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Using reference trajectories for predicting uncertain systems exemplified for a power plant

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Abstract— **This paper presents a method for prediction of uncertain closed loop systems, where the uncertainties are depending on operating points. Such model uncertainties are often present when complicated non-linear systems are predicted. The method uses precomputed mean and variances of the prediction error depending on the operation point given by references and disturbances. These uncertainty models are stored in a model bank, linear interpolation is applied to the elements of the model bank in order to predict uncertainty bounds on the predictions using the statistics of the past prediction uncertainties. It is as well proposed to update the uncertainty prediction models on-line. The potential of the method is illustrated by an example from a coal-fired power plant. This example shows prediction of the uncertainties as a bounded region in which the given system variable can be assumed to be contained in. In the example these successfully bound the system variables while in comparison applying a simple prediction diverges from the system.**

I. INTRODUCTION

Prediction of dynamic system performance is of interest in numerous fields like meteorology, biology, economics, physics, medicine etc, see [1], [2], [3], [4], [5] and [6]. In control engineering this prediction of dynamic systems represented by models is a key part of model predictive control, see [7]. In all this work a model is used to represent the real system. The model is subsequently used to predict the future performance of the system. In most of the work on prediction of the dynamic system the focus is on finding good models representing the system, and using these to predict the expected time series of the future behavior of the system, see [8], [9], [10], [11] and [12]. Little focus has been on prediction of uncertain models where uncertainties and uncertain system conditions are taken into account in the prediction. Robust model predictive control is an exception, where worst case model uncertainties are included in the optimization of the control law, see [13], [14], [15], [16], [17], [18], [19] and [20]. These do not, however, directly give a prediction of the uncertainty of the system performance. Instead they compute a control law, which guarantees acceptable worst case system performance.

Adaptation of the prediction model does not solve this prediction problem, since the model structure is given, and the uncertainties cannot always be described only by varying parameters in this model. This means it can be assumed that they will derivate from the system with time. The expected time series behavior of the system is only out of a number of possible system trajectories, and using only one can be misleading. E.g. in this case where the operator of the plant could be interested in a “what if” scenario, where the task is to predict how the plant will behave given certain disturbances and reference changes, where the plant is still subject to given constraints on its outputs. The operator can subsequently take actions accordingly to these predictions of the future performance. It is of high importance to know that the required state value can be achieved within time with a certain probability, or to predict the upper and lower bounds of the proper performance, given a certain probability.

In [21] a method was presented, which used statistics of past prediction errors for a prespecified number of samples to predict upper and lower bounds on the uncertainty of the expected prediction for k-step predictions. The statistics of k-step prediction was computed as mean and variance of a number of most recent k-step predictions. These are computed for all $k \in \{1 \cdots K\}$ where K is the maximal required prediction horizon. These arrays are updated after each sample where new residuals are computed. In order to relate the states in the prediction model to the present time/sample an observer is introduced. This observer can as well be used to estimate disturbances into the system using an internal model representing the disturbance.

This representation of the uncertainties is only relevant in cases where the uncertainties are due to time varying model parameters and not uncertainties due to non-linearities where different operating points result in different model uncertainties. Instead it is proposed to represent the model uncertainties as statistics depending on references and disturbances. Meaning that the uncertainty models are stored in a data base representing samples of the definition set of reference and disturbances. Linear interpolation is subsequently used to compute the uncertainty model for reference and disturbance values not directly represented in the uncertainty model base. A further option is to adapt the stored uncertainty models then a larger number of samples at the given set of operating points are present.

The system in question is subsequently described in Section II. In Section III prediction of the expected value is described together with the prediction of the uncertainty. In this section the interpolation scheme and the adaptive update of the uncertainty model are given as well. The

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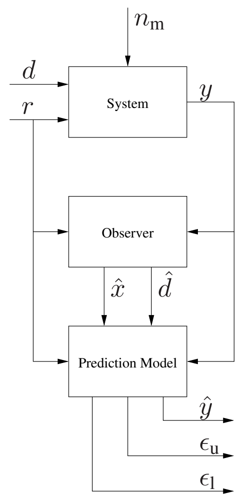

Fig. 1. Overview of the predictor structure, where the observer estimates the present states and in some cases the disturbances as well. The prediction model predicts the system behavior and uncertainties k steps into the future.

scheme is applied to a power plant example in Section IV. In Section V the conclusions are drawn.

