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Bumpless transfer between observer-based gain scheduled controllers

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This paper deals with bumpless transfer between a number of observer-based controllers in a gain scheduling architecture. Linear observer-based controllers are designed for a number of linear approximations of a non-linear system in a set of operating points, and gain scheduling control can subsequently be achieved by interpolating between each controller. The Youla-Jabr-Bongiorno-Kucera (YJBK) parameterization is used to achieve a smooth scheduling between the controllers. This approach produces a scheduled controller as a linear fractional transformation between a fixed controller and a scheduling parameter. The approach is tested on a simple, but highly non-linear model of a fossil fuel power plant.

1. Introduction

Gain scheduling control is a popular approach to tracking control of ''well-behaved'' non-linear systems, whose behaviour can be adequately described using a selection of local linearized models. These linearizations are typically evaluated in a set of operating points that capture the key modes of operation. Linear controllers are then synthesized for the system in these points, for instance using classical PID controllers or robust and/or optimal design methods, ensuring that some relevant performance specifications are met in the vicinity of the operating point in question. The individual controllers are parameterized according to one or more *scheduling variables*, signals that indicate where the system state is currently situated relative to the operating points. The resulting family of controllers is then implemented as a single overall control law whose parameters are changed in accordance with the scheduling variables. Alternatively, a number of controllers can be operated in parallel, by letting the actual control be a weighted combination of the outputs of the individual controllers. The weights are chosen in a way such that if the system state is close to an operating point then the corresponding controller will dominate the control signal. Such classical gain scheduling control has been employed in numerous practical applications in diverse fields, such as power plant control (Hangstrup *et al.* 1999, Kallappa and Ray 2000, Parlos *et al.* 2000), flight control systems (Hyde and Glover 1993, Nichols *et al.* 1993), automotive control (Hrovat and Tran 1993), and process control (Crisafulli and Pierce 1999). Rugh and Shamma (2000) provides a general survey of gain scheduling applications and methods.

A recent paper on the anti-windup and bumpless transfer problem is Zaccarian and Teel (2002). In this work, the emphasis has been on systems with saturations, where an $\mathcal{L}_2$ optimization based approach is suggested.

The present paper relies on coprime factorization techniques. A coprime factorization approach to anti-windup problems was pioneered in (Kothare *et al.* 1994).

An interesting application of coprime factor techniques to bumpless transfer can be found in Ford and Glover (2000). This paper addresses the spark ignition engine idle speed control problem, where the initial state of an off-line controller is conditioned in order to reduce the bump when it is turned on-line.

In Edwards and Poslethwaite (1998), an $\mathcal{H}_\infty$ type method was proposed for anti-windup/bumpless transfer

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problems, partly in order to circumvent the assumption that the controller should be minimum-phase, as introduced in the classical work (Hanus et al. 1987).

The bumpless transfer problem has been treated in a linear quadratic framework in Turner and Walker (2000). In this work, the input of an off-line controller is modified in order to minimize bumps in a linear quadratic sense.

In this paper, we present a novel method for bumpless transfer between observer-based multi-variable gain scheduled controllers. To make matters precise, we will define bumpless transfer in the following way within the scope of this paper. Consider a simple, PI-controller designed to control a system in the vicinity of an operating point

$$u(t) = K_{p1}e(t) + K_{I1}x_1(t)$$

with $\dot{x}_1(t) = x_1(t) + e(t)$ representing the integral state, $u(t)$ the control signal and $e(t)$ the control error. K_{p1} and K_{I1} are the proportional and integral gains, respectively. Now assume that we wish to switch to another controller for the same system given by K_{p2} and K_{I2} at some instant $\bar{t}$. Simply replacing K_{p1} by K_{p2} and K_{I1} by K_{I2} without concern for the integral state is very likely to cause a large change in $u(\bar{t})$ if K_{I2} is significantly different from K_{I1} or if the integral state of the new PI-controller starts at some arbitrary value at $\bar{t}$. The straightforward way of achieving bumpless transfer is instead to reset the integral state such that a smooth control action is achieved:

$$K_{p1}e(\bar{t}) + K_{I1}x_1(\bar{t}) = K_{p2}e(\bar{t}) + K_{I2}x_2(\bar{t}) \quad \Leftrightarrow$$
$$x_2(\bar{t}) = \frac{(K_{p1} - K_{p2})e(\bar{t}) + K_{I1}x_1(\bar{t})}{K_{I2}}.$$

Gain scheduling can obviously be achieved by switching, i.e., by using the gains of one controller as long as the system state is close to the corresponding operating point and simply replacing them by the gains of the next controller when the system state is sufficiently close to the next operating point. The controller gains can also be mixed in a smooth manner, for instance via linear interpolation. To continue our simple PI example, a gain scheduled control law could be chosen as

$$u(t) = (\alpha(t)K_{p1} + (1 - \alpha(t))K_{p2})e(t)$$
$$\quad + (\alpha(t)K_{I1} + (1 - \alpha(t))K_{I2})x(t)$$
$$\dot{x}(t) = e(t),$$

where $\alpha \in [0, 1]$ is a scheduling variable. More than two controllers can be combined into one control law,

in which case some controllers that are designed for operating points far away from the current plant state may be disengaged entirely (reset to 0 or run in 'open loop').

It is important to note that, even if two stabilising controllers K_1 and K_2 are designed for the same linear system, there is no guarantee that a simple linear combination of the two controllers $K = \alpha K_1 + (1 - \alpha)K_2$ stabilizes the system for $0 < \alpha < 1$. An example of the contrary can be found in Niemann et al. (2004) Obviously, the least one should require from a gain scheduled control loop is that stability is ensured for every possible combination of controllers in the same operating point. In this paper, we present a controller construction that guarantees stability for any $\alpha \in [0; 1]$ when applied to one linear system. While it does not guarantee stability in between the operating points, it is still an improvement compared to the simple linear interpolation. Furthermore, as pointed out in Shamma and Athans (1990), in the classical gain scheduling control setup there is a "hidden feedback" induced by the scheduling itself, which may cause the plant-controller loop to lose stability if it is forced to change operating point too rapidly. The approach presented in this paper does not address the hidden feedback and hence requires that the scheduling must happen relatively slowly. However, it is worth noting that the proposed scheme allows for arbitrarily fast scheduling of controllers if the system remains in a fixed operating point.

Linear parameter varying control (Apkarian and Gahinet 1995, Packard 1994, Scherer 2001), which is a form of gain scheduling, can be used to design controllers with stability guarantees for fast parameter and/or operating point changes, but they require an exact model of the plant in between the operating points. Such models are not always available or may be difficult and costly to obtain—for instance in systems involving thermal and fluid components where highly non-linear material data makes model-based non-linear control design hard, but where local linear models may nonetheless be available for control purposes.

Niemann and Stoustrup (1999) provided a framework for gain scheduling control based on the Youla-Jabr-Bongiorno-Kucera (YJBK) parameterization of all stabilizing controllers (refer to e.g. Anderson (1998) and the references therein). Bendtsen et al. (2003) elaborated upon this idea by demonstrating how to switch between different nominal controllers in a continuous gain scheduling scheme once a new operating point has been reached.

In this paper we will go into greater detail with the work done by Bendtsen et al. (2003). We employ the YJBK theory to change from one controller designed in one operating point to another controller designed

in a different operating point of a non-linear system. We shall exploit some specific properties of the state space implementation of the YJBK parameterization to transfer not only parameters but also state information from one controller to another, thus achieving bumpless transfer.

It is noted that, in some previous approaches such as Graebe and Ahlén (1996), bumpless transfer is achieved by introducing a feedback that continuously forces the output of the 'next' controller to stay close to the actual controller output. In the present approach, a similar structure is introduced based on the YJBK parameterization, which allows for handling stability in a more systematic way.

We also go into details with how to handle integrators in this framework. Furthermore, it is explained how *integral quadratic constraints* (Megretski and Rantzer 1997) can be used to analyse stability when some reasonable assumption is made about how the plant behaves in between the operating points. The gain scheduling approach proposed in this paper is tested on a simple, but highly non-linear model of a fossil fuel power plant, validated against experimental data.

