Accommodation of Repetitive Sensor Faults—Applied to Surface Faults on Compact Discs

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***Abstract*—Surface defects such as scratches and fingerprints on compact discs (CDs) can cause CD players to lose focus and tracking on the discs. A scheme for handling these defects has previously been proposed. In this brief, adaptive and predictive versions of this scheme are developed. The adaptive scheme can be used to adapt the accommodation to specific surface defects on specific discs, while the predictive scheme can be used to jump between tracks with surface defects on the disc. Sufficient and necessary stability conditions for the proposed accommodation schemes are derived as well. Performance of the accommodation scheme is discussed. Both proposed methods show their potentials through simulations with a CD player playing a CD with a surface defect (scratch).**

***Index Terms*—Adaptive control, compact disc (CD) players, fault tolerant control, predictive control, surface defects.**

I. Introduction

FEEDBACK control is used to position the optical pick-up in the compact disc (CD)-player such that it is focused and radially tracked. A number of different control strategies have been applied to the CD problem. Some examples are: the first application of a μ-controller used in a CD player was reported in [1], which was based on DK-iterations. An adaptive repetitive method was suggested in [2]. Reference [3] improves on the repetitive control scheme for reaction to the repetitive reference. The surface defects like scratches and finger prints often cause problems for the positioning controllers. The defects introduce faulty components in the position measurements, this leads to possible losses of focus and tracking. In [4] and [5], an accommodation scheme is presented which approximates the surface defect components and subsequently removes the faulty component at the next defect encounter by subtracting the approximation of the faulty component. The variation from one encounter of the surface defect to the next encounter is negligible, whereas the faults develop slowly over hundreds of defect encounters.

Since this defect accommodation scheme, previously denoted feature-based control scheme, uses a precomputed approximating basis of the surface defects for computing the approximation of the defect component, it could encounter problems if the scheme meets a defect not well supported by this approximating basis. Consequently, it would be better to adapt the approximating basis to the faults which are actually met. In [6], such an adaptive version of the scheme handling the surface defect/sensor fault problem on a CD-player is presented. In this brief, this adaptive method is presented and improved. A predictive version of the scheme is proposed for handling jumping among tracks where surface defects are problematic. Stability and performance of the schemes are analyzed resulting in a number of stability criteria for the scheme, and a method for performance analysis of the accommodation. The developed adaptive and predictive schemes are applied to the application. These simulations show the potential of these accommodation schemes.

In Section II, the CD player and the problem of the surface defects are described. In Section III, the previously developed accommodation scheme called feature-based control is described. This leads to an adaptive version of the repetitive sensor defect accommodation scheme in Section IV. Deterministically propagating repetitive surface defects are dealt with in Section V. In Section VI, three stability criteria for the accommodation methods are derived, as well as formula for performance analysis. In Section VII, the proposed schemes are applied to the CD player playing discs with surface defects. Finally, in Section VIII, a conclusion is drawn.

II. Description of the CD Player and the Problem

The CD player has two interesting control loops which are used for positioning of the optical pick-up relative to the information track storing the information of the CD. These two controls called focus and radial tracking, have linear electromagnetic actuators which are designed to be orthogonal to each other. The two directions are illustrated in Fig. 1. Fig. 1 shows the movement directions of the OPU enabling the positioning of the OPU correctly on the track. The optical pick-up generates two sensor signals which are approximations of the position errors in the focus and radial direction. More information concerning the CD player is given in [7] and [8]. Surface defects such as scratches and fingerprints introduce faulty components on these position measurements which can lead to loss of focus and tracking of the pick-up. The frequency content of the surface faults is partly in the region of the disturbances as well as the measurement noises, i.e., these surface faults are, consequently, handled as sensor faults.

In addition to the position estimate, the optical pick-up generates a pair of residuals which can be used to detect the surface defects, alternatively the residuals presented in [9] could be used. The two control loops in the CD-player are decoupled by construction, implying that the system can be examined as two system-input system-output (SISO) systems. The focus and radial models are very similar, they can both be modeled by

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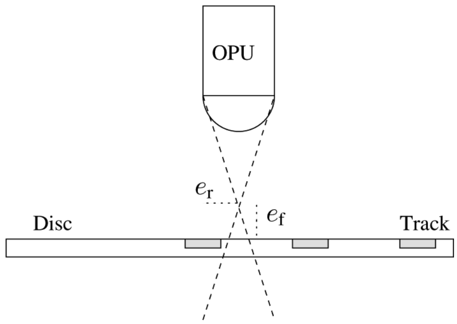

Fig. 1. Focus error e_f is the distance from the focus point of the laser beam to the reflection layer of the disc, the radial error is the distance from the center of the laser beam to the center of the track.

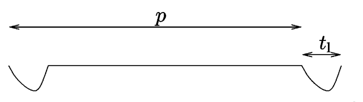

Fig. 2. Illustration of the relation between the period of the repetition of the fault $\check{e}$ denoted p and the length of the fault t_1.

second-order models (see [7], [8], [10], and [11]). A combined discrete time model for both loops is expressed as

$$\mathbf{x}[n+1] = \mathbf{A}\mathbf{x}[n] + \mathbf{B}\mathbf{u}[n] + \mathbf{E}\mathbf{d}[n] \tag{1}$$
$$\mathbf{y}[n] = \mathbf{C}\mathbf{x}[n] \tag{2}$$
$$\mathbf{y}_\mathrm{m}[n] = \mathbf{C}\mathbf{x}[n] + \mathbf{f}[n] + \mathbf{n}_\mathrm{m}[n] \tag{3}$$

where $\mathbf{x}[n]$ is a vector of the discrete time states, $\mathbf{u}[n]$ is a vector of the control signals to the system, $\mathbf{y}[n]$ is a vector of physical focus and radial positions, $\mathbf{y}_\mathrm{m}[n]$ is a vector of the measured focus and radial positions, $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{E}$ are the system matrices. $\mathbf{d}[n]$ is a vector of disturbances. $\mathbf{n}_\mathrm{m}[n]$ is a vector of the measurement noise. $\mathbf{f}[n]$ is a vector of the repetitive surface faults, which is zero in the defect free case. During a fault the frequency content of $\mathbf{f}[n]$ might be partly in the frequency region of $\mathbf{d}[n]$ and thereby in the region where low sensitivity is required of the control loops. This means that these surface defects cannot be viewed as being measurement noise. It is assumed that $\mathbf{f}[n]$ does not change dramatically from encounter to encounter. The interval, in which $\mathbf{f}[n]$ takes values significantly larger than zero, called length of the fault, t_1, is much smaller than the time between defect encounters p. This is illustrated in Fig. 2. In addition, it is also assumed that $p \gg \tau$, where $(1/\tau)$ is the real part of the smallest closed-loop system eigenvalue. The specific coefficients for the used model can be found in [5]. In regard of accommodating the surface defects, fault tolerant control schemes are often used (see [12]).

