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Parametric Fault Estimation Based on H_∞ Optimization in A Satellite Launch Vehicle

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***Abstract*—**

Correct diagnosis under harsh environmental conditions is crucial for space vehicles' health management systems to avoid possible hazardous situations. Consequently, the diagnosis methods are required to be robust toward these conditions. Design of a parametric fault detector, where the fault estimation is formulated in the so-called standard set-up for $\mathscr{H}_\infty$ control design problem, is addressed in this paper. In particular, we investigate the tunability of the design through the dedicated choice of the fault model. The method is applied to the model of turbopump as a subsystem of the jet engine for the satellite launch vehicle and the results are discussed.

I. Introduction

Reliability is an essential topic within many industrial sectors, in particular the aerospace industry as no possible and foreseeable fault should interrupt the mission objectives of a space craft (or a launch vehicle). In this regards, having the capability of continuous monitoring of the system states e.g., the ability to diagnose the system's dynamics behavior, is a necessity in order to implement fault tolerant strategies.

Fault diagnosis has since the 1980s been an active research topic. Depending on the models that have been used to describe the systems, model based (linear/nonlinear) or others, different approaches have been proposed [1], [2], and [3]. One of the important problems that has attracted attention of most in this research community is the robustness issue that arises due to the fact that there is some mismatch (however small) between the derived model and the real system dynamics.

The particular focus in this paper is on employing methods for fault diagnosis which have been inspired by and derived from the area of robust control theory, or in wider generality of optimization based control synthesis methods. An early paper, which suggested combining methods for diagnosis and control was [4]. [5] suggested to use $\mathscr{H}_\infty$ optimization to design fault diagnosis filters. The methods that used dedicated and specialized filter structures were presented in [6], [7], [8], [9]. Parametric faults are here of main interests as the real nature of many faults is in fact parametric. A fault diagnosis approach for systems with parametric faults has been used. Such an approach was presented in [10], [11], and [12]. However, very few applications of this method has been reported [13].

In this paper, systems with parametric faults are studied in more details and extra notes are added such as introducing the fault model as a design factor to improve the performance of $\mathscr{H}_\infty$ optimization-based method and a practical algorithm for estimating the uncertain fault parameter. The main results of this paper are applied to the launch vehicle simulator.

A key element for the re-usability and maintainability of a space vehicle is provided by *health management system* (HMS) that is an integral part of the system design [14]. Part of the health management system's responsibility is to perform diagnosis on different parts of the launch vehicle's dynamic behavior, herein the engines. The HMS shall be able to diagnose faults of which the effect is hardly recognizable due to system uncertainties (unpredictable environmental conditions or system parameters). In addition, as the dynamics of the engines are highly nonlinear and varies depending on flight phases it is required that the corresponding diagnosis algorithms are sufficiently robust in order to avoid false-detection scenarios.

The paper is arranged as follows: In Section II, dynamics of the turbopumps of the engine is explained. In Section III, the employed method is discussed, the problem has been formulated into the standard set-up, and the fault detector for the system has been designed regarding the design factor. In section IV, an algorithm to estimate the uncertainty is proposed and the fault estimation results of the obtained filter for the different designs are compared. Finally, section V provides the conclusion of the paper.

II. Problem Formulation

A. Turbopump Model

The assembly of a turbine with one or more pumps is called a turbopump. Its purpose is to raise the pressure of the following propellant. Its principal subsystems are a hot gas powered turbine and one or two propellant pumps. It is a high precision rotation machine, operating at high shaft speed with severe thermal gradients and large pressure changes, it usually is located next to a thrust chamber, which is a potential source of noise and vibration. The principal components of the engine with turbopump system is shown in the simplified diagram of Fig. 1.