The outline of the paper is as follows. § 2 provides an overview of the YJBK parameterization and controller scheduling framework. In § 3 we present the main contribution of this work, a novel approach to gain scheduling with bumpless transfer, and discuss the actual implementation of the gain scheduling control method in detail. The control design is based on observer-based state space formulations, and we show how it is possible to design observer and feedback gains independently of one another, i.e., that the well-known separation principle of standard observer-based output feedback control carries over to our proposed controller structure. §§4 and 5 illustrate the usage of the method on a simulation model of a fossil fuel power plant and outline how to analyse stability using integral quadratic constraints, respectively. Finally, § 6 sums up the conclusions of the work.

Notation: The notation is mostly standard. We use plain capital letters for systems described by standard transfer functions

$$H = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

which should be understood as a system mapping an input signal $u \in \mathbb{R}^m$ to an output signal $y \in \mathbb{R}^p$ with a state space realization

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du.$$

We use calligraphic letters to denote (block) *two-port systems*, e.g.,

$$\mathcal{H} = \left[\begin{array}{c|cc} A & B & B_1 \\ \hline C & D & D_{12} \\ C_1 & D_{21} & D_{22} \end{array} \right]$$

mapping two vector input signals to two vector output signals.

$\mathcal{RH}_\infty$ is the set of all proper, stable, real rational transfer functions. $\star$ denotes the Redheffer star product, i.e. interconnection of systems; note that if a two-port system is connected with a one-port system, the star product reduces to a linear fractional transformation (Zhou *et al.* 1996).

A (block) two-port system can be reduced to a (block) one-port system by selecting the proper input and output channels or by interconnection with a zero system, i.e.

$$\begin{bmatrix} I & 0 \end{bmatrix} \mathcal{H} \begin{bmatrix} I \\ 0 \end{bmatrix} = \mathcal{H} \star 0 = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] = H.$$

2. Basic controller parameterisation

In this section, we provide a brief review of the framework established in Niemann and Stoustrup (1999), on which we base our controller synthesis. We note that all results shown in both this and the next section can easily be extended from continuous to discrete time (Bendtsen *et al.* 2003).

Consider the system $\bar{\mathcal{G}}$ depicted in the left part of figure 1, and assume that it has the state space realization

$$\bar{\mathcal{G}} = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_v & D_{vw} & D_{vu} \\ C_y & D_{yw} & D_{yu} \end{array} \right]. \tag{1}$$
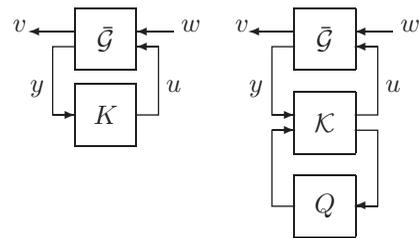
Figure 1. Left: The interconnection of the system $\bar{\mathcal{G}}$ and the observer-based controller K. Right: K is implemented as $K(Q) = \mathcal{K} \star Q$.

This system maps the input signals $u \in \mathbb{R}^{m_u}$ (controls) and $w \in \mathbb{R}^{m_w}$ (disturbances, noise and command signals) to the output signals $y \in \mathbb{R}^{p_y}$ (measurements) and $v \in \mathbb{R}^{p_v}$ (the signal to be controlled, which may coincide with y). If the subsystem $G_{yu} = [0 \quad I]\bar{\mathcal{G}}\begin{bmatrix} 0 \\ I \end{bmatrix}$ given by the matrices (A, B_u, C_y, D_{yu}) is stabilizable and detectable, $\bar{\mathcal{G}}$ can be stabilized by an observer-based feedback controller mapping y to u of the form; see e.g. Zhou (1996)

$$K = \left[\begin{array}{c|c} A + B_u F + L C_y + L D_{yu} F & -L \\ \hline F & 0 \end{array} \right]. \quad (2)$$

Furthermore, it is a simple matter to add integral action to this controller if desired; the controller state is augmented with a state that integrates the control error, and a feedback gain can be designed for the augmented state. The explicit state space formulae derived in the sequel will be based on such controllers with integral action. This is motivated from the general observation that integral action is useful in a huge class of practical applications.

Let $G_{yu} = C_y(sI - A)^{-1}B_u + D_{yu}$ be written using coprime factorization as

$$G_{yu} = NM^{-1} = \tilde{M}^{-1}\tilde{N} \quad (3)$$

with $N, M, \tilde{M}, \tilde{N} \in \mathcal{RH}_\infty$. Further, let a number of controllers for G_{yu} be given by

$$K_i = U_i V_i^{-1} = \tilde{V}_i^{-1}\tilde{U}_i, \ i = 0, \ldots, \nu \quad (4)$$

where $U_i, V_i, \tilde{U}_i, \tilde{V}_i \in \mathcal{RH}_\infty$. These coprime factorizations can be chosen to satisfy the double Bezout identity

$$\begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_i \\ N & V_i \end{bmatrix} = \begin{bmatrix} M & U_i \\ N & V_i \end{bmatrix} \begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ \tilde{N} & \tilde{M} \end{bmatrix}$$
$$= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

for $i = 0, \ldots, \nu$. All stabilizing controllers for G_{yu} based on any particular stabilising, *nominal* K_0 can now be written according to the YJBK parameterization

$$K(Q) = \mathcal{K} \star Q = K_0 + \tilde{V}_0^{-1} Q (I + V_0^{-1} N Q)^{-1} V_0^{-1},$$
$$Q \in \mathcal{RH}_\infty \quad (5)$$

i.e., the linear fractional transformation setup depicted in the right part of figure 1, which, due to the Bezout identity, also can be implemented as in figure 2. We then have the following result, adapted from Zhou *et al.* (1996).
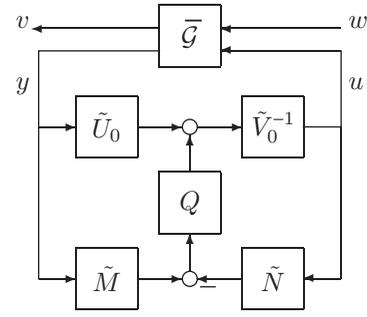
Figure 2. The controller is composed of coprime factorizations of the controller and system.

Lemma 1: *Let a number of stabilizing controllers* (4) *be given for a system* (3). *Then* $K_i, i = 0, \ldots, \nu$ *can be implemented as* $K(Q_i) = \mathcal{K} \star Q_i$, *with* $Q_i \in \mathcal{RH}_\infty$ *given by*

$$Q_i = \tilde{U}_i V_0 - \tilde{V}_i U_0 = \tilde{V}_i(K_i - K_0)V_0.$$

Proof: Follows by inserting $Q_i = \tilde{V}_i(K_i - K_0)V_0$ in (5), rewriting the expression as

$$K(Q_i) = K_0 + \tilde{V}_0^{-1}\tilde{V}_i(I + (K_i - K_0)N\tilde{V}_i)^{-1}(K_i - K_0)$$

and using the Bezout identity to show that $I + (K_i - K_0)N\tilde{V}_i = \tilde{V}_0^{-1}\tilde{V}_i$. $\square$

Lemma 1 states that it is possible to implement a controller as a function of a stable parameter system Q based on another stabilizing controller, as depicted in the right part of figure 1. As stated in Niemann and Stoustrup (1999) this implies that it is possible to change between two controllers online, say, from a nominal controller K_0 to another controller K_i, in a smooth fashion *without losing stability* by scaling the Q_i parameter by a factor $\alpha \in [0; 1]$.

