

A Novel Method for Control of Systems with Costs Related to Switching: Applications to Air-Condition Systems

Honglian Deng and Lars Larsen and Jakob Stoustrup and Henrik Rasmussen

Abstract—The objective of this paper is to investigate a control method for systems with discrete inputs that have switch related cost. For such systems, the control objective often is a trade off between the deviation from the reference (performance) and the number of switches (weariness, energy efficiency etc.). For such systems a steady state might never be attained, but rather the optimal behavior might be constituted by a limit cycle. In this paper we consider the problem of finding and controlling the system towards an optimal limit cycle. A low complexity approach will be proposed.

I. INTRODUCTION

Within the recent years optimal control of hybrid systems has attained a lot of focus. Among the recent results can be mentioned [1], [2], [3], [4], [5], [6], [7]. One reason is that numerous industrial applications have the features of a hybrid system. In this paper we will however only consider systems with discrete inputs, which can be categorized as a subclass of hybrid systems. Examples of such systems in the industry are numerous. Here we will focus on air conditioning system (AC-system) with a discrete compressor capacity, for motivating the analysis and demonstrate the proposed method.

A method for synthesizing optimal control for hybrid systems is hybrid model prediction control using the mixed logical dynamical framework (MLD)[4], but also other MPC methods based on solving finite horizon optimization problems exists such as [8] and [9]. Common for these methods is that they typically produce rather complex controllers requiring large computational power. Furthermore, if special care is not taken using finite horizon methods on systems with discrete inputs, it can lead to rather poor performance,[6],[10]. There exist also other methods based on steady state optimization of limit cycles, for instance [7] on page 253. Some of these methods give good results for particular applications, but are complex and difficult to apply especially in the lower levels of the control hierarchy where sufficient computational power is not available. This motivates the development of much less complex methods overcoming the potentially poor performance that finite horizon methods leads to. This is the focus of this paper.

The paper is organized in the following way. Modeling of the system is briefly presented in Section II. An analysis of performance function nature for discrete input system are carried out in Section III. A novel method is presented in Section IV. Some tests of the method on a second order

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system system is explained. Comparison of the developed method and hysteresis control are made in Section V. The conclusion is given in Section VI.

II. SYSTEM MODELING

A simplified first order room model with an air conditioning system (AC-system) is shown below. In this paper the AC-system is not modeled in detail, but presented as an energy input to a simple room model.

$$\frac{dT_{room}}{dt} = \frac{\dot{Q}_{a2r} - \dot{Q}_e}{Cp_{air}m_{air}},$$

$$\dot{Q}_{a2r} = \frac{(T_{amb} - T_{room})}{Res}$$

where T_{room} , is the temperature of room air.

\dot{Q}_{a2r} , is the heat flow from ambient to the room through wall.

\dot{Q}_e , is the cooling energy.

Res is the thermal resistances from ambient to the room air.

III. COST FUNCTION ANALYSIS

In AC-systems, the objective is usually to control the indoor temperature sufficient close to the reference. At a certain load, where the system can not run continuously, a discrete input has to be used, and the result will be a limit cycle instead of a steady state solution. Let us assume that the AC-system with lowest possible speed can remove 300W. Hence, in case 150W needs to be removed, the system can only run between on (300W) and off (0W) to achieve an average close to the reference temperature. In this paper a definition of load is used. It is a percentage and it means that a system running with this percentage of the high limit power (300W in this example) will be controlled at the reference. In this example, 50% load means that running with 150W (50%·300) will make the system stay at the reference.

For a small temperature variation, a fast switch is needed which is at the cost of AC-system components wearing out fast and a low efficiency due to frequent starts and stops often, and vice versa. Hence the optimal control of AC-system is in fact a trade-off between comfort (small temperature deviation from reference) and economic cost (number of switches) and can be formulated as Equation (1), which is the comfort and economic cost per time unit under a stable limit cycle. This formulation represents a control objective of a discrete input system for which switch costs have to be taken into consideration.

The comfort error accumulation is divided into on ($0 - \alpha T$) and off ($\alpha T - T$) periods (1), where α is the duty

cycle and T is the cycle period. x_{ref} is the room set-point temperature. Through the user specification, the comfort and economic objectives can be balanced by assigning the two weight factors (Q and R in (1)).

