A Sensor Fusion Approach for Exploiting New Measurements in an Existing Controller

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Abstract— **This paper considers the problem of improving an original controller once a number of new measurements become available. We consider the situation, where for commissioning reasons or other reasons, it is required that the original controller stays in place (e.g. in case it is integrated in the actuator). Therefore, the new measurements can be used only to improve the inputs of the original controller. To that end, a sensor fusion approach is taken. An observer based architecture is proposed. For this observer based approach, a separation theorem is proven, which under certain necessary and sufficient conditions specialize to a double separation principle. The main novelty in the paper is the incremental approach, which is obtained by combining a couple of standard techniques. A numerical example is given, which illustrates the potentials for improvements of performance.**

I. INTRODUCTION

An advanced (typically model based) control system is usually expensive to implement for a large scale system, as e.g. obtaining and verifying the model consumes significant time, and so does the controller design, verification and testing.

One of the blocking factors for spreading advanced control technology even more widely is the fact that real plants are not static. Most large plants are constantly being re-engineered. Components will be replaced with other components that are not identical. Subsystems might be added or removed. Instrumentation might be changed in terms of adding or removing sensors and/or actuators.

Unfortunately, it has been studied only very sparsely in the control community how to design control systems, that are robust to the kind of changes outlined above. This means, that the expenses to the control system will accumulate, as the control system needs to be re-engineered along with the dynamic changes of the plant itself. To some industries, this added cost might be preventive.

In a number of recent papers, this challenge has been addressed, see [1], [2], [3], [4], [5], [6], [7], [8]. In these papers, it is in particular studied, how an advanced control system can be equipped with intelligence, that facilitates that it adjusts itself automatically in the face of changes e.g. in instrumentation.

Along this line of research, in the present paper it is studied how to modify a controller, once it is allowed to exploit a larger number of sensors. In particular, it is assumed that the control signal should still be generated by the existing (original) controllers. Thus, only the inputs to the controllers are allowed to change with the new measurements. This could be relevant in a number of cases, e.g. if the existing controller is integrated in the actuator hardware, or if the existing control algorithm is embedded in a block of software which is inaccessible for various reasons, e.g. for practical reasons, for reasons of software vendor legal issues, or due to quality management rules.

The approach taken will be sensor fusion based, see e.g. [9], [10]. In a wide range of literature it is described how to implement controllers based on sensor fusion, see e.g. [11] and references therein. In the present paper, however, we specifically address the situation of fusing new measurements with existing ones in order to modify the inputs to an existing controller, such that the overall performance increases.

To that end, an observer based architecture will be proposed below. For this architecture, a (single) separation principle will be demonstrated, allowing the added observer poles to be designed separately. Further, it will be shown that if certain constraints are imposed on the design parameters, a double separation principle will hold, where all the involved poles can be designed separately as either observer poles or state feedback poles.

A numerical example illustrates both a design procedure based on the single and double separation principle. It turns out, that the general method in most cases

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can lead to better performance than the method based on the double separation principle.

II. AN OBSERVER BASED APPROACH

In the sequel, we shall describe an approach to replacing the inputs to a controller with new such, based on additional measurements. The approach will be based on the architecture illustrated in Figure 1. In this architecture, the inputs to the original controller, which is assumed to be a full order observer based controller, are generated as outputs from a new observer, including a direct feed-through term from the measurements.

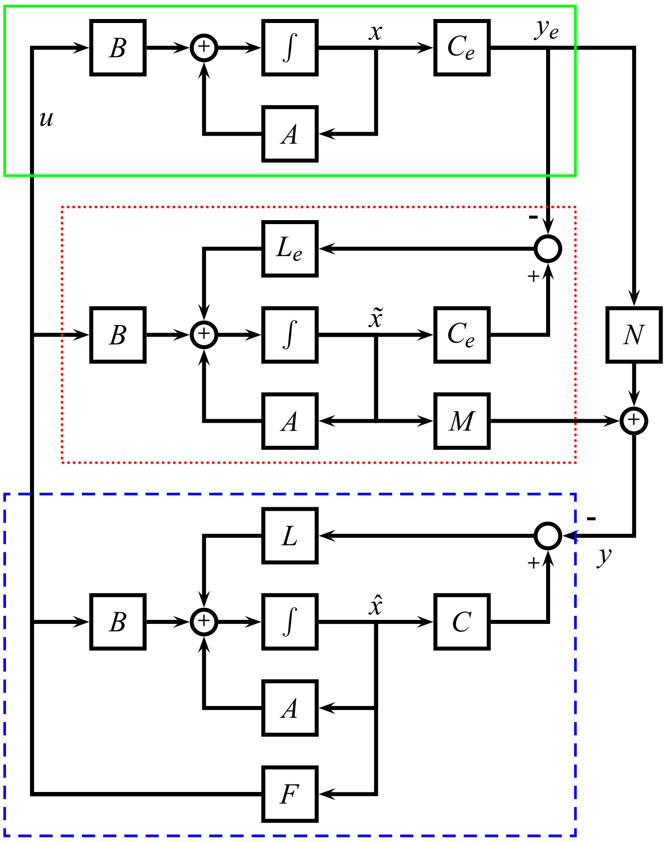

Fig. 1. Proposed architecture with additional observer. Solid box: system; Dotted box: new observer; Dashed box: original controller.

