Thermal Analysis of an HVAC System with TRV Controlled Hydronic Radiator

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Abstract— **A control oriented model for an HVAC system is derived in this paper. The HVAC system consists of a room and a hydronic radiator with a temperature regulating valve (TRV) which has a step motor to adjust the valve opening. The heating system and the room are simulated as a unit entity for thermal analysis and controller design. A discrete-element model with interconnected small scaled elements is proposed for the radiator. This models the radiator more precisely than that of a lumped model in terms of transfer delay and radiator gain. This precise modeling gives us an intuition into a regular unwanted phenomenon which occurs in low demand situations. When flow is very low in radiator and the supply water temperature and the pressure drop across the valve is constant, oscillations in room temperature frequently occur. The model derived in this paper demonstrates that the oscillations are in part due to the large gain of the radiator in low demand conditions compared to the high demand situations. The simulation model of radiator is optimized in terms of approximating the small signal gain of radiator in all operating points accurately. The controller designed for high demand weather conditions is applied to the opposite conditions to illustrate the oscillatory condition more apparently. Suggestions to alleviate this situation are proposed.**

I. Introduction

In recent years, there has been a growing interest in thermal comfort analysis and control of heating, ventilation and air conditioning (HVAC) systems. Efficient control of HVAC systems has a great influence on the productivity and satisfaction of indoor residents. The other important objective of a well designed control strategy is energy savings, mainly because of the growth of energy costs, consumptions and also correlated environmental effects.

Design of more sophisticated HVAC control systems requires development of more advanced modeling and analysis tools to be able to assess the performance of new systems. For instance, consider the following example:

In a typical Danish residential building, hydronic radiators are used to provide heat demands. Each radiator is equipped with a temperature regulating valve (TRV) that maintains the temperature of each individual room close to its respective desired value. Such flexibility serves both above mentioned objectives. Thermal analysis of the experimental results of renovating a villa built before 1950 has demonstrated that energy savings near 50% were achieved by mounting TRVs on all radiators and fortifying thermal envelop insulation [1].

In this paper, we have modeled an HVAC system consisting of a room and a radiator with a TRV valve. We analyzed the thermal behavior of the radiator integrated into the room to achieve more comprehensive perception of the radiator behavior.

Generally, there are two different approaches to find a comprehensive model for HVAC systems. The first one is based on using physical characteristics and energy balance equations of the air, structural mass and other components of the system. For instance, [2] has proposed a methodology for building thermal control and energy analysis to make a unified approach in this area. Using thermal network models, Laplace transfer functions of the building are obtained. The frequency where short-term and long-term dynamics of convective loads can be separated is obtained in this work. Another example where the first approach was employed is to find a simple lumped capacitance model analogous to an RC electrical circuit in residential buildings [3] and also in commercial buildings [4] where the thermal capacitance is higher than the residential ones. By simulating the first and second order linear models in Simulink®, [3] and [4] have reported that there is no tangible advantage of using higher order models in short term simulations.

The alternative approach to find an integrated system model is to use building measured data with inferential and statistical methods for system identification. A grey box modeling approach is used in [5] to derive a simple stochastic continuous-time model for an experimental data set. It is explicitly described how the measurement and model errors enter into the model. Other methods based on experimental data include: utilizing pseudorandom binary sequences of the input to derive the heat dynamics [6], and employing an inverse grey box thermal network model for transient building load prediction [7].The main drawbacks of this approach are that they require a significant amount of training data and may not always reflect the physical behavior [8].

The TRVs give good comfort under normal operating conditions. To follow reference in a high load, a high controller gain is required. A minor drawback arises when the heat demand is low and the pressure drop across the valve is constant with constant supply water. In this

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circumstance, oscillation occurs due to the high system gain. This problem is addressed in [9] for central heating systems with gas-expansion driven TRVs. They control the differential pressure across the valve to keep the TRV in a suitable operating point using an estimate of the valve position.

In this paper, we have used the lumped capacitance model for a single room having one radiator as the only heat source. The TRV of the radiator has a step motor that adjusts the valve opening with a good resolution. Our goal is to study the thermal behavior of the radiator and the integrated HVAC system.