Furthermore, as pointed out in Niemann and Stoustrup (1999) it is not only possible to change from K_0 to K_i, but indeed from any K_i to any $K_j, i, j = 0, \ldots, \nu$. In this case, we may compute the parameter Q as the following linear combination of the Q_i's given in Lemma 1

$$Q = \sum_{i=0}^{\nu} \alpha_i Q_i, \quad \sum_{i=0}^{\nu} \alpha_i = 1, \ \alpha_i \in [0; 1].$$

Then the resulting controller is given as

$$K(Q) = \left(\sum_{i=0}^{\nu} \alpha_i \tilde{V}_i \right)^{-1} \sum_{i=0}^{\nu} \alpha_i \tilde{U}_i. \quad (6)$$

This controller stabilizes the system depicted in figure 1, giving rise to a closed loop transfer function T_{vw} from w to v given by

$$T_{vw} = G_{vw} + G_{vu} M \left(\sum_{i=0}^{v} \alpha_i \tilde{U}_i \right) G_{vw} \qquad (7)$$

where G_{vw}, G_{vu} and G_{yw} represent the transfer functions of the subsystems of (1) from w to v, u to v, and w to y, respectively. Note that stability is guaranteed for all linear combinations of the form (6). In fact, the linear combinations do not even have to be convex; again, see Niemann and Stoustrup (1999) for details.

3. State space controller parameterization

The controller implementation proposed in equation (6), has one weakness, however; if controllers have been designed in many operating points, the order of the controller $K(Q)$ may become prohibitively large. If there are $v + 1$ controllers in $v + 1$ operating points, all of order n, the order of $K(Q)$ would typically be $2(v + 1)n$, due to the parallel couplings of the Q_i parameter systems. Even with the relatively low-order parameterization suggested in the following section, the order would still be $(2v + 1)n$ if all v Q-systems were to be run in parallel. Thus, it is clearly desirable to devise a way to keep the number of Q's low by switching to a new nominal controller, whenever the plant state has reached a new operating point, and base further gain scheduling on this new controller. In that way, the order of $K(Q)$ would be maintained at $3n$ at all times, at the expense of having to replace $\mathcal{K}$ and Q during operation.

This section presents the main contribution of this work, a state space formulation of an observer-based gain scheduled output feedback control law with the desirable qualities outlined above with explicit formulae for the systems comprising the controller. We show how to include integral action in the controller and prove that the observer can be designed independently of the feedback and integral feedback gains. Furthermore, we present a procedure for bumpless transfer between such controllers that preserves controller state information from one pair of controllers to the next.

3.1. *Integrator factorization*

Now assume we wish to construct a gain scheduled controller like the one referred to above, which includes integral action in order to remove any steady state errors that might arise from unmodelled dynamics, etc. In the following, we will assume without loss of generality that $D_{yu} = 0$; if this is not the case, the effect of the direct feedthrough can be compensated for by including a corresponding extra direct feedthrough in the controller. For convenience, we will also assume that the system is square, i.e., that the number of inputs is equal to the number of outputs (if the number of outputs is greater than the number of inputs, a state feedback design with integrators for all outputs is not feasible in the first place; if the number of inputs is greater than the number of outputs, similar results can be obtained, but the formulae become somewhat more complicated).

The integral action is included in the controller by augmenting the system model by extra states defined as the integrals of the control error $e = y - y_{\mathrm{ref}}$, which corresponds to placing open-loop poles in $s = 0$. However, we observe that both of the coprime factors $\tilde{U}_i$ and $\tilde{V}_i$ in (4) must be stable. This means that including an integrator on either side of the summation points in figure 2 will add a pole in $s = 0$, violating the conditions for Lemma 1 to hold. However, by choosing any scalar $r > 0$ we can factorize the integrator into the following coprime factorization

$$\frac{1}{s} = \left(\frac{s}{rs + 1} \right)^{-1} \left(\frac{1}{rs + 1} \right) = \tilde{V}_I^{-1} \tilde{U}_I \qquad (8)$$

yielding $\tilde{V}_I, \tilde{U}_I \in \mathcal{RH}_\infty$.

A controller with control action composed of observer based feedback and integral action can now be designed according to the usual separation principle paradigm. The observer gain is designed for the original system parameters, an (extended) feedback gain is obtained for an extended system model including the integrator, and the two actual feedback gain matrices—state feedback and integral feedback—are obtained by partitioning the extended feedback matrix consistently with the extended state space model.

3.2. *Finding the YJBK parameter*

Next, we present how to find Q once a number of stabilizing controllers have been found in individual operating points. That is, we have designed feedback and observer gains F_i, F_{Ii} and L_i such that the matrices

$$\begin{bmatrix} A + B_u F_i & B_u \\ F_{Ii} C_y & 0 \end{bmatrix}$$

and $A + L_i C_y$ are Hurwitz. The following calculations should then be carried out for each pair of adjacent operating points, between which gain scheduling takes place.
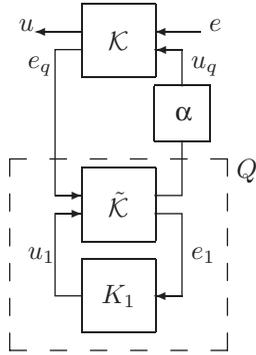
Figure 3. The construction of the controller $K(\alpha Q) = \mathcal{K} \star (\alpha Q) = \mathcal{K} \star (\alpha \tilde{\mathcal{K}} \star K_1)$.

Figure 3 illustrates the interconnection of the controller and the stable parameter system Q, whose output will be scaled by the scheduling parameter α. $K(\alpha Q)$ is formed as a linear fractional transformation of $\mathcal{K}$ and Q scaled by α, i.e., $K(\alpha Q) = \mathcal{K} \star (\alpha Q) = \mathcal{K} \star (\alpha \tilde{\mathcal{K}} \star K_1)$.

Let $\mathcal{K}$, the augmented controller on which we base the YJBK scheduling, be given as

$$\mathcal{K} = \left[\begin{array}{cc|cc} A + B_u F_0 + L_0 C_y & B_u & -L_0 & B_u \\ 0 & 0 & F_{I0} & rI \\ \hline F_0 & I & 0 & I \\ C_y & 0 & -I & 0 \end{array} \right] \quad (9)$$

where $rI \in \mathbb{R}^{p \times p}$, $r > 0$ represents the integrator factorizations included for each measurement output channel, as described in (8). The augmented state vector of this system is denoted $\xi_0 = [x_0^T \ x_{I0}^T]^T$. $\mathcal{K}$ takes the signals e (from the plant) and u_q as inputs and yields the outputs u, which is applied to the plant, and $e_q = C_y x_0 - e$, which is connected to Q. Note that, for $\alpha = 0$, the resulting controller becomes

$$[I \ \ 0] \mathcal{K} \begin{bmatrix} I \\ 0 \end{bmatrix} = \left[\begin{array}{cc|c} A + B_u F_0 + L_0 C_y & B_u & -L_0 \\ 0 & 0 & F_{I0} \\ \hline F_0 & I & 0 \end{array} \right] = K_0$$
$$(10)$$

which can be recognized as a standard observer-based controller with integral action.

When $\alpha = 1$ we must have

$$K(Q) = K_1 = \left[\begin{array}{cc|c} A + B_u F_1 + L_1 C_y & B_u & -L_1 \\ 0 & 0 & F_{I1} \\ \hline F_1 & I & 0 \end{array} \right].$$
$$(11)$$

The augmented state vector of this system is denoted $\xi_1 = [x_1^T \ x_{I1}^T]^T$.

As indicated in figure 3, we may now construct Q as $Q = \tilde{\mathcal{K}} \star K_1$, where $\tilde{\mathcal{K}}$ is chosen such that $\mathcal{K} \star \tilde{\mathcal{K}}$ is an identity system. We propose

$$\tilde{\mathcal{K}} = \left[\begin{array}{cc|cc} A & 0 & L_0 & B_u \\ F_{I0} C_y - r F_0 & -rI & -F_{I0} & rI \\ \hline -F_0 & -I & 0 & I \\ C_y & 0 & -I & 0 \end{array} \right] \quad (12)$$

with the augmented state vector $\tilde{\xi} = [\tilde{x}^T \ \tilde{x}_I^T]^T$. If we set $\alpha = 1$ in figure 3, we can calculate the transfer function of the two upper blocks:

$$\mathcal{K} \star \tilde{\mathcal{K}} = \left[\begin{array}{cccc|cc} A + B_u F_0 + L_0 C_y & B_u & -B_u F_0 & -B_u & -L_0 & B_u \\ 0 & 0 & -r F_0 & -rI & F_{I0} & rI \\ L_0 C_y & 0 & A & 0 & -L_0 & B_u \\ -F_{I0} C_y & 0 & F_{I0} C_y - r F_0 & -rI & F_{I0} & rI \\ \hline F_0 & I & -F_0 & -I & 0 & I \\ -C_y & 0 & C_y & 0 & I & 0 \end{array} \right].$$
$$(13)$$

This system can, indeed, be seen to be an identity system, i.e., $e_1(t) = e(t), u(t) = u_1(t), \ \forall t \geq 0$ and zero initial conditions. This is obviously equivalent to the plant being controlled by K_1.