II. The system

The system is a closed loop system, which from the prediction point-of-view is represented by a state space of the form

$$\mathbf{x}_{\mathrm{m}}[n+1] = \mathbf{f}_{\mathrm{m}}\left(\mathbf{x}_{\mathrm{m}}[n], \mathbf{r}[n], \mathbf{d}[n]\right), \tag{1}$$

$$\mathbf{y}_{\mathrm{m}}[n] = \mathbf{g}_{\mathrm{m}}\left(\mathbf{x}_{\mathrm{m}}[n]\right), \tag{2}$$

where $\mathbf{f}_{\mathrm{m}}()$ and $\mathbf{g}_{\mathrm{m}}()$ are the nonlinear model mappings, $\mathbf{x}_{\mathrm{m}}[n]$ is the model state vector, $\mathbf{y}[n]$ is a vector of model outputs, $\mathbf{d}[n]$ is a vector of the disturbances, and $\mathbf{r}[n]$ is a vector of the references to the close loop system.

A residual is defined as in (3) to represent the deviation between the system output and the model output.

$$\xi[n] = \mathbf{y}[n] - \mathbf{y}_{\mathrm{m}}[n]. \tag{3}$$

III. The predictor

The structure of this suggested predictor is illustrated by Fig. 1, where the system inputs and outputs are used to estimate the present state values. These are fed to the predictor together with system inputs and outputs in order to predict the expected values as well as the uncertainty bounds. $\hat{\mathbf{x}}[n]$ and $\hat{\mathbf{d}}[n]$ denote the estimated state and disturbance vectors for the time instance n. $n_{\mathrm{m}}[n]$ is the measurement noise. $\hat{\mathbf{y}}[n]$ is the vector of the predicted system output for the time n, $\epsilon_{\mathrm{u}}[n]$ and $\epsilon_{\mathrm{l}}[n]$ denote respectively upper and lower bounds on system prediction for the time n.

The observer and predictor (prediction model) will subsequently be described in more details.

The closed loop model is uncertain with respect to the real system. Consequently an observer is introduced in order to estimate the value of the states at the sample time n.

$$\hat{\mathbf{x}}[n] = \mathbf{\Gamma}\left(\hat{\mathbf{x}}[n-1], \mathbf{u}[n], \mathbf{y}[n]\right), \tag{4}$$

where $\mathbf{\Gamma}$ is an operator representing the observer, and $\hat{\mathbf{x}}[n]$ is the estimated state vector at time n.

The estimated states can be used to predict the state and the output vectors a number of samples/steps into the future. In some cases the reference is partly known in the future due to the prediction of the required plant production, such as power plants since the general power production is planned one day ahead. The disturbance might be known up to time n, e.g. by estimation. Subsequently these are denoted: $\hat{\mathbf{r}}[n]$ and $\hat{\mathbf{d}}[n]$. The k-step predictor of the output, $\mathbf{y}[n+k|n]$, and states, $\mathbf{x}[n+k|n]$, are computed by

$$\mathbf{x}[n+1|n] = \mathbf{f}_{\mathrm{m}}\left(\hat{\mathbf{x}}[n], \hat{\mathbf{r}}[n], \hat{\mathbf{d}}[n]\right), \tag{5}$$

where

$$\mathbf{x}[n+2|n] = \mathbf{f}_{\mathrm{m}}\left(\mathbf{x}[n+1|n], \hat{\mathbf{r}}[n], \hat{\mathbf{d}}[n]\right). \tag{6}$$

Continue this recursive process until $\mathbf{x}[n+k|n]$ is computed, and then compute

$$\mathbf{y}[n+k|n] = \mathbf{g}_{\mathrm{m}}\left(\mathbf{x}[n+k|n], \hat{\mathbf{r}}[n], \hat{\mathbf{d}}[n]\right). \tag{7}$$

Now where the k-step predictor is defined, it is possible to define a k-step prediction error residual.

$$\xi[n+k|n] = \mathbf{y}[n+k] - \mathbf{y}_{\mathrm{m}}[n+k|n]. \tag{8}$$

This prediction residual defined by (8) can of course only be computed earliest at sample n.