In the gas generator cycle, the turbine inlet gas comes from a separate gas generator. This cycle is relatively simple; the pressure in the liquid pipes and pumps are low but the

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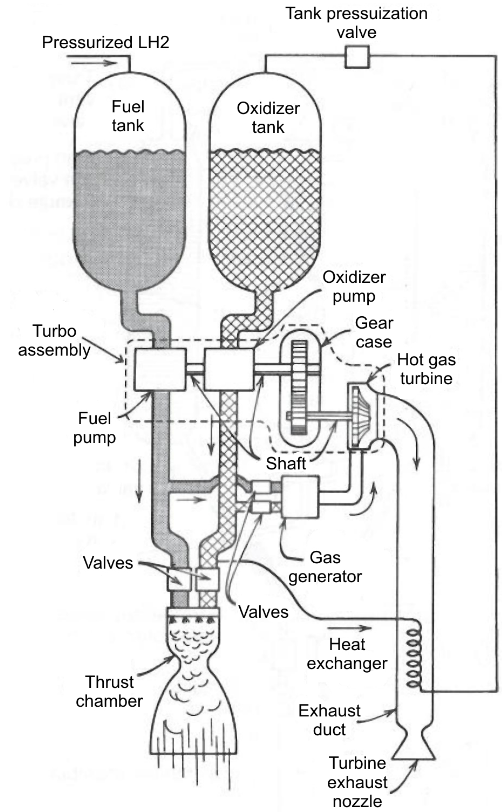

Fig. 1. Simple diagram of liquid propellant engine containing turbopump feed system and gas generator [15].

pressure ratio across the turbine is relatively high; however, the turbine or gas generator flow is small compared to the closed cycle.

Cryogenic propellants use LOX-LH2 (Liquid Oxygen-Liquid hydrogen). Liquid hydrogen LH2 is sub-cooled below its normal boiling point to increase its density (propellant densification) and to reduce vapor pressure and correspondingly also tank pressure, tank size, tank mass, and turbopumps power demand. For the same reasons liquid oxygen LOX is sub-cooled. The required pump flow is established by the design for a given thrust, effective exhaust velocity, propellant densities, and mixture ratios.

The dynamic model of LH2 turbopump includes three important elements: the pump speed R_h, the pump flow Q_h, and the mixture ratio R_{oh}. In a simplified way, the dynamics of LH2 turbopump is as follows

$$\dot{R}_h = \frac{a_h Q_h^2}{R_{oh}} + b_h Q_h R_h + c_h R_{oh} R_h^2 + b T_h, \qquad (1)$$

where a_h, b, b_h, and c_h are constant coefficients depending on the design of the turbopump and T_h is the LH2 turbine torque. The same model can be used for LOX turbopump while to avoid repeating design procedure, we only continue with LH2 turbopump.

B. Fault Discussion

Efficiency loss (δ_h) has been considered as a parametric fault for LH2 turbopump. The more efficiency loss, the less change in speed. i.e., the dynamic equation is satisfied only for no fault case ($\delta_h = 0$). The fault augmented model is hence,

$$\dot{R}_h = \left(\frac{a_h Q_h^2}{R_{oh}} + b_h Q_h R_h + c_h R_{oh} R_h^2\right)(1-\delta_h) + b T_h. \qquad (2)$$

The linear representation of the LH2 pump dynamics is formulated as

$$\dot{R}_h = (-aR_h - cQ_h)(1-\delta_h) + bT_h, \qquad (3)$$

where a, b, and c are constant coefficients of the linear term of Taylor series about the operating point of the non-linear system.

III. Method

A. Robust Parametric FDI in A Standard Set-up

A general concept of parametric fault detection architecture in a robust standard set-up is proposed in [11]. The approach is to model a potentially faulty component as a nominal component in parallel with a (fictitious) error component. The optimization procedure suggested here then tries to estimate the ingoing and outgoing signals from the error component. This works only well in cases where the component is reasonably well excited, but on the other hand, if the component is not active at all, there is absolutely no way to detect whether it is faulty. The considered plant is described by the model

$$\begin{cases} \dot{x} = A_\Delta x + B_u u \\ y = C_y x + D_{yu} u \end{cases} \qquad (4)$$

