ &= G_{ed} + G_{eu}M(\tilde{U} + Q\tilde{M})\tilde{M} \\ &\quad \times (I - \theta N_u(\tilde{U} + Q\tilde{M}))^{-1}(I+\theta)G_{yd} \end{aligned} \qquad (35)$$

for parametric faults at the output, respectively.

Using a standard setup formulation, we get the following open loop transfer functions for the design of the Q controller in the two cases (see figure 6 for

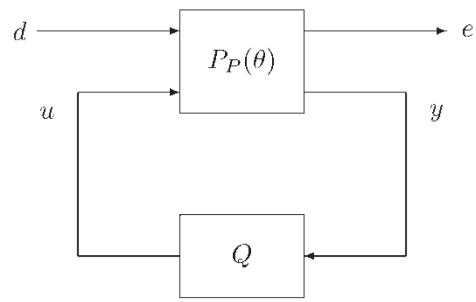

Figure 6. The standard setup for design of Q for systems with parametric faults.

the standard setup). For the input fault case, we have

$$P_{P,I}(\theta) = \begin{pmatrix} \bar{G}_{ed}(\theta) & \bar{G}_{eu}(\theta) \\ \bar{G}_{yd}(\theta) & \bar{G}_{yu}(\theta) \end{pmatrix} = \begin{pmatrix} G_{ed} + G_{eu}(I+\theta)U(I - \tilde{N}_u\theta U)^{-1}\tilde{M}G_{yd} & G_{eu}(I+\theta)(I - U\tilde{N}_u\theta)^{-1}M \\ (I - \tilde{N}_u\theta U)^{-1}\tilde{M}G_{yd} & (I - \tilde{N}_u\theta U)^{-1}\tilde{N}_u\theta M \end{pmatrix} \quad (36)$$

and for the output fault case, we have

$$P_{P,O}(\theta) = \begin{pmatrix} \bar{G}_{ed}(\theta) & \bar{G}_{eu}(\theta) \\ \bar{G}_{yd}(\theta) & \bar{G}_{yu}(\theta) \end{pmatrix} = \begin{pmatrix} G_{ed} + G_{eu}M\tilde{U}(I - \theta N_u\tilde{U})^{-1}(I+\theta)G_{yd} & G_{eu}M(I - \tilde{U}\theta N_u)^{-1} \\ \tilde{M}(I - \theta N_u\tilde{U})^{-1}(I+\theta)G_{yd} & \tilde{M}\theta N_u(I - \tilde{U}\theta N_u)^{-1} \end{pmatrix} \quad (37)$$

respectively.

As in the additive fault case, it is possible to combine CR-FTC with fault isolation. It is then possible to design a number of Q controllers, one for every single fault case and then select a specific Q controller when a fault appears in the system. A system setup including a CR-FTC controller for 3 potential parametric faults is shown in figure 3, where $Q_{\mathrm{CR},i}$ are the CR-FTC part and $Q_{\mathrm{FDI},i}$ are the residual generators for the FDI part.

5.3. *FTC for systems with structural changes*

This is the most relevant problem in connection with FTC. From a feedback point of view, a fault in a closed-loop system will in most cases change the structure of the system. However, in many cases, these structural changes can be described by using LFTs as considered in the parametric fault case.

In the following, let us just consider the system given by transfer functions described by (5). It is further assumed that the system can only be in the normal (nominal) mode and in one abnormal mode. The abnormal mode is given by:

$$\Sigma_S : \begin{pmatrix} \tilde{G}_{ed} & \tilde{G}_{eu} \\ \tilde{G}_{yd} & \tilde{G}_{yu} \end{pmatrix}. \quad (38)$$

The closed loop transfer function for the nominal system Σ and Σ_S when the feedback controller in (2) is applied is given by

$$\left.\begin{aligned} T_{cl} &= G_{ed} + G_{eu}K(I - G_{yu}K)^{-1}G_{yd} \\ T_{S,cl} &= \tilde{G}_{ed} + \tilde{G}_{eu}K(I - \tilde{G}_{yu}K)^{-1}\tilde{G}_{yd} \end{aligned}\right\} \quad (39)$$

respectively.