As previously stated the model is assumed to be uncertain in relation to the real system. It also means that $y[n+N+1|n]$ is more certain than $y[n+N+2|n]$ validated in terms of the variance of $\xi[n+N+1|n]$ is smaller than the variance of $\xi[n+N+2|n]$. In other words the prediction is expected to be more uncertain as longer into the future the prediction is made. This is illustrated by Fig. 2. The predicted system value is drawn with the dashed line, (from sample $n+1$ to sample $n+8$), the measured system output value is drawn with solid line (sample $n-2$ to n). The uncertainty of the predicted values are marked by the vertical markings, with the small horizontal lines at the ends, the distance between these end markings represent the uncertainty for the specific predicted system value.

In the context mentioned in the introduction (see Section I), it is as well interesting to predict a region in which output can be expected to be in. E.g. in order to verify if it is possible for the system to reach a specific state value in a given time period, e.g. whether the required performance of the system during a load change can be fulfilled given the specified operation conditions. This situation is illustrated by Fig. 3, in which the required system output region

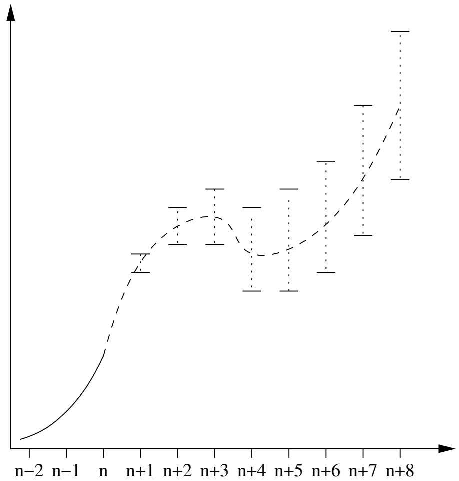

Fig. 2. Illustration of the uncertainty in the prediction. The uncertainty at each prediction step is increased as the number of prediction steps increases.

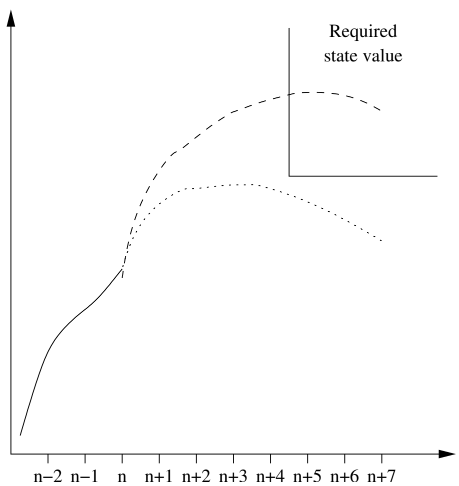

Fig. 3. Illustration of situation where a specific output value is required to be reach with a given time. This figure shows two possible future behaviors of the system, where one reaches the required value in time and the other do not.

shall be reached at sample $n+5$, two different possible system behaviors are shown, where the dashed line reaches the required value in time and the dotted line does not. The restarting of the plant is a costly process, meaning it would be preferable if it could be known beforehand if it is possible at all to reach the requested values in time. This means it is interesting to predict the uncertainty of the prediction as well.

A. The uncertainty predictor

The uncertainty of the prediction can be represented in a number of ways. In this approach the prediction error residuals are assumed to be a normal random process, with a specific variance and mean depending on the number of prediction steps. $\xi[n+k|n]$ is the prediction uncertainty at sample $n+k$ given estimate at n.

$$\xi[n+k|n] = \Phi\left(\sigma^k_{\mathbf{r}[n+k],\mathbf{d}[n+k]}, \mu^k_{\mathbf{r}[n+k],\mathbf{d}[n+k]}\right), \quad (9)$$

where Φ is the normal distributed random process, $\sigma^k_{\mathbf{r}[n+k],\mathbf{d}[n+k]}$ is the variance of the k-step prediction error for reference at time $n+k$ and disturbance at time $n+k$, $\mu^k_{\mathbf{r}[n+k],\mathbf{d}[n+k]}$ is the mean of the k-step prediction error for reference at time $n+k$ and disturbance at time $n+k$.

B. The uncertainty model parameter bank

The uncertainty model parameters are stored in a database, ordered accordingly to the depending variables, e.g. $\mathbf{r}[n+k]$ and $\mathbf{d}[n+k]$ for k-step prediction. For each element in this model parameter bank, a set of model parameters are attached.