where A_Δ is the deviated matrix from the nominal value (A) by a dependency to the fault where the dependency can be nonlinear. The fault should not change B and C. When this is the case (as in our plant and many other applications where sensor and actuator faults are supposed to be detected), it is possible to model such faults in the setup given by (4) with an input/output filter by introducing fast dynamics for the filter such as

$$\dot{x}_u = -W x_u + W c Q_h. \qquad (5)$$

The possibly nonlinear parameter dependency of A_Δ is approximated with a polynomial. Therefore,

$$A_\Delta = A + p(\delta)A, \qquad (6)$$

where p is a polynomial or rational function of the parameter δ satisfying $p(0) = 0$ (the non-faulty operation mode).

Finally, the model (4) is written in linear fractional transformation form. As a result we get a system of the form

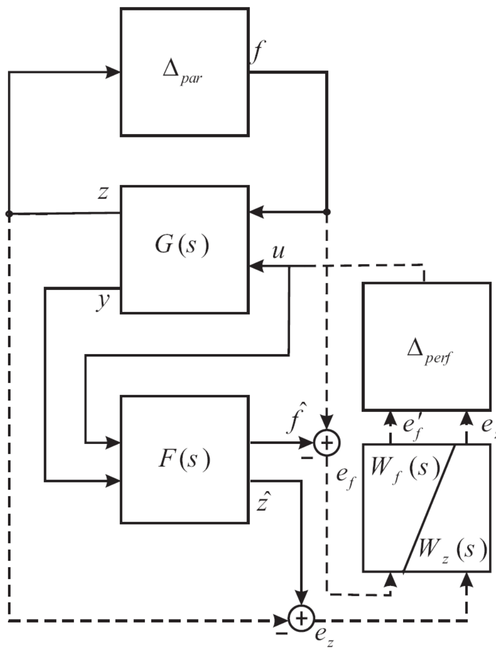

Fig. 2. Standard problem set-up for parametric fault detection combined with fictitious performance block (The dashed lines are the connections which are artificially assumed only for the design and they do not exist in implementation).

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_f & B_u \\ \hline C_f & D_{zf} & 0 \\ C_y & 0 & D_{yu} \end{array}\right] \begin{bmatrix} x \\ f \\ u \end{bmatrix} \tag{7}$$

where z is the external output, f is the fault input signal, the matrix D_{zf} is well-posed (LFT's are normally used), and the connection between z and f is given by

$$\begin{aligned} f &= \Delta_{par} z \\ \Delta_{par} &= \delta I. \end{aligned} \tag{8}$$

The next step in setting up the fault estimation problem as a standard problem is to introduce two fault estimation errors e_f ad e_z as

$$\begin{cases} e_f = f - \hat{f} \\ e_z = z - \hat{z} \end{cases}$$

where $\hat{f}$ and $\hat{z}$ are the estimation of f and z to be generated by the filter respectively.

Fig. 2 shows the setup for this approach. In order to design a filter F such that applying F to u and y provides the two desired estimates $\hat{f}$ and $\hat{z}$ one additional step is required, which is the introduction of a fictitious performance block Δ_{perf} ; suggesting that the input u was generated as a feedback Δ_{perf} from the outputs $\begin{bmatrix} e_f \\ e_z \end{bmatrix}$

$$u = \Delta_{perf} \begin{bmatrix} e_f \\ e_z \end{bmatrix}. \tag{9}$$

Therefore, two filters are introduced to make sure that the norm of $\frac{\|e_f\|}{\|f\|}$ is minimized in the frequency area of interest. (For incipient faults a low frequency filter is used.)