Following the line from the above section, we can again calculate S as a function of the system changes. The structural changes of G_{yu} can be described in the following way:

$$\begin{aligned} \tilde{G}_{yu} &= G_{yu} + (\tilde{G}_{yu} - G_{yu}) \\ &= G_{yu} + \theta. \end{aligned}$$

From table 1, we have that

$$S = \tilde{M}\theta(I - U\tilde{M}\theta)^{-1}M \quad (40)$$

Using $\theta = \tilde{G}_{yu} - G_{yu}$ in S, we get directly

$$S = (\tilde{M}\tilde{G}_{yu} - \tilde{N}_u)(\tilde{V} - \tilde{U}\tilde{G}_{yu})^{-1}.$$

If S given by (40) is unstable, the controller needs to be modified by using the Q feedback controller for stabilizing the system in the abnormal mode. Based on this fact, Problem 6 for systems with structural changes is then given by

Problem 13: The CR-FTC problem for system with structural changes is defined as the problem of designing, if existent, a feedback controller Q, such that $\tilde{S}(Q)$ given by

$$\tilde{S}(Q) = (I - QS)^{-1}$$

is stable, where S is given by

$$S = \tilde{M}\theta(I - U\tilde{M}\theta)^{-1}M$$

with $\theta = \tilde{G}_{yu} - G_{yu}$.

Now, let us consider the closed loop transfer function from d to e given by (39). Let the system given in the abnormal mode be described as additive changes of the nominal transfer functions, i.e.

$$\begin{pmatrix} \tilde{G}_{ed} & \tilde{G}_{eu} \\ \tilde{G}_{yd} & \tilde{G}_{yu} \end{pmatrix} = \begin{pmatrix} G_{ed} & G_{eu} \\ G_{yd} & G_{yu} \end{pmatrix} + \begin{pmatrix} \theta_{ed} & \theta_{yd} \\ \theta_{eu} & \theta_{yu} \end{pmatrix}. \quad (41)$$

In the general case, the θ parameters defined in (41) will be function of a single θ parameter, i.e.

$$\begin{pmatrix} \theta_{ed} & \theta_{yd} \\ \theta_{eu} & \theta_{yu} \end{pmatrix} = \begin{pmatrix} \theta_{ed}(\theta) & \theta_{yd}(\theta) \\ \theta_{eu}(\theta) & \theta_{yu}(\theta) \end{pmatrix}$$

due to the fact that every system change is caused by a single fault. The closed loop transfer function $T_{S,cl}$ is given by

$$\begin{aligned} T_{S,cl} &= \tilde{G}_{ed} + \tilde{G}_{eu}K(I - \tilde{G}_{yu}K)^{-1}\tilde{G}_{yd} \\ &= (G_{ed} + \theta_{ed}) + (G_{eu} + \theta_{eu})K(I - (G_{yu} + \theta_{yu})K)^{-1} \\ &\quad \times (G_{yd} + \theta_{yd}). \end{aligned} \quad (42)$$

In the special case where G_{yu} does not change in the abnormal mode, i.e. $T_{S,cl}$ in (42) is given by

$$\begin{aligned} T_{ed,S}(s) &= (G_{ed} + \theta_{ed}) \\ &\quad + (G_{eu} + \theta_{eu})K(I - G_{yu}K)^{-1}(G_{yd} + \theta_{yd}) \end{aligned}$$

In this case, the stability of the closed loop system will not be affected by the system change. The system change will only affect the performance of the closed loop system. This is equivalent to the additive fault case, where the design of Q turns out to be an open loop design problem. Note that a change in G_{ed} and/or G_{eu} might not be detectable from the measurement signal y, which can make it impossible to do any compensation for the fault in the system. This case will not be discussed further.