The simplest way to use these model parameters is to use the instance in the model bank, which is nearest to specified, depending variables.

C. Linear interpolation of model bank

Another way to find model parameters between elements in the model bank is to use linear interpolation between the represented points. Define the mean and variance which are requested interpolated as: $\rho^k_{(\mathbf{r},\mathbf{d})}$ and $\mu^k_{(\mathbf{r},\mathbf{d})}$, and define the operating points which they are closest to as $\rho^k_{(\mathbf{r}_c,\mathbf{d}_c)}$ and $\mu^k_{(\mathbf{r}_c,\mathbf{d}_c)}$.

$\rho^k_{(\mathbf{r},\mathbf{d})}$ is interpolated as

$$\rho^k_{(\mathbf{r},\mathbf{d})} = \rho^k_{(\mathbf{r}_c,\mathbf{d}_c)} + \sum_{n=1}^{R}(r_n - r_{c_n}) \cdot \left(\rho^k_{\mathbf{r}_n,\mathbf{d}_c} - \rho^k_{\mathbf{r_c},\mathbf{d}_c}\right), \quad (10)$$
$$+ \sum_{n=1}^{D}(r_n - r_{c_n}) \cdot \left(\rho^k_{\mathbf{r}_c,\mathbf{d}_n} - \rho^k_{\mathbf{r_c},\mathbf{d}_c}\right),$$

where

$$\mathbf{r}_n = \begin{bmatrix} r_{c_1} & \cdots & r_n & \cdots & r_{c_R}\end{bmatrix}^T, \quad (11)$$
$$\mathbf{d}_n = \begin{bmatrix} d_{c_1} & \cdots & d_n & \cdots & d_{c_D}\end{bmatrix}^T. \quad (12)$$

$\mu^k_{(\mathbf{r},\mathbf{d})}$ is interpolated as

$$\mu^k_{(\mathbf{r},\mathbf{d})} = \mu^k_{(\mathbf{r}_c,\mathbf{d}_c)} + \sum_{n=1}^{R}(r_n - r_{c_n}) \cdot \left(\mu^k_{\mathbf{r}_n,\mathbf{d}_c} - \mu^k_{\mathbf{r_c},\mathbf{d}_c}\right), \quad (13)$$
$$+ \sum_{n=1}^{D}(r_n - r_{c_n}) \cdot \left(\mu^k_{\mathbf{r}_c,\mathbf{d}_n} - \mu^k_{\mathbf{r_c},\mathbf{d}_c}\right),$$

where

$$\mathbf{r}_n = \begin{bmatrix} r_{c_1} & \cdots & r_n & \cdots & r_{c_R}\end{bmatrix}^T, \quad (14)$$
$$\mathbf{d}_n = \begin{bmatrix} d_{c_1} & \cdots & d_n & \cdots & d_{c_D}\end{bmatrix}^T. \quad (15)$$

D. Adapting the model bank

The reference and disturbance depending reference models might change with time. Consequently with time the uncertainty model parameters should preferably be updated. A simple method for updating the uncertainty models are, subsequently, presented. When more than N samples in the given operational region have been sampled, then compute mean and variance of these, denote these μ_{new} and ρ_{new}. Subsequently, update the model parameters as in (16-17).

$$\mu = A \cdot \mu_{\text{old}} + B \cdot \mu_{\text{new}}, \tag{16}$$

$$\rho = A \cdot \rho_{\text{old}} + B \cdot \rho_{\text{new}}, \tag{17}$$

where

$$A = \frac{\gamma \cdot N_{\text{old}}}{\gamma \cdot N_{\text{old}} + N}, \tag{18}$$

$$B = \frac{N}{\gamma \cdot N_{\text{old}} + N}, \tag{19}$$

γ is a training factor and N_{old} are the number of element used to compute the old set of model parameters.

