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Optimal Model-Based Control in HVAC Systems

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***Abstract*— This paper presents optimal model-based control of a heating, ventilating, and air-conditioning (HVAC) system. This HVAC system is made of two heat exchangers: an air-to-air heat exchanger (a rotary wheel heat recovery) and a water-to-air heat exchanger. First dynamic model of the HVAC system is developed. Then the optimal control structure is designed and implemented. The HVAC system is splitted into two subsystems. By selecting the right set-points and appropriate cost functions for each subsystem controller the optimal control strategy is respected to gaurantee the minimum thermal and electrical energy consumption. Finally, the controller is applied to control the mentioned HVAC system and the results show that the expected goals are fulfilled.**

I. INTRODUCTION

The consumption of energy by heating, ventilating, and air conditioning (HVAC) equipment in industrial and commercial buildings constitutes a great part of the world energy consumption [1]. In spite of the advancements made in microprocessor technology and its impact on the development of new control methodologies for HVAC systems aiming at improving their energy efficiency, the process of operating HVAC equipment in commercial and industrial buildings is still an inefficient and high-energy consumption process.

It has been estimated that by optimal control of HVAC systems almost 100 GWh energy can be saved yearly in Denmark (five million inhabitants) [2]. It shows that a huge amount of energy can be saved and according to the current energy prices it will be reasonable to invest a little bit more in the first cost of HVAC systems.

In this paper, an integrated control system is developed. That is, in the proposed control system there is no need for an expilicit supervisory layer to make the system work in its optimal conditions. The optimal control strategy that has been developed in [2] is implemented here. So, the controller follows the optimal control strategy while it tracks the set-point. In Section II, the dynamic model of the HVAC system is described. The controller design is presented in Section III. Finally, the results of applying the proposed control system is shown in Section IV.

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TABLE I
NOMENCLATURE

q_a	inlet or outlet air flow (m^3/h)
$TE21$	outdoor air temperature (^oC)
$TE22$	temperature of outdoor air after heat recovery (^oC)
$TE11$	room air temperature (^oC)
$TE12$	temperature of room air after heat recovery (^oC)
q_{wt}	water flow of the tertiary circuit (l/h)
q_{ws}	water flow of the supply (primary/secondary) circuit (l/h)
$Twin$	tertiary supply water temperature (^oC)
$Twout$	tertiary return water temperature (^oC)
$Tinlet$	temperature of the supply air (^oC)
$Tpin$	primary/secondary supply water temperature (^oC)
$Tpout$	primary/secondary return water temperature (^oC)
η_{t2}	air-to-air heat exchanger temperature efficiency ($\eta_{t2} = \frac{TE22-TE21}{TE11-TE21}$)
ρ_w	water mass density (Kg/m^3)
C_{pw}	water specific heat ($J/Kg\ ^oC$)
ρ_a	air mass density (Kg/m^3)
C_{pa}	air specific heat ($J/Kg\ ^oC$)
wrf	wheel rotation factor ($1 \geq wrf \geq 0$)
n	rotation speed of the wheel ($10\ rpm \geq n \geq 0\ rpm$)
$\acute{A_s}$	heat transfer surface area of one tube (m^2)
U	mean air velocity in the tube (m/s)
$T_m(x,t)$	matrix temperature (^oC)
$T_a(x,t)$	air temperature (^oC)
h	convective heat transfer coefficient ($W/m^2\ ^oK$)
L	wheel length (m)
A_m	cross sectional area of one tube of matrix (m^2)
A_a	cross sectional area of one tube for air (m^2)
K_m	matrix thermal conductivity ($W/m\ ^oK$)
P	exposion time (half of the period) ($sec.$)
M_m	total matrix mass (Kg)
$\dot{m}_a$	air mass flow rate (Kg/h)
A_s	heat transfer surface area on the supply or exhaust side (m^2)
C_{pm}	matrix specific heat ($J/Kg\ ^oC$)
C_r^*	$\frac{M_m\ C_{pm}\ n}{\dot{m}_a\ C_{pa}}$
NTU	$\frac{h\ A_s}{\dot{m}_a\ C_{pa}}$
C_{pc}	specific heat of the coil ($J/Kg\ ^oC$)
$\dot{m}_{ws}$	supply water mass flow rate (Kg/h)
$\dot{m}_{wt}$	tertiary water mass flow rate (Kg/h)
m_{cw}	effective mass of the region of the coil at an average temperature equal to outlet water temperature (Kg)
m_{ca}	effective mass of the region of the coil at an average temperature equal to outlet air temperature (Kg)
C_w	$C_{pc}\ m_{cw}$ ($J/^oC$)
C_a	$C_{pc}\ m_{ca}$ ($J/^oC$)
N	pump speed (rpm)