As a closing of this section, we will give the $\mathcal{H}_\infty$, CR-FTC performance design problem for system with structural changes. To do this, $K(Q)$ is applied. It is further assumed that $\theta_{yu}(\theta) = \theta$. This assumption is without loss of generality. Problem 7 is then given by

Problem 14: For a given number $\gamma > 0$, the $\mathcal{H}_\infty$ CR-FTC performance problem for system with structural changes is defined as the problem of designing, if existent, a feedback controller Q, such that the closed loop transfer function $T_{S,cl}$ is stable and the $\mathcal{H}_\infty$ norm of $T_{S,cl}$ is less than or equal to γ, where $T_{S,cl}$ is given by

$$T_{S,cl} = \mathcal{F}_l(P_S, Q)$$

and P_S is given by

$$P_S = \begin{pmatrix} (G_{ed} + \theta_{ed}) + (G_{eu} + \theta_{eu})U\tilde{M}(I - \theta U\tilde{M})^{-1}(G_{yd} + \theta_{yd}) & (G_{eu} + \theta_{eu})(I - U\tilde{M}\theta)^{-1}M \\ \tilde{M}(I - \theta U\tilde{M})^{-1}(G_{yd} + \theta_{yd}) & \tilde{M}\theta(I - U\tilde{M}\theta)^{-1}M \end{pmatrix}.$$

6. Example

A simple servo system is considered below. The system includes two faults, an additive sensor fault and an internal parametric fault. The focus in this example will be on the CR-FTC part of the FTC controller. The FDI part will only consist of a simple fault detector. The reason is that this part has not been considered in this paper. It is important to point out that the general FTC architecture does not limit the FDI block to a simple fault detector as will be used in this paper.

Let the nominal servo system be given by the following state space realization:

$$\begin{aligned} \dot{x} &= Ax + B_d d + B_u(r_{ref} - u) \\ y &= C_y x, \end{aligned}$$

where r_{ref} is the reference input, d is a disturbance load, u is the control input and y is the measurement signal. The state space matrices are given by

$$A = \begin{pmatrix} 0 & \dfrac{1}{k_0} \\ 0 & -\dfrac{1}{\tau}(1 + k_1 k_2 \alpha_0) \end{pmatrix} \quad B_u = \begin{pmatrix} 0 \\ \dfrac{1}{\tau}k_1 k_2 \end{pmatrix}$$

$$B_d = \begin{pmatrix} 0 \\ \dfrac{1}{\tau}k_1 \end{pmatrix} \quad C_y = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

$$k_0 = 12.5 \quad k_1 = 25 \quad k_2 = 9 \quad \tau = 0.05 \quad \alpha_0 = 0.04,$$

α_0 is the tacho gain in an internal feedback loop. It is assumed that the tacho gain α_0 can be reduced due to a fault in the tacho. This fault will result in a parametric fault. Further, the measurement signal can be affected by bias in the sensor. The bias is modelled as an additive fault at the measurement signal. The system including the two faults then take the following form:

$$\dot{x} = Ax + B_w w + B_u(r_{ref} - u)$$
$$z = C_z x$$
$$y = C_y x + f_y,$$

where f_y is the additive sensor fault. Further, B_w and C_z are given by

$$B_w = \begin{pmatrix} 0 \\ \frac{1}{\tau} k_1 k_2 \alpha_0 \end{pmatrix} = B_u \alpha_0$$
$$C_z = \begin{pmatrix} 0 & 1 \end{pmatrix}.$$

The loop from w to z is closed by

$$w = \theta_\alpha z, \theta_\alpha \in [0, 1]$$

with $\theta_\alpha = 0$ as the nominal value.

Based on this setup, we get directly the following transfer functions for the system:

$$\begin{pmatrix} G_{zw} & G_{zf} & G_{zu} \\ G_{yw} & G_{yf} & G_{yu} \end{pmatrix} = \begin{pmatrix} C_z(sI - A)^{-1} B_u \alpha_0 & 0 & C_z(sI - A)^{-1} B_u \\ C_y(sI - A)^{-1} B_u \alpha_0 & I & C_y(sI - A)^{-1} B_u \end{pmatrix}.$$

Let us use an observer based controller with state feedback gain F such that $A + B_u F$ is stable and an observer gain L such that $A + LC_y$ is stable. The two gains are given by

$$F = \begin{pmatrix} -3.3333 & 0.0196 \end{pmatrix}$$
$$L = \begin{pmatrix} -109.71 \\ -615670 \end{pmatrix}.$$