For instance,

$$\begin{aligned} \dot{x}_{ef} &= A_{ef} x_{ef} + B_{ef} e_f \\ \acute{e}_f &= C_{ef} x_{ef} + D_{ef} e_f \end{aligned} \tag{10}$$

so $\acute{e}_f = W_f(s) e_f$. The same procedure for e_z will be

$$\begin{aligned} \dot{x}_{ez} &= A_{ez} x_{ez} + B_{ez} e_z \\ \acute{e}_z &= C_{ez} x_{ez} + D_{ez} e_z \end{aligned} \tag{11}$$

i.e., $\acute{e}_z = W_z(s) e_z$. It should be noticed that these filters are only considered in the design phase, but they are not used in the implementation. In fact, we introduce these filters to handle the high excitation level of the inputs. Finally we introduce

$$\Delta = \begin{bmatrix} \Delta_{par} & 0 \\ 0 & \Delta_{perf} \end{bmatrix}. \tag{12}$$

The significance of the Δ_{perf} block is the following. According to the small gain theorem, the $\mathscr{H}_\infty$ norm of the transfer function from u to $\begin{bmatrix} \acute{e}_f \\ \acute{e}_z \end{bmatrix}$ is bounded by γ if and only if the system in Fig. 2 is stable for all Δ_{perf}, $\| \Delta_{perf} \|_\infty < \gamma$. Hence, the problem of making the norm of the fault estimation error bounded by some quantity has been transformed to a stability problem. Eventually, the main result for FDI problem with parametric fault is provided by the following [11]:

Theorem 1: Let $F(s)$ be a linear filter applied to the system as in Fig. 2 as $\begin{bmatrix} \hat{f} \\ \hat{z} \end{bmatrix} = F \begin{bmatrix} u \\ y \end{bmatrix}$, and assume that $F(s)$ satisfies:

$$\| \mathscr{F}_l(G_{\tilde{z}\tilde{w}}, F) \|_\infty < \gamma, \tag{13}$$

where $\tilde{z} = \begin{bmatrix} z \\ \acute{e}_f \\ \acute{e}_z \end{bmatrix}$, $\tilde{w} = \begin{bmatrix} f \\ u \end{bmatrix}$, and $\mathscr{F}_l(.)$ is the lower Linear Matrix Transformation (LFT) representation of the two connected blocks [16]. Then the resulting fault estimation error is bounded by

$$\| \begin{bmatrix} \acute{e}_f \\ \acute{e}_z \end{bmatrix} \|_\infty < \gamma N \tag{14}$$

where N is the excitation level of the system i.e., $\| u \|_\infty = N$.

B. Design of The Fault Detector for Turbopump

The important fact which is emphasized in this paper is that the result we get from our design is fairly tunable by the model of the fault we consider in (6). This is the place we investigate in more details and finally perform the system (7). (For convenience, we avoid using the index h for δ_h and apply this to the end of the paper.)

Considering the fact that our uncertain parameter (as efficiency loss) in (2) changes from 0 to 1, it is obvious that the $\mathscr{H}_\infty$ design will be conservative because we only use half the range we considered in our design ($-1 < \delta < 1$). Thus, the basic assumptions are used for the fault model $p(\delta)$ are that it should satisfy the boundary conditions (in addition to $p(0) = 0$) as

$$p(-1) = 1$$

and

$$p(1) = 1.$$

A fast search for such functions is relatively easy by choosing polynomials as the structure of such function. To reduce the number of degrees of freedom and complexity of the system it is suggested to choose a low order polynomial. In the case of choosing a second order polynomial, there is only one unique function satisfying the conditions ($p(\delta) = \delta^2$) so we consider the third order case which has more generality and degree of freedom.

A third order polynomial $p(\delta) = \beta_3\delta^3 + \beta_2\delta^2 + \beta_1\delta + \beta_0$ which satisfies the mentioned conditions has the form

$$p(\delta) = \lambda\delta^3 + \delta^2 - \lambda\delta, \tag{15}$$

where we have one degree of freedom to tune our design when varying λ. The upper LFT of the polynomial should be composed into (7). The representation from robust control is

$$p(\delta) = \mathscr{F}_u(M, \delta I_3), \tag{16}$$

where $M = \begin{bmatrix} 0 & -\lambda & 1 & \lambda \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Finally, the system is formulated in a standard form as