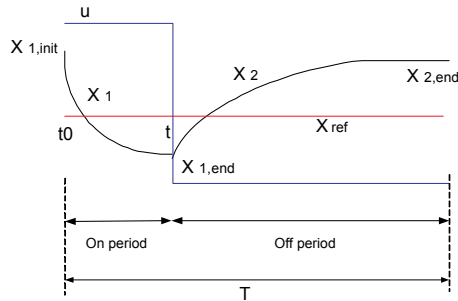


Fig. 1. one period

$$J = \frac{Q \cdot \left(\int_0^{\alpha T} (x_1 - x_{ref})^2 dt + \int_{\alpha T}^T (x_2 - x_{ref})^2 dt \right) + R \cdot J_{sw}}{T} \quad (1)$$

T is the total period. Q and R are the weight factors which balances the comfort and the switch terms.

The AC-system state space model is as Equation (2) where E is a disturbance, and u is a power input. For the on period, the model becomes Equation (3), and for the off period where u is 0, the model becomes Equation (4). $x_{1,init}$ is the state when the system switches from off to on.

$$\dot{x} = Ax + Bu + E \quad (2)$$

$$x_1 = \frac{Bu + E}{-A} + \left(x_{1,init} + \frac{Bu + E}{A} \right) \cdot e^{At} \quad (3)$$

$$x_2 = \frac{E}{-A} + \left(x_{1,end} + \frac{E}{A} \right) \cdot e^{A(t-\alpha T)} \quad (4)$$

Under steady state conditions, the system is periodical, which means that the states at the start of a cycle is the same as the end of the cycle, which is described in Equation (5). $x_{2,end}$ denotes the state at the end of the off period, and can be got by applying $t = T$ to Equation (4). $x_{1,end}$ in Equation (4) can be got by applying $t = \alpha T$ to Equation (3). The result can be seen in Equation (6) and (7). So now x_1 and x_2 are functions of α, T, t . Therefore Equation (1) becomes a function of only α, T .

$$x_{1,init} = x_{2,end} \quad (5)$$

$$x_{1,init} = \frac{\frac{E}{-A} + \frac{Bu}{-A} e^{A \cdot (1-\alpha)T} + \frac{Bu+E}{A} e^{A \cdot T}}{1 - e^{A \cdot T}} \quad (6)$$

$$x_{1,end} = \frac{\frac{Bu+E}{-A} + \frac{E}{A} \cdot e^{A \cdot T} + \frac{Bu}{A} \cdot e^{A \cdot \alpha T}}{1 - e^{A \cdot T}} \quad (7)$$

For simplicity, the above idea is tested with a first order system example, which is a simplification of the AC-system

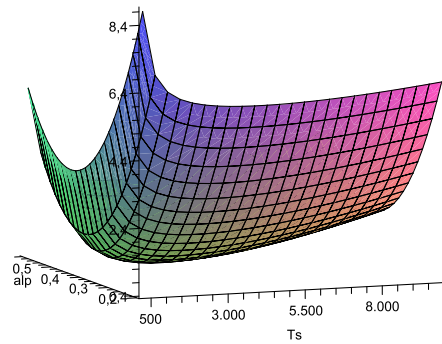


Fig. 2. Cost J in period T_s and duty cycle α , load 45%

model. The parameters are:

$$A = -2.00 \times 10^{-4}$$

$$B = -4.40 \times 10^{-6}$$

$$D = 5.73 \times 10^{-3}$$

The corresponding plot of (1) (function J) can be seen in Fig.2. From the figure it can be seen that there is only one optimum.

For a system with 90% load, the optimal period is 6447 seconds, and the duty cycle is 91%, which is different from the load (90%). The cost J for running with the optimal solution is 0.242. If the system runs with 90% duty cycle, which gives the smallest cost for 90% duty cycle is 6082 seconds, and the cost is 0.249. There it shows that the optimal solution actually runs with longer period ((6447-6082)/6082=6%) but at the same time gives smaller cost.