III. Description of the Previous Proposed Accommodation Scheme

An overview of the previously proposed accommodation scheme (see [4]) can be seen in Fig. 3. The plant represents the controlled CD player, including disturbances, sensor defects, and measurement noise, i.e., it is represented by (1)–(3).

The defect detector detects the occurrence of the defect, e.g., based on residuals generated by measurements and models. The defect detector sets its output $f_\mathrm{d}[n]$ equal to 1 when a surface defect is detected and zero elsewhere.

The defect approximator computes a block approximation of the defect component at the most recent defect encounter which can be used to accommodate the next defect encounter. The defect component is approximated by a projection of the lifted measurement block containing the defect onto an approximating basis $\mathbf{P} = \mathbf{K_f} \cdot \mathbf{K_f}^T$, i.e., $\hat{\mathbf{f}}[v] = \mathbf{K_f} \cdot \mathbf{K_f}^T \cdot \mathbf{y}_\mathrm{m}[v]$, where v is an interval of the time series containing the defect. $\mathcal{P}(\,\cdot\,)$ denotes subsequently an operator projecting a time series block by the projection matrix $\mathbf{P}$. $\mathbf{K}_\mathrm{f}$ is the approximating basis.

The defect accommodator uses the present defect approximation to correct the defect during its occurrence. This means that during a defect, a cleaned measurement $\hat{\mathbf{y}}$ is computed by subtracting the block approximation from the measurement. While no defect is present the cleaned signal is equal to the measured signal. The controller computes $\mathbf{u}[n]$ based on $\hat{\mathbf{y}}[n]$. Since the defect component is removed from the measurement a nominal controller can be used, which is designed for handling the defect free situation.

A. Defect Approximation

The approximating basis used in the scheme can be achieved in a number of ways. However, some requirements to the approximation can be stated by the following:

$$\hat{\mathbf{y}}[n] = \mathbf{y}_\mathrm{m} - \mathcal{P}(\mathbf{y}_\mathrm{m}) \approx \mathbf{y} + \mathbf{n}_\mathrm{m}. \tag{4}$$

This will be fulfilled if

$$\mathbf{f} \approx \mathcal{P}(\mathbf{y}_\mathrm{m}) \Rightarrow \tag{5}$$
$$\mathcal{P}(\mathbf{n}_\mathrm{m}) \approx 0 \wedge \mathcal{P}(\mathbf{y}) \approx 0 \wedge \mathbf{f} \approx \mathcal{P}(\mathbf{f}). \tag{6}$$

References [4] and [5] use a Karhunen–Loève approximation of the defect component based on a data set of sampled defects. This basis has the advantage that it represents the general signal structure with a few basis vectors (see [13] and [14]). This method will also be used here. An example on the approximating properties of the Karhunen–Loève basis can be seen in Fig. 4, where a vector (of length 256) containing a defect, noise, and disturbances is approximated by the most and the four most approximating Karhunen–Loève basis vectors. From Fig. 4 it can be seen that these few basis vectors supports the general signal trends, which is the defect component, and they do not support the noises and disturbances in the signal.

1) Karhunen–Loève Basis: The Karhunen–Loève basis has a desirable property in this context. It supports the general signal trends in a matrix in which each column vector is an occurrence of one of the signals. The remaining basis vectors support the noise in the signals. This implies that if a Karhunen–Loève basis is computed of a set of $\mathbf{f}[n]$ during different surface defects, a few most approximating basis vectors will support the general trends in these signals, which can be assumed to be the defect component, i.e., if a $\mathbf{f}[n]$ sequence is subsequently projected onto these approximating basis vectors, the defect component can be approximated (see [13] and [14]).

The Karhunen–Loève basis is computed based on $\mathbf{X}$. First of all, it is assumed that the column vectors in $\mathbf{X}$ have zero mean, otherwise, a preliminary step might be introduced in order to fulfill that assumption. The Karhunen–Loève basis $\mathcal{K}$ which can be defined as