This particular implementation of the YJBK parameterization has the following surprising properties.

Theorem 1: *Assume that the system given by* (1) *is controlled by the controller* $K(\alpha Q) = \mathcal{K} \star \alpha(\tilde{\mathcal{K}} \star K_1)$ *where* $\mathcal{K}$, $\tilde{\mathcal{K}}$, *and* K_1 *are given by* (9), (12) *and* (11) *respectively. Then the poles of the resulting closed loop system are identical to the eigenvalues of the matrices*

$$\begin{bmatrix} A + B_u F_0 & B_u \\ F_{I0} C_y & 0 \end{bmatrix}, \begin{bmatrix} A + B_u F_1 & B_u \\ F_{I1} C_y & 0 \end{bmatrix},$$

$A + L_0 C_y, A + L_1 C_y$ *and* $-rI$, *for any* α.

Proof: Combining (9), (12) and (11) with the system equations

$$\dot{x} = Ax + B_u u, \quad e = C_y x$$

yields a closed-loop equation for the system-controller interconnection of the form $\dot{\chi} = P\chi$ where $\chi = [x^T \ \xi_0^T \ \tilde{\xi}^T \ \xi_1^T]^T = [x^T \ x_0^T \ x_{I0}^T \ \tilde{x}^T \ \tilde{x}_I^T \ x_1^T \ x_{I1}^T]^T$ denotes the collection of augmented state vectors of the system, $\mathcal{K}$, $\tilde{\mathcal{K}}$, and K_1, respectively, and P is given in equation (14).

$$P = \begin{bmatrix} A & B_uF_0 & B_u & -\alpha B_uF_0 & -\alpha B_u & \alpha B_uF_1 & \alpha B_u \\ -L_0C_y & A+B_uF_0+L_0C_y & B_u & -\alpha B_uF_0 & -\alpha B_u & \alpha B_uF_1 & \alpha B_u \\ F_{I0}C_y & 0 & 0 & -\alpha rF_0 & -\alpha rI & -\alpha rF_1 & \alpha rI \\ -L_0C_y & L_0C_y & 0 & A & 0 & B_uF_1 & B_u \\ F_{I0}C_y & -F_{I0}C_y & 0 & F_{I0}C_y-rF_0 & -rI & rF_1 & rI \\ -L_1C_y & L_1C_y & 0 & -L_1C_y & 0 & A+B_uF_1+L_1C_y & B_u \\ F_{I1}C_y & -F_{I1}C_y & 0 & F_{I1}C_y & 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

We then apply the state transformation $\bar{\chi} = \Phi\chi$, where Φ is given as

$$\Phi = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I & 0 & I \\ 0 & 0 & 0 & 0 & -I & 0 & I \\ I & -I & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \\ I & -I & 0 & I & 0 & -I & 0 \\ I & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

This yields $\dot{\bar{\chi}} = \Phi P\Phi^{-1}\bar{\chi} = \bar{P}\bar{\chi}$, where $\bar{P}$ is upper block triangular with the block diagonal consisting of the matrices

$$\begin{bmatrix} A+B_uF_0 & B_u \\ F_{I0}C_y & 0 \end{bmatrix}, \quad -rI, \quad \begin{bmatrix} A+B_uF_1 & B_u \\ F_{I1}C_y & 0 \end{bmatrix},$$

$A+L_1C_y$ and $A+L_0C_y$ (in sequence from the upper left to the lower right corner). Since a similarity transformation does not change the eigenvalues of the mapping it is applied to, the claim follows immediately. $\square$

Remark 1: It is noted that the bottom block of the transformed states, $\bar{\chi}$, represents the state estimation error $x - x_0$. Since $A+L_0C_y$ is a Hurwitz matrix, we see that x_0 converges to x, just as in the case of a traditional, non-gain scheduled observer-based state feedback controller.

Remark 2: Note also that the stability of the closed-loop system does not depend on the value of $\alpha(t)$. There are no requirements on the rate of change of α. Thus, for a fixed system transfer function as above, the parameterization allows for arbitrarily fast controller scheduling or even discontinuities in $\alpha(t)$, as long as the signal remains bounded. This is similar to results stated in Hespanha and Morse (2002), which deals with controller switching for a fixed system.

In the bumpless transfer algorithm presented below, we shall use the above separation property by means of the following properties of the controller states.

Theorem 2: *Let ξ_0 and $\tilde{\xi}$ denote the state vectors of the two systems given by* (9) *and* (12), *respectively. If, for*

some time $\bar{t}$, $\alpha = 1$ for $t \geq \bar{t}$, then $\tilde{\xi}$ will converge to ξ_0, i.e.,

$$\bar{\xi}(t) = \xi_0(t) - \tilde{\xi}(t) \to 0 \quad for \quad t \to \infty \quad (14)$$

at a rate governed by the eigenvalues of the matrix

$$\begin{bmatrix} A+B_uF_0 & B_u \\ F_{I0}C_y & 0 \end{bmatrix}$$

independently of e and u_1.

Proof: For the sake of brevity, we define

$$J_0 = \begin{bmatrix} A+B_uF_0+L_0C_y & B_u \\ 0 & 0 \end{bmatrix}, \quad \tilde{J} = \begin{bmatrix} A & 0 \\ F_{I0}C_y-rF_0 & -rI \end{bmatrix}$$

and (15) above. We set $\alpha = 1$ and use the system equations (9) and (12) directly:

$$\dot{\bar{\xi}} = J_0\xi_0 + \begin{bmatrix} -L_0 & B_u \\ F_{I0} & rI \end{bmatrix}\begin{bmatrix} e \\ u_q \end{bmatrix}$$

$$- \tilde{J}\tilde{\xi} - \begin{bmatrix} L_0 & B_u \\ -F_{I0} & rI \end{bmatrix}\begin{bmatrix} e_q \\ u_1 \end{bmatrix}$$

$$= J_0\xi_0 + \begin{bmatrix} -L_0 & B_u \\ F_{I0} & rI \end{bmatrix}\begin{bmatrix} e \\ -[F_{I0} \ \ I]\tilde{\xi}+u_1 \end{bmatrix}$$

$$- \tilde{J}\tilde{\xi} - \begin{bmatrix} L_0 & B_u \\ -F_{I0} & rI \end{bmatrix}\begin{bmatrix} -[C_y \ \ 0]\xi_0-e \\ u_1 \end{bmatrix}$$

$$= J_0\xi_0 - \begin{bmatrix} B_uF_0 & B_u \\ rF_0 & rI \end{bmatrix}\tilde{\xi}$$

$$- \tilde{J}\tilde{\xi} - \begin{bmatrix} L_0C_y & 0 \\ -F_{I0}C_y & 0 \end{bmatrix}\xi_0$$

$$= \begin{bmatrix} A+B_uF_0 & B_u \\ F_{I0}C_y & 0 \end{bmatrix}(\xi_0-\tilde{\xi}) \qquad \square$$

Proposition 1: *Let* ξ_0, $\tilde{\xi}$, *and* ξ_1 *denote the state vectors of the three systems given by* (9), (12), *and* (11), *respectively. If* $\bar{\xi} = \xi_0 - \tilde{\xi} = 0$ *at some time instance* t^*, *the control signal* $u(t^*)$ *generated by the controller* $K(\alpha) = \mathcal{K} \star (\alpha \mathcal{K} \star K_1)$ *is given by*

$$u(t^*) = (1 - \alpha) [F_0 \quad I] \, \xi_0 + \alpha [F_1 \quad I] \xi_1.$$

Proof: The control law is calculated from the state space forms

$$
\begin{aligned}
u &= [F_0 \quad I]\xi_0 + u_q \\
&= [F_0 \quad I]\xi_0 - \alpha[F_0 \quad I]\tilde{\xi} + \alpha u_1 \\
&= [F_0 \quad I]\xi_0 - \alpha[F_0 \quad I]\tilde{\xi} + \alpha[F_1 \quad I]\xi_1
\end{aligned}
$$

The claim then follows by letting $\tilde{\xi} = \xi_0$ in this expression.   □

The point of this proposition becomes clear in a situation where the scheduling variable α has been equal to one for a while. Theorem 2 then shows that $\tilde{\xi}$ converges to ξ_0 (even in the presence of non-zero reference and/or disturbance inputs), which implies that the control signal transmitted to the plant then becomes equal to the control signal generated by K_1 in a smooth manner.