Two models for radiators are presented in this paper. The first one is a simple lumped capacitance model which frequently has been used in literature. This method models the radiator as a single lump with a constant temperature along its surface. The other model is a discrete-element model that consists of *N* lumps and considers the temperature distribution along the radiator surface. The latter model provides us with a more precise dynamical description of radiator by considering the transfer delay of each element of the radiator separately. To have a precise simulation model of radiator, the latter model is optimized in terms of the number of elements.

The rest of the paper is as follows: Section II describes the model of each component of the HVAC system. Section III finds the best simulation model through simulations. The drawback of TRVs is brought up in this section and two PI controllers are designed, one for low demand and the other for high demand situations. Suggestions for alleviating the situation are recommended in this part. Finally the conclusions are given in Section IV.

Table I.
Symboles and Subscripts

Nomenclature and Subscripts	
A	surface area (m^2)
C	thermal capacitance (kJ/kg°C)
c_w	thermal capacitance of water (kJ/kg°C)
d_v	degree of valve opening
h_r	convection heat coefficient (kW/m^2°C)
K	constant coefficient
N	number of radiator elements
q_m , q	flow of water (kg/s)
Qe	internal heat gains
Q_s	Solar radiation through glazing
Q_f	heat supplied by floor heating
Q_{rad}	heat supplied by radiator
T	temperature (°C)
U	thermal transmittance (kW/m^2°C)
V	volume (m^3)
ρ_w	density of water (kg/m^3)
w	wall
f	floor
r	room
rad	radiator
sen	sensor
trv	TRV

II. Dynamic Model of the Room and Hydronic Thermostatic Radiators

The integrated system includes the zone which is a single room and a radiator as the heating system. Lumped models for both zone and radiator are presented in this section. The lumped model of the room is mainly based on the model used in [4] with some modifications.

All symbols and subscripts are introduced in the Table I.

A. Room Model

The lumped capacitance model is analogous to the building thermal network. This model has been frequently used in the literature both for residential and commercial-scaled buildings [3,4,10,11]. The validity of the model has been extensively investigated through experiments in the literature. The energy balance equations can be derived applying Kirchhoff's current law in each node of the network. [10].

Energy balance equations are derived based on the analogous electrical circuit of the HVAC system which is depicted in Fig. 1. There are some minor changes in the presented model with regard to the original model in [4]. In this model all walls, ceiling and glazing are assumed to have the same characteristics which generally we call envelop parameters and show them in the equations with the state variable T_w . Also, two separate temperature layers for floor are considered in the model to make the model extendable to the situation where floor heating is available as another heat source.

Energy balance on the envelopes gives:

$$C_w \dot{T}_w = U_{wo} A_w (T_{out} - T_w) + U_{wi} A_w (T_r - T_w) + (1-p) Q_s \quad (1)$$

Energy balance equations on two layers of floor:

$$\begin{aligned} C_{f_1} \dot{T}_{f_1} &= h_r A_f (T_r - T_{f_1}) + U_{f_1} A_f (T_{f_2} - T_{f_1}) + p Q_s \\ C_{f_2} \dot{T}_{f_2} &= U_{f_1} A_f (T_{f_1} - T_{f_2}) + U_{f_2} A_f (T_g - T_{f_2}) + Q_f \end{aligned} \quad (2)$$

Energy balance equations of the indoor air:

$$C_r \dot{T}_r = U_{wi} A_w (T_w - T_r) + h_r A_f (T_{f_1} - T_r) + Q_{rad} + Q_e \quad (3)$$

The equivalent state-space model can be written as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (4)$$

where x is a vector of state variables which contains room temperature T_r, envelop temperature T_w, and floor temperatures $T_{f1}$and T_{f2}. A and B are the matrices of coefficients. u is the vector including system inputs that includes outdoor ambient temperature T_{out}, the heat supplied by the radiator Q_{rad}, floor heating Q_f, internal heat gains Q_e, solar radiations through glazing $Q_s$which p fraction of it absorbed by floor and the rest by walls. Matrix C indicates which of the state variables are measurable.

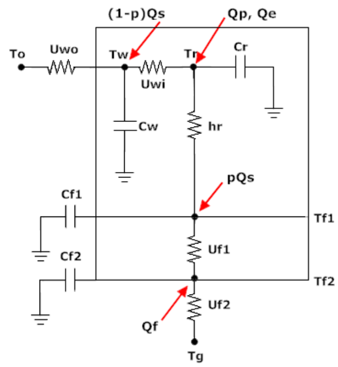

Fig. 1. Lumped capacitance model of the room

B. Radiator Model

The lumped capacitance model of the radiator is depicted in Fig 2. If we assume an ideal radiator, with enough surface area, it can transfer the heat of hot water very well to the ambient air in the room. Then, the outlet water temperature would be around the room temperature.