Two experiments were designed to study the above phenomena. One is to run with different load to check if the load has influence on the different between the load and optimal duty cycle. The other is to change penalty ratio Q and R . The results show that for duty cycles α or $1 - \alpha$ close 1, there is a difference between optimal duty cycle and load. When the load is close to 1, the optimal duty cycle is always bigger than the load, but when the load is close to 0, the optimal duty cycle is always smaller than the load. The reason is that, the longer half period error accumulation (either on or off period) has smaller gradient in the longer period than the short half period error accumulation in the short half period. Therefore the optimal will always prolong the longer half period which results in a duty cycle bigger than load. The value of the difference depends on the ratio of Q and R . With the increase of R , the difference will become bigger. For 50% load, both on and off period has the same gradient over the corresponding period, therefore, the optimal duty cycle is the same as the load. (More details can be found in [10])

The computational load of finding the optimum grows fast with increase of the system size.

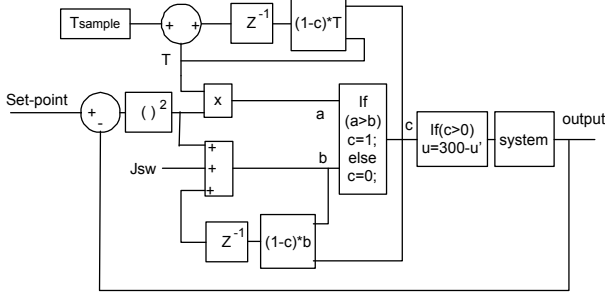


Fig. 3. Illustration of the control method. T_{sample} is the sampling time. T is the current adapting period

IV. A NOVEL METHOD

In last section, we have mentioned that there is only one optimum for the cost function J assuming the first order limit cycle. The challenge is to find a method that automatically drives the system towards the optimal limit cycle. In the following we make the assumption that the optimal limit cycle can be found in the set of first order limit cycles. An analysis of finding the optimum of cost function (1) has been presented, and following that the system will be controlled to the optimal period and duty cycle. Computing the optima explicitly is difficult for higher order systems due to the heavy computational load. What we are really looking for here is a simple method which requires little computational power and at the same time overcomes the fundamental problem of the finite prediction horizon. An idea for such a simple method is presented below.

We have known that cost function J has only one optimum, which should fulfill Equation (8). The analytical solution is difficult to find, but (8) can be rewritten as (9) and it has 3 terms, the cost of current output, the integration of error in a period, and the switch. If we rearrange the three terms as in (9), it can be understand in this way: the left side involves only the current output, while the right side involves the accumulated comfort error and a fixed switch cost. The two sides can only become equal when the period and duty cycle are optimal, since the cost function J has no local optimum. Hence, it can be used to find the optimum. For simplicity, we rename both sides of (9) as in (10) and (11). The controller always compare b (which is the error accumulation from the beginning of a period) and a with the value from last sample at each time sample to check if the switch should happen at the current time sample. If $a-b=0$ (in reality if the sign of $a-b$ is different from last sample, then a switch should happen.), the system should switch now and start a new period (T is reset to 0). Otherwise, the system should update T ($T = T + T_{sample}$) and then wait for next sample time and repeat the above procedure. The control diagram of this idea is shown in Fig.3 where u is the control input to the system u' is the control input from last sampling time. From Fig.3 it can be seen that the algorithm is very simple and requires very little computational power.

$$\frac{\partial J}{\partial T} = 0 \tag{8}$$

$$Q \cdot (x_{t0+T} - x_{ref})^2 T = Q \cdot \int_{t0}^{t0+T} (x_{t0+T} - x_{ref})^2 dt + R \cdot J_{sw} \tag{9}$$

$$a = Q \cdot (x_{t0+T} - x_{ref})^2 T \tag{10}$$

$$b = Q \cdot \int_{t0}^{t0+T} (x_{t0+T} - x_{ref})^2 dt + R \cdot J_{sw} \tag{11}$$

The basic idea of the method above has been explained, but some adjustments have to be made in order to apply it in this application, because, with this method, only one decision of switch can be made for the current time sample, but in a period, there are two switches. A straight forward way to apply it is to just take **half period** into consideration (we name it 'half period adaptation') and using (10) directly. The equations become (12) and (13, where T_{now} means half period (either on or off). By doing this, the correlation between on/off periods are not taken into account by the controller, and therefore we expects to see solutions that are suboptimal.