The original system to be controlled is described by a state space model of the form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

It is thus assumed that the system has no direct feedthrough term. As usual, a direct feedthrough term, D, can be handled by subtracting the signal Du in the observers described below.

After adding new sensors, the system will be described by a a state space model of the form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y_e &= C_e x \end{aligned} \tag{2}$$

where typically C_e will take the form:

$$C_e = \begin{pmatrix} C \\ C_{\text{new}} \end{pmatrix}$$

The existing compensator is assumed to be a full-order observer based controller:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(C\hat{x} - y) \\ u &= F\hat{x} \end{aligned} \tag{3}$$

This assumption is fairly mild, in fact under mild conditions, any controller can be written as an observer based controller, see [12]. In order to exploit the new outputs, an additional observer is introduced:

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + Bu + L_e(C_e\tilde{x} - y_e) \\ y &= M\tilde{x} + Ny_e \end{aligned} \tag{4}$$

where L_e, M and N are design parameters, see below.

To summarize the properties of the proposed architecture, we have the following separation principle.

Theorem 1: Consider the configuration illustrated by Figure 1, where a system given by the state space model (2) is controlled by an observer based compensator, designed for an original system (1), where the input to the controller is generated by an additional observer of the form (4).

This closed loop system has poles given by the eigenvalues of the two matrices:

$$A + L_eC_e \quad \text{and} \quad \begin{pmatrix} A+BF & BF \\ L(C-M-NC_e) & A+LC \end{pmatrix}$$

In the special case, where M and N are chosen to fulfill

$$M + NC_e = C \tag{5}$$

then the closed loop system satifies a 'full' separation principle, i.e. the closed loop poles are given by the eigenvalues of the three matrices:

$$A + BF\,, \quad A + L_eC_e \quad \text{and} \quad A + LC$$

which means that observer and feedback gains can be designed independently, if only the closed loop poles are of concern.

Proof: The proof follows directly from manipulations of the closed loop state space formulation, which is given by:

$$\dot{\begin{pmatrix} x \\ \tilde{x} \\ \hat{x} \end{pmatrix}} = \begin{pmatrix} A & 0 & BF \\ -L_eC_e & A+L_eC_e & BF \\ -LNC_e & -LM & A+LC+BF \end{pmatrix} \begin{pmatrix} x \\ \tilde{x} \\ \hat{x} \end{pmatrix}$$

Applying the state space transformation:

$$x_{\text{new}} = \begin{pmatrix} I & 0 & 0 \\ I & I & 0 \\ I & 0 & I \end{pmatrix} \begin{pmatrix} x \\ \tilde{x} \\ \hat{x} \end{pmatrix}$$

yields a new state space model for the closed loop system of the form:

$$\dot{x}_{\text{new}} = A_{\text{new}} x_{\text{new}}$$

where

$$A_{\text{new}} = \begin{pmatrix} A+BF & 0 & BF \\ 0 & A+L_eC_e & 0 \\ L(C-M-NC_e) & -LM & A+LC \end{pmatrix}$$

From this it is clearly seen, that the set of closed loop eigenvalues can be separated into the union of the sets of eigenvalues of the following two matrices:

$$A+L_eC_e \quad \text{and} \quad \begin{pmatrix} A+BF & BF \\ L(C-M-NC_e) & A+LC \end{pmatrix}$$

In the special case, where M and N are chosen to fulfill

$$M+NC_e = C$$

then the second matrix above specializes to:

$$\begin{pmatrix} A+BF & BF \\ 0 & A+LC \end{pmatrix}$$

from which it is seen that we have a full separation of the closed loop eigenvalues into the eigenvalues of the three matrices:

$$A+BF\,, \quad A+L_eC_e \quad \text{and} \quad A+LC$$

■

The intuition for the condition (5) is that the new input to the original controller is generated as a interpolation between the original measurements and an estimate of the original measurements based on the original and the new measurements. Therefore, if the new measurements are of a poor quality, (5) will specialize to

$$NC_e \approx C\,, \quad M \approx 0$$

On the other hand, if the new measurements are highly superior to the original measurements, (5) will specialize to

$$M \approx C\,, \quad N \approx 0$$

Although Theorem 1 suggests that the new observer can be designed independently of the existing controller, it should be noted, however, that the new observer can introduce a significant phase shift, which should be taken into consideration in the design process. In fact, practical experience with the method shows that better results can in general be achieved, if M and N are chosen, such that (5) is *not* satified.