3.3. *A bumpless transfer procedure*

In § 3.2 a global linear model was intrinsically assumed in order to establish the theoretical results. Their practical use is based on robustness properties of the results, i.e. that stability etc. is preserved in an open neighbourhood of the system parameters. In this section, we will present a practical procedure specifying how to update parameters from one controller to the next for a system with linearizations that depend on the operating points.

In particular, we shall use the controller structure from § 3.2 with the modification that the system parameters (A, B_u, C_y) are replaced by (A_0, B_{u0}, C_{y0}) (the original system parameters, for which a controller K_0 has been designed) in (9) and in (12), whereas the next controller K_1 will be based on the new system parameters, such that (A, B_u, C_y) are replaced by (A_1, B_{u1}, C_{y1}) in (11). The procedure is illustrated in figure 4. A gain scheduling controller K_{01} allowing scheduling between K_0 and K_1 via the YJBK parameter Q_{01} is in use as long as the system operates between operating points (A_0, B_{u0}, C_{y0}) and (A_1, B_{u1}, C_{y1}). When the system reaches operating point (A_1, B_{u1}, C_{y1}), the control law is replaced such that the controller K_{12} allowing scheduling between K_1 and K_2 via the YJBK parameter Q_{12} becomes active. Furthermore, it is possible to achieve bumpless transfer, as explained below.
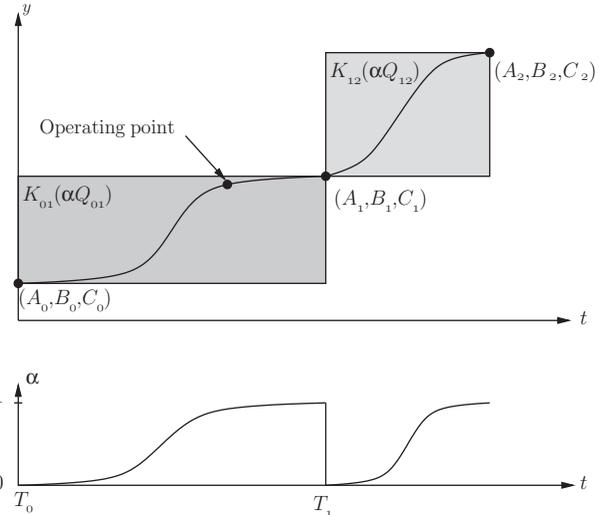
Figure 4.   Illustration of the proposed scheduling algorithm. Controllers K_0, K_1 and K_2 are designed for operating points (A_0, B_{u0}, C_{y0}), (A_1, B_{u1}, C_{y1}) and (A_2, B_{u2}, C_{y2}), respectively. A gain scheduling controller K_{01} allowing scheduling between K_0 and K_1 via the YJBK parameter Q_{01} is in use as long as the system operates between operating points (A_0, B_{u0}, C_{y0}) and (A_1, B_{u1}, C_{y1}) (the dark area in the top plot). When the system reaches operating point (A_1, B_{u1}, C_{y1}), the control law is replaced at $t = T_1$, such that the controller K_{12} allowing scheduling between K_1 and K_2 via the YJBK parameter Q_{12} becomes active (the light area in the top plot). The bottom plot shows the evolution of the scheduling parameter α. α is reset to 0 at the switching time T_1.

Procedure 1: *Assume that the transfer has to take place between time* T_0 *and time* T_1, *that the system parameters have changed from* (A_0, B_{u0}, C_{y0}) *to* (A_1, B_{u1}, C_{y1}) *during that time interval, and that the next controller in line is also an integral observer-based controller with feedback gains* F_2 *and* F_{I2} *and observer gain* L_2 *corresponding to the system parameters* (A_2, B_{u2}, C_{y2}). *Then* $\alpha(\cdot)$ *should be chosen as a continuous function of time such that* $\alpha(T_0) = 0$ *and* $\alpha(T_1) = 1$, *and such that* $\alpha(t) \approx 1$ *for* $T_1 - t_\varepsilon < t < T_1$. *Here,* t_ε *should be chosen sufficiently large for the controller states* $\tilde{\xi}$ *to converge to* ξ_0.

At time T_1 *the transfer is performed by the following substitutions of parameters and states*

$$
\begin{aligned}
(A_0, B_{u0}, C_{y0}) &\longrightarrow (A_1, B_{u1}, C_{y1}) \\
(A_1, B_{u1}, C_{y1}) &\longrightarrow (A_2, B_{u2}, C_{y2}) \\
(F_0, F_{I0}, L_0) &\longrightarrow (F_1, F_{I1}, L_1) \\
(F_1, F_{I1}, L_1) &\longrightarrow (F_2, F_{I2}, L_2) \\
\xi_0 &\longrightarrow \xi_1 \\
\tilde{\xi} &\longrightarrow \tilde{\xi}^* \\
\xi_1 &\longrightarrow \xi_1^*
\end{aligned}
$$

and $\alpha(T_{1+})$ *is reset to 0.* $\tilde{\xi}^*$ *and* ξ_1^* *are steady-state values of the states of Q assuming a constant input from the new controller with index 0.*

Under the assumptions above, Procedure 1 will guarantee bumpless transfer to the new controller at $t = T_1$. The intuition behind the procedure is that the state convergence property ($\tilde{\xi} \to \xi_0$) shown in Theorem 2 is exploited together with Proposition 1 to guarantee

$$u = [F_1 \quad I]\xi_1 \qquad (16)$$

at $t = T_{1-}$. Now, replacing ξ_0 by ξ_1, F_0 by F_1 and resetting α to zero, ensures that the control signal is still given by (16) at $t = T_{1+}$.

The new values of the states of Q, $\tilde{\xi}^*$ and ξ_1^*, are steady-state values calculated assuming a constant input e_q^* from the new controller with index 0 after the switch. The reason for this choice of new states is to avoid inducing unnecessary disturbances if α changes from 0 quickly after the switch, in which case the output from Q may not have settled yet. By choosing steady-state values as initial values for the states of Q, the effect of any undesired transients in Q are suppressed.

As can be seen, the procedure requires advance knowledge of the next operating point (A_1, B_{u1}, C_{y1}) in line, which will be the case if the reference is known in advance. If such advance knowledge is not available, it is necessary to implement the YJBK parameter as a combination of Q's as in (6), where the Q_i correspond to the operating 'above' and 'below' the current one.

Finally, we note that it is advantageous to add the steady-state control signal corresponding to the operating point in which the controller is designed, to the output from the controller. This control signal should be scaled according to the scheduling parameters before being fed to the plant. That is, assume u_0° and u_1° are the steady-state control signals required to maintain the undisturbed plant state in operating point 0 and 1, respectively. Then the actual control signal should consist of the feedback control contribution described in Proposition 1 plus a steady-state contribution $u^\circ = (1 - \alpha)u_0^\circ + \alpha u_1^\circ$.