$$\mathcal{K} = \{v_1, \ldots, v_m\} \tag{7}$$

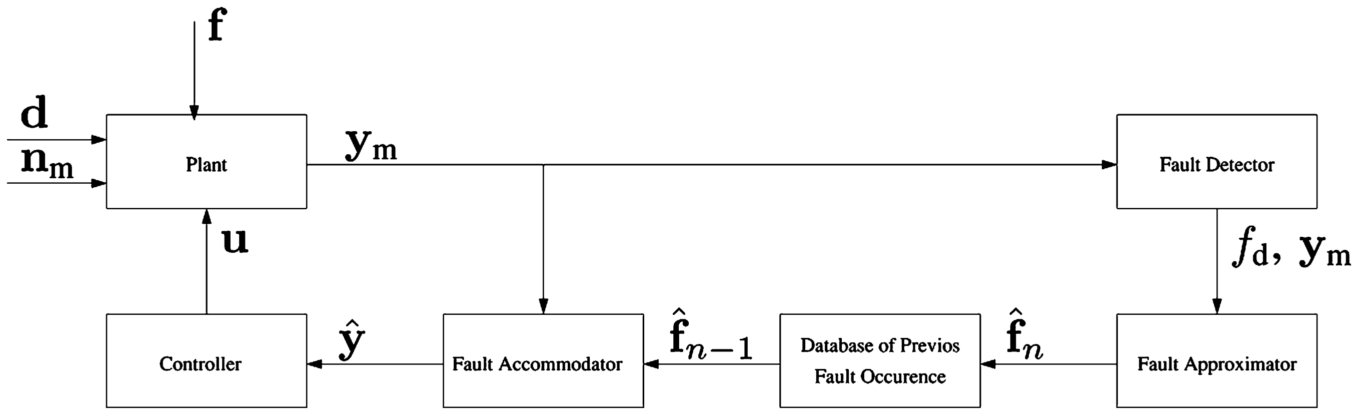

Fig. 3. Illustration of the defect accommodation scheme for accommodating repetitive surface defects (sensor faults), in which $\mathbf{y}_{\mathrm{m}}[n]$ is the vector of the measured error signals, $f_{\mathrm{d}}[n]$ is the vector of the fault detection signal indicating a detected defect or note, $\mathbf{f}[n]$ is a vector of the defects, $\hat{\mathbf{f}}[n]$ is the vector of the approximations of $\mathbf{f}[n]$, and the index n denotes the defect occurrence number. $\hat{\mathbf{y}}[n]$ is the vector of the corrected measurement from which the defect approximation has been subtracted. $\mathbf{u}[n]$ is the vector of the control signal generated by the controller for controlling the plant. $\mathbf{d}[n]$ is a vector of the disturbances, $\mathbf{n}_{\mathrm{m}}[n]$ is a vector of the measurement noise.

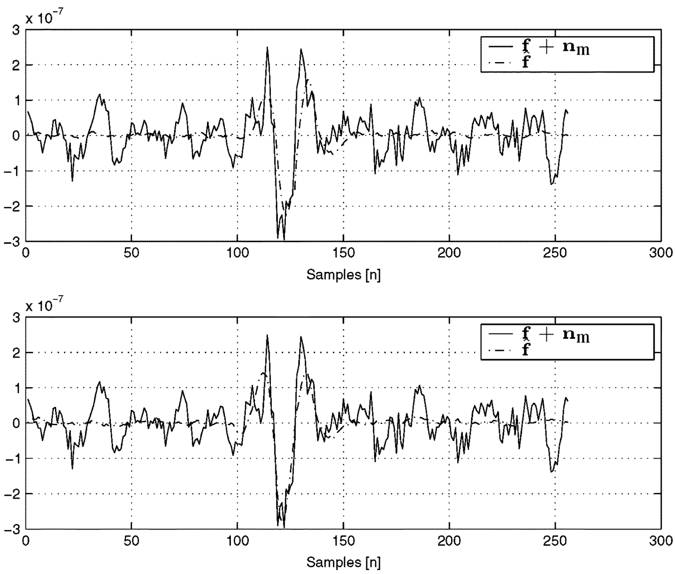

Fig. 4. Illustration of the Karhunen–Loève approximation of the defect component due to a scratch on a CD. $\mathbf{f} + \mathbf{n}_{\mathrm{m}}$ (defect and measurement noise of the signals) which contains a typical scratch. The approximation is denoted with $\hat{\mathbf{f}}$. The first approximation is based on the most approximating coefficient. The second approximation is based on the four most approximating coefficients.

is an orthonormal basis of eigenvectors of the matrix $\mathbf{X}\mathbf{X}^T$, ordered in such a way that v_n is associated with the eigenvector λ_n, and $\lambda_i \geq \lambda_j$ for $i < j$. A matrix of the k most approximating basis vectors can be defined as follows:

$$\mathbf{K}^{\mathrm{L}} = [v_1, v_2, \ldots, v_k]. \tag{8}$$

So in other words the Karhunen–Loève basis are the eigenvectors of the autocorrelation of $\mathbf{X}$. The eigenvalues of the autocorrelation have the values of the variances of the related Karhunen–Loève basis vectors. The approximating properties of the Karhunen–Loève basis vectors are sorted in increasing numerical order of their corresponding eigenvalues, which means that if the basis consists of k vectors the basis vector v_k is the most approximating basis vector.

Two practical problems arise with this chosen approximation basis. The preliminary step in which measurements are made zero mean is performed by subtracting the mean of each measurement vector. Choosing the number of approximating bases is challenging as well. In [4], it was found that the four most approximating bases gave a good support for the surface defect components and did not support the disturbances. Consequently, the four most approximating bases vectors are used in this brief as well.

IV. Proposed Adaptive Accommodation Method

In Section III and in [4], the accommodation method was presented assuming that an approximating basis of the defect component can be precomputed. The set of all possible surface defects is huge, meaning it would be practically impossible to compute a basis supporting every possible surface defect. In addition, the defects develop slowly over a high number of encounters, i.e., an online adaptation of the basis would be beneficial.

In [6], the accommodation method is made adaptive by recomputing the Karhunen–Loève basis after encounters of the defect, just before the new approximation is computed. In order to compute the basis a data matrix is formed consisting of historical encounters of defects. It is, however, important only to use "open-loop" data for this basis computation. In order to achieve this a Kalman estimator is used to estimate and eliminate the system response to the control signal. In [9], such a Kalman estimator is designed such that the position depending on the control signals is estimated. The estimated position signal is denoted $\tilde{\mathbf{y}}[n]$ and computed using a Kalman estimator on the signals $\mathbf{y}_{\mathrm{m}}[n]$ and $\mathbf{u}[n]$. Now define a signal $\tilde{\mathbf{f}}[n]$ as

$$\tilde{\mathbf{f}}[n] = \mathbf{y}_{\mathrm{m}} - \tilde{\mathbf{y}}[n]. \tag{9}$$

Define a vector of elements from $\tilde{\mathbf{f}}[n]$ containing the qth defect encounter as $\tilde{\mathbf{f}}[v_q]$. The matrix of historical defect encounters is denoted $\mathbf{X}$. This matrix is subsequently augmented with the newest defect encounter

$$\mathbf{X}_q = [\mathbf{X} \quad \tilde{\mathbf{f}}[v_q]]. \tag{10}$$

The approximating Karhunen–Loève basis for the qth defect encounter can subsequently be computed as the k most approximating Karhunen–Loève basis vectors of $\mathbf{X}_q$. This basis update step is included in the accommodation scheme (see [4]). The update consists of removing dc components from the most recent estimated defect encounter, and subsequently computing the eigenvector of the matrix $\mathbf{X}_q$.

A. Algorithm of the Adaptive Scheme

The surface defect/adaptive repetitive sensor defect accommodation scheme can be presented as an algorithm as follows.