Using an observer-based feedback controller, it is possible to write up the coprime factorization of the system and the controller directly. One possible way to construct the eight stable coprime matrices in (10) is then Tay *et al*. (1997):

$$\left.\begin{aligned} \begin{pmatrix} M & U \\ N & V \end{pmatrix} &= \left(\begin{array}{c|cc} A + B_u F & B_u & -L \\ \hline F & I & 0 \\ C_{yF} & D_{yu} & I \end{array}\right) \\ \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} &= \left(\begin{array}{c|cc} A + LC_y & -B_{uL} & L \\ \hline F & I & 0 \\ C_y & -D_{yu} & I \end{array}\right) \end{aligned}\right\} \quad (43)$$

with $C_{yF} = C_y + D_{yu} F$ and $B_{uL} = B_u + LD_{yu}$. Note that $D_{yu} = 0$ for the servo system.

It is also possible to write up the coprime matrices based on the general state space description of the feedback controller. This can be found in Tay *et al*. (1997).

As a detector, a simple varians detector is applied. The detector is given by

$$\delta = \int r^2 dt.$$

The occurrence of an abrupt fault, an additive fault or a parametric fault, in the system will either give a change in the increasing rate of δ or a step in δ. Together with information of the ampliture of the residual signal r, it is possible to detect and also isolate the occurring of abrupt faults in the system.

First, let's consider a simulation of the FDI part of the FTC architecture when faults occur in the system. The simulations are derived with an additive or a parametric fault occuring at $t = 2.5$ sec. The simulation results are shown in figures 7–9 for the nominal case, the system with an additive fault or with a parametric fault. The additive fault is given by $f_y = 0.1$ and the parametric fault is given by $\theta_\alpha = 0.25$.

It can be seen from figure 7, that the additive fault only has a minor effect on the output, i.e. the output is almost identical with the output from the nominal system. A parametric fault in the system has a larger effect on the output. It is clear that the system has lost performance due to the parametric fault.

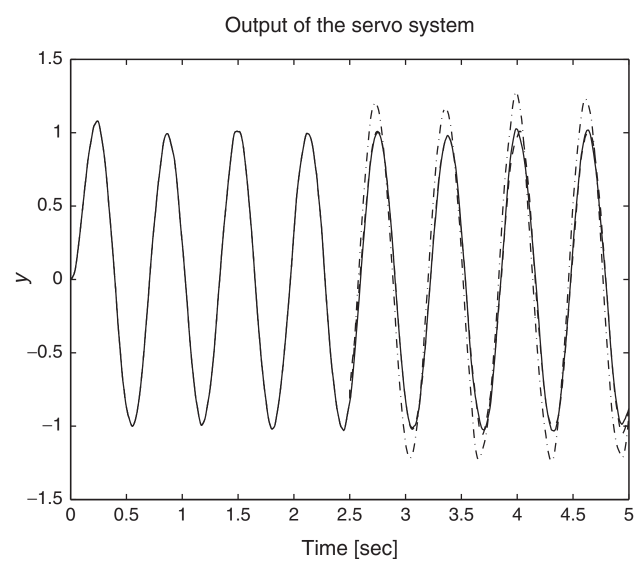

Figure 7. The time response of the output y of the servo system. The solid line is the output for the nominal system, the dashed line is for the system with an additive fault and the dashdot line is for the system with a parametric fault.

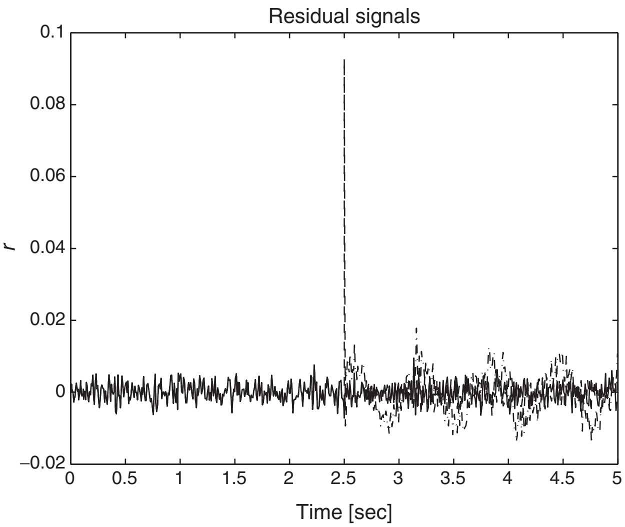

Figure 8. The residual signal r of the servo system. The solid line is the output for the nominal system, the dashed line is for the system with an additive fault and the dashdot line is for the system with a parametric fault.