4. Power plant control simulation

Originally, the work presented in this paper was motivated by electric power production control at fossil fuel power plants. Power plant processes are complex, of high order, highly non-linear, and noisy, which implies the necessity for employing multivariable control principles in order to obtain good stability and performance (Mølbak 1999); they are thus precisely of the type referred to in the introduction, where exact model-based control is, at best, hard. Conventionally, power plants have been operating for extended periods of time in, or close to, steady state, and the transitions from one operating point to another, when required, are typically fairly slow. However, in the power generation industry, the current trend toward market deregulation, coupled with increasing demands for maximization of the efficiency of utilization of natural resources and minimization of environmental impact, places greater and greater focus on efficient plant-wide operation and control systems. Load following, i.e., the ability of the power plant to meet the power production demands at all times without causing excessive material stress, is becoming a major concern due to the growing competition between power companies and other market forces; see also Garduno-Ramirez and Lee (2001) and the references therein.

Thus, with the increasing demand for load following capability, the ability of the power plant to perform stable and fast transitions between different operating points is becoming more and more important, a task that must be addressed by the control system. In this section we will demonstrate the usefulness of the proposed scheduling method on a simple simulation model of a power plant.

Figure 5 illustrates how the considered power plant works. Water is pumped from a feedwater tank through a preheater (economizer) and then into the evaporator, where the water evaporates, producing steam. The temperature is further increased in the superheaters. The superheated steam is then expanded through the turbines, which drive a number of generators producing electricity. After the turbines the steam is condensed into water, which is led back to the feedwater tank.

Figure 6 illustrates the simplified model of the boiler used here. The gas in the furnace and the steam in the evaporator are lumped together into single average states. Assuming that the mass flow of the flue gas equals the mass flow of coal and air, just three state variables are left: the temperature and density of the steam, T_s and ρ_s, along with the temperature of the
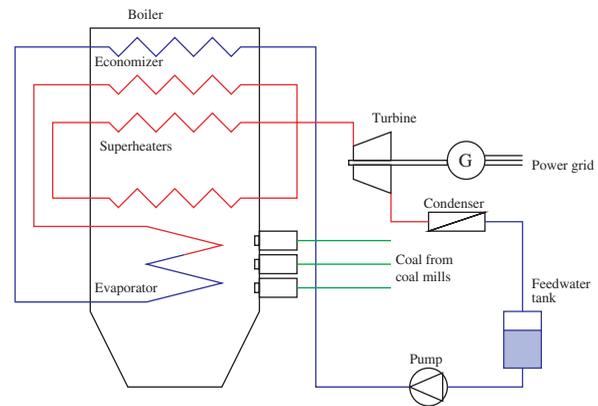
Figure 5. Fossil fuel power plant with once-through boiler.
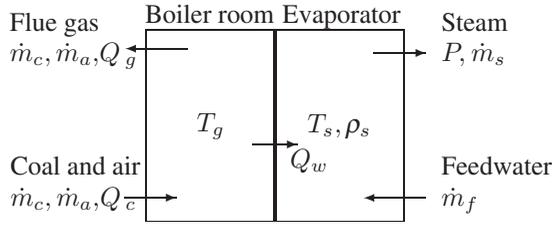
Figure 6.   Simplified model of boiler.

flue gas, T_g. The controlled inputs are the mass flow of coal, $\dot{m}_c$, and the mass flow of the feed water, $\dot{m}_f$. The air flow is governed by the mass flow of the coal.

The heat flux from the coal and air is modelled as

$$Q_c = \dot{m}_c h_c + \dot{m}_a h_a,$$

where h_c and h_a are the specific enthalpies of the coal and air, and $\dot{m}_a$ is the mass flow of air. The heat flux of the flue gas is modelled as

$$Q_g = (\dot{m}_c + \dot{m}_a) c_g T_g,$$

where c_g is the specific heat capacity of the flue gas. This gives the following time derivative of T_g

$$\frac{dT_g}{dt} = \frac{1}{c_g m_g}(Q_c - Q_g - Q_w), \qquad (17)$$

where m_g is the mass of the flue gas (and other byproducts) and Q_w is the heat flux through the evaporator wall, modelled as

$$Q_w = \alpha_w(T_g - T_s) + \epsilon_w(T_g^4 - T_s^4).$$

Here, α_w and ϵ_w are heat transfer coefficients of the wall. The time derivative of T_s is modelled as

$$\frac{dT_s}{dt} = \frac{\rho_s(\partial h_s/\partial \rho_s)(\dot{m}_s - \dot{m}_f) + \dot{m}_f(h_f - h_s) + Q_w}{C_w + V\rho_s(\partial h_s/\partial T_s)}, \quad (18)$$

where $h_s(T_s, \rho_s)$ is the enthalpy of the steam, h_f is the enthalpy of the feed water, C_w is the heat capacity of the wall, V is the volume of the evaporator, and $\dot{m}_s$ the mass flow of steam out of the evaporator modelled as (Lu and Hogg 2000)

$$\dot{m}_s = \beta_v \sqrt{(P^2 - P_0^2)/T_s},$$

where $P(T_s, \rho_s)$ is the pressure of the steam, P_0 is the pressure in the tank, and β_v is a flow coefficient.
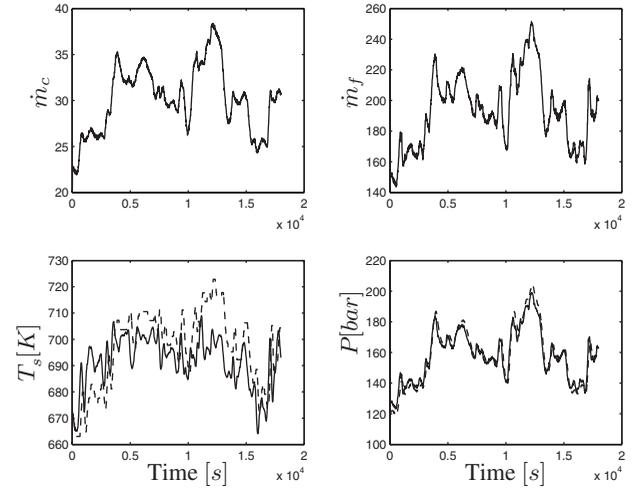
Figure 7.   Comparison of model and measured data, low pressure operating point.

The final time derivative needed is that of ρ_s which is simply given by

$$\frac{d\rho_s}{dt} = \frac{\dot{m}_f - \dot{m}_s}{V}. \qquad (19)$$

By assuming

$$
\begin{aligned}
&h_c = 25\,\text{MJ/kg}, &&h_a = 570\,\text{kJ/kg},\\
&c_g = 1280\,\text{J/(kgK)}, &&m_g = 1677\,\text{kg},\\
&\alpha_w = 12\,\text{kW/K}, &&\epsilon_w = 0.00068\,\text{W/K}^4,\\
&C_w = 103\,\text{MJ/K}, &&V = 28.3\,\text{m}^3,\\
&h_f = 1400\,\text{kJ/kg}, &&P_0 = 6.2\,\text{MPa},
\end{aligned}
$$

and $\beta_v = 0.00031\,\text{kgK}^{1/2}/(\text{sPa})$ to be constants we have a third order dynamical model given by equations (17), (18), and (19). With $\dot{m}_a$ calculated as a function of $\dot{m}_c$ the model has two control inputs, $\dot{m}_c$ and $\dot{m}_f$.

The values of the constants were found by fitting the model to measurement data from an actual 400 MW power plant. Figures 7 and 8 compare the responses of the real plant to those of the simulation model under different operating conditions (varying steam pressure). The fitted model shows surprisingly good agreement with the measurements, considering its simplicity. The only significant discrepancy between the model and the actual power plant behaviour appears to be the steam temperature at high steam pressure, where the model predicts too low temperature. Nonetheless, close inspection reveals that the transient behaviour is replicated quite well.