IV. Examples

The proposed scheme for predicting the uncertain closed loop system is illustrated by a simulation of a power plant. The purpose of the simulations is to validate that this method for prediction of uncertain systems can be used to predict the uncertainty bounds on the predicted performance of the system. The model found in [22] and [23] is extended with a coal mill model, and an uncertainty model. An overview of the model structure can be seen in Fig. 4. The coal mill pulverizes and dries the coal dust, before it is blown into the furnace. Two disturbances are influencing the coal mill: outside temperature, $T_{\text{o}}[n]$, and coal moisture content, $\gamma[n]$. The temperature of the primary air, $T_{\text{pa}}[n]$, which is used to dry and lift the coal dust into the furnace, is used to keep the coal dust temperature, $T_{\text{m}}[n]$ at 100°C. In the furnace the coal dust is burned and the hot flue gas is used to heat water to pressurized steam. The steam temperature, $T_{\text{s}}[n]$, and pressure, $p_{\text{s}}[n]$, are used to control the plant. This control results in coal flow, primary air flow and feed water flow requirements, $\dot{m}_{\text{c,ref}}[n]$, $\dot{m}_{\text{pa,ref}}[n]$ and $\dot{m}_{\text{f}}[n]$.

A simple three state model is made approximating this system, which is in contrast with the simulation model's 10 states. This reduction in states will consequently result in large residual between prediction and simulation models.

The prediction method is subsequently applied, with a prediction horizon of 80 samples. First the prediction without linear interpolation is used. a good represent of the predictions of the data, starts at sample 500 and predicts 80 samples. The value of three variables are predicted, these are flue gas temperature, $T_g[n]$, steam temperature, $T_s[n]$, and steam pressure, $p_s[n]$. It is assumed that the disturbance can be estimated at the time of the prediction, and the references are assumed to be known during the prediction period. Notice in the plots of the prediction that the uncertainty bounds do not start together, this is due to changes in the operating points in the beginning of the prediction as well as the mean of the prediction errors are included in the uncertainty models. These predictions can be seen in Figs. 5-7. It is also seen that the prediction for all variables are following the system well. From these it can be seen that the prediction of the uncertainties have some large jumps in the uncertainty prediction due to large difference in the model parameter values.

Instead linear interpolation is used to interpolate between points in the model set. These predictions can be seen in Figs. 8- 10. The use of the linear interpolation has removed the large jumps in uncertainty bounds prediction. These plots also show that the system behavior is well bounded by the uncertainty bounds, even though the prediction of the expected value diverges from the system output.

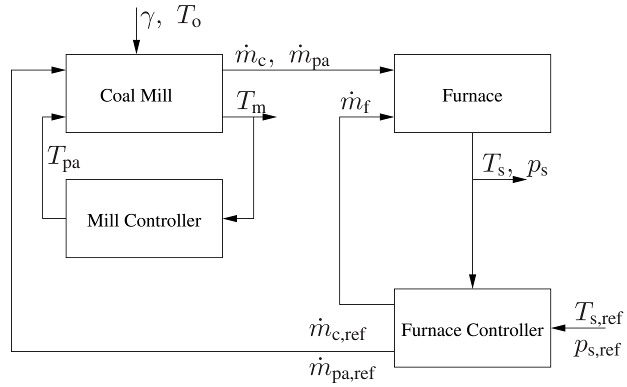

Fig. 4. Illustration of model structure

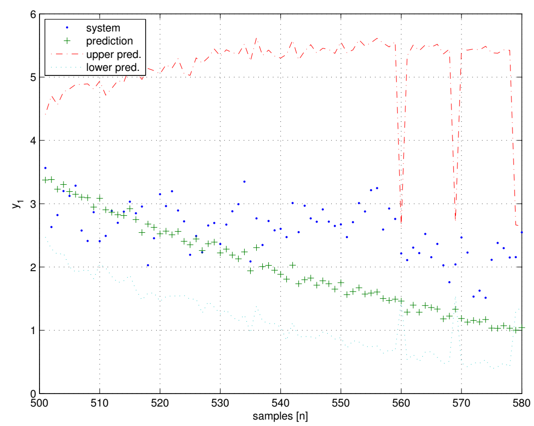

Fig. 5. Plot of the predicted and "real" output T_g, without linear interpolation. The three spikes at the bounds at time 560, 570, 580 is due to change operating points.