$$\begin{aligned}
\dot{x} &= -ax - x_u + bT_h + \lambda f_1 + f_2 - \lambda f_3 \\
\dot{x}_u &= -Wx_u + WcQ_h \\
\dot{x}_{ef} &= A_{ef}x_{ef} + B_{ef}(\lambda f_1 + f_2 - \lambda f_3 - \hat{f}) \\
\dot{x}_{ez} &= A_{ez}x_{ez} + B_{ez}(\lambda z_1 + z_2 - \lambda z_3 - \hat{z}) \\
z_1 &= ax + x_u \\
z_2 &= f_1 \\
z_3 &= f_2 \\
\acute{e}_f &= C_{ef}x_{ef} + D_{ef}e_f \\
\acute{e}_z &= C_{ez}x_{ez} + D_{ez}e_z \\
y_1 &= x \\
y_2 &= T_h \\
y_3 &= Q_h.
\end{aligned} \tag{17}$$

The standard model (with $D_{ef} = D_{ez} = 0$) becomes

$$\begin{bmatrix} \dot{x} \\ \dot{x}_u \\ \dot{x}_{ef} \\ \dot{x}_{ez} \\ \cdots \\ z_1 \\ z_2 \\ z_3 \\ \acute{e}_f \\ \acute{e}_z \\ \cdots \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} A_1 & \vdots & B_1 & B_f & \vdots & B_2 \\ \cdots & & \cdots & \cdots & \cdots & \cdots \\ C_1 & \vdots & D_{11} & D_{1f} & \vdots & D_{12} \\ \cdots & & \cdots & \cdots & \cdots & \cdots \\ C_2 & \vdots & D_{21} & D_{2f} & \vdots & D_{22} \end{bmatrix} \begin{bmatrix} x \\ x_u \\ x_{ef} \\ x_{ez} \\ \cdots \\ T_h \\ Q_h \\ f_1 \\ f_2 \\ f_3 \\ \cdots \\ \hat{z}_1 \\ \hat{f}_1 \end{bmatrix}, \tag{18}$$

where the matrix values are as bellow

$$A_1 = \begin{bmatrix} -a & -1 & 0 & 0 \\ 0 & -W & 0 & 0 \\ 0 & 0 & A_{ef} & 0 \\ a\lambda B_{ez} & \lambda B_{ez} & 0 & A_{ez} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} T_h & 0 & 0 & 0 \\ 0 & Wc & 0 & 0 \end{bmatrix}^T,$$

$$B_f = \begin{bmatrix} \lambda & 1 & -\lambda \\ 0 & 0 & 0 \\ \lambda & 1 & -\lambda \\ 0 & B_{ez} & -\lambda B_{ez} \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -B_{ef} \\ -B_{ez} & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} a & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C_{ef} & 0 \\ 0 & 0 & 0 & C_{ez} \end{bmatrix},$$

$$D_{11} = 0_{5\times 2},$$

$$D_{1f} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_{12} = 0_{5\times 2},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

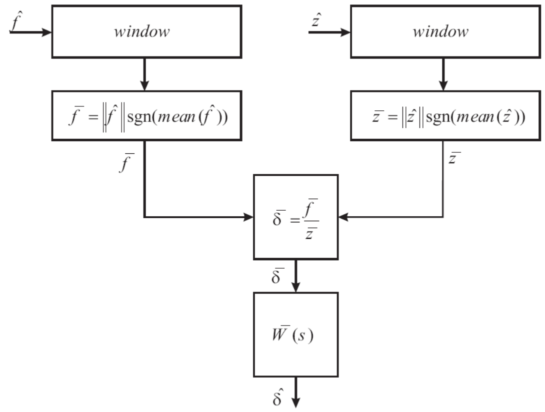

Fig. 3. Block diagram of the algorithm for the estimation of δ.

$$D_{2f} = 0_{3\times 3},$$

and $D_{22} = 0_{3\times 2}$.

Finally, a $\mathscr{H}_\infty$ filter F, which estimates $\hat{f}$ and $\hat{z}$ and takes u and y as inputs, is designed using *hinfsyn* in **MATLAB**. This filter results in e_f and e_z vanishing to zero as time goes to infinity.