II. DYNAMIC MODELING

The HVAC system that will be considered consists of two heat exchangers: an air-to-air heat exchanger and a water-toair heat exchanger. In this section these components will be described and their dynamic models will be developed. Finally the overall nonlinear model of the HVAC system will be linearized. This linear model will be used to design the controller later.

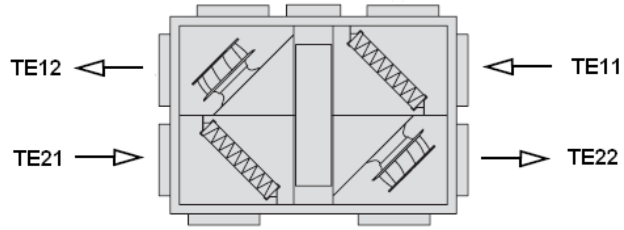

Fig. 1. The Air-to-air Heat Exchanger Scheme

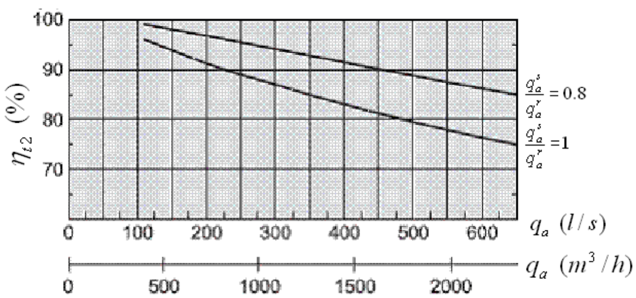

Fig. 2. Dependency of η_{t2} on q_a while n=10 rpm; q_a^s and q_a^r represent supply air flow and return air flow, respectively.

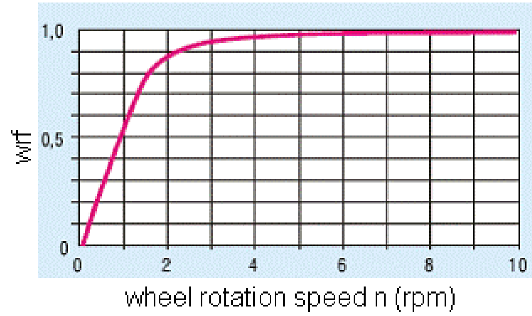

Fig. 3. Normalized Dependency of η_{t2} on n

A. Air-to-air Heat Exchanger

The air-to-air heat exchanger is a rotary heat exchanger in aluminium, with low pressure loss (shown in Fig. 1). The rotor control comprises a gear motor with frequency converter. Two fans are installed to produce the desired inlet and outlet air flow.

1) Steady State Gain Determination: Here, it is supposed that the ratio of the supply air flow to the return air flow is one. Therefore, η_{t2} will be a function of air flow (q_a), that is the same for both supply and return air, and the rotation speed of the wheel (n). In this context, results of testing the rotary heat exchanger that was performed according to European Standard for laboratory testing of air-to-air heat recovery devices (EN 247, EN 305, EN 306, EN 307, EN 308) will be used. According to results of the test, it is possible to specify η_{t2} as a multiplication of two functions. Fig. 2 and 3 illustrate these functions [2]. Therefore, η_{t2} can be described as following:

$$\eta_{t2} = (-1.0569 \cdot 10^{-4}\ q_a + 0.9943) \cdot wrf(n) \tag{1}$$

As we know, η_{t2} definition is as following:

$$\eta_{t2} = \frac{TE22 - TE21}{TE11 - TE21} \tag{2}$$

Combining recent equations (equations 1 and 2) will result in the steady state gain for the wheel model:

$$TE22 = TE21 + wrf(n) \cdot (TE11 - TE21) \cdot$$
$$(-1.0569 \cdot 10^{-4} q_a + 0.9943) \tag{3}$$