$$a = Q \cdot (x_{t0+T_{now}} - x_{ref})^2 T_{now} \tag{12}$$

$$b = Q \cdot \int_{t0}^{t0+T_{now}} (x_{t0+T_{now}} - x_{ref})^2 dt + R \cdot J_{sw} \tag{13}$$

Some experiments with different loads have been done with the half period adaptation, and the results can be seen in Fig.4. Comparing the period cost and duty cycle from the half period adaptation (the curve marked with 'Half') with the optimal solution (the curve marked with 'Optimal'), it can be seen that close to 50% load, the results are very close to the real optimal, but when the load is away from 50%, this method gives a deviation. For instance, with 90% load, the period from this method is about 7796 seconds, but real optimal is 6447 seconds, which is about 20% error. It demonstrates that our prediction of a method based on considering only a half period will not be able to make the system reach the real optimal solution.

An idea now is to extend the method to take both on and off periods into account (we name it full period adaptation). There could be two ways of doing it. One is to introduce a prediction for the next period (either on or off). Since the method is very simple, which only uses the accumulated square error information from the past and the current output, we would like to keep the simplicity of the method therefore a prediction based approach which requires a fairly accurate model is not preferred. An easy way of avoiding taking in models but still using the full period adaptation is to use the last period (either on or off) which has just been found, together with the current period to compose a full period, which can be expressed as Equation (14). The algorithm of (14) can be seen in Fig. 5, where after a switch, the accumulated error is set to the accumulated square error for the last period, while with half period adaptation it is reset to zero. Another

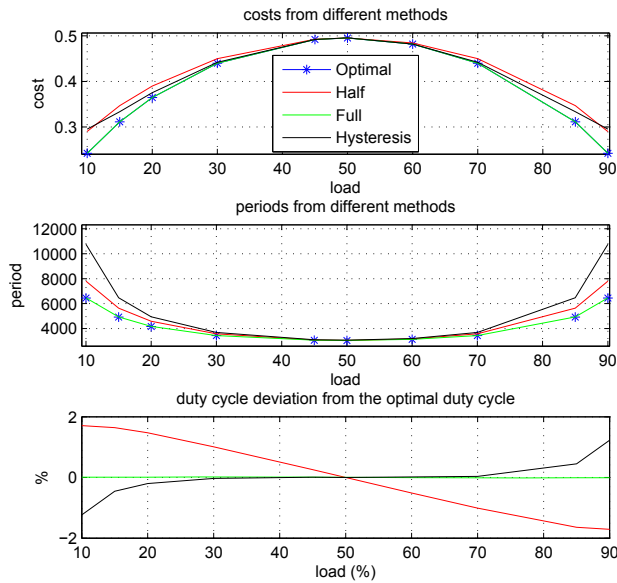


Fig. 4. Comparison of solutions from the different methods. The initial room temperature $x_0 = 22.8^\circ\text{C}$ 'Optimal': the optimal solution. 'Half': half period adaptation. 'Full': full period adaptation. 'Hysteresis': hysteresis control.

difference is that T_{last} is added to the a term. It should be noted that when we calculate for the first switch with the full period adaptation, the error accumulated from earlier period is 0, and the switch cost should be only half of the later calculation because the calculation is only for a half period for the first switch. The switch cost in (9) and (14) are different and their relations are, for the first switch $J'_{sw} = \frac{1}{2}J_{sw}$, otherwise $J'_{sw} = J_{sw}$.

Since the algorithm takes the information from past, it might be sensitive to the initial conditions because the adaptation will carry the error (between the initial condition and the reference) all the way. An analysis of the significance of this sensitivity is given below

The same experiments with the same initial condition as for half period adaptation has been carried out with the full period adaptation, and the results are shown in Fig.4 (green curve). It can be seen that the full period adaptation results reach the optimal solution at different loads, but the half period adaptation gives only good results when the load is close to 50%. Therefore we would like to have the full period adaptation.

$$\begin{aligned} & (x_{t_0+T} - x_{ref})^2 (T_{now} + T_{last}) \\ &= \int_{t_0}^{t_0+T_{now}} (x_{t_0+T_{now}} - x_{ref})^2 dt + err + J'_{sw} \end{aligned} \quad (14)$$