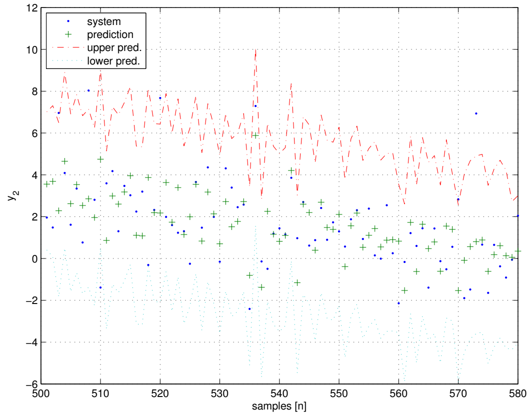

Fig. 6. Plot of the predicted and "real" output T_s, without linear interpolation.

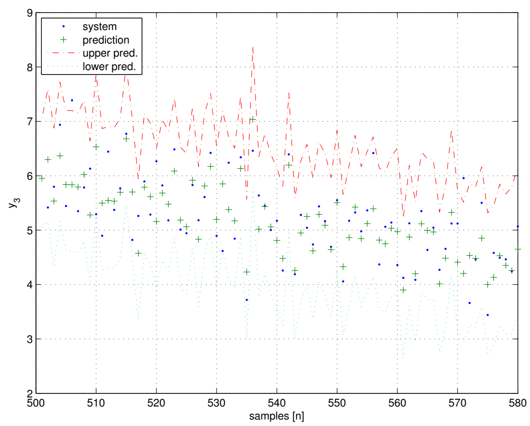

Fig. 7. Plot of the predicted and "real" output p_s, without linear interpolation.

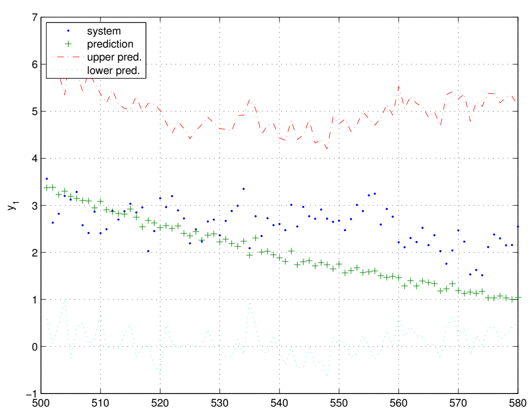

Fig. 8. Plot of the predicted and "real" output T_g.

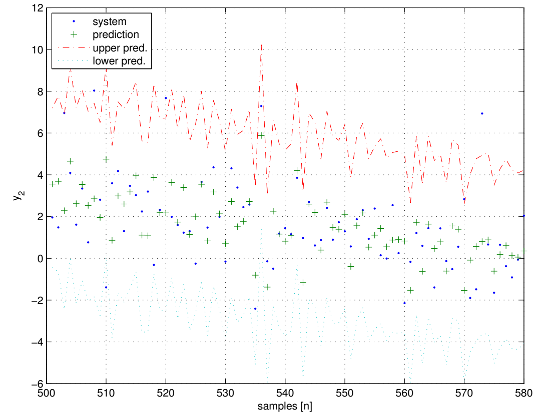

Fig. 9. Plot of the predicted and "real" output T_s.

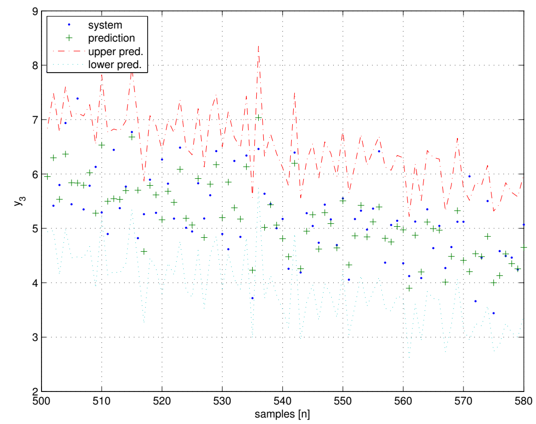

Fig. 10. Plot of the predicted and "real" output p_s.

V. CONCLUSION

This paper presents a method for prediction of uncertain closed loop systems. The method uses precomputed mean and variances of the prediction error depending on the operating point given by references and disturbances. These uncertainty models are stored in a model bank, which by using linear interpolation is used to predict uncertainty bounds on the prediction using the statistics of the past prediction uncertainties. It is as well proposed to update the uncertainty prediction models on-line. The potential of the method is illustrated by an example from a coal-fired power plant.

VI. ACKNOWLEDGMENT

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