2) Dynamic Behavior: Fig 4 shows an energy wheel operating in a counter flow arrangement. Under typical operating conditions, warm air enters the tube during the supply part of the cycle and transfers energy to the matrix. This energy is then transferred from the matrix to the air during the exhaust part of the cycle. The half plane of the matrix tube is assumed impermeable and adiabatic and the bulk mean temperatures of air are used in the model. The formulation is therefore one dimensional and transient with space (x) and time (t or $\theta = w \cdot t$) as the independent variables. The governing equations for heat transfer (energy equations) in energy wheel for air and matrix include energy storage, convection, conduction based on the usual assumptions are as fllows respectively:

$$\rho_a C_{pa} A_a \frac{\partial T_a}{\partial t} + U \rho_a C_{pa} A_a \frac{\partial T_a}{\partial x} + h \frac{\acute{A_s}}{L}(T_a - T_m) = 0 \tag{4}$$

$$\rho_m C_{pm} A_m \frac{\partial T_m}{\partial t} - h \frac{\acute{A_s}}{L}(T_m - T_a) = \frac{\partial}{\partial x}(K_m A_m \frac{\partial T_m}{\partial x}) \tag{5}$$

It is reasonable to suppose that the conductivity has a samll share in heat transfer through the matrix [3]. Thus, equation (5) can be rewritten as following:

$$\frac{\partial T_m}{\partial t} + \frac{NTU}{C_r^* P} T_m = \frac{NTU}{C_r^* P} T_a \tag{6}$$

Equation (6) shows that air temperature (T_a) can be assumed as the input for the matrix temperature (T_m) differential equation. It means the matrix temperature as a function of time (t) will perform as an output of the ordinary first order differential equation. Another point that should be emphasized is that the time constant ($\frac{C_r^* P}{NTU}$)in the differential equation is fixed. That is, the time constant depends on matrix (wheel) properties. Thus, it is a design parameter not a control parameter. It is claimed that air stream temperature has the same behavior as matrix temperature [4]. So, The air stream temperature shows first order dynamic behavior.

3) Dynamic Model of The Air-to-air Heat Exchanger: According to our earlier debate the wheel behavior can be modeled by a first order transfer function. So, we will have:

$$TE22(s) = \frac{TE21}{\tau\ s + 1} +$$

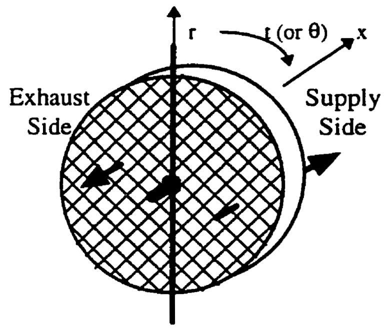

Fig. 4. Counter flow energy wheel

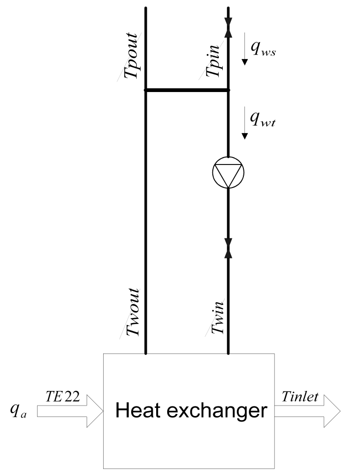

Fig. 5. The Water-to-air Heat Exchanger Scheme

$$\frac{wrf(s)\cdot(TE11-TE21)(-1.0569\times10^{-4}q_a+0.9943)}{\tau\ s+1} \quad (7)$$

The first part on the right side of the equation (7) will be treated as disturbance. That is, the transfer function from $TE22$ to wrf is as following:

$$\frac{TE22(s)}{wrf(s)}=\frac{(TE11-TE21)(-1.0569\times10^{-4}q_a+0.9943)}{\tau\ s+1} \quad (8)$$

As it was discussed, the time constant (τ) is fixed and according to the experiments, it is 28.0374 seconds.