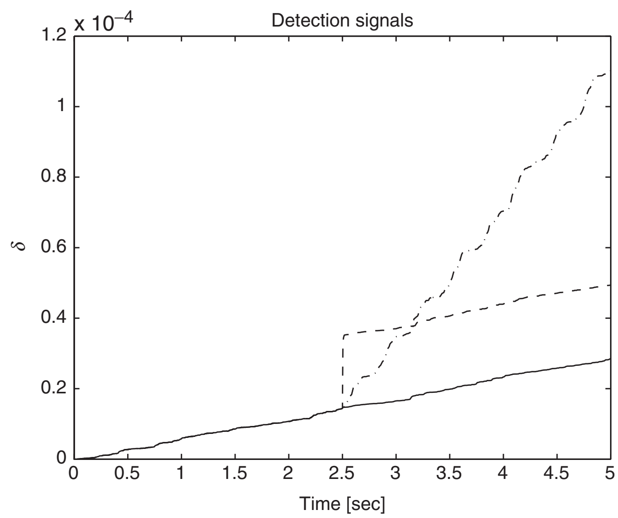

Figure 9. The detection signal δ for the servo system. The solid line is the output for the nominal system, the dashed line is for the system with an additive fault and the dashdot line is for the system with a parametric fault.

In figure 8, the residual signals are shown. The abrupt change of the additive fault can be seen very clearly at the residual signal as a spike. The parametric fault result in an oscillation with the same frequency as the input signal.

The three detection signals δ are shown in figure 9. The detection signal for the nominal system will increase with a fixed rate. The additive fault will result in a jump of δ, whereas a parametric fault will change the increasing rate of δ. Based on these observations, it can be seen that both the additive fault as well as the parametric fault can be detected. Further, due to the different effect on δ, it is also possible to isolate the two faults. Only the parametric fault will be considered in the rest of this example.

Calculating the dual YJBK parameter S given by (28) can now be done by using the coprime factorization in state space form. $S(\theta_\alpha)$ is then given by

$$\begin{aligned} S(\theta_\alpha) &= C_y(sI - A - LC_y)^{-1} B_u \alpha_0 \theta_\alpha \\ &\quad \times \Big(I - C_z(sI - A - B_u F)^{-1} \\ &\qquad \times B_u\Big(I - F\big(sI - A - LC_y\big)^{-1} B_u\Big)\alpha_0\theta_\alpha\Big)^{-1} \\ &\quad \times C_z(sI - A - B_u F)^{-1} B_u \\ &= \tilde{N}_u \alpha_0 \theta_\alpha (I - N_z \tilde{V} \alpha_0 \theta_\alpha)^{-1} N_z \end{aligned}$$

by using that $G_{zw} = N_z M^{-1}$.

Based on the above observer based feedback controller, the poles for S can now be calculated. It turns out that S is unstable when the tacho gain zero loop, i.e. $\theta_\alpha = 1$. In this case, the poles of S are given by

$$\text{poles}(S) = \begin{pmatrix} -141.76 & \pm 240.49i \\ 20.91 & \pm 25.67i \end{pmatrix}$$

i.e. S is not stable for all $\theta_\alpha \in [0,\ 1]$. Hence, a Q controller needs to be applied for stabilizing the system in this case.

Let's consider the Problem 9 for the design of a stabilizing Q controller. Q must be designed such that

$$\tilde{S}(Q) = (I - QS(\delta))^{-1}, \quad \theta_\alpha \ \in [0,\ 1]$$

is stable. In this case, a constant Q controller can be applied to stabilize $\tilde{S}$, given by

$$Q(s) = q, \quad q \in [-291\ \ -50.4].$$

The implementation of the Q controller must be done based on the YJBK parameterized controller given by (14) and (15).