1) Detect the defect and locate its position in time, when the defect is detected at sample n, $f_{\mathrm{d}}[n] = 1$. If this is the first encounter of the defect jump to step 3), else continue to step 2).
2) If $f_{\mathrm{d}}[n] = 1$, compute the cleaned signal estimation by subtracting the defect approximation from the position measurements
$$\hat{\mathbf{y}}[n] = \mathbf{y}_{\mathrm{m}}[n] - \hat{\mathbf{f}}[\iota]$$
where ι is a counter used to locate the given sample relative to the defect correction block.
3) When the defect has been passed, compute the samples where the defect begins and ends, and compute the defect length l_{f}.
4) Recompute the approximating Karhunen–Loève basis, $\mathbf{K}_{\mathbf{f}_q}$. Update the data matrix $\mathbf{X}$ to include the present defect encounter. Then compute the k most approximating Karhunen–Loève basis vectors of $\mathbf{X}$.
5) Compute the defect correction block by: $\hat{\mathbf{f}}[\upsilon] = \mathbf{K}_{\mathbf{f}_q} \cdot \mathbf{K}_{\mathbf{f}_q}^T \cdot \mathbf{y}_{\mathrm{m}}[\upsilon]$, where υ is the interval of L samples in which the defect is present.

B. New Adaptive Method

Instead of storing $\mathbf{X}$, the general characteristics of the historical data is supported by the k most approximating Karhunen-Loève basis vectors and their corresponding eigenvalues. It can be assumed that the remainder of Karhunen–Loève basis vectors supports noise and disturbances in the data vectors. This means that in order to compute the k most approximating basis vectors of $\mathbf{X}_q$, it can be approximated as

$$\mathbf{X}_q \approx [\lambda_1 \cdot v_1 \quad \cdots \quad \lambda_k \cdot v_k \quad \tilde{\mathbf{f}}[\upsilon_q]] \tag{11}$$

where v_1 is the most approximating Karhunen–Loève basis vector, λ_1 is the corresponding eigenvalue, v_k is the kth most approximating Karhunen–Loève basis vector, and λ_k is the corresponding eigenvalue. The adaptiveness of the new basis can be controlled by introducing an updating factor, as used in iterative system identification (see [15]). The introduction of an updating factor, β would modify (11) and (12)

$$\mathbf{X}_q \approx [\lambda_1 \cdot v_1 \quad \cdots \quad \lambda_k \cdot v_k \quad \beta \cdot \tilde{\mathbf{f}}[\upsilon_q]] \tag{12}$$

where $\beta \in [0 \cdots 1]$. The optimal choice of β depends on a balance of how fast a defect develops and how much any given defect can vary from the known ones, not unlike the way an observer balances process and measurement noise.

Subsequently, the approximating Karhunen–Loève basis is computed of $\mathbf{X}_q$. This means that the only step in the algorithm which is altered is step 4), where $\mathbf{X}_q$ is approximated by (12).

V. Predictive Accommodation Method

The previously mentioned accommodation schemes require a model of the present defect. The adaptive scheme can be used to adapt to a surface defect as it evolves through the tracks. However, if one would like the CD player to jump to another track, this might be problematic if a surface defect has grown large at that track. By using deterministic propagation it might be possible to model the propagation of the defect. This model can be used in order to predict how the defect would propagate in the next encounter or number of encounters. Using this propagation model jumping between track with defects is made possible.

Define a function approximating the surface defect at encounter q for $\Phi_q[n] = \mathbf{K}_q \cdot \mathbf{K}_q$ and at encounter $q+1$ for $\Phi_{q+1}[n]$. This means the prediction of encounter $q+1$ based on encounter q, should be close to the approximation of encounter $q+1$. A simple way to predict the surface defect at encounter $m+1$ is to scale the approximation of encounter m in time and amplitude

$$\Phi_{q+1}[n] \approx c_q \cdot \Phi_q[a_q \cdot n] \tag{13}$$

where c_q is an amplitude scaling coefficient, and a_q is the time scaling coefficient. The length of the scratch of the different encounters is computed by the detection algorithm. Denote l_q as the length of the defect at encounter q. It is clear that

$$a_q = \frac{l_q}{l_{q+1}} \tag{14}$$

due to the small variations from encounter to encounter it can be assumed that

$$a_q \approx a_{q-1} = \frac{l_{q-1}}{l_q}. \tag{15}$$

The amplitude scaling coefficient can be predicted in the same way

$$c_q \approx c_{q-1} = \frac{||\Phi_q n]||}{||\Phi_{q-1} n]||}. \tag{16}$$

A. Algorithm

1) Detect the defect and locate its position in time, when the defect is detected at sample n, $f_{\mathrm{d}}[n] = 1$.
2) If $f_{\mathrm{d}}[n] = 1$:
$$\bar{\mathbf{e}}[n] - [\tilde{\check{\mathbf{e}}}_{\mathrm{f}}[\iota] \quad \tilde{\check{\mathbf{e}}}_{\mathrm{r}}[\iota]]^T$$
where ι is a counter used to position the correction in relation to the actual defect.
3) Compute the focus approximating function by: $\Phi_{\check{\mathbf{e}}_{\mathrm{f}}} = \{\text{eigenvector } (\mathbf{X}_{\mathrm{f}} \cdot \mathbf{X}_{\mathrm{f}}^T)\}\{N\}$, and the radial approximating function by: $\Phi_{\check{\mathbf{e}}_{\mathrm{r}}} = \{\text{eigenvector } (\mathbf{X}_{\mathrm{r}} \cdot \mathbf{X}_{\mathrm{r}}^T)\}\{N\}$.
4) Check stability using Lemma 2. If the system is stable use the newly computed approximating basis, if not use the latest computed stable basis.
5) Compute a_q and c_q by (15) and (16), respectively.