The method presented in §3 is now applied to the simulation model of the power plant. The control objective is to maintain T_s at 700 K while keeping P at a
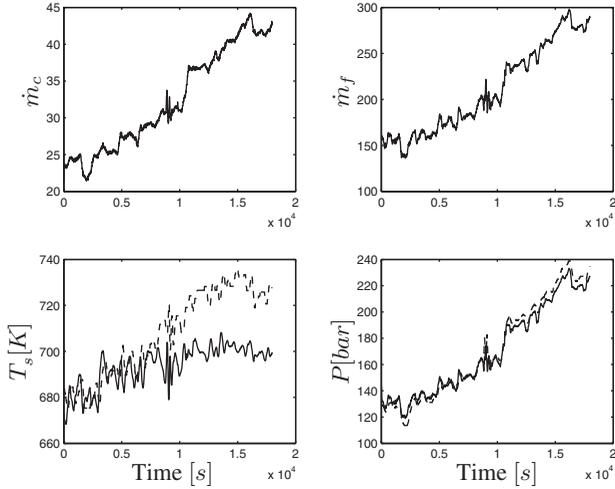
Figure 8. Comparison of model and measured data, high pressure operating point.

desired reference value using the control inputs $\dot{m}_f$ and $\dot{m}_c$. The operating point is determined according to the desired steam pressure, $P_{ref} \in [112.5; 315]$ *bar*. Three operating points are chosen: w_1: $P_{ref} = 315$ *bar*, w_2: $P_{ref} = 206$ *bar*, and w_3: $P_{ref} = 112.5$ *bar*. In each of these three points a discrete-time linearized model of the plant is obtained with a sampling period of 5 s and a discrete time LQR/LQE controller with integral action is designed for this model with emphasis on disturbance rejection. The controllers are denoted K_1, K_2, and K_3. Now the three controllers are scheduled according to P_{ref} using the method presented in §3. This is done by finding the Q that schedules between each controller pair, i.e., finding Q_{ij} and K_{ij} such that $K_{ij}(0) = K_i$ and $K_{ij}(Q_{ij}) = K_j$, $1 \leq i, j \leq 3$. Figure 9 shows the simulation going through the three operating points. At the dotted lines the controller pairs are switched. Initially $K_{12}(0)$ is used. As P_{ref} ramps to w_2, α is ramped from 0 to 1 making $K_{12}(\alpha Q_{12})$ go from K_1 to K_2. At the first dotted line a bumpless transition to K_{23} is performed and α is set to 0. At the next dotted line we switch to K_{32}, and at the last line we switch to K_{21}. Before each transition α has been 1 for a while to ensure the bumpless transfer. As can be seen, the transfer is indeed completely bumpless and the performance during the relatively fast ramping is good. If we simply use the controller designed for w_2 in the entire operating range, the control loop performance is degraded in w_1 and w_3, as seen in figure 10. The steam temperature control is seen to be tighter for the scheduled case than for the unscheduled case in both operating points, while the pressure control clearly shows different behaviours in the two operating points for the unscheduled control. In w_3 there is some undershoot and oscillatory behaviour, whereas in w_1 there is a clear overshoot.

The scheduled controller, on the other hand, yields the same, slightly overdamped response in both operating points.

5. Stability analysis

Once the gain scheduled control law has been designed, we are faced with the question of whether the control loop remains stable when the system state is not in the vicinity of any of the operating points. A thorough analysis of the stability of the closed loop obviously requires an exact model of the system behaviour in between the operating points—knowledge, which is not assumed to be available in the design. Instead of attempting to obtain such knowledge, we will assume a specific parameter variation for the system between the different operating points using the dual YJBK parametrization. Assuming that such a parameter variation describes the system adequately well, it is possible to analyse the stability of the closed loop between the operating points using *integral quadratic constraints* in a particularly simple manner.

Consider a feedback connection

$$v = M(s)w, \qquad w = \Delta(v)$$

where the uncertainty Δ is a bounded causal operator mapping $v \in \mathcal{L}_\infty^{n_v}[0, \infty)$ to $w \in \mathcal{L}_\infty^{n_w}[0, \infty)$ and M is a linear dynamical system of appropriate dimensions.

v and w are said to satisfy the *integral quadratic constraint (IQC)* defined by Π if

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix} d\omega \geq 0 \qquad (20)$$

in which $(\hat{\cdot})$ denotes the Fourier transform of $(\cdot)$ and Π is a (frequency-dependent) multiplier. An IQC can be used to analyse stability of the closed-loop interconnection:

Theorem 3: (Megretski and Rantzer 1997)**:** *Let $M(s) \in \mathcal{RH}_\infty$ and let Δ be a bounded causal operator. Suppose that*

1. *For every $\tau \in [0, 1]$, the interconnection of M and $\tau\Delta$ is well-posed (i.e., $I - M\tau\Delta$ is causally invertible);*
2. *For every $\tau \in [0, 1]$, the IQC (20) defined by Π is satisfied by $\tau\Delta$;*
3. *There exists some $\epsilon > 0$ such that*

$$\begin{bmatrix} M(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} M(j\omega) \\ I \end{bmatrix} \leq -\epsilon I, \quad \forall \omega \in \mathbb{R} \qquad (21)$$

Then the interconnection of M and Δ is stable.
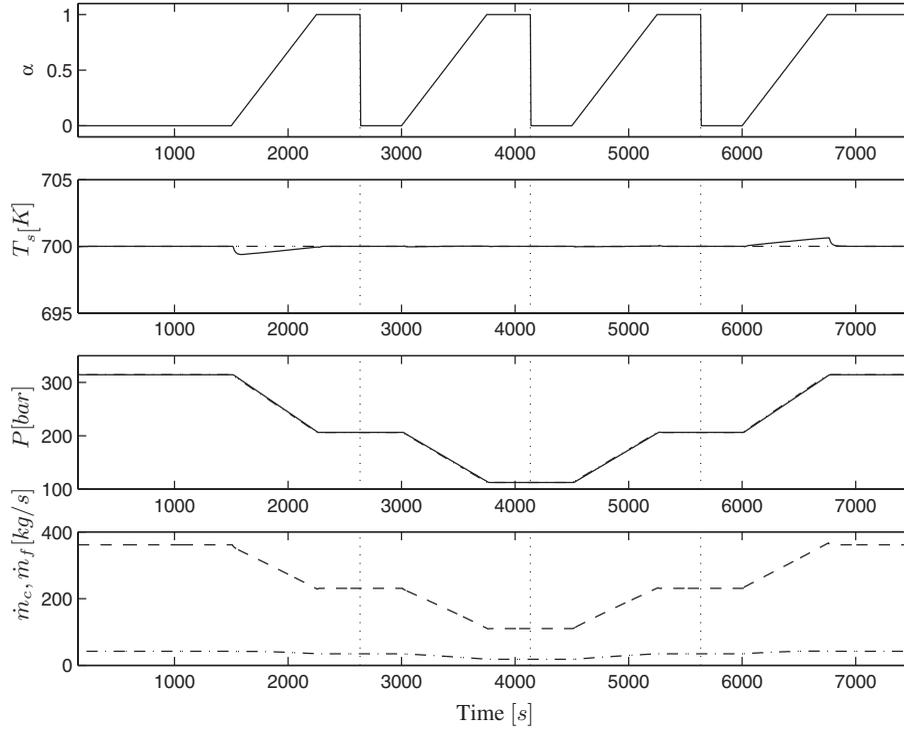
Figure 9. Simulation in the entire operating range. The controllers are switched at the vertical dotted lines. The figures show from top to bottom: Scheduling weight, steam temperature reference $(- \cdot -)$ and simulated output $(—)$, steam pressure reference $(- \cdot -)$ and simulated output $(—)$, and mass flows of coal $(- \cdot -)$ and feed water $(- -)$.
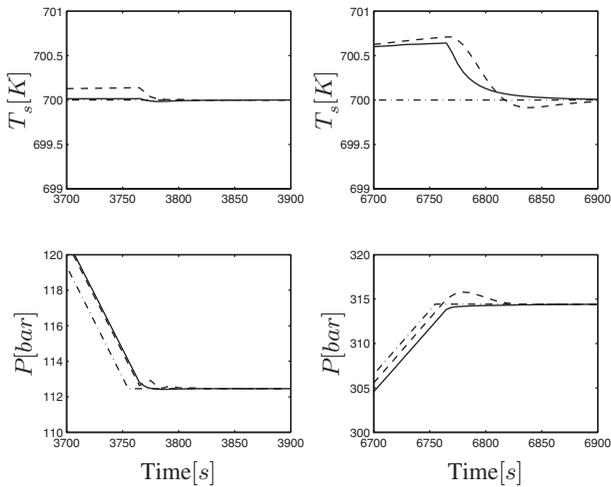
Figure 10. Simulation showing reference values $(- \cdot -)$ and controlled outputs with $(—)$ and without $(- -)$ gain scheduling. Top: steam temperature, bottom: steam pressure; left: low pressure operating point (w_3), right: high pressure operating point (w_1).