IV. Results

The detector filters have been implemented in nonlinear Launch Simulator.

In order to obtain an estimation for δ, an algorithm has been employed as follows (See Fig. 3).

1) A window has been located on the latest samples of $\hat{f}$ and $\hat{z}$. The length of the window is 5 samples and the sampling frequency is $67Hz$.
2) The second norm of the sampled data has been computed and multiplied by the sign of their mean values.
3) The results from the above blocks has been used to calculate $\bar{\delta} = \frac{\|\bar{f}\|}{\|\bar{z}\|}$.
4) A low-pass filter $\bar{W}(s)$ is used to reduce the effect of the noise on the estimated fault.

Two series of comparison regarding the change in γ and λ values are considered here to show that the choice of the model plays an important role to improve the results. In these experiments, the injected parametric fault has been raised from 0 to 1 at $25s$.

In Fig. 4 the comparison of estimation results among four different values of γ ranging from 0.005 to 0.1 for three constant values of λ is shown. In fact, it confirms that increasing γ is equivalent to pay less attention to the condition (13) and consequently changing the problem into a **Kalman Filter** optimization problem. This results in amplifying of the effect of the disturbances in the fault estimation and less robustness. On the other hand, decreasing γ reduces the effect of the disturbances and noise inputs to the estimation and increases the robustness. For example, in the case for $\lambda = 0.1$ we can see that the performance of the estimation with $\gamma = 0.005$ is better than those for $\gamma = 0.008$ and $\gamma = 0.01$.

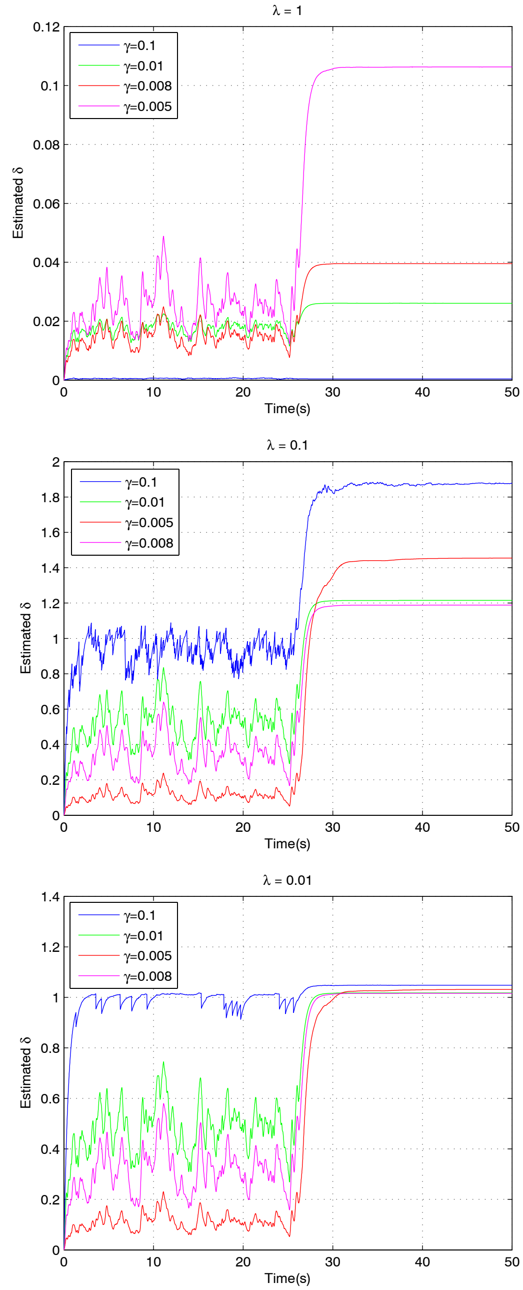

Fig. 4. Estimation of efficiency loss δ in LH2 pump for γ comparison while λ is constant.