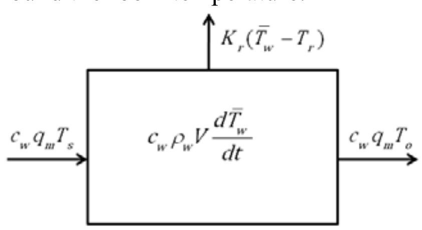

Fig. 2. Lumped model of Radiator

Knowing the supply and outlet water temperature, the energy balance of the model is

$$c_w \rho_w V \dot{\bar{T}}_w = c_w q_m T_s - c_w q_m T_o - K_r(\bar{T}_w - T_r) \tag{5}$$

In which $\bar{T}_w$ is the spatially averaged water temperature of the radiator. K_r denotes the equivalent heat transfer coefficient which depends on the temperature difference between radiator water and ambient temperature that is $K_r = F(\bar{T}_w - T_r)^{\alpha}$. α is a characteristic coefficient of the radiator which usually equals 1.3 [12]. F is the radiator surface area. With this definition K_r is not constant and makes the energy balance equation of the radiator nonlinear. A simplifying assumption is presuming the same temperature for radiator surface as the water temperature inside radiator. By this assumption, the heat from the radiator is transferred to the ambient only by convection. In this case, equivalent heat transfer coefficient can be described as $K_r = h_r A_{rad}$ in which h_r is the convection heat transfer coefficient of the room air. Consequently the energy balance equation of the radiator gives a bilinear model for radiator.

The power transferred to the room can be calculated using the average water temperature $\bar{T}_w$ as below:

$$Q_{rad} = K_r(\bar{T}_w - T_r) \tag{6}$$

The lumped model does not consider the transfer time of water along the radiator. For high flows, the transfer delay is very small which is not considerable with respect to the settling time of the room temperature. But this parameter becomes more important when the demand for heat and consequently the flow is very low.

A discrete-element model for radiator considers the transfer delay of water separately for each element and gives a more precise description of the temperature profile along the radiator. In this paper we proposed a discrete-element model which differs from what is presented by [5]. The proposed model is an extension of the lumped model. Considering N elements in this model, the energy balance equation for the water of each element is as follows:

$$V_n c_w \rho_w \dot{T}_n = c_w q(T_{n-1} - T_n) + K_r(T_r - T_n) \tag{7}$$

in which T_n is the temperature of the water in the n^{th} element of radiator, $n = 1, \ldots, N$ and N is the number of elements knowing $T_0 = T_s$ supply water temperature. The state space equations of radiator can be written as:

$$\dot{\bar{T}} = A(q)\bar{T} + B(q)w \tag{8}$$

with

$$\begin{aligned} \bar{T} &= [T_1, T_2, \ldots, T_N]^T \\ w &= [T_s, T_r]^T \end{aligned} \tag{9}$$

$$A = \begin{bmatrix} \left(-k - \frac{q}{\rho_w V_n}\right) & 0 & \cdots & 0 \\ \frac{q}{\rho_w V_n} & \left(-k - \frac{q}{\rho_w V_n}\right) & & \vdots \\ 0 & \ddots & \ddots & 0 \\ \vdots & 0 & \frac{q}{\rho_w V_n} & \left(-k - \frac{q}{\rho_w V_n}\right) \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} \frac{q}{\rho_w V_n} & k \\ 0 & k \\ \vdots & \vdots \\ 0 & k \end{bmatrix} \tag{11}$$

in which $V_n = \frac{V}{N}$ and $k = \frac{K_r}{V_n c_w \rho_w}$. Vectors $\bar{T}$ and w are the states and disturbances for the radiator system. The disturbances are T_s, supply water and T_r, room temperature. The input of the system is q which is the flow of the water.

C. Model of TRV

The TRV has a step motor to control the valve opening. The static relation between opening degree of valve and the flow can be modeled statically by a three order polynomial equation with the coefficients $[-0.00033562, 0.75391, 0]$.