Unfortunately, it has been proved that the full period adaptation is sensitive to the initial conditions, while the half period adaptation is insensitive to the initial conditions in [10]. It means that the full period adaptation can lead to some

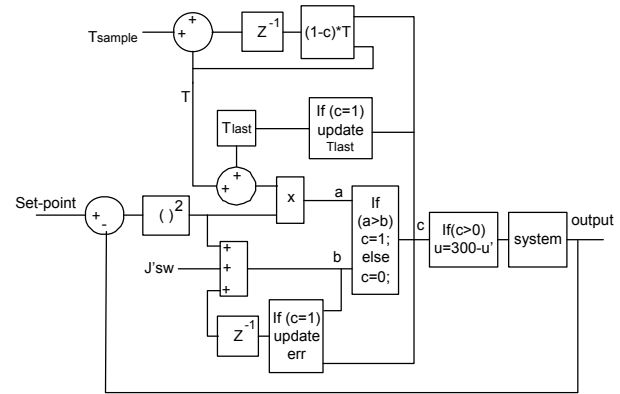


Fig. 5. Illustration of the full period control method. T_{sample} is the sampling time. T is the current adapting period

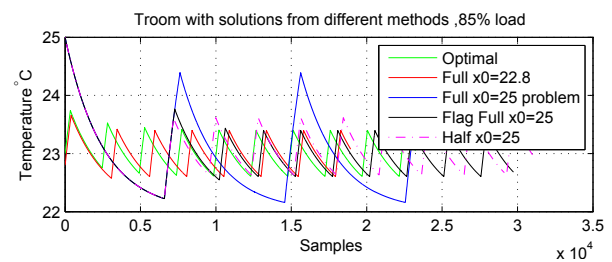


Fig. 6. Comparison of room temperature from the different methods. 'Optimal': the optimal solution. 'Half': half period adaptation. 'Full': full period adaptation. 'Flag Full': full period adaptation method with a fix. ' x_0 ': the initial room temperature

unsatisfactory results under some of initial conditions. Some experimental results show that if the initial conditions is on the track of or close to the real optimal room temperature, the temperature converges to the optimal limit cycle, otherwise it converges to a larger one. This can be seen in Fig.6 (the curve marked with Full, $x_0 = 22.8^\circ\text{C}$ and $x_0 = 25^\circ\text{C}$). The experiments condition is: 85% load. It is obvious that the full period adaptation with initial 22.8°C gives optimal results (the temperature variation is between 22.6 and 23.4°C , the red curve), but the full period method with initial 25°C (the blue curve) results in a much larger limit cycle (between 22.2 and 24.4°C). This means that 25°C is a 'bad' initial condition for the full adaptation method. Therefore our concern about the full period adaptation on initial conditions is not unnecessary.

The full period adaptation results are very promising when the initial conditions are good, but the problem of depending on the initial conditions really degrades the method. It is natural to ask a question: is it possible to get rid of the problem with initial conditions for the full period adaptation? or to combine the two methods - first start with the half period method to get rid of the bad initial condition and then switch to the full period method to get to the real optimal solution?