Now, let the parametric fault θ_α in the system be given as a constantly increasing function with $\theta_\alpha = 0$ at $t = 2.5$ sec and $\theta_\alpha = 10$ at $t = 4.0$ sec. The resulting servo system will be unstable, if nothing is done. The nominal controller is reconfigurated by a stabilizing Q_{stab}. This reconfiguration is based on a detection of a change in the system by using a bound on the detection signal δ. This detection will take place around $t = 3.5$ sec. The Q_{stab} controller is given by

$$Q_{\text{stab}} = -100$$

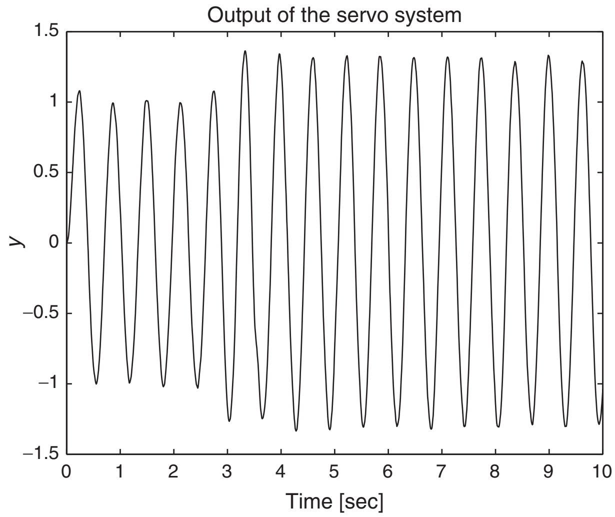

Figure 10. The time response of the output y of the servo system.

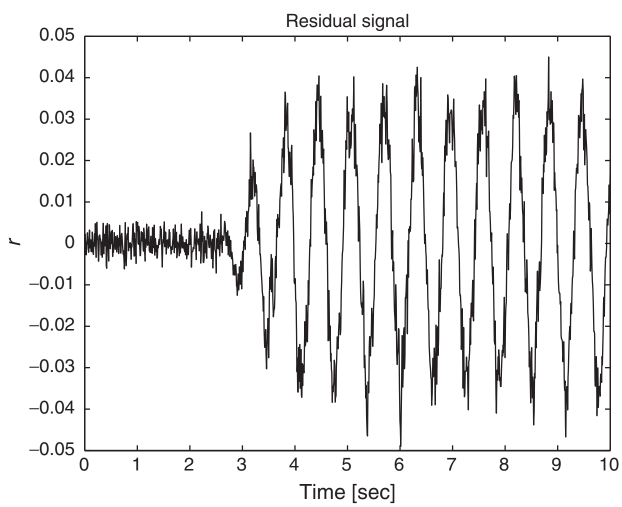

Figure 11. The residual signal r of the servo system.

for stabilizing the system. The simulation results are shown in figures 10–12.

It can be seen from figure 10 that the feedback controller including a reconfiguration at $t = 3.5$ sec stabilizes the system. The system is stabilized by the applied FTC controller, but there is a degradation of the performance. The residual signal r is shown in figure 11. The parametric fault results in a major increase of the residual signal r. Equivalent to figure 12, the detection signal δ gives also a very clear indication of a fault in the system, i.e. the increasing rate changes in the invertal $t = 2.5$ sec to 4.0 sec.

As it turns out from figure 10, the performance is not preserved with the selected Q_{stab}. Instead of considering

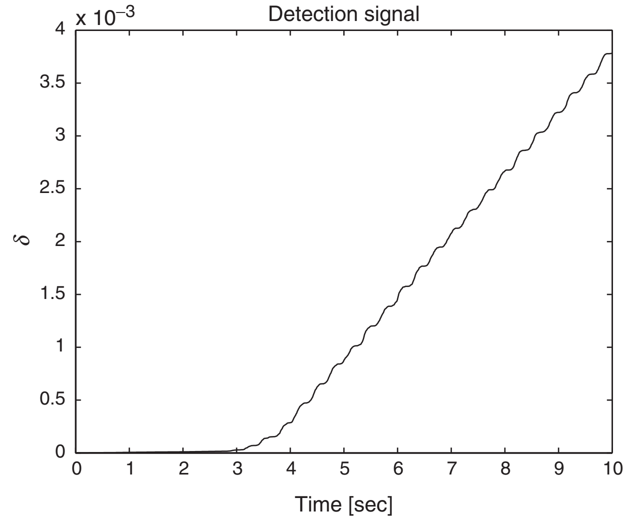

Figure 12. The detection signal δ for the servo system.

