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Optimal Set-point Synthesis in HVAC Systems

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Abstract— **This paper presents optimal set-point synthesis for a heating, ventilating, and air-conditioning (HVAC) system. This HVAC system is made of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger. The objective function is composed of the electrical power for different components, encompassing fans, primary/secondary pump, tertiary pump, and air-to-air heat exchanger wheel; and a fraction of thermal power used by the HVAC system. The goals that have to be achieved by the HVAC system appear as constraints in the optimization problem. To solve the optimization problem, a steady state model of the HVAC system is derived while different supplying hydronic circuits are studied for the water-to-air heat exchanger. Finally, the optimal set-points and the optimal supplying hydronic circuit are resulted.**

I. INTRODUCTION

Classical HVAC control techniques such as the ON/OFF controllers (thermostats) and the proportional-integral-derivative (PID) controllers are still very popular because of their low cost. However, in the long run, these controllers are expensive because they operate at a non-optimal efficiency. So, there is a high potential to apply advanced control methods to save large amount of energy. For example, by optimal control of HVAC systems almost 100 GWh energy can be saved yearly in Denmark (five million inhabitants)[1].

A common method used to maintain an industrial plant at its optimal operating condition is to calculate optimal values of feedback controller set-points, employing a steady-state mathematical model of the process [2], [3]. Steady-state optimization of an industrial process often considers that the overall control is performed within a two-layer hierarchical structure. The lower layer performs direct regulatory control, where the aim is to maintain selected process variables at their desired set-point values, and the upper layer, known as the supervisory layer, has the task of detremining the set-points of the regulatory controllers to obtain optimal steady-state performance.

This kind of two layer control strategy has been applied before to a cooling system and a refrigeration system and has shown great results [4], [5]. Implementing the supervisory layer through genetic algorithms in the cooling system case showed saving energy by 19.5%. In the refrigeration system case it was proved that by using this control configuration it was possible to derive the set-points close to the optimum and thus reduce the energy consumption with up to 20%.

In this paper, the supervisory layer of the overall control of a HVAC system is considered. In Section II, the HVAC system used in analysis is described. Section III presents formulation of the problem. Determination of optimal set-points through solving the defined problem is presented in Section IV. Section V presents conclusions and final comments.

TABLE I

NOMENCLATURE

q_a	inlet or outlet air flow (m^3/h)
$TE21$	outdoor air temperature (^oC)
$TE22$	temperature of outdoor air after heat recovery (^oC)
$TE11$	room air temperature (^oC)
$TE12$	temperature of room air after heat recovery (^oC)
q_{wt}	water flow of the tertiary circuit (l/h)
q_{ws}	water flow of the supply (primary/secondary) circuit (l/h)
$Twin$	tertiary supply water temperature (^oC)
$Twout$	tertiary return water temperature (^oC)
$Tinlet$	temperature of the supply air (^oC)
$Tpin$	primary/secondary supply water temperature (^oC)
$Tpout$	primary/secondary return water temperature (^oC)
η_{t1}	water-to-air heat exchanger temperature efficiency ($\eta_{t1} = \frac{Tinlet-TE22}{Twin-TE22}$)
η_{t2}	air-to-air heat exchanger temperature efficiency ($\eta_{t2} = \frac{TE22-TE21}{TE11-TE21}$)
ρ_w	water mass density (Kg/m^3)
C_w	water specific heat ($J/Kg\ ^oC$)
ρ_a	air mass density (Kg/m^3)
C_a	air specific heat ($J/Kg\ ^oC$)
wrf	wheel rotation factor ($1 \geq wrf \geq 0$)
n	rotation speed of the wheel ($10\ rpm \geq n \geq 0\ rpm$)
k	$k = 1000 \cdot \frac{\rho_a\ C_a\ q_a}{\rho_w\ C_w}$

II. THE HVAC SYSTEM DESCRIPTION

The HVAC system that will be considered consists of two heat exchangers: an air-to-air heat exchanger and a water-to-air heat exchanger. In this section the temperature efficiency of these two heat exchangers, which can be used as a steady state model of heat exchangers, will be described.