A new method

The problem and good properties of the full period adapta-

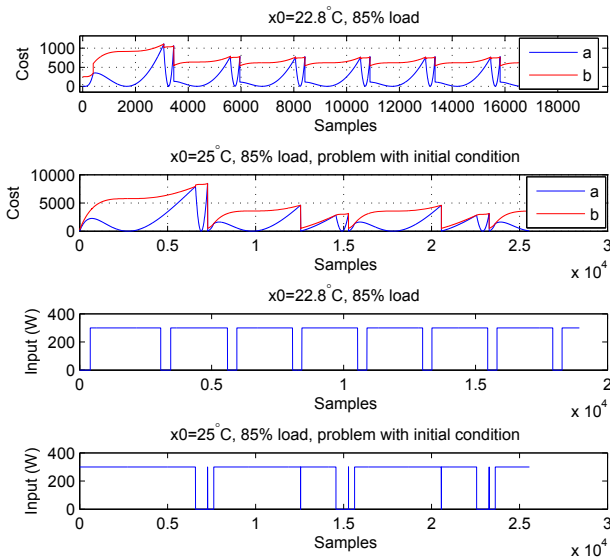


Fig. 7. Problem of full period method

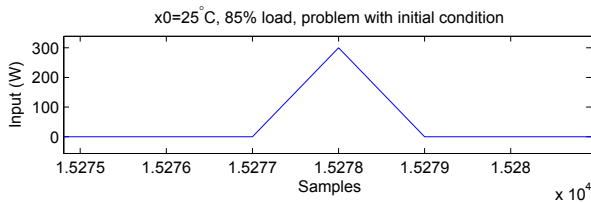


Fig. 8. Amplification of u at problem area

tion with initial conditions has been shown in last subsection. The interest here is to find out what is the reason that causes the problem and how it can be fixed.

The full period adaptation is based on (14), and the switch criteria is that the left side of the equation (a) equals to the right side (b). Before a switch, the left side is smaller than the right side. For comparison, results from full period adaptation with two initial conditions are plotted. One of them converges to the real optimal cycle and the other converges to a larger limit cycle. The left and right side of (14) are plotted for each initial condition in Fig.7. The inputs for the two initial conditions are also plotted in Fig.7.

The a and b curves from 22.8C initial condition are very much different from 25°C initial condition and so are the input curves. It looks strange with the input from 25°C initial condition - there are some peaks. The period of the cycle (disregarding the peaks) is too long compared with the input from 22.8°C, which of course result in large temperature variation as shown in Fig.6.

Amplification of the input switch with initial 25°C around the peak at 16000 sample is shown in Fig.8. At sample 16003, the system switches from off to on, but at sample

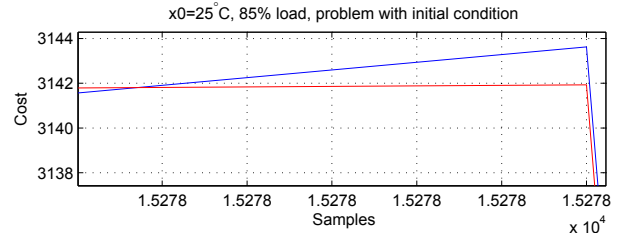


Fig. 9. Amplification of cost

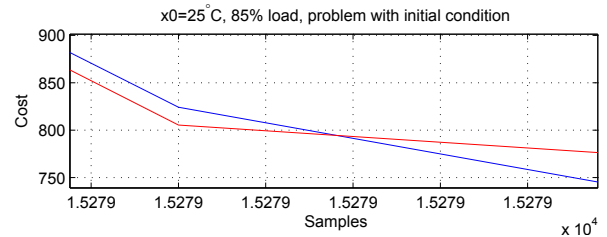


Fig. 10. Amplification of cost

16004, the system switches from on to off again. This short switch will not cause much difference in temperature, but constitute a heavy cost in switching. This is going away from the control purpose of reducing switches - this switch is unnecessary, but what is the reason it happened?

According to the original idea, the switch should only happen when a becomes equal to or larger (discrete) than b , and after the switch, a should become smaller than b again and a new period adaptation starts again. Amplification of a and b plot for initial 25°C at around 16003 and 16004 are shown as Fig.9 and Fig.10. The system switches on at sample 16003, where the cost a is larger than b as shown in Fig.9 but the strange thing is that at sample 16004, the cost a is still larger than b , which is the reason why the system switch again. After the switch at sample 16004, a becomes smaller than b . The reason here is that, the room temperature change after the switch at sample 16003 is very little (it might be caused by quantization error), which was not able to make a smaller than b .

One easy way to solve the problem is to use a larger sampling time, this has been tried with 20 seconds in stead of 2 seconds. The problem can be solved, but it is not preferred because we do not exactly know what is the limit of the sampling time that will cause this problem.