In other words, if we can find a multiplier Π satisfying both (20) and (21) for all possible Δ, we can guarantee stability of the feedback loop.

We will employ the method above to analyse the stability of the gain scheduled control system when moving between two given operating points. In order to do so, it is necessary to have a model of the plant behaviour between these operating points. So far we have only assumed the availability of linear models in the operating points.

To employ IQC analysis in our setting, we first exploit the existence of a dual YJBK parametrization to characterize all systems stabilised by the controller K via a stable dual parameter system S, as indicated in the left diagram in figure 11. Let $\mathcal{G}$ denote an augmented system which, when interconnected with a zero mapping, yields the linearization G_0 in operating point 0, i.e., $0 \star \mathcal{G} = G_0$. Note that we have some freedom in choosing the parts of $\mathcal{G}$ that are connected to S. In a similar manner as when constructing Q in §3.2, S is now constructed in such a way that if S is interconnected with $\mathcal{G}$ we obtain the linearization in operating point 1, G_1. This idea is illustrated in the middle diagram in figure 11, where a scaling variable $\beta \in [0, 1]$, which represents the linearizations moving between the operating points, has been inserted between $\mathcal{G}$ and S. S is constructed as an interconnection between G_1 and a system $\tilde{\mathcal{G}}$, which, when interconnected with $\mathcal{G}$, yields an identity system; that is,

$$\tilde{\mathcal{G}} \star \mathcal{G} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \Rightarrow S \star \mathcal{G} = G_1 \star \tilde{\mathcal{G}} \star \mathcal{G} = G_1.$$
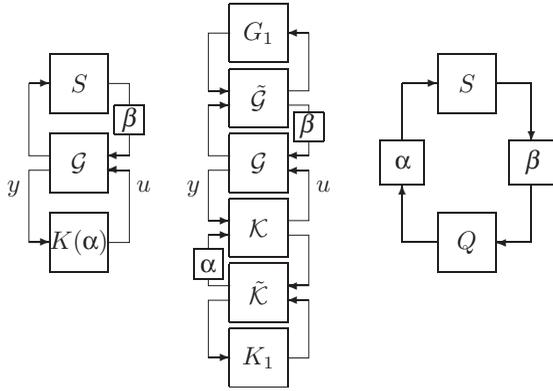
Figure 11. Left: The interconnection of all systems stabilized by the controller K parametrized via a nominal system $\mathcal{G}$ and a stable parameter S. Middle: The controller structure introduced in §3.2 is mirrored in the construction of S. $\alpha, \beta \in [0, 1]$ are scheduling parameters. Right: The resulting closed loop used in the stability analysis.

Due to the observer-based structure of the control law employed in the control design and the freedom we have in choosing the parts of $\mathcal{G}$ connected to S, a surprisingly simple choice of parametrization exists

$$\mathcal{G} = \tilde{\mathcal{K}} \tag{22}$$

$$\tilde{\mathcal{G}} = \mathcal{K} \tag{23}$$

This choice of $\mathcal{G}$ and $\tilde{\mathcal{G}}$ yields the closed loop depicted in the right block diagram in figure 11. With this assumption on the plant behaviour we can now find values of α and β for which this interconnection is stable. The interconnection can easily be put into the IQC framework with

$$M = \begin{bmatrix} 0 & Q \\ S & 0 \end{bmatrix} \quad \text{and} \quad \Delta = \begin{bmatrix} \alpha I & 0 \\ 0 & \beta I \end{bmatrix}, \quad \alpha, \beta \in [0, 1].$$

We can now view Δ as a constant or a time-varying uncertainty, which will allow for different classes of multipliers.

We illustrate the approach outlined above on the gain scheduled control loop designed for the power plant model presented in §4. For the numerical analysis we use the MatLab toolbox IQC-β (Kao *et al.* 2001), which provides IQC multiplier classes for various uncertainties.

The stability of the loop in figure 11 is analysed with $Q = Q_{ij}, S = S_{ij}$ for the four cases $(i = 1, j = 2)$, $(i = 2, j = 1)$, $(i = 2, j = 3)$, and $(i = 3, j = 2)$, where i, j indicate the operating points of the power plant model. We check stability for constant (time invariant, TI), arbitrarily fast varying (time varying, TV), and slowly time varying (STV) values of the scalar gains
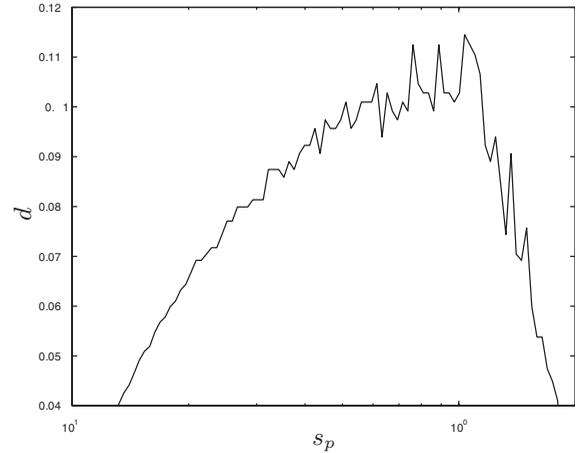
Figure 12. Maximum rate limit as a function of the pre-assigned multiplier pole.

$\alpha, \beta \in [0; 1]$. We do not assume any correlation between the gains, such as $\alpha \approx \beta$. For TI gains we can apply frequency-dependent multipliers, but in this case it was simpler to guarantee stability for all four controllers simply by dividing this interval into a small number of overlapping intervals and assuring stability for TV gains for each of these. For $(i = 1, j = 2)$ and $(i = 2, j = 1)$ stability was guaranteed for TV gains in the entire interval $[0 ; 1]$.

For $(i = 2, j = 3)$ (and $(i = 3, j = 2)$) it was necessary to assume a bound on the rate of variation, i.e. $|d\alpha/dt|, |d\beta/dt| < d$. In general, bounds on the rate of variation of the uncertainty allows for frequency-dependent multipliers. Unfortunately, this makes the IQC problem non-convex. In IQC-β this is circumvented by pre-assigning multiplier poles, making the problem convex but at the same time introducing conservatism. By pre-assigning a single multiplier pole in $s = s_p$ we could guarantee stability for the rate limits shown in figure 12. With the least conservative choice of s_p we can guarantee stability for $|d\alpha/dt|, |d\beta/dt| < 0.11$ corresponding to a change between the two operating points in only 10 seconds, which is much faster than required for a power plant.

6. Discussion

A procedure for bumpless transfer has been proposed, which, under assumptions of mild non-linearities, is able to guarantee stability and to ensure that the entire state of the new controller is aligned with the former state. The stability is established by virtue of the Youla-Jabr-Bongiorno-Kucera parameterization of all stabilizing controllers (and its dual), which in the particular implementation in the present paper provides a novel and interesting separation principle.

An interesting application of the bumpless transfer scheme is for smooth phasing-in of a new control system to supplement and/or replace an existing control system at a large-scale plant, where down-time is expensive and safety is important. That is, the gain scheduling scheme could be implemented at the medium-to-high level of the control hierarchy, where the computational demands can be met easily, and ensure a smooth transition to a new and (hopefully) better performing closed-loop system.

Finally, even if a gain-scheduling control design method with bumpless transfer is guaranteed to stabilize a large model class, there is never any guarantee for stabilization of the real plant. However, at least such a design method should guarantee stability while scheduling between controllers for a *fixed* linear model. This is satisfied for the proposed method in contrast to several of the classical methods. In addition, the method updates the whole controller state vector, which the authors consider to be a sound approach.

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