A. The Air-to-air Heat Exchanger

The air-to-air heat exchanger is a rotary heat exchanger in aluminium, with low pressure loss (shown in Fig. 1). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow. Here, it is supposed that the ratio of the

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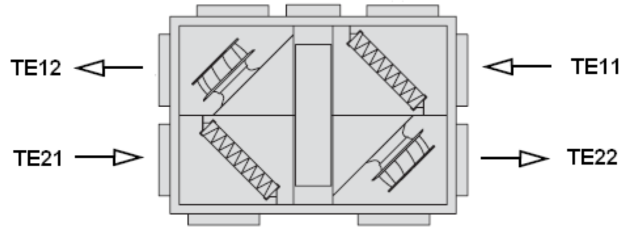

Fig. 1. The Air-to-air Heat Exchanger Scheme

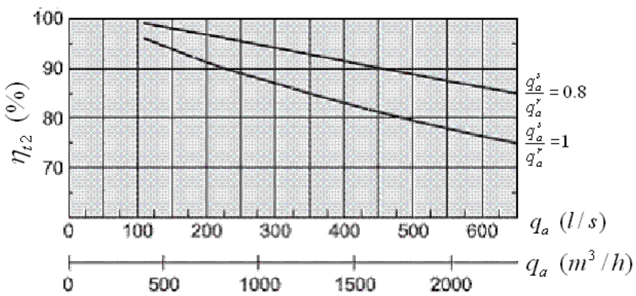

Fig. 2. Dependency of η_{t2} on q_a while n=10 rpm; q_a^s and q_a^r represent supply air flow and return air flow, respectively.

supply air flow to the return air flow is one. Therefore, η_{t2} will be a function of air flow (q_a) [6] , that is the same for both supply and return air, and the rotation speed of the wheel (n).

In this context, results of testing the rotary heat exchanger that was performed according to European Standard for laboratory testing of air-to-air heat recovery devices (EN 247, EN 305, EN 306, EN 307, EN 308) will be used.This European Standard is intended to be used as a basis for testing heat recovery devices for HVAC systems, which as specified in EN 247 consist of the heat exchanger itself installed in a casing having the necessary air duct connecting elements and in some cases the fans and pumps, but without any additional components of the HVAC system.

According to results of the test, it is possible to specify η_{t2} as a multiplication of two functions. Fig. 2 and 3 illustrate these functions. Therefore, η_{t2} can be described as following:

$$\eta_{t2} = (-1.0569 \cdot 10^{-4}\ q_a + 0.9943) \cdot wrf(n) \qquad (1)$$

B. The Water-to-air Heat Exchanger

The water-to-air heat exchanger is shown in Fig. 4. As can be seen, a primary/secondary-tertiary hydronic circuit supplies the heat exchanger with hot water. The air flow that passes the hot coil is controllable by changing the speed of the fan installed in the air-to-air heat exchanger.

Here, temperature efficiency (η_{t1}) is a function of hot water flow (q_{wt}) and air flow (q_a). To obtain this function several experiments were done. Results are illustrated in Fig. 5. Again, it is possible to describe η_{t1} as a multiplication of two functions that the first one depends only on air flow (q_a) and the second one depends only on water flow (q_{wt}):