A better way to solve it is to add an extra condition for the switch criterion. The extended switch criterion is at the last sample $a - b < 0$ and at current sample $a - b \geq 0$, which earlier was only $a - b \geq 0$ at the current sample. This extra condition helps to avoid the switch right after another switch due to the slow change of the room temperature or the quantization error. The result of the full period adaptation algorithm with such a correction (we call the full period adaptation with adaptation 'the new adaptation method' later in this paper.) is shown in Fig.6 the black curve, where the

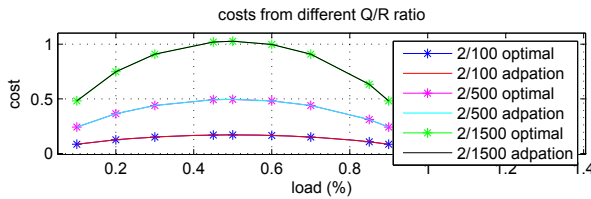


Fig. 11. Experiment result of the new method with different Q/R ratio compared with the optimal solution.

'bad' initial 25°C the room temperature converges to the optimal solution (with legend 'flag full $x_0=25^{\circ}$ '). Now we have found a method, which gives results very close to the optimal solution and which is not sensitive to the initial condition. For the initial condition 22.8°C , the full period result with or without this correction is the same, because $a - b \geq 0$ happens only at the switch sample, otherwise $a - b < 0$.

From the example shown in Fig.6, it can be seen that with good initial condition (22.8°C), the results converges to the optimal solution after one optimal period length and with a 'bad' initial condition (25°C), the room temperature converges to the optimal solution after three optimal period length.

Different Q/R ratio has also been tested with this new adaptation method, and the costs are compared with the optimal solution for different load. The results can be seen in Fig.11. It can be seen that the new adaptation method gives optimal solution under different Q/R and different load.

We have suggested also another idea using half and full period together to solve the problem, but it will not be studied here because the fixed full period method works very good, and that if we first use half period and then switch, we will not get a faster converging rate.

A note has to be made here that, with simple modification the algorithm can be made to work even under the conditions corresponding to infinitely long cycle times. An example is to start with a room temperature that is far from the reference for example 28°C (assume that the outdoor is also 28°C). The adaptation method does not work, because the error accumulation is very big at the beginning due to the difference between initial and reference and therefore the term a can not catch up with the term b . In this case, much higher power ($>300\text{w}$) will be continuously (corresponding to infinitely long cycle times) used to bring the room temperature down close to the reference at the beginning, and later when the load is less than 100%, (300W) running continuous later, then the system will switch to this control algorithm.

V. EXPERIMENTS WITH 2^{nd} ORDER SYSTEM

In last section, a method of controlling switch system has been developed based on a 1^{st} order system, but the AC-system behavior is more like a second order system. Therefore we would like to see how this method work with a 2^{nd} order system. Some investigations have been done and they show that this method only works with 2^{nd} order system

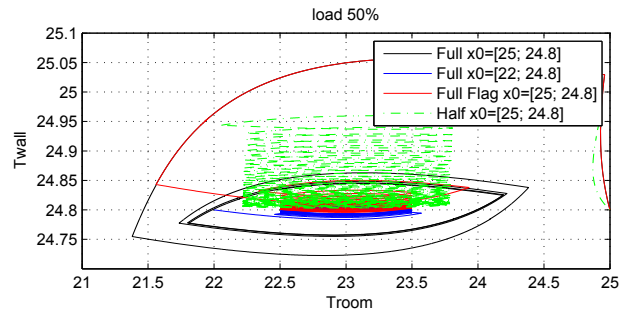


Fig. 12. Phase plot of second order example.

with real poles. Therefore here we focus only on 2^{nd} system with real poles. The reason we toyed with the first order example was that it has much less computational load, and that the results are easy to visualize graphically.

By experiments, we have found out that for a second order system using the full period adaptation gives better solution than the half period, and that the full period adaptation method is sensitive to the initial conditions without the extra switch condition. These are the same results as for the first order systems. The problem with initial condition for the full period adaptation is the same as for first order systems - it results in a second order limit cycle with some initial conditions, and it has the same problem in the a and b values where after a switch, the sign of $a - b$ is not changed because of the reaction on temperature is slow or the quantization error. The result of applying the new adaptation method (full period with the extra switch condition) is experimented and it gives a solution which seems to be the optimal. The results from different methods are shown in Fig.12. The initial condition x_0 here is the temperature of room and wall.