B. Water-to-air Heat Exchanger

The water-to-air heat exchanger is shown in Fig. 5. As can be seen, a primary/secondary-tertiary hydronic circuit supplies the heat exchanger with hot water. The air flow that passes the hot coil is controllable by changing the speed of the fan installed in the air-to-air heat exchanger.

The hydronic circuit that is used for supplying the water-to-air heat exchanger is a primary/secondary-tertiary circuit isolated from each other by a bypass pipe. The supply water flow (q_{ws}) is controlled by the motorized primary/secondary valve. A variable speed pump and a valve is installed in the tertiary circuit. The tertiary valve is used to set the desired maximum flow rate through the variable speed pump. By changing the speed of the tertiary pump, it is possible to sweep the desired interval for the tertiary water flow (q_{wt}).

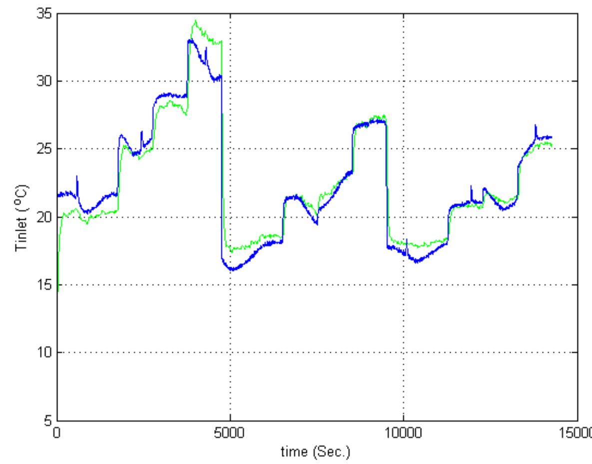

Fig. 6. Coil Model Verification, Blue Curve: Real Output, Green Curve: Simulated Output

1) Dynamic Model of The Coil: Here, the nonlinear coil model that was developed by Underwood and Crawford [5] will be applied. According to their model, the differential equations, resulted from energy balance equations, which describe the coil behavior are as follows:

$$\begin{aligned}&[(-C_{pw}-b/2)\dot{m_{wt}}(t)-d/2\dot{m}_a(t)-a/2]\ T_{wout}(t)+\\&[(C_{pw}-b/2)\dot{m_{wt}}(t)-d/2\dot{m}_a(t)-a/2]\ T_{win}(t)\\&+(b\ \dot{m_{wt}}(t)+d\ \dot{m}_a+a)\ TE22(t)=C_w\frac{d}{dt}T_{wout}(t)\end{aligned} \quad (9)$$

$$\begin{aligned}&-\dot{m}_a(t)C_{pa}Tinlet(t)+[(C_{pa}-d)\dot{m}_a(t)-b\dot{m_{wt}}(t)-a]\cdot\\&TE22(t)+(a/2+b/2\dot{m_{wt}}(t)+d/2\dot{m}_a(t))T_{win}(t)\\&+(a/2+b/2\dot{m_{wt}}(t)+d/2\dot{m}_a(t))T_{wout}(t)=C_a\frac{d}{dt}Tinlet(t)\end{aligned} \quad (10)$$

where a, b, d, C_{pa}, C_{pw}, C_a, and C_w are unknown parameters that have to be identified through the experiments.

The unknown parameters have identified through some experiments on the coil. Fig 6 show verification of the model along with identified parameters.

2) Dynamic Model of The Water-to-air Heat Exchanger: According to the hydronic circuit configuration $T_{win}(t)$ will be as following:

$$T_{win}(t)=\frac{T_{pin}(t)\ \dot{m}_{ws}+T_{wout}(t)\ (\dot{m_{wt}}-\dot{m}_{ws})}{\dot{m_{wt}}} \quad (11)$$

where it is supposed that $\dot{m_{wt}}\geq\dot{m_{ws}}$.