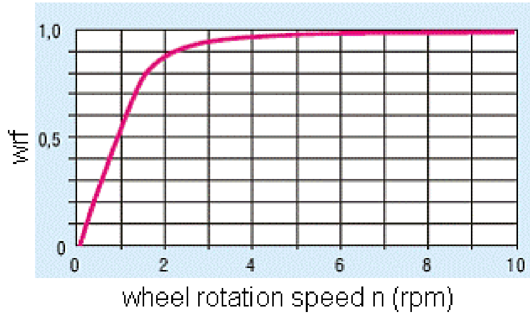

Fig. 3. Normalized Dependency of η_{t2} on n

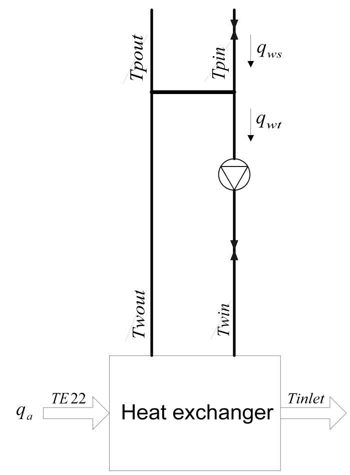

Fig. 4. The Water-to-air Heat Exchanger Scheme

$$\eta_{t1} = \frac{1}{0.3215}\ (a\ q_{wt}^4 + b\ q_{wt}^3 + c\ q_{wt}^2 + d\ q_{wt} + e) \cdot$$
$$(A\ q_a^3 + B\ q_a^2 + C\ q_a + D) \qquad (2)$$

where:

$a = -5.399 \cdot 10^{-12}$	$A = 1.0665 \cdot 10^{-10}$
$b = 1.0733 \cdot 10^{-8}$	$B = -1.643 \cdot 10^{-7}$
$c = -7.887 \cdot 10^{-6}$	$C = -2.880 \cdot 10^{-4}$
$d = 2.7199 \cdot 10^{-3}$	$D = 0.6927$
$e = 8.3711 \cdot 10^{-4}$	

III. PROBLEM FORMULATION

As it was mentioned, the aim of this paper is to find the optimal set-points for the described HVAC system. Thus, an objective function is needed to formulate the problem. The HVAC system mission can be described as constraints for the defined objective function.

A. Objective Function

The desired objective function is defined as following:

$$J = P_{pt} + P_{pp} + P_f + P_w + \Phi/2.5 \qquad (3)$$

subject to:

$$q_a = q_{a_0}$$

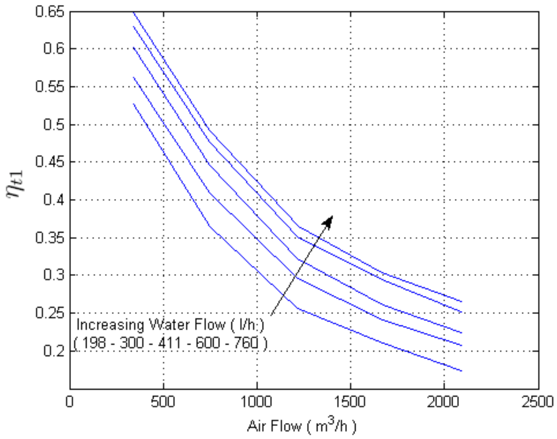

Fig. 5. Result of Experiments on Water-to-air Heat Exchanger

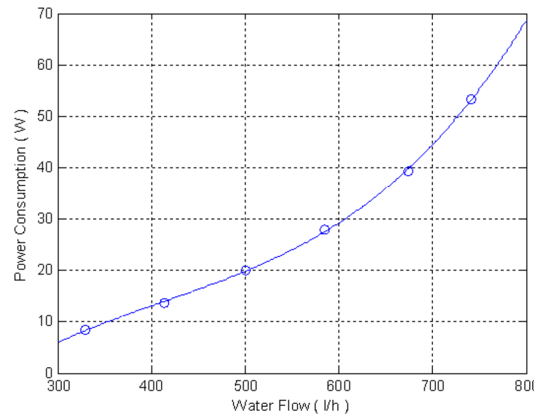

Fig. 6. Tertiary Pump Power vs q_{wt}

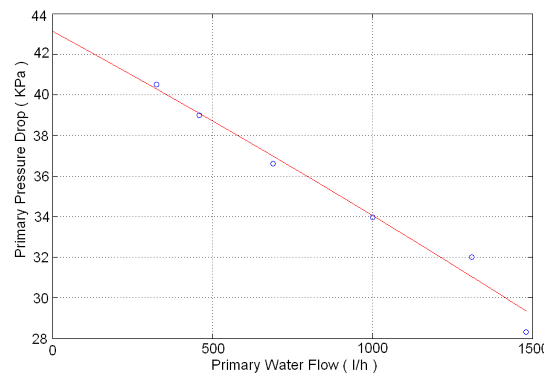

Fig. 7. Primary Pressure Drop vs q_{ws}

$$Tinlet = 19$$

$$Tpout \leq 40$$

and,

$$0 \leq q_{wt} \leq 743$$

$$0 \leq q_{ws} \leq 1400$$

$$300 \leq q_a \leq 2200$$

$$0 \leq n \leq 10$$

where:

P_{pt} , P_{pp}, P_f , P_w , and Φ are tertiary pump power, primary pumping power, fan power, wheel rotation power, and thermal power, respectively.

q_{a_0} is a constant that will be determined in accordance with the required ventilation [7]. This formulation discuss a typical HVAC system used for ventilation purposes.