It should also be noted, that if (5) is not satisfied, then $\tilde{x}$ is still an estimate of x, whereas $\hat{x}$ can not be assumed to be an estimate of x. Thus, if the original controller to some extent relies on having a reliable estimate, then the M and N should be chosen to satisfy (5).

It is not in itself surprising that a better result can be achived, if (5) is not imposed as a constraint. In fact, in that case, the combined new controller, consisting of the original controller and the new observer, is allowed to increase the gains of the system, based on the improved measurement situation. The main disadvantage of pursuing a design that does not satisfy (5) is that the link between design parameters and design objectives becomes more complicated, and that typically an optimization procedure will be involved in the design.

III. Example

In this section, we consider a random third order example, described by the following state space description:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

where

$$\begin{aligned} A &= \begin{pmatrix} -0.5277 & -0.0244 & -0.8930 \\ -0.5113 & -0.6620 & -1.3970 \\ 0.3326 & -1.1656 & 0.4068 \end{pmatrix}, \\ B &= \begin{pmatrix} -1.3676 \\ -1.1816 \\ 1.5603 \end{pmatrix}, \\ C &= \begin{pmatrix} 2.1565 & 2.2956 & -0.6431 \end{pmatrix} \end{aligned}$$

For this system, an observer based controller is designed such that the feedback poles are $\{-1,-2,-3\}$ and the observer poles are $\{-4,-5,-6\}$. With positive feedback convention (see Figure 1), this is achieved by:

$$F = \begin{pmatrix} -8.6858 & 6.4133 & -6.1004 \end{pmatrix},$$

$$L = \begin{pmatrix} 11.6089 \\ -14.6535 \\ 8.7287 \end{pmatrix}$$

giving rise to the following nominal controller

$$K(s) = \frac{248.1s^2 + 279.8s - 193.3}{s^3 + 20.22s^2 - 1493s - 2216} \tag{6}$$

(which is likely to cause implementation challenges, as it is open loop unstable and non-minimum phase. That is, however, besides the point).

The control objective is disturbance attenuation, i.e. to have $y(t) \approx 0$, for $t > 0$.

A new sensor is introduced, such that the extended system is described by:

$$\begin{array}{rcll} \dot{x} & = & Ax & + \; Bu \\ y_e & = & C_e x & \end{array}$$

where

$$C_e = \left(\begin{array}{ccc} 2.1565 & 2.2956 & -0.6431 \\ \hdashline -1.4058 & 1.3159 & -1.4420 \end{array} \right)$$

In the sequel, we shall illustrate two design approaches that design a new input for the nominal controller (6) using the additional sensor by means of a new observer, introduced between the plant and the original controller.

As a first design approach, we shall try to use the full separation approach of Theorem 1. In this case, we will use a fully filtered input to the original controller, meaning that we choose $N = 0$. From the necessary and sufficient condition for separation (5), we then obtain:

$$M = C - NC_e = C$$

The new observer is designed to have observer poles in $\{-70, -80, -90\}$, which is obtained by the observer gain:

$$L_e = 10^3 \times \left(\begin{array}{rr} -0.5476 & 0.0454 \\ 0.8009 & -0.0521 \\ 1.2697 & -0.0361 \end{array} \right)$$

Figure 2 shows a simulation of this case, where the system is subjected to an input disturbance:

$$d = 2\sin(2\pi t) + w_1\,, \quad w_1 \in N(0,1)$$

and each of the measurements are subjected to independent noise sources that are normally distributed with unit variance.

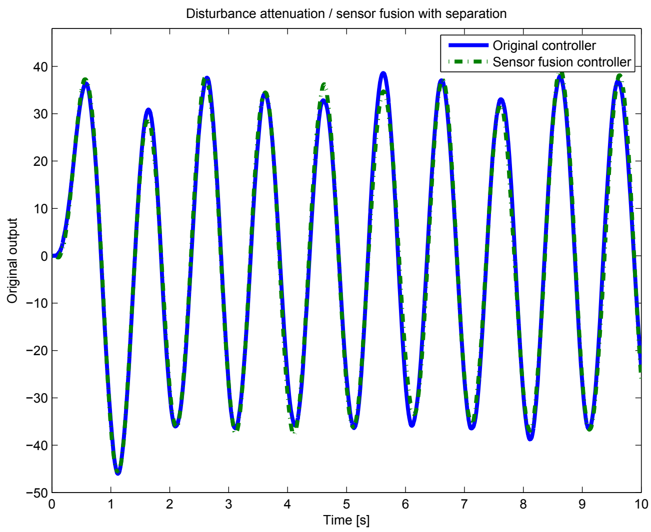

Fig. 2. Simulation of a disturbance attenuating controller using a new output based on a design with full separation. The improvement of the variance is marginal, and can hardly be discerned.