The recent formula for T_{win} should be placed in coil model (equations (9) and (10))to have the water-to-air heat exchanger model versus real inputs $\dot{m}_w$ and $\dot{m_{wp}}$. Therefore, final water-to-air heat exchanger model will be as following:

$$[k_1 - b\dot{m}_{wt} - k_2\dot{m}_{ws} + k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{wout}(t) + [k_2\dot{m}_{ws} - k_3\frac{\dot{m}_{ws}}{\dot{m}_{wt}}]T_{pin}(t)$$
$$+[a + b\ \dot{m_{wt}} + d\ \dot{m_a}(t)]TE22 = C_w\frac{d}{dt}T_{wout}(t) \tag{12}$$

$$-\dot{m}_a C_{pa} Tinlet(t) + [(C_{pa} - d)\dot{m}_a - b\dot{m_{wt}} - a]TE22 + [-k_1 +$$
$$b\dot{m_{wt}} - b/2\dot{m}_{ws} - k_3\frac{\dot{m}_{ws}}{\dot{m_{wt}}}]T_{wout}(t) + [b/2\dot{m}_{ws} + k_3\frac{\dot{m}_{ws}}{\dot{m_{wt}}}]T_{pin}(t)$$
$$= C_a\frac{d}{dt}Tinlet(t) \tag{13}$$

where:
$k_1 = -a - d\ \dot{m}_a(t)$
$k_2 = C_{pw} - b/2$
$k_3 = d/2\ \dot{m}_a(t) + a/2$

C. Linearization of The Nonlinear HVAC System Model

The model of the whole HVAC system consists of the air-to-air and water-to-air heat exchanger models which were described by equations (8), (12) and (13). The nonlinear model of the HVAC system can be described as following:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \\ T\dot{E}22 \end{bmatrix} = f(Tinlet, Twout, TE22, \dot{m_{ws}}, \dot{m_{wt}}, wrf) \tag{14}$$

where: $Twout$, $Tinlet$, and $TE22$ are states of the HVAC system. $\dot{m_{ws}}$, $\dot{m_{wt}}$, and wrf are inputs of the HVAC system.

The linearized model of the HVAC system will have the following shape:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \\ T\dot{E}22 \end{bmatrix} = \begin{bmatrix} a_4 & a_3 & a_5 \\ 0 & a_1 & a_2 \\ 0 & 0 & a_6 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \\ TE22 \end{bmatrix} +$$
$$\begin{bmatrix} b_3 & b_4 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & b_5 \end{bmatrix} \cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} \\ wrf \end{bmatrix}$$
$$y = [1\ \ 0\ \ 0] \cdot \begin{bmatrix} Tinlet \\ Twout \\ TE22 \end{bmatrix} \tag{15}$$

where:
$a_1 = (-a - b\ \dot{m_{wt}} - d\ \dot{m_a} - C_{pw}\ \dot{m_{ws}} + b/2\ \dot{m_{ws}} + a/2\ \frac{\dot{m_{ws}}}{\dot{m_{wt}}} + d/2\ \frac{\dot{m_{ws}}\ \dot{m_a}}{\dot{m_{wt}}})/C_w$
$a_2 = (a + b\ \dot{m_{wt}} + d\ \dot{m_a})/C_w$
$a_3 = (a + b\ \dot{m_{wt}} + d\ \dot{m_a} - b/2\ \dot{m_{ws}} - a/2\ \frac{\dot{m_{ws}}}{\dot{m_{wt}}} - d/2\ \frac{\dot{m_{ws}}\ \dot{m_a}}{\dot{m_{wt}}})/C_a$
$a_4 = -C_{pa}\ \dot{m_a}/C_a$
$a_5 = (C_{pa}\ \dot{m_a} - a - b\ \dot{m_{wt}} - d\ \dot{m_a})/C_a$
$a_6 = -1/\tau$
$b_1 = (C_{pw} - a/2\ \frac{1}{\dot{m_{wt}}} - b/2 - d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_w\ Tpin + (-C_{pw} + a/2\ \frac{1}{\dot{m_{wt}}} + b/2 + d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_w\ Twout$
$b_2 = (a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} + d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_w\ \ Tpin + (-b - a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} - d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_w\ Twout + b\ TE22$
$b_3 = (a/2\ \frac{1}{\dot{m_{wt}}} + b/2 + d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_a\ Tpin + (-a/2\ \frac{1}{\dot{m_{wt}}} - b/2 - d/2\ \frac{\dot{m_a}}{\dot{m_{wt}}})/C_a\ Twout$
$b_4 = (-a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} - d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_a\ \ Tpin + (b + a/2\ \frac{\dot{m_{ws}}}{(\dot{m_{wt}})^2} + d/2\ \frac{\dot{m_a}\dot{m_{ws}}}{(\dot{m_{wt}})^2})/C_a\ Twout - b\ TE22$
$b_5 = \frac{1}{\tau}\ (-1.0569 \times 10^{-4}\ \frac{\dot{m_a}}{\rho_a} + 0.9943)\ (TE11 - TE21)$