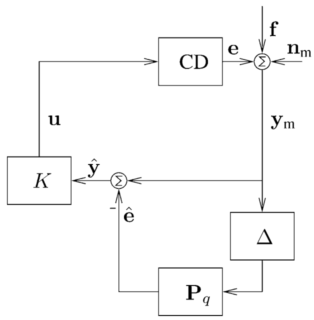

Fig. 5. Illustration of the closed-loop with the feature-based correction $\mathbf{P}_q$ at encounter q, for the nonadaptive feature-based control scheme $\mathbf{P}_m = \mathbf{P}$ and time invariant. K is the controller, and CD is the CD player. Δ is the unit revolution delay. $\mathbf{u}$ is a vector of the control signals, $\mathbf{y}$ is a vector of focus and radial distances, $\mathbf{f}$ is a vector of faulty sensor components due to the surface defect, $\hat{\mathbf{f}}$ is a vector of the estimates of the faulty sensor components due to the surface defect. $\mathbf{y}_\mathrm{m}$ is a vector of the measured distance signals and $\mathbf{n}_\mathrm{m}$ is a vector of the measurement noises.

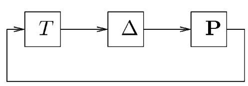

Fig. 6. Closed-loop of the feature-based control system.

6) Compute the focus defect correction block by: $\tilde{\hat{e}}_\mathrm{f} = c_m \cdot \Phi_\mathrm{f}[a_q \cdot n]$, and the radial defect correction block by: $\tilde{\hat{e}}_\mathrm{r} = c_q \cdot \Phi_\mathrm{r}[a_q \cdot n]$.

VI. Stability and Performance of the Methods

The proposed schemes for accommodating these repetitive defects can, under certain conditions, destabilize the closed-loop system. In this section, stability criteria for the closed-loop system are derived.

A. Stability of the Schemes

In [4], a necessary and sufficient condition for stability of the closed-loop system is given, assuming that system gain is not changed by the defect, the defect component approximation block is synchronized with the defect component, and that the system response on the defect component has died out before the next defect encounter. The system can be represented by Fig. 5.

Fig. 5 illustrates the feature-based control scheme. This can be transformed to Fig. 6, where T denotes the complementary sensitivity of the nominal servo system, and Δ is a delay of one defect occurrence (see Fig. 6).

The lifted $\mathbf{P}$ can be computed by

$$\mathbf{P}^\mathrm{L} = \mathbf{K_f} \cdot \mathbf{K_f}^T \tag{17}$$

and the lifted representation of the complementary sensitivity is

$$\mathbf{T}^\mathrm{L} = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & & \vdots \\ \vdots & & \ddots & \\ h_{L-1} & \cdots & & h_0 \end{bmatrix} \tag{18}$$

where $\mathbf{h} = [h_0 \ h_1 \ \cdots \ h_{L-1}]$ is a time series of L samples of the impulse response of T. By lifting the system illustrated in Fig. 6, one gets a set of discrete difference equations of the form

$$\xi[N+1] = \mathbf{\Phi}\xi[N] + \mathbf{Ku}[N] \tag{19}$$

where $\mathbf{\Phi} = \mathbf{T}^\mathrm{L}\mathbf{P}^\mathrm{L}$. These definitions make it possible to formulate Lemma 1, which states when the linear system is stable.

Lemma 1: The scheme for accommodating repetitive surface defects defined by Fig. 5 is stable if and only if:

$$\max(|\mathrm{eig}(\mathbf{T}^\mathrm{L}\mathbf{P}^\mathrm{L})|) < 1$$

where $\mathbf{P}^\mathrm{L}$ is defined in (17) and $\mathbf{T}^\mathrm{L}$ is defined in (18).

Proof: Necessary and Sufficient Conditions: The stability of the closed-loop system shown in Fig. 5 is equivalent to stability of the system in (19), which is a standard linear time invariant (LTI) discrete time system, from which the result follows, due to the standard Schur condition. ■

In order to analyze whether the assumption that system response of the past defect encounter has died out before the next defect is met, the attention should address the lifted complementary sensitivity $\mathbf{T}^\mathrm{L}$. This situation can be analyzed by introducing elements from the impulse response in an upper triangular matrix. The distance in samples between two successive defect encounters is not known beforehand, consequently, it is required to analyze every possible modified $\mathbf{T}^\mathrm{L}$. Denote the set of all possible altered complementary sensitivity matrix with $\mathcal{T}^\mathrm{L}$, and it is defined as in (20) shown at the bottom of the page. The length of the analyzed defect response is denoted L in (20). It can subsequently be checked if the closed-loop system is stable even though the system response on the defect is still present at the next defect encounter. The stability of every matrix in the set $\mathcal{T}^\mathrm{L}$ is checked using the criteria in Lemma 1. This leads to Lemma 2.

Lemma 2: Assume that a second defect is encountered before the previous defect response has died out. When the scheme for accommodating repetitive sensor defects defined by Fig. 5 is stable if and only if

$$\max(|\mathrm{eig}(\mathbf{T}^\mathrm{L}\mathbf{P}^\mathrm{L})|) < 1 \text{ for all } \mathbf{T}^\mathrm{L} \in \mathcal{T}^\mathrm{L}$$

where $\mathbf{P}^\mathrm{L}$ is defined in (17) and $\mathbf{T}^\mathrm{L}$ is defined in (20).

$$\mathcal{T}^\mathrm{L} \in \left(\begin{bmatrix} h_0 & 0 & \cdots & 0 & h_L \\ h_1 & h_0 & & & 0 \\ \vdots & & \ddots & & \vdots \\ h_{L-1} & & \cdots & & h_0 \end{bmatrix} \cdots \begin{bmatrix} h_0 & h_{L-1} & \cdots & h_2 & h_1 \\ h_1 & h_0 & & & h_2 \\ \vdots & & \ddots & & \vdots \\ h_{L-1} & & \cdots & & h_0 \end{bmatrix} \right) \tag{20}$$

Proof: Necessary and Sufficient Conditions: Stability for one $\mathbf{T}^{\mathrm{L}}$ is proven by Lemma 1. The system is consequently stable even though the response of the past defect has not died out before the next defect is encountered, if conditions of Lemma 1 are fulfilled for all $\mathbf{T}^{\mathrm{L}}$ in $\mathcal{T}^{\mathrm{L}}$. ■

The last stability issue to address here is the stability of the adaptive scheme. In this situation, a new $\mathbf{P}_q^{\mathrm{L}}$ matrix is computed at the qth encounter. The closed-loop system can subsequently be represented by

$$\xi[N+1] = \mathbf{\Phi}_q \xi[N] + \mathbf{K}\mathbf{u}[N] \tag{21}$$

where $\mathbf{\Phi}_q = \mathbf{T}^{\mathrm{L}}\mathbf{P}_q^{\mathrm{L}}$.