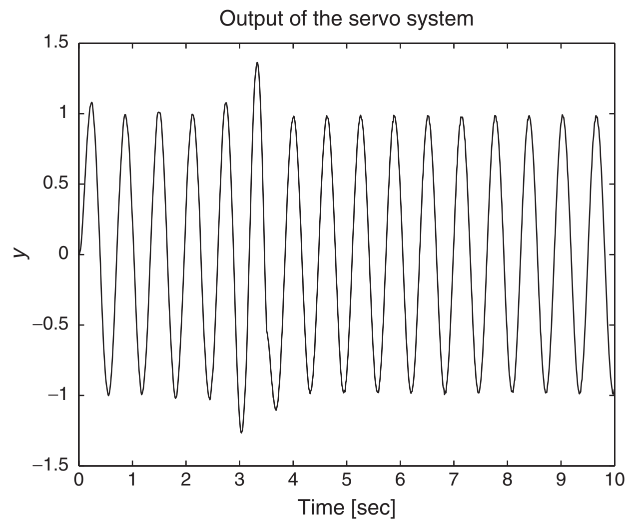

Figure 13. The time response of the output y of the servo system.

the Problem 9, we can design Q with respect to both stability and performance, i.e. use Problem 12. Optimizing a constant Q with respect to stability and performance, i.e. with respect to the specified input signal, gives the following optimal Q_{perf}

$$Q_{\text{perf}} = -175.$$

The resulting response of the system is shown in figure 13.

It is clear from figure 13 that the peformance is recovered after the controller is reconfigurated by using Q_{perf}. It is also clear that the servo system loss performance from the time that the fault occurs in the system until the controller is reconfigurated, in the time interval from $t = 2.5$ sec to around $t = 3.5$ sec.

It is important to point out that the optimal Q_{perf} applied in this simulation is only optimal with respect to the specified input signal and for $\theta_\alpha = 1$. The first problem can be handled by designing a dynamic Q_{perf}, such that the reconfigurated controller is optimal in a specified frequency range. Here, Problem 12 must be used directly. The second problem dealing with optimality only for a specified parametric fault is a little more complex. The solution is to design a number of optimal controllers Q_{perf}, and then select the best one based in the fault diagnosis of the parametric fault. This will require that it is both possible to detect, isolate and also come up with a reasonable estimate of the parametric fault. This is possible in this example, because it is possible to use the increasing rate of δ as a measure of the parametric fault. However, this will not in general be the case.

7. Conclusion

An architecture for fault tolerant control has been proposed. This architecture relies on a common framework for fault modelling based on linear fractional transformations, which facilitates modelling of additive faults, parametric faults, as well as faults that change the model structure.

By applying the (primary) YJBK parameterization, an additional controller parameter has been introduced as the main tool to achieve fault tolerance. A feature of the YJBK parameterization is that it automatically includes a diagnostic signal.

Systematic design procedures to obtain numerical values for the correction parameter have been indicated, which rely on optimization-based control design techniques.

In order to quantify the fault tolerance of a given configuration, the dual YJBK parameterization has been introduced. The magnitude of the corresponding parameter reflects the magnitude of faults that can be handled by the FTC system without losing e.g. stability or performance.

Although faults leading to structural changes of a system in principle calls for *ad hoc* solutions, it has still been possible to give general formulae for fairly rich and important classes of structural changes.

The example demonstrated how the method can be used to maintain stability for a simple servo loop, even if the tacho constant is faulty. Both the FDI-FTC part as well as the CR-FTC part have been considered in the example. It is shown that the stabilization problem using FTC controllers is easy compared with the problem of recovering the closed-loop performance.