B. Tertiary Pump Power (P_{pt})

The hydronic circuit that is used for supplying the water-to-air heat exchanger is a primary/secondary-tertiary circuit isolated from each other by a bypass pipe. The bypass pipe is a short lenghth of full bore piping. The pressure drop across the bypass pipe is then small compared to the pressure drop in the tertiary circuit and through the supply circuit [8]. The supply water flow (q_{ws}) is controlled by the motorized primary/secondary valve. A variable speed pump and a valve is installed in the tertiary circuit. The tertiary valve is used to set the desired maximum flow rate through the variable speed pump. By changing the speed of the tertiary pump, it is possible to sweep the interval between 330 (l/h) and 743 (l/h) for the tertiary water flow (q_{wt}). If the tertiary water flow has to be less than 330 (l/h), the pump will be pulsed. Fig. 6 illustrates power of the tertiary pump as a function of q_{wt}. This curve is approximated by the following polynomial:

$$P_{pt} = A_p\ q_{wt}^3 + B_p\ q_{wt}^2 + C_p\ q_{wt} + D_p \tag{4}$$

where:

$A_p = 5.1873 \cdot 10^{-7}$ $\quad B_p = -6.4260 \cdot 10^{-4}$
$C_p = 3.2906 \cdot 10^{-1}$ $\quad D_p = -48.8641$

When the pump is pulse width modulated, it is assumed that the power of the pump is the duty cycle fraction of the pump power while it is running at its minimum speed, i.e. it is proportional to the pump working period.

C. Primary Pumping Power(P_{pp})

The primary/secondary pumping power has to be measured implicitly because there is no direct access to the primary/secondary pump. Therefore, it is supposed that the efficiency of the pump in converting electrical power to hydraulic power is 50%. The curve indicating required pressure drop versus primary/secondary water flow (q_{ws}) is shown in Fig. 7. A second order polynomial is used to represent this curve. As we know, multiplying water flow by head results in hydraulic power. So, primary pumping power can be expressed as follows:

$$P_{pp} = \frac{2}{3600}\, q_{ws} \cdot \left(-4.8131 \cdot 10^{-7}\, q_{ws}^2 - 8.5955 \cdot 10^{-3}\, q_{ws} + 43.1390\right) \tag{5}$$

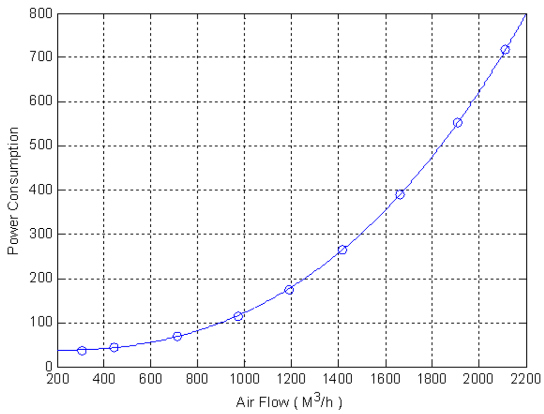

Fig. 8. Fan Power vs Air Flow (q_a)