Lemma 3: The adaptive feature-based control system defined by Fig. 3, is stable if: $\max_q(\bar{\sigma}(\mathbf{T}^{\mathrm{L}}\mathbf{P}_q^{\mathrm{L}})) < 1$.

Proof: Sufficient Conditions: The stability of the closed-loop system shown in Fig. 5, which is equivalent to stability of the system in (21), which is a linear time varying discrete time system, from which the result follows, the system is stable if $\max_m(\bar{\sigma}(A_q)) = \max_q(\bar{\sigma}(\mathbf{T}^{\mathrm{L}}\mathbf{P}_q^{\mathrm{L}})) < 1$. The system response through $\mathbf{T}^{\mathrm{L}}\mathbf{P}_q^{L}$ will converge towards zero if the maximum singular value of $\mathbf{T}^{\mathrm{L}}\mathbf{P}_q^{L}$ is strictly less than one for all q, meaning that the system represented by $\mathbf{T}^{L}\mathbf{P}_q^{L}$ is stable if $\max_q(\bar{\sigma}(\mathbf{T}^{\mathrm{L}}\mathbf{P}_q^{\mathrm{L}})) < 1$. ■

B. Performance of the Repetitive Sensor Defect Accommodation Scheme

In [4], the performance of the proposed algorithm is analyzed by analyzing how well the defect is approximated, and to what degree the defect approximation only relies on the defect directly and not on the closed-loop response of the system and the controller. In order to inspect the performance of the scheme it is needed to take the closed loop into account, i.e., to determine the influence from the closed loop on the approximation of the surface defect. It was found that the approximation of the surface defect at encounter 1 $\vartheta = 1$ depends on the defect directly and through the closed-loop sensitivity

$$\hat{\mathbf{f}}_1^L = \mathbf{P}^{\mathrm{L}}(\mathbf{S}^{\mathrm{L}})\mathbf{f}_0^L \tag{22}$$

where $\mathbf{S}^{\mathrm{L}}$ is the lifted sensitivity of the servo. Another important issue to verify is whether the performance converges over a number of defect encounters. In [4], the approximation is computed for encounters $\vartheta = 2$ and $\vartheta = 3$ as well

$$\hat{\mathbf{f}}_2^L = \mathbf{P}^{\mathrm{L}}(\mathbf{S}^{\mathrm{L}} - \mathbf{T}^{\mathrm{L}}\mathbf{P}^{\mathrm{L}}\mathbf{S}^{\mathrm{L}})\mathbf{f}_1^L \tag{23}$$

and the same scheme is repeated again for computing the estimate at encounter $\vartheta = 3$

$$\hat{\mathbf{f}}_3^L = \mathbf{P}^{\mathrm{L}}(\mathbf{S}^{\mathrm{L}} - \mathbf{T}^{\mathrm{L}}\mathbf{P}^{\mathrm{L}}\mathbf{S}^{\mathrm{L}})\mathbf{f}_2^L + \mathbf{P}^{\mathrm{L}}\mathbf{T}^{\mathrm{L}}\mathbf{P}^{\mathrm{L}}\mathbf{T}^{\mathrm{L}}\mathbf{S}^{\mathrm{L}}\mathbf{f}_2^L. \tag{24}$$

The estimates of the these three encounters have now been computed. This means that the influence from the closed loop can be determined by computing the energy of $\hat{\mathbf{f}}_{1,2,3}$ over the energy of $\mathbf{P}^{\mathrm{L}}\mathbf{f}[n_\vartheta], \vartheta \in \{1,2,3\}$, i.e.,

$$\frac{||\mathbf{f} - \hat{\mathbf{f}}||}{||\mathbf{f}||}. \tag{25}$$

In order to analyze the performance under the changed conditions, the respective $\mathbf{P}^{\mathrm{L}}$, $\mathbf{S}^{\mathrm{L}}$, and $\mathbf{T}^{\mathrm{L}}$ need to be changed according to the changed conditions like in the stability analysis.

The three signals in case of the adaptive scheme would be

$$\hat{\mathbf{f}}_1^L = \mathbf{P}_1^{\mathrm{L}}(\mathbf{S}^{\mathrm{L}})\mathbf{f}_0^L \tag{26}$$

$$\hat{\mathbf{f}}_2^L = \mathbf{P}_2^{\mathrm{L}}\left(\mathbf{S}^{\mathrm{L}} - \mathbf{T}^{\mathrm{L}}\mathbf{P}_1^{\mathrm{L}}\mathbf{S}^{\mathrm{L}}\right)\mathbf{f}_1^L \tag{27}$$

$$\hat{\mathbf{f}}_3^L = \mathbf{P}_3^{\mathrm{L}}\left(\mathbf{S}^{\mathrm{L}} - \mathbf{T}^{\mathrm{L}}\mathbf{P}_2^{\mathrm{L}}\mathbf{S}^{\mathrm{L}}\right)\mathbf{f}_2^L + \mathbf{P}_3^{\mathrm{L}}\mathbf{T}^{\mathrm{L}}\mathbf{P}_1^{\mathrm{L}}\mathbf{T}^{\mathrm{L}}\mathbf{S}^{\mathrm{L}}\mathbf{f}_2^L. \tag{28}$$

The adapted estimates of the these three encounters have now been computed. This means that the influence from the closed loop can be determined by computing the energy of $\hat{\mathbf{f}}_{1,2,3}^L$ over the energy of $\mathbf{P}_\vartheta^{\mathrm{L}}\hat{\mathbf{f}}[n_\vartheta], \vartheta \in \{1,2,3\}$, i.e.,

$$\frac{||\mathbf{f}^L - \hat{\mathbf{f}}^L||}{||\mathbf{f}^L||}. \tag{29}$$

1) Performance While Correction is Out-of-Synchronization: The attention is subsequently turned to the performance of the scheme if the defect correction is out of synchronization with the real defect component. In order to analyze this performance it is needed to extend the matrices used. The extended complementary sensitivity is defined as

$$\bar{\mathcal{T}}^{\mathrm{L}} = \begin{bmatrix} h_0 & h_{3\cdot L-1} & \cdots & h_3 & h_2 \\ h_1 & h_0 & & & h_3 \\ \vdots & & \ddots & & \vdots \\ h_{3\cdot L-1} & & \cdots & & h_0 \end{bmatrix} \tag{30}$$

$$\bar{\mathcal{S}}^{\mathrm{L}} = \mathbf{I} - \bar{\mathcal{T}}^{\mathrm{L}}. \tag{31}$$