Appendix A. Calculation of $S(\Delta)$

Let us consider the transfer function from u to y in (4) when parametric faults appear in the system. The transfer function is given by

$$G_{yu}(\theta) = \mathcal{F}_u(G_{unc}, \theta), \tag{44}$$

where

$$G_{unc} = \begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{pmatrix}$$

S needs to satisfy

$$G_{yu}(S) = G_{yu}(\theta), \tag{45}$$

where $G_{yu}(S)$ is given by (16) or (17). Equation (45) given now directly that:

$$\tilde{M}^{-1} S(I + M^{-1} US)^{-1} M^{-1} = G_{yw}\theta(I - G_{zw}\theta)^{-1} G_{zu}.$$

Rewriting this equation gives us

$$\begin{aligned} S &= (I - \tilde{M} G_{yw}\theta(I - G_{zw}\theta)^{-1} G_{zu} U)^{-1} \\ &\quad \times \tilde{M} G_{yw}\theta(I - G_{zw}\theta)^{-1} G_{zu} M \\ &= \tilde{M} G_{yw}\theta(I - (I - G_{zw}\theta)^{-1} G_{zu} U \tilde{M} G_{yw}\theta)^{-1} \\ &\quad \times (I - G_{zw}\theta)^{-1} G_{zu} M \\ &= \tilde{M} G_{yw}\theta(I - G_{zw}\theta - G_{zu} U \tilde{M} G_{yw}\theta)^{-1} G_{zu} M \\ &= \tilde{M} G_{yw}\theta(I - (G_{zw} + G_{zu} U \tilde{M} G_{yw})\theta)^{-1} G_{zu} M \end{aligned}$$

Appendix B. Calculation of T_{cl}

Closing the upper loop in (4) with Δ gives

$$\begin{pmatrix} G_{ed}(\Delta) & G_{eu}(\Delta) \\ G_{yd}(\Delta) & G_{yu}(\Delta) \end{pmatrix} = \begin{pmatrix} G_{ed} + G_{ew}\Delta(I - G_{zw}\Delta)^{-1} G_{zd} & G_{eu} + G_{ew}\Delta(I - G_{zw}\Delta)^{-1} G_{zu} \\ G_{yd} + G_{yw}\Delta(I - G_{zw}\Delta)^{-1} G_{zd} & G_{yu} + G_{yw}\Delta(I - G_{zw}\Delta)^{-1} G_{zu} \end{pmatrix}.$$

Let the controller $K(Q)$ be given by

$$K(Q) = K_0 + \tilde{V}^{-1}Q(I + V^{-1}N_uQ)^{-1}V^{-1} = (U + \tilde{V}^{-1}Q(I + V^{-1}N_uQ)^{-1})V^{-1}.$$

The closed-loop transfer function from e to d, T_{cl} is then given by

$$\begin{aligned}T_{cl}(\Delta, Q) &= G_{ed}(\Delta) + G_{eu}(\Delta)K(Q) \\ &\quad \times (I - G_{yu}(\Delta)K(Q))^{-1}G_{yd}(\Delta) \\ &= G_{ed}(\Delta) + G_{eu}(\Delta)(U + \tilde{V}^{-1}Q \\ &\quad \times (I + V^{-1}N_uQ)^{-1})V^{-1}(I - G_{yu}(\Delta) \\ &\quad \times (U + \tilde{V}^{-1}Q(I + V^{-1}N_uQ)^{-1})V^{-1})^{-1}G_{yd}(\Delta) \\ &= G_{ed}(\Delta) + G_{eu}(\Delta)(U(I + V^{-1}N_uQ) + \tilde{V}^{-1}Q) \\ &\quad \times (V(I + V^{-1}N_uQ) - G_{yu}(\Delta)U(I + V^{-1}N_uQ) \\ &\quad - G_{yu}(\Delta)\tilde{V}^{-1}Q)^{-1}G_{yd}(\Delta) \\ &= G_{ed}(\Delta) + G_{eu}(\Delta)(U + UV^{-1}N_uQ + \tilde{V}^{-1}Q) \\ &\quad \times (V + N_uQ - G_{yu}(\Delta)U - G_{yu}(\Delta)V^{-1}N_uQ \\ &\quad - G_{yu}(\Delta)\tilde{V}^{-1}Q)^{-1}G_{yd}(\Delta) \\ &= G_{ed}(\Delta) + G_{eu}(\Delta)(U + MQ)(V - G_{yu}(\Delta)U \\ &\quad + (N_u - G_{yu}(\Delta)M)Q)^{-1}G_{yd}(\Delta).\end{aligned}$$

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