D. Model of Temperature Sensor

A first order model is considered for the temperature sensor with the time constant τ_s as follows:

$$\tau_{sen} \dot{T}_{sen} = T_r - T_{sen} \tag{12}$$

III. Simulation and Controller Design

The HVAC model is implemented in Matlab/Simulink. A single room with the dimensions of 4×7×2 m^3 and a radiator with $5m^2$ area and water capacity of $0.005m^3$ are considered. The constructions materials and properties are chosen based on the ASHRAE handbook [13]. In simulations solar heat, internal heat and floor heating are ignored. Outside temperature and heat from the radiator are the only disturbances that affect the system.

In the first experiment we find the best number of elements for simulation model of the radiator. The bigger the *N* the more accurate radiator model is; but in order to reduce the simulation time we need to choose the smallest *N* which gives an acceptable precision.

In order to choose the suitable *N*, we introduce the small signal gain of radiator which will be utilized further in subsequent parts. The gain can be defined as small perturbation in radiator temperature due to a small change in flow. The other more general measurement of radiator performance could be the small variation in radiator power due to a small change in radiator flow. Based on the first definition the gain $\frac{\hat{T}_n}{\hat{q}}$ can be calculated by considering a small perturbation saying $(\hat{q}, \hat{T}_n)$ around the operating point $(\bar{q}, \bar{T}_n)$ in steady state, as follows:

$$c_w(\bar{q}+\hat{q})(\bar{T}_{n-1}+\hat{T}_{n-1}-\bar{T}_n-\hat{T}_n)+K_r(T_r-\bar{T}_n-\hat{T}_n)=0 \quad (13)$$

Subtracting this equation by

$$c_w(\bar{q})(\bar{T}_{n-1}-\bar{T}_n)+K_r(T_r-\bar{T}_n)=0 \quad (14)$$

We have

$$\frac{\hat{T}_n}{\hat{q}}=\frac{c_w(\bar{T}_{n-1}-\bar{T}_n)}{c_w\bar{q}+k_r} \quad (15)$$

in which T_n is the temperature of the n^{th}element.

Following the second definition, and defining the power of radiator by $Q_{rad}=\sum_{n=1}^{N}k_r(T_r-T_n)$, the small signal gain is:

$$\frac{\hat{Q}_{rad}}{\hat{q}}=c_w(T_s-\bar{T}_N) \quad (16)$$

in which T_s is the supply water temperature and $\bar{T}_N$ is outlet water temperature.

In the second experiment the drawback of temperature regulating valve is illustrated by simulation. While the TRV is working well in the high demand situation, the room temperature oscillates in low demand conditions. This will raise the power consumption and in the case of motor driven TRV, the life time of batteries will decrease which is not desirable.

The open loop system of HVAC implemented in Simulink is depicted in Fig. 3.

A. Experiment 1

We investigate the radiator model in this part independent of the room dynamics. We assume that room temperature is fixed to 20°C and the supply water temperature in fixed to 70°C. Scanning the flow from very small values to its maximum value, the small signal gain of radiator defined by (17) for different number of elements is shown in Fig. 4. Since the transferred power is the only input that room receives from radiator, the second definition for gain is chosen.