The projection matrix $\mathbf{P}$ is expanded such that it passes the signal through outside the correction area

$$\bar{\mathbf{P}}^{\mathrm{L}} = \begin{bmatrix} \mathbf{I}_{L\times L} & \mathbf{0}_{L\times L} & \mathbf{0}_{L\times L} \\ \mathbf{0}_{L\times L} & \mathbf{P}^{\mathrm{L}} & \mathbf{0}_{L\times L} \\ \mathbf{0}_{L\times L} & \mathbf{0}_{L\times L} & \mathbf{I}_{L\times L} \end{bmatrix}. \tag{32}$$

Now define an extended defect vector which is out-of-synchronization with w samples

$$\mathbf{f}_w = \begin{bmatrix} \mathbf{0}_{(L+w)\times 1} & \mathbf{f} & \mathbf{0}_{(L+w)\times 1} \end{bmatrix}^T. \tag{33}$$

The performance of the approximation scheme can now be computed for $w \in \{-L \cdots L\}$ by

$$\frac{||\mathbf{f}_w - \bar{\mathbf{P}}^{\mathrm{L}} \cdot \bar{\mathbf{S}}^{\mathrm{L}}\mathbf{f}_w||}{||\mathbf{f}_w||}. \tag{34}$$

VII. Experimental Results

The specific CD player is described in more details in [5], where model parameters can be found as well. In this example, the length of the approximation vectors in $\mathbf{K}_{\check{e}}$ is 256, since typical surface defects are shorter than 256 samples, see [4], [5], and [16]. The set of surface defects on CDs is a large set, where some defects are more alike than others, these can be grouped into subclasses of surface defects. In [16], a method is suggested for classifying the surface defects into three different classes of surface defects. The approximating bases are computed for each

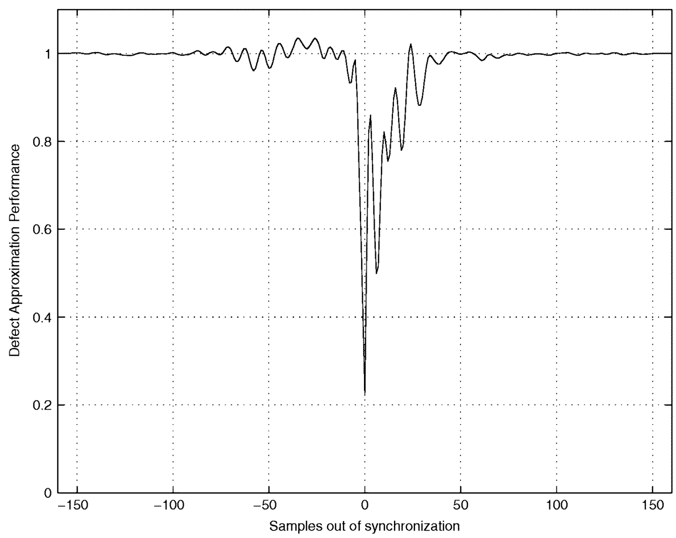

Fig. 7. Illustration of the focus loop performance depending on the number of samples that the correction is out of synchronization.

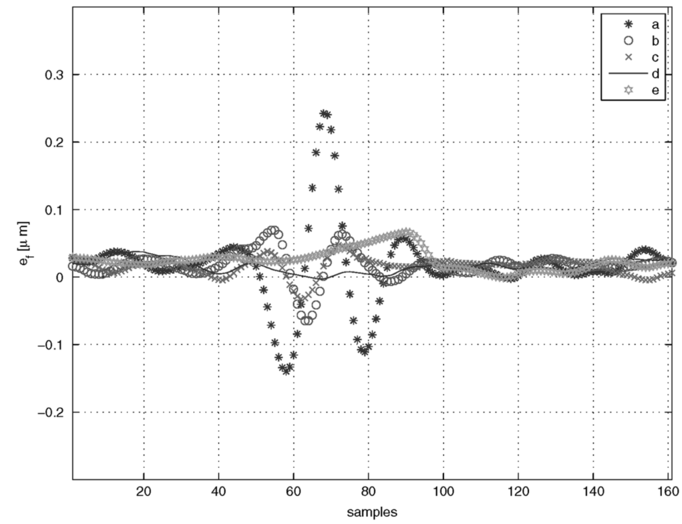

Fig. 8. Zoom on focus distance during the handling at the same defect in five different ways. (a) In case of no correction. (b) Handled by the feature-based control scheme. (c) Handled by the adaptive scheme after four encounters of the defect. (d) Handled by the adaptive scheme after nine encounters of the defect. (e) Standard industrial defect handling method.

of the classes and the approximating basis of the chosen class is used in the following.

Based on the models and the controllers and the Karhunen–Loève bases, it is possible to verify that $\mathrm{eig}(\mathbf{T}^{\mathrm{l}} \cdot \mathbf{P}_{\mathrm{l}}) < 1$. The computed value in the focus case is 0.689 and the computed value in the radial case is 0.499, i.e., it can be concluded that the stability criteria is fulfilled for both servo loops. The difference between the two values for each of the control loops, is due to two factors. The first factor is how well the defect is approximated, and the second is the amplification of the system dynamics through the approximations.

In [5] and [4], the accommodation scheme is tested on the modeled CD player playing a disc with a scratch in a class for which the approximation base has been proven stable. These experiments showed a dramatic improvement of the handling of the surface defects. In [4], a successful synchronization scheme for the accommodation was proposed. The importance of the correctness of this synchronization is, in this brief, investigated by using the method presented in Section VI-B1 The results can be seen in Fig. 7. It can be seen that just a few samples out of synchronization can deteriorate the performance with a factor of five. This factor can actually deteriorate the systems defect handling capability instead of improving it. It was as well verified that the defect response at one encounter could be assumed to have died out before its next encounter.