It is seen from Figure 2 that the improvement is very marginal. This discouraging result is due to two effects:

- Adding the new observer increases the phase lag of the controller
- Adding the new output also introduces more noise

It should be mentioned, however, that it is easy to come up with examples where the advantage is more obvious, e.g. in a series connected system, where a more upstream measurement is added. For random examples, though, the situation depicted here is fairly generic.

We now proceed to a design, where we deviate from the principle of full separation. Intuitively, this means that we are allowed to increase the gain of the controller, as the measurements become more reliable. Again we design a fully filtered input to the original controller, meaning that we pick $N = 0$. In this case M was found through a small optimization based on a loop transfer recovery condition. This was done by studying the loop gains from disturbance and noise, respectively, following the transients for which *uniform* improvements were obtained. As a result, the following parameter values were found:

$$M = \left(\begin{array}{ccc} 2.6 & 3.4 & -1.2 \end{array} \right)$$

A simulation result based on this design with the same inputs as in the former design case, is shown in Figure 3.

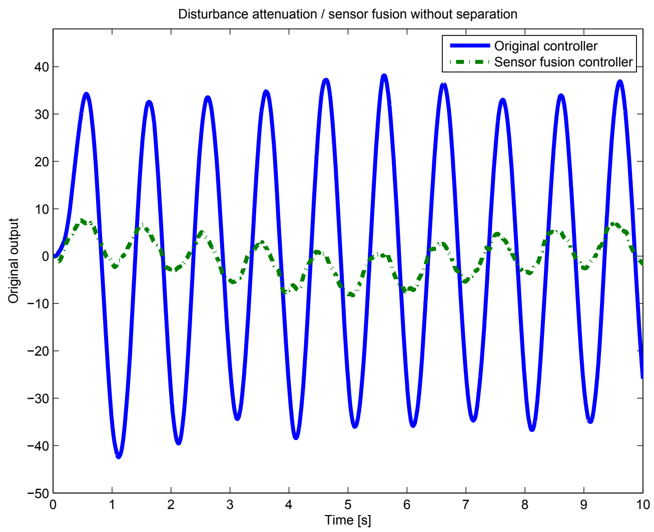

Fig. 3. Simulation of a disturbance attenuating controller using a new output based on a design with partial separation only. A radical improvement of the variance is seen, and the attenuation of the sinusoidal disturbance is significant.

This time, a significant performance improvement is seen. This is due in part to an improved estimate, and in part to slightly increased gains (approx. 15%).

IV. Conclusions

This paper has studied the problem of improving the control of a system, once a number of new measurements become available. A precondition has been that the original controller should stay in place, only its inputs could be changed.

To that end, an architecture has been presented, which introduces an additional observer between the outputs of the system and the inputs of the original controller, and is thus essentially relying on sensor fusion between the original and the new measurements.

It has been demonstrated that the proposed scheme satisfies a separation principle, such that the poles of the new observer can be designed separately. In fact, by introducing a constraint between some of the design parameters, a complete separation principle will hold, i.e. if the original controller was observer based, then the closed loop poles will be those of the original feedback, the original observer, and of the new observer.

By virtue of an illustrative example, two designs were given. In the first design, the full separation property was established. In that case, however, the performance improvement was marginal. Other examples can be found, where the improvement is larger. However, simulation experience shows that for random systems, the modest results presented here are typical.

In the second design, however, where only the new observer poles could be designed separately, a significant performance improvement was accomplished, and the output variance was decreased by almost an order of magnitude.

Against the proposed approach, it could be argued that it is better to replace the original controller altogether if new instrumentation is introduced. This is true, although it is not always admissible to remove the original controller, as it can be hardware integrated, or embedded in a larger software structure, where it can not be removed for practical or legal reasons.

Also, it could be argued that as rather than introducing an observer between the plant and the existing controller, one could simply perform an additional controller design, where the series connection of the plant and the original controller is considered to be a new 'plant' to be controlled. This can definitely be done, but has the drawback that the new design is not explicitly built on an observer. Furthermore, e.g. an optimal controller design will lead to a controller order twice that of the one suggested in this paper, as its order would be the sum of the plant order and the original controller order.

The results given in this paper have been formulated for continuous time systems. As all derivations rely on simple separation results only, all the results hold in (single rate) discrete time as well without modification.

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