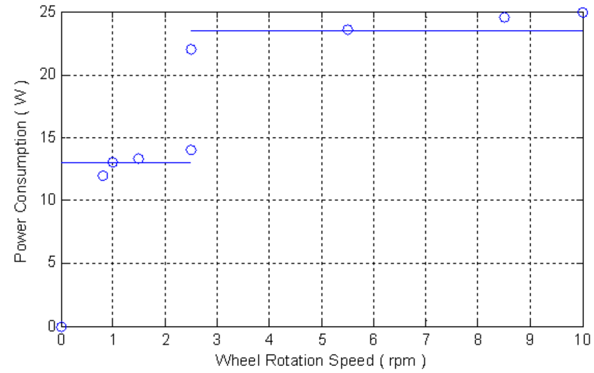

Fig. 9. Wheel Power Consumption vs n

D. Fan Power(P_f)

The HVAC system structure is assumed fixed during the entire work. As a result, the path for the air does not change. So, it is possible to have fan power as a function of air flow (q_a). Fig. 8 illustrates this function. The curve is approximated by a third order polynomial as follows:

$$P_f = A_f\ q_a^3 + B_f\ q_a^2 + C_f\ q_a + D_f \tag{6}$$

where:

$A_f = 4.7354 \cdot 10^{-8}$ $\quad B_f = 6.705 \cdot 10^{-5}$
$C_f = -3.2527 \cdot 10^{-2}$ $\quad D_f = 40.3043$

E. Wheel Rotation Power(P_w)

The electrical power input to the wheel as a function of rotation speed of the wheel is sketched in Fig. 9. The step change in the figure is due to the frequency converter used to control rotation speed of the wheel.

As Fig. 9 reveals, it is feasible to assume that the electrical power of the wheel is composed of three parts:

$$\begin{aligned} P_w &= 23.5\ W & 10\ rpm \geq n > 2.5\ rpm & \quad (1 \geq wrf > 0.9) \\ P_w &= 13\ W & 2.5\ rpm \geq n > 0\ rpm & \quad (0.9 \geq wrf > 0) \\ P_w &= 0\ W & n = 0\ rpm & \quad (wrf = 0) \end{aligned} \tag{7}$$

F. Thermal Power (Φ)

Φ is the thermal power that is being used by the water-to-air heat exchanger:

$$\Phi = \rho_w\ q_{wt}\ C_w\ (Twin - Twout) \tag{8}$$

In steady state conditions:

$$\rho_w\ q_{wt}\ C_w\ (Twin - Twout) = \rho_a\ q_a\ C_a\ (Tinlet - TE22) \tag{9}$$

According to the definitions, we have:

$$Tinlet = TE22 + \eta_{t1}\ (Twin - TE22) \tag{10}$$

$$TE22 = TE21 + \eta_{t2}\ (TE11 - TE21) \tag{11}$$

Substituting (10) and (11) in (9) will result in:

$$\begin{aligned} \Phi = {} & \rho_a\ q_a\ C_a\ \eta_{t1}\ (Twin - TE21) + \\ & \rho_a\ q_a\ C_a\ \eta_{t1}\ \eta_{t2}\ (TE21 - TE11) \end{aligned} \tag{12}$$

This formula will be used for computing the thermal power consumption. Thermal power is devided by 2.5 in the objective function because thermal power is 2.5 time cheaper than electrical power according to building regulations in Denmark.

IV. DETERMINING OPTIMAL SET-POINTS

To obtain optimal set-points, the optimization problem that was defined in the previous section has to be solved.

In this section solving the defined optimization problem is presented while two different cases are assumed for the hydronic circuit. In the first case it is assumed that $q_{ws} \leq q_{wt}$. In the second case we will have $q_{ws} \geq q_{wt}$. These two cases are selected because they are the most general cases.

A. Computing Optimal Set-points While $q_{ws} \leq q_{wt}$

According to the discussion so far, the optimization problem consists of a four-variable (q_a, q_{ws}, q_{wt}, n) objective function along with two equality constraint and two inequality constraints. Because q_a will be determined in accordance with required ventilation, actually we have to deal with a three-variable optimization problem along with an equality constraint and two inequality constraints. Thus, in the sequel by optimization problem we mean the latter statement. For convenience we will deal with $TE22$ instead of n in the procedure of solving.