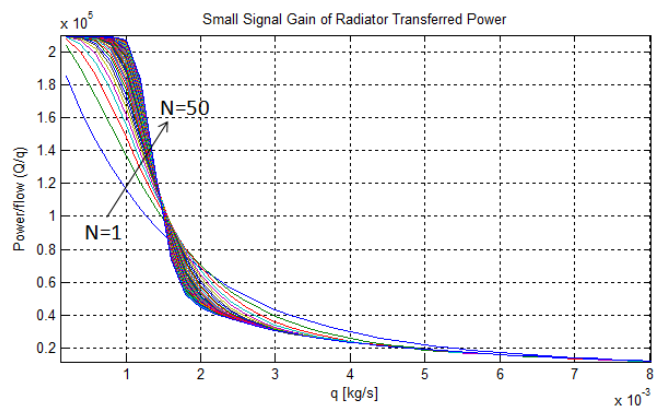

Fig. 4. Small signal gain for different number of elements

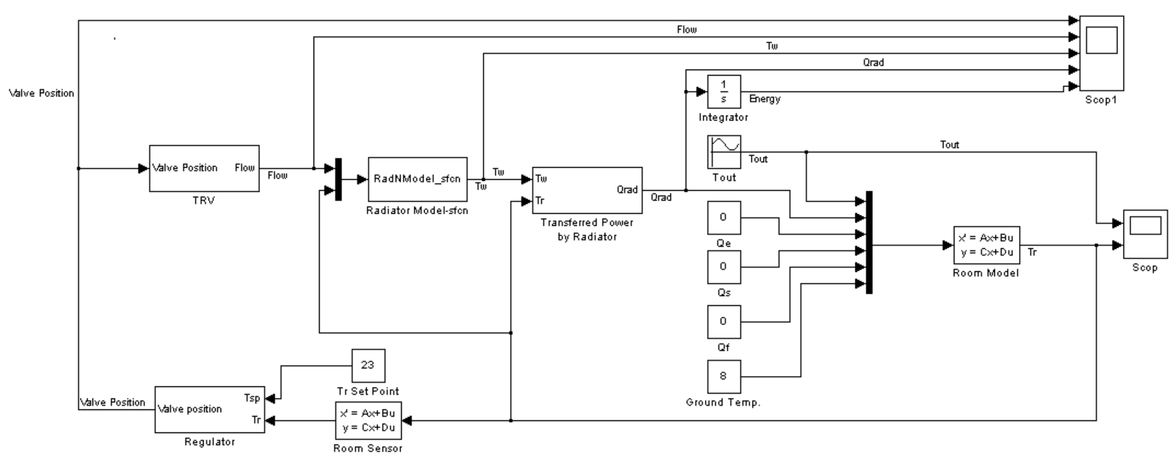

Fig. 3. Control system of a HVAC system including a room and a hydronic radiator with TRV

To choose the best N we introduce the integrated absolute error as the following:

$$IAE_n = \int_{q=0}^{q=q_{max}} |g_N(q) - g_n(q)| dq$$

The maximum error is related to the lumped model which we call it IAE_{max}. The best n is chosen where the $IAE_n \leq 0.01\, IAE_{max}$ which gives *N=45*.

The small signal gain with both definition of (16) and (17) for *N=45* is depicted in Fig. 5.

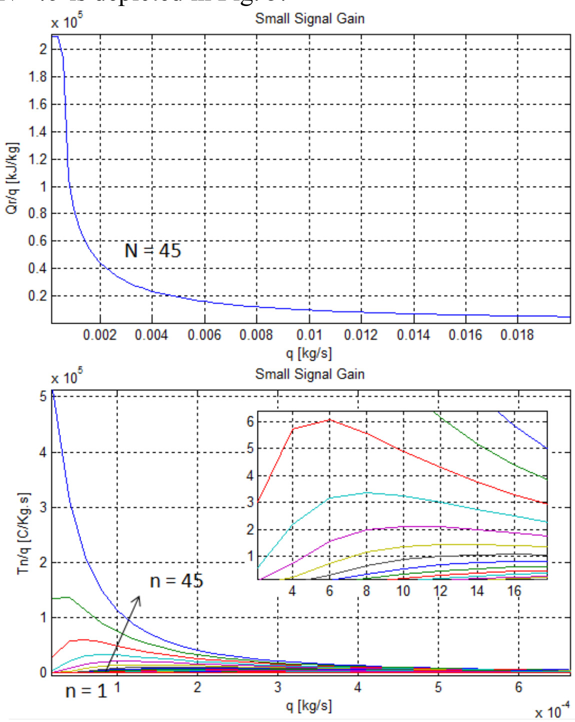

Fig. 5. Small signal gain from flow to transferred power (first), and from flow to radiator temperature for each element (second)

It can be interpreted from the first figure in Fig. 5 that in lower flows, the radiator gain is 40 times the gain in the highest flows. This may result in oscillations in low demand situations.

B. *Experiment 2: Controller Design*

In this section we close the feedback loop and tune a PI controller to regulate the temperature of the room. Using the precise simulation model (*N=45*) achieved in previous part, a PI controller is adjusted for two operating areas. These are high and low demand situations when the outdoor temperature is much lower than the set point or close to the set point.

We used Ziegler-Nichols frequency response to find the initial values for controller parameters. Then parameters are modified to seek desired overshoot and settling time for the controlled system.

Another issue is alleviating the effect of wind up caused by integrator term. Control signal here is the valve position which should be restricted to a determined range. Hence we use saturation to make the signal of controller limited to an area; but the integration of error continuous and the integrator value becomes very large. The control error then has to be of the opposite sign for a long time to bring the controller back to its steady-state. This results in a large overshoot and a high settling time. In this paper we have used the standard structure of a tracking anti-windup PID controller based on what proposed in [14].