This linear model will be used in the control section to design the controller.

III. Optimal Model-based Control

A. Control Strategy

It is shown that to make the system perform optimally the control strategy has to be defined in a way that the following conditions are satisfied:

1) The maximum possible exploitation of the air-to-air heat exchanger is achieved.
2) In the steady state conditions supply water flow (q_{ws}) must be equal to the tertiary water flow (q_{wt}). That is, it is optimal to make the system work in a way that no water passes through the bypass pipe. It should be noted that it is not possible to eliminate the bypass pipe because it makes the tertiary hydronic circuit hydraulically decoupled and it is necessary to keep the bypass to remove fast disturbances.

If the control strategy respects the mentioned conditions the HVAC system will perform in such a way that it will result in minimum thermal and electrical energy consumption [2].

B. Controller Design

The mentioned HVAC system is going to be used for ventilation purposes. It means that the air flow (q_a) will be determined in accordance with the required ventilation and the inlet air temperature ($Tinlet$) has to be kept at 19^oC. So, the optimal controller task is to track the set-point for the inlet air temperature while satisfying the conditions that were described in the control startegy section to gaurantee the optimal performance of the system. The traditional way to design the control system that works in this way is applying the two-layer hierarchical control system. The lower layer performs direct regulatory control, where the aim is to maintain selected process variables at their desired set-point values, and the upper layer, known as the supervisory layer, has the task of detremining the setpoints of the regulatory controllers to obtain optimal steady state performance.

Looking at the linear model that was developed before reveals that the HVAC system can be splitted into two decoupled subsystems as follows:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3 & b_4 \\ b_1 & b_2 \end{bmatrix}$$
$$\cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$
$$y = [1\ \ 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{16}$$

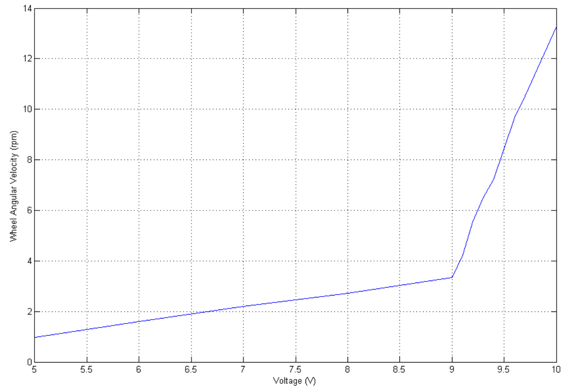

Fig. 7. Wheel Speed vs. Voltage

$$T\dot{E}22 = a_6 \cdot TE22 + b_5 \cdot wrf \tag{17}$$

It means that control of the HVAC system can be considered as control of the air-to-air heat exchanger and control of the water-to-air heat exchanger separately. It should be noted that $TE22$ acts as disturbance for the water-to-air heat exchanger in this new formulation.

If the set-point for temperature of the fresh air that leaves the wheel ($TE22$) is defined as the set-point for temperature of the inlet air (19^oC) we will be sure that the air-to-air heat exchanger has its maximum contribution to warm up the fresh air. Thus, the first condition for optimality will be met. The second condition for optimality can be included in the cost function that will be defined for the water-to-air heat exchanger controller. Therefore, there is no need to design an explicit supervisory layer.