Because, in this case, supply water flow (q_{ws}) is always less than or equal to the tertiary water flow (q_{wt}), mixing between the supply water flow that enters the tertiary circuit and a part of the tertiary return water flow occurs. So, the temperature of the water that enters the heat exchanger is as follows:

$$Twin = \frac{q_{ws}\ \cdot\ Tpin\ +\ (q_{wt} - q_{ws})\ \cdot\ Twout}{q_{wt}} \tag{13}$$

Actually, the mission of the motorized primary/secondary valve is controlling $Twin$ by changing the supply water flow (q_{ws}).

As it was mentioned, to solve the optimization problem here we have to deal with three variables q_{ws}, q_{wt}, and $TE22$. One of these variables is dependent due to equality constraint (Tinlet = 19):

$$Tinlet \;=\; (1-\eta_{t1}) \;\cdot\; TE22+\eta_{t1} \;\cdot\; Twin \qquad (14)$$

As we know, the energy balance equation in a water-to-air heat exchanger is as follows:

$$\rho_w \, q_{wt} \, C_w \, (Twin-Twout) = \rho_a \, q_a \, C_a \, (Tinlet-TE22) \quad (15)$$

Substituting the two recent equations in equation (13) will result in a formula for q_{ws} versus q_{wt} and $TE22$:

$$q_{ws} = \frac{k \;\; \eta_{t1} \;\; q_{wt} \;\; (19-TE22)}{\eta_{t1} q_{wt}(Tpin-TE22)+(q_{wt}-k\eta_{t1})(TE22-19)} \qquad (16)$$

Also, in this case $q_{ws} \leq q_{wt}$; Therefore, substituting equation (16) in the recent inequality results in an inequality as following:

$$TE22 \geq \frac{19-Tpin \;\, \eta_{t1}}{1-\eta_{t1}} \qquad (17)$$

Combining equations (14) and (15) results in a formula for $Twout$:

$$Twout = \frac{19q_{wt}+TE22(-q_{wt}+\eta_{t1}q_{wt}+k\eta_{t1})-19k\eta_{t1}}{\eta_{t1}q_{wt}} \qquad (18)$$

In this case, $Twout$ is equal to $Tpout$ because the supply water flow is less than or equal to the tertiary water flow. Thus, the inequality constraint ($Tpout \leq 40$) can be translated into the following inequality:

$$(-q_{wt}+\eta_{t1}q_{wt}+k\eta_{t1}) \; TE22 \leq 40\eta_{t1}q_{wt}+19k\eta_{t1}-19q_{wt} \qquad (19)$$

Finally, the optimization problem transferred to an objective function of two variables (q_{wt} and $TE22$) with two inequality constraints (inequalities (17) and (19)). The typical feasible region of this optimization problem is shown in Fig. 10 (assuming $TE21 = -12$, $Tpin = 80$ and $q_a = 2104.9$). Optimal set-points in different conditions as a result of solving the optimization problem can be found in Table II.

B. Computing Optimal Set-points While $q_{ws} \geq q_{wt}$

We have to deal with a three-variable optimization problem along with an equality constraint and two inequality constraints again. The only difference is the fact that supply water flow is greater than or equal to teriary water flow. The impact of keeping supply water flow higher than or equal to the tertiary water flow on the system is that the supply tertiary water flow to the water-to-air heatexchanger will not be mixed water. So, $Twin$ will be equal to the $Tpin$. Actually,

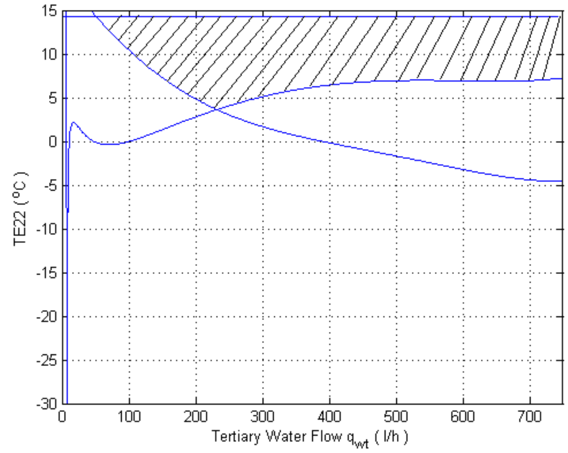

Fig. 10. Feasible Region While $q_{ws} \leq q_{wt}$ ($TE21 = -12$, $Tpin = 80$ and $q_a = 2104.9$)

water mixing occur between the return tertiary water flow (q_{ws}) and the hot water passes the balance pipe ($q_{ws} - q_{wt}$). Therefore, we have:

$$Tpout = \frac{q_{wt} \;\cdot\; Twout+(q_{ws}-q_{wt}) \;\cdot\; Tpin}{q_{ws}} \qquad (20)$$