1) Adaptive Scheme: The adaptive scheme is tested by simulations, using a simulation model of a CD player playing a CD with a surface defect (see [17]). In order to make the simulations challenging the variance of the defect from encounter to encounter is increased with a factor of 3 compared to measured defect variations. In the simulation model the surface defects are generated based on statistics of measured defects transformed by the approximating basis. In simulation, the basis vectors used for the defect generation are modified such that the standard approximating basis is not a good representation anymore. The β factor is chosen to a value of 0.05. Surface defects like scratches varies notably with approximately 50 encounters.

The output of this simulation is illustrated by Fig. 8, which shows five zooms on the system's reaction on the defect, in five different situations: no correction, the standard feature-based control scheme, the adaptive feature-based control scheme after four and nine defect encounters and the standard industrial defect handling method. From this it can be seen that the feature-based scheme improves the defect handling, and further improvements are seen from the two curves representing the adaptive scheme which is better than the industrial method. The adaptive handling improves with the number of encounters. However, in the simulations the improvements seem to converge at the ninth defect encounter. The plotted results are chosen, since they are good representatives of a number of different simulations of both focus and radial loops. It can thereby be concluded that an adaptive feature-based control scheme is an improvement of the feature-based scheme in terms of CD players handling surface defects on the disk surface. This means that the adaptive scheme is an improvement if the defect is not well supported by the precomputed basis. In all the simulations made of the adaptive feature-based control scheme, the stability criterion has been fulfilled at all encounters.

2) Predictive Scheme: The predictive scheme can handle situations, where the user requests the CD player to jump to another "track" and a surface defect has grown large on this specific track. Using the prediction of defect development, jumps can either be handled by taking the jump in smaller steps or only one step and the defect at each step is predicted in order to accommodate it. In the simulations, the defects are simulated with increasing amplitude and time duration. The algorithm responses are compared both for the focus and radial loops and simulated accommodated by the adaptive and predictive scheme.

A zoom of the focus responses can be seen in Fig. 9 and a zoom of the radial responses can be seen in Fig. 10. In both plots, the upper plots are the defect handled by the adaptive feature-based control scheme and in the lower plot the defect is handled

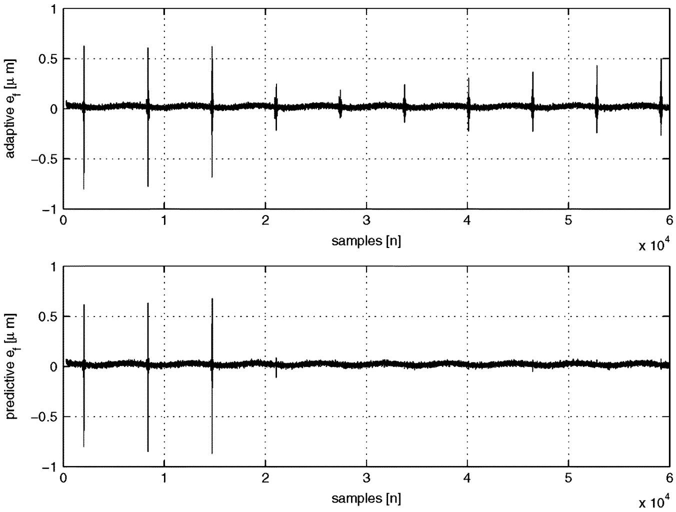

Fig. 9. Plot of the simulated $e_{\rm f}$ simulating the track jump. The upper plot shows a zoom on the handling by the adaptive feature based control scheme, the lower plot shows a zoom on the handling by the predictive scheme.

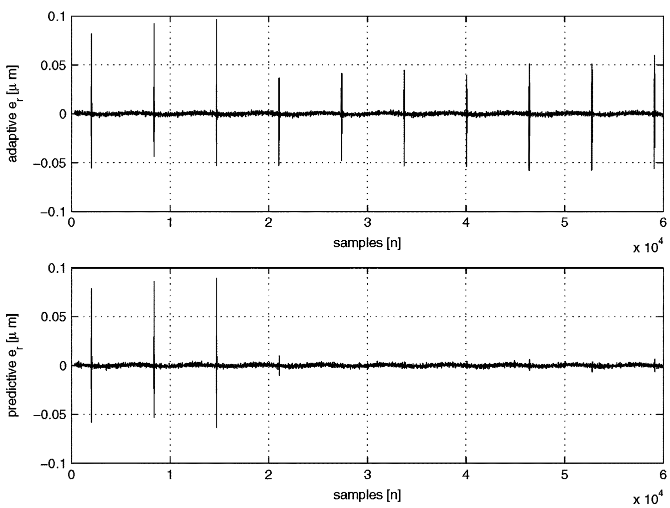

Fig. 10. Plot of the simulated $e_{\rm r}$ simulating the track jump situation. The upper plot shows a zoom on the handling by the adaptive feature-based control scheme, the lower plot shows a zoom on the handling by the predictive scheme.

by the predictive scheme proposed in this brief. From these plots it can be seen that the predictive feature-based control scheme handles these jumps better than the adaptive. This means that the predictive version of the accommodation scheme can be used to jump between tracks.

VIII. Conclusion

Accommodation of surface defects on CDs can be viewed as dealing with repetitive sensor defects. A scheme for handling these has previously been proposed. In this brief, adaptive and predictive versions of this scheme are developed. Both use approximations of repetitive surface defects in order to remove these defects from the sensor signals when a surface defect/sensor defect is located. The defect components are estimated using a Karhunen–Loève basis. In the adaptive scheme the approximating Karhunen–Loève basis is recomputed after each defect encounter, and in the predictive scheme, deterministic propagation of the defect is taken into account. Sufficient and necessary stability conditions for the proposed accommodation scheme are derived as well. The proposed scheme is subsequently applied to the example of CD players playing CDs with scratches and other surface defects. Both the standard and adaptive repetitive defect accommodation schemes show improvements compared with a standard industrial scheme. The predictive defect accommodation scheme can be used to handle jumps between tracks, where the final track contains a large surface defect. The accommodation scheme can handle this by predicting the defect development and maybe using a number of intermediate steps in the jumping. All in all, this accommodation scheme for handling repetitive sensor defects shows a large potential for handling scratches and other surface defects on CDs and other optical discs.

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