1) Air-to-air Heat Exchanger Controller: To design controller for the rotary wheel we need to model the wheel actuators. To do so, several experimets were done. Fig 7 shows the relation between the voltage and the wheel speed. As can be seen in the Fig 7, the curve describing the relation between the voltage and the wheel speed can be approximated by two lines ($\frac{speed}{V} = 1/5$ for $0 \leq speed \leq 3$ and $\frac{speed}{V} = 10$ for $3 \leq speed \leq 10$). It should be noted that there is also a time delay varying from 6 seconds to 22 seconds while the speed of the wheel is going to change.

Fig 3 showed the normalized curve that describes the effect of the wheel speed on the efficiency of the wheel. This nonlinear curve also will be approximated by three lines ($\frac{wrf}{speed} = 8/15$ for $0 \leq speed \leq 1.6$, $\frac{wrf}{speed} = 4/55$ for $1.6 \leq speed \leq 3$ and $\frac{wrf}{speed} = 7/1000$ for $3 \leq speed \leq 10$).

Therefore, the rotary wheel along with actuators can be modeled as follows:

$$\frac{TE22}{V} = \frac{k(TE11 - TE21)(-1.0569 \times 10^{-4} q_a + 0.9943)}{\tau\ s + 1} e^{-Ts} \tag{18}$$

where:
$k \in \{8/75, 4/275, 7/100\}$
$6 \leq T \leq 22$

It should be reminded that the outdoor air temperature ($TE21$) will perform as disturbance through a first order system on the wheel. Thus, the model of the rotary wheel is a first order system along with varying gain, varying delay and disturbance. The input (v) is also constrained. These conditions remark that model predictive controller (MPC) is a good choice for the control.

The control problem can be formulated as follows:

$$\min_{v[k/k]} \quad \sum_{i=1}^{6} \|TE22[k+i/k] - 19\|^2_{I(i)}$$

subject to:

$$0 \leq v[k/k] \leq 10 \tag{19}$$

Sampling time for the controller is supposed to be 15 seconds. The gain and delay for internal model of the MPC controller are 1.7 and 22, respectively.

2) Water-to-air Heat Exchanger Controller: To control the water-to-air heat exchanger we have to deal with constrainted inputs. We also have to penalize inputs in a way that in the steady state conditions no water passes through the the bypass pipe. So, again MPC is a good candidate for this control problem. To design the MPC controller we need to modify equation (16) as follows:

$$\begin{bmatrix} \dot{T}inlet \\ \dot{T}wout \end{bmatrix} = \begin{bmatrix} a_4 & a_3 \\ 0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} + \begin{bmatrix} b_3 + b_4 & b_4 \\ b_1 + b_2 & b_2 \end{bmatrix}$$
$$\cdot \begin{bmatrix} \dot{m_{ws}} \\ \dot{m_{wt}} - \dot{m_{ws}} \end{bmatrix} + \begin{bmatrix} a_5 \\ a_2 \end{bmatrix} \cdot TE22$$
$$y = [1 \quad 0] \cdot \begin{bmatrix} Tinlet \\ Twout \end{bmatrix} \tag{20}$$

where:

$$a1 = -0.0352, a2 = 0.0310, a3 = 0.0564$$
$$a4 = -0.5961, a5 = 0.4833, b1 = 17232$$
$$b2 = 46628, b3 = 227635, b4 = -199119$$

Thus, the control problem can be described as follows:

$$\min_{\dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]} \quad \sum_{i=1}^{6} \|Tinlet[k+i/k] - 19\|^2_{I(i)}$$
$$+ \sum_{i=0}^{1} \|\dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k]\|^2_{(0.2 \times I(i))}$$

subject to:

$$0 \leq \dot{m_{ws}}[k+i/k], \dot{m_{wt}}[k+i/k] - \dot{m_{ws}}[k+i/k] \leq 250 \tag{21}$$

The variable speed pump that is installed in the tertiary hydronic circuit will provide the required(q_{wt}). According to the pump affinity laws we have:

$$N = \frac{N_0}{q_{wt0}} \cdot q_{wt} \tag{22}$$

It means that by adding a gain it is possible to model the pump. Here the gain is 11.02 ($\frac{N_0}{q_{wt0}}$).