According to the equality constraint ($Tinlet = 19$) and the fact that $Twin$ is always equal to $Tpin$, it is possible to have $TE22$ as a function of q_{wt}:

$$TE22 = \frac{19-Tpin \;\, \eta_{t1}}{1-\eta_{t1}} \qquad (21)$$

Substituting equations (15) and (21) in equation (20) will result in the desired formula for $Tpout$:

$$Tpout = \frac{-(Tpin-19) \;\, k \;\, \eta_{t1}+Tpin \;\, q_{ws}-Tpin \;\, \eta_{t1} \;\, q_{ws}}{(1-\eta_{t1}) \;\, q_{ws}} \qquad (22)$$

Using the recent formula for $Tpout$, the inequality constraint ($Tpout \leq 40$) can be translated into the following inequality:

$$q_{ws} \leq \frac{(Tpin-19) \;\, k \;\, \eta_{t1}}{(Tpin \;-\; 40) \;\, (1-\eta_{t1})} \qquad (23)$$

To summarize, the optimization problem transferred to an objective function of two variables (q_{wt} and q_{ws}) with two inequality constraint (inequality (23) and $q_{ws} \geq q_{wt}$). The typical feasible region in this case is sketched in Fig. 11 (supposed $TE21 = -30$, $Tpin = 60$ and $q_a = 1674.1$).

Solving the optimization problem in this case results in the same optimal values obtained in the previous case. That is, Table II represents optimal set-points also in this case.

C. Consideration of Results

Regarding results of solving the optimization problem in different conditions (Table II) revealvs that in all conditions supply water flow (q_{ws}) and tertiary water flow (q_{wt}) are equal. Therefore, from energy point of view the optimal hydronic circuit for supplying water-to-air heat exchanger

TABLE II

OPTIMAL SET-POINTS IN DIFFERENT CONDITIONS WHILE $q_{ws} \leq q_{wt}$

Air Flow (q_a)	Set-points	$Tpin$ = 80 $TE21$ = -12	80 −20	80 −30	70 −12	70 −20	70 −30	60 −12	60 −20	60 −30
2104.9	q_{wt}	51.6	73.9	104.7	62.7	90.8	131.4	80.2	118.6	179.0
	q_{ws}	51.6	73.9	104.7	62.7	90.8	131.4	80.2	118.6	179.0
	wrf	1	1	1	1	1	1	1	1	1
	$TE22$	14.2	12.4	10.1	14.2	12.4	10.1	14.2	12.4	10.1
	$Twout$	24.2	26.1	28.8	24.2	26.2	29.2	24.2	26.5	30.1
1674.1	q_{wt}	29.6	44	62.8	35.8	53.3	76.5	45	67.6	98.3
	q_{ws}	29.6	44	62.8	35.8	53.3	76.5	45	67.6	98.3
	wrf	1	1	1	1	1	1	1	1	1
	$TE22$	15.8	14.3	12.5	15.8	14.3	12.5	15.8	14.3	12.5
	$Twout$	27.9	28.9	30.2	26.9	27.9	29.2	25.8	26.8	28.2
979.9	q_{wt}	4	9.3	15.9	4.8	11.2	19.1	6.1	14	23.9
	q_{ws}	4	9.3	15.9	4.8	11.2	19.1	6.1	14	23.9
	wrf	1	1	1	1	1	1	1	1	1
	$TE22$	18.3	17.4	16.3	18.3	17.4	16.3	18.3	17.4	16.3
	$Twout$	29.7	31.8	32.5	28.5	29.9	30.5	27.0	28	28.4
308.3	q_{wt}	0	0	0	0	0	0	0	0	0
	q_{ws}	0	0	0	0	0	0	0	0	0
	wrf	0.94	0.96	0.98	0.94	0.96	0.98	0.94	0.96	0.98