Among the green (half period method), blue (full period $x_0=[22^{\circ}\text{C} \ 24.8^{\circ}\text{C}]$) and black (full period, $x_0=[25^{\circ}\text{C} \ 24.8^{\circ}\text{C}]$), the blue curve represents the smallest limit cycle. The black curve converges to a much larger one, which is caused by the 'bad' initial condition. The red curve uses the new adaptation method (full period method with the extra switch condition), and it proceeds very close to the limit cycle illustrated by the blue curve. This example shows that the new adaptation method adapts to a limit cycle like the optimal solution.

VI. COMPARISON BETWEEN THE NOVEL METHOD AND HYSTERESIS CONTROL

A comparison of the adaptation method and a traditional hysteresis controller is carried out to find out how much we can gain by using the new developed adaptation method.

First step is to tune the hysteresis controller at 50% load, such that the best possible hysteresis bounds can be found, i.e which gives the same cost as the optimal solution. The results are shown in Fig.13. It is obvious that 0.5°C deviation (i.e. hysteresis bounds equals the reference $\pm 0.5^{\circ}\text{C}$) gives the cost closest to the real optimal solution at 50% load.

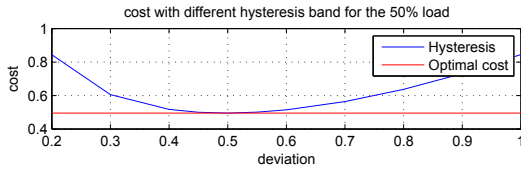


Fig. 13. Tuning of the hysteresis controller.

The next step is to run the tuned hysteresis controller with the AC-system under different loads. The results can be seen in Fig.4 (black curve), where the cost, period, duty cycle from different loads are plotted. Obviously the hysteresis controller tuned at 50% load has results very close to the optimum at this load. Away from 50% load, the results are deviating from the optimal solution, for example at 90% load, the cost of hysteresis control has 20% higher cost than the optimal solution. It can be seen that the new adaptation method attains the optimal solution as stated previously. Further more the adaptation method converges quite fast which has been discussed in last section.

VII. CONCLUSION

The paper has proposed a novel low complexity adaptation method for systems with discrete inputs that have costs related to switching. It should also be noted that the method currently works only for single input and single output systems. The method has been tested with first order systems which achieves the optimal solution and second order systems which in general also seems to attain the optimal solution reasonably fast. A comparison between the developed adaptation method and traditional hysteresis control was made. It showed that the performance of the hysteresis controller tuned at a nominal working condition gives suboptimal results when the working condition changes. In this example, the cost from the hysteresis controller is 20% higher than the optimal solution. The new adaptation method gives the optimal solution regardless at different load and different initial conditions. The complexity of the adaptation method is not increasing dramatically compared to the hysteresis control. In fact it requires only three states more than the hysteresis controller, the period for the last half period, the comfort error accumulation for the last half period, and the current period comfort error accumulation.

Further studies on this method will be focused on first order system with delay. In reality systems are often delayed or high order. The complexity of the presented method increases dramatically with system size, but often higher order system can be approximated as first order system and delay. Experiment will be carried to validate the method.

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J	cost per time unit
J_{sw}	cost for two switches in one period
J'_{sw}	$J'_{sw} = \frac{1}{2} J_{sw}$ before the first switch. $J'_{sw} = J_{sw}$ after the first switch.
Q	penalty on temperature deviation
R	penalty on switch
a	current error
b	accumulated error and switch cost
T	cycle period
α	duty cycle
T_{last}	last half period
T_{now}	the current half period
x_1	temperature when AC-system is on
$x_{1,init}$	temperature at the beginning of on period under stable limit cycle
$x_{1,end}$	temperature at the end of on period under stable limit cycle
x_2	temperature when AC-system is off
$x_{2,end}$	temperature at the end of the off period under stable limit cycle
$x_{t0+T_{now}}$	current measured temperature
T_{sample}	sampling time
T_{room}	room temperature
T_{amb}	ambient temperature
Q_{a2r}	energy from ambient to the room
Q_e	cooling energy from AC-system
Res	thermal resistance from ambient to the room

TABLE I
NOMENCLATURE.

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