In Fig. 5 the comparison of estimation results among three different values of λ ranging from 0.01 to 1 for four constant values of γ is shown. Increasing the value of λ results in very weak estimation, however, decreasing λ does not mean that the estimation is perfect. Indeed, for $\lambda = 0.1$ the estimation of the δ is converging to 1 which corresponds to the injected value of δ. Therefore, one could say neither the increase of λ, nor the decrease of γ will give the best estimation results. In fact, there is an optimal point λ which gives the best estimation, however, we did not consider a methodological way to obtain this optimal value but this example was a witness for existence of such point which can be considered in the future developments.

Eventually, any alternative optimization method could be considered to solve the problem e.g., numerical algorithms for μ optimization. However, by presenting this method and finding a significant model for the fault there is still the advantage of solving a convex optimization problem by $\mathscr{H}_\infty$ method compared to μ optimization.

V. Conclusions

The uncertainties/faults in turbopumps, a subsystem of the engine, have been modeled as parametric faults in this paper. This model has been formulated in a standard set-up which is compatible with $\mathscr{H}_\infty$ control design formulation. The designed $\mathscr{H}_\infty$ filter for different fault models are implemented to this system. The output of the filter processed in a way to produce the estimation of possible fault. Finally, the method has been verified in launch simulator and the results for different design factors have been compared then a trade off in the design has been demonstrated.

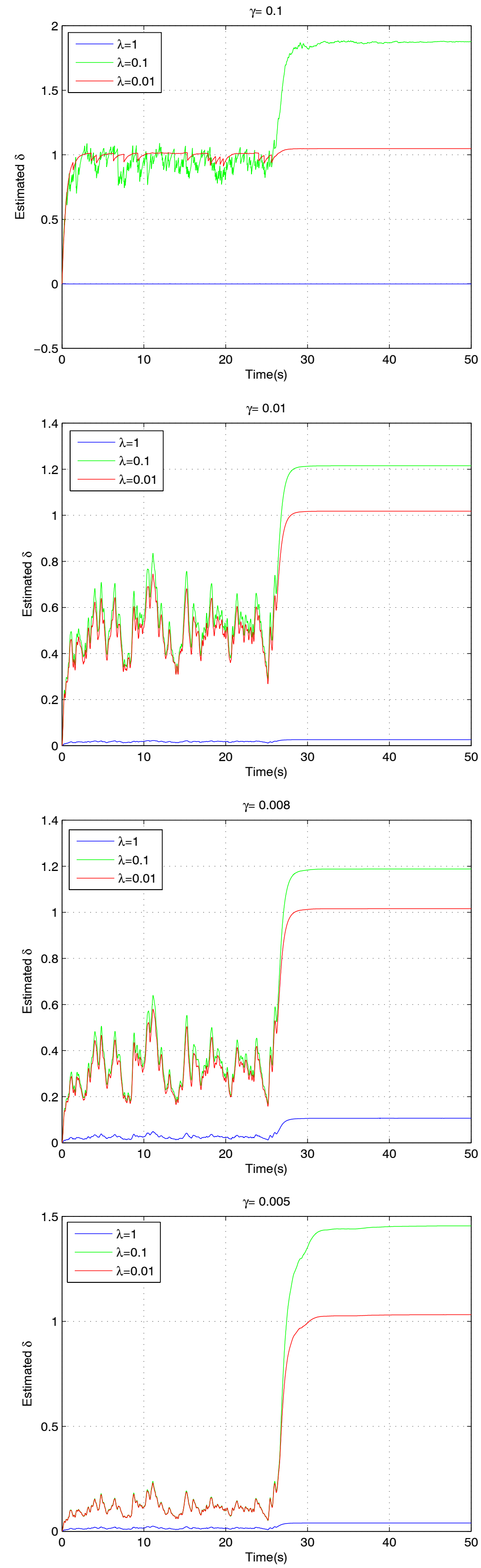

Fig. 5. Estimation of efficiency loss δ in LH2 pump for λ comparison while γ is constant.

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