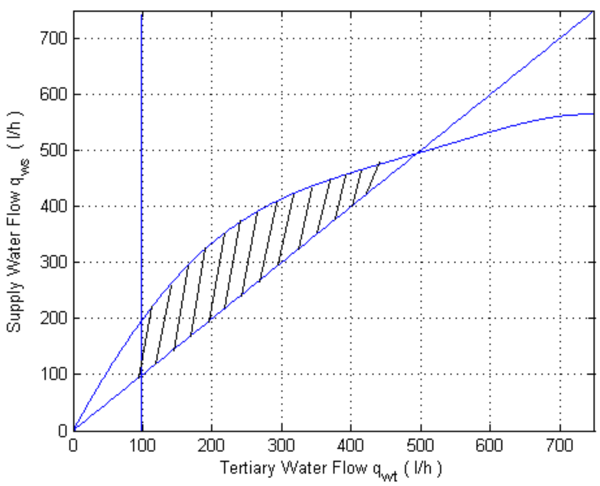

Fig. 11. Feasible Region While $q_{ws} \geq q_{wt}$ ($TE21 = -30$, $Tpin = 60$ and $q_a = 1674.1$)

is variable-primary flow circuit [9]. It also shows that using air-to-air heat exchanger to produce required heat is cheaper than using water-to-air heat exchanger. That is, the control strategy must be designed in such a way that maximum exploitation of the air-to-air heat exchanger is achieved.

V. CONCLUSIONS

Optimal set-point synthesis for a HVAC system was presented in this paper. The HVAC system was a typical HVAC system consisted of an air-to-air heat exchnager and a water-to-air heat exchanger. To derive the optimal set-points, an objective function composed of electrical power of different components in the HVAC system and a fraction of thermal power used by the system was defined. The goals defined for the HVAC system were treated as constraints to the objective function. Finally, the defined optimization problem was solved using the steady state model of the system. Analysis of the obtained results revealed that in all conditions supply water flow was equal to tertiary water flow. Thus, the varying-primary flow system was the optimal hydronic circuit to supply the water-to-air heat exchanger. The synthesis done here can be applied as a supervisory layer for the two layer control of the HVAC system to make the system work at its optimal set-points.

VI. ACKNOWLEDGMENTS

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REFERENCES

[1] P. Svendsen and H. Andersen, "Energy Efficient Pump Coupling in HVAC Systems", *Technical Report*, Danish Technological Institute (Industry and Energy Section), 2005.

[2] J. Lin, S. Chen, and P.D. Roberts, "Modified Algorithm for Steady-state Integrated System Optimization and Parameter Estimation", *IEE Proceedings*, Vol. 135, March 1988.

[3] S. Gros, B. Srinivasan, and D. Bonvin, "Static Optimization via Tracking of The Necessary Conditions of Optimality Using Neighboring Extremals", *American Control Conference Proceeding*, June 2005, pp 251-255.

[4] N. Nassif, S. Kajl, and R. Sabourin, "Two-Objective On-Line Optimization of Supervisory Control Strategy", *Building Services Engineering Research and Technology*, 2004, pp 241-251.

[5] L.F.S. Larsen, "Model Based Control of Refrigeration Ssytems", *Ph.D. Thesis*, Aalborg University, Denmark, 2005.

[6] M. Wetter, "Simulation Model: Air-to-air Heat Exchnager", *Technical Report*, Lawrence Berkeley National Laboratory, 1999.

[7] N. Nassif, S. Kajl, and R. Sabourin, "Ventilation Control Strategy Using Supply CO_2 Concentration Set Point", *International Journal of HVAC & R Research*, 2005, pp 239-262.

[8] R.H. Green, "An Air-Conditioning Control System Using Variable-speed Water Pumps", *ASHRAE Transaction:Research*, part 1, 1994, pp 463-470.

[9] W.P. Bahnfleth, "Varying Views on Variable-primary Flow", *Chilled Water Engineering J.*, March 2004.