A valve will control the supply water flow (q_{ws}). Here the sampling time for the controller is 15 seconds too. Thus, transient behavior of the valve is not important and it can be modeled as a single gain. The valve has nonlinear characteristic curve in steady state conditions. So, an average value for this gain is selected. The controller is robust enough to tolerate this approximation.

IV. RESULTS

Fig 8 shows the result of applying the designed control system to the HVAC system. It reveals that the controller keeps perfect tracking of the set-point. At time 660 sec. a step disturbance adds to the supply hot water temperature and the temperature dorps from 80^oC to 75^oC. As can be seen, the control system compensates for this disturbance and can track the set-point again. At time 1090 sec. a step disturbance adds to the outdoor air temperature and the temperature rises 5^oC. Here also the control system show perfect compensation and tracking.

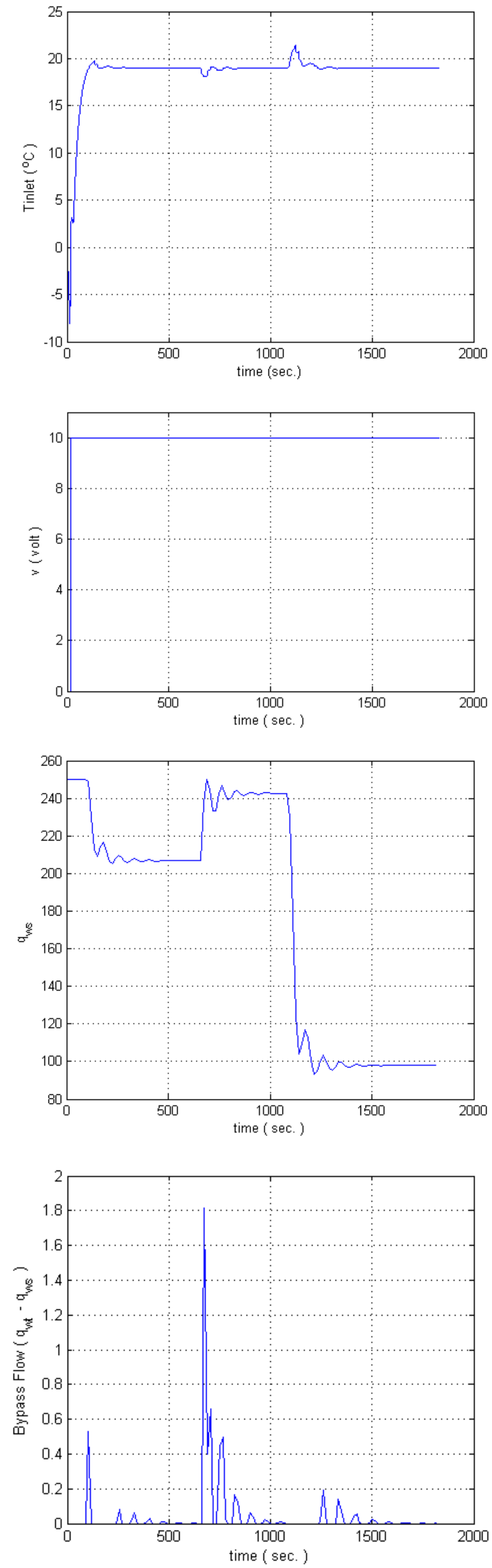

Fig. 8. The Controller Performance

V. CONCLUSIONS

Optimal model-based control for a heating, ventilating, and air-conditioning (HVAC) system was presented in this paper. The HVAC system was a typical HVAC system consisted of an air-to-air heat exchnager and a water-to-air heat exchanger. Dynamic model of the system was developed through dynamic modeling of different components of the system. Derived nonlinear model was linearized to design the controllers. The HVAC system was splitted into two subsystems and the set-points and cost functions for each subsystem controller were defined in a way that optimal control strategy which had been proposed in [2] was followed. The results of applying the developed control system showed that the system respected optimal control policy while it had the perfect tracking of the set point for the inlet air temperature.

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