To simulate the high demand situation, the ambient temperature is set to -10°C. The parameters of the tuned PI controller are $K = 0.013$ and $T_i = 90$. The initial temperature for the room and radiator is 18°C. The result depicted in Fig. 6 shows that the steady state flow is 89% of q_{max}. According to figure 5, the corresponding gain of radiator in this flow is 5800.

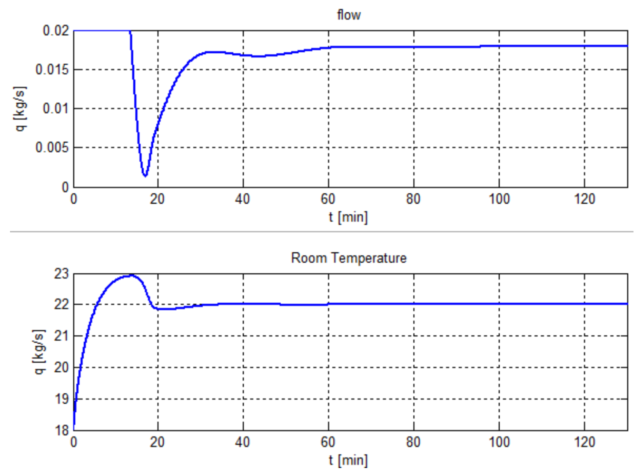

Fig. 6. Response of the flow controlled HVAC system in high demand situation

When the tuned PI is applied to a low demand situation with the ambient temperature 21°C, the response of the system with the same initial conditions is depicted in Fig. 7.

The gain of radiator corresponding to the current low demand condition is 2.093×10^5 which is 36 times the gain in high demand of last simulation. This is one of the reasons of oscillatory behavior which we observe in Fig. 7. These oscillations will increase the power consumption which is not desirable. In our case, another drawback is that the battery of the motor driven TRV will wear out much faster than in the normal operating conditions.

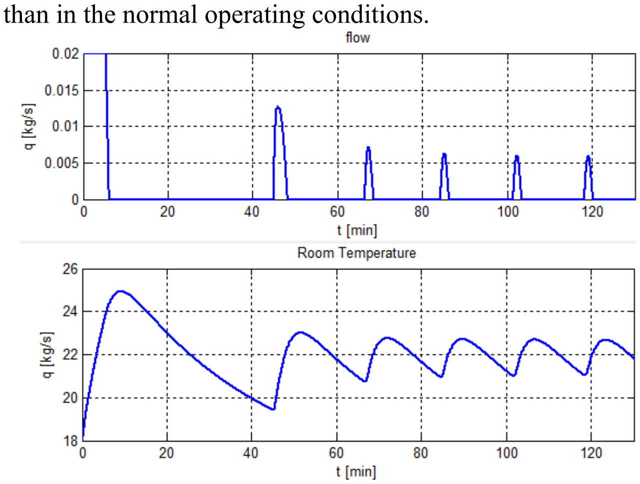

Fig. 7. PI tuned for the high demand condition is applied to a low demand situation

The tuned PI for low demand condition has the parameters as $K = 0.0008$ and $T_i = 500$. The controlled HVAC system response is shown in Fig.8.

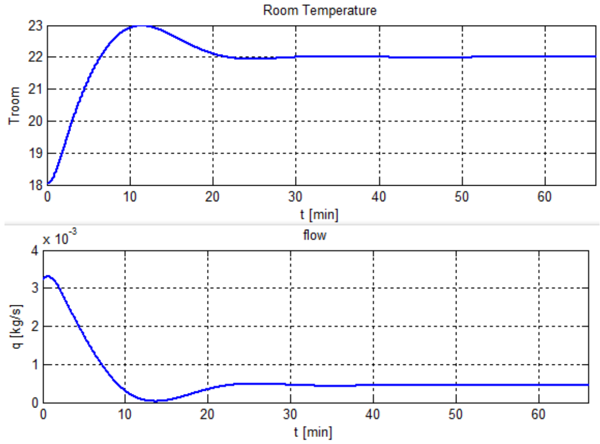

Fig. 8. Response of the flow controlled HVAC system in Low demand situation

If the PI tuned for the low demand condition applies to the high demand situation, the response of the system will be very slow as shown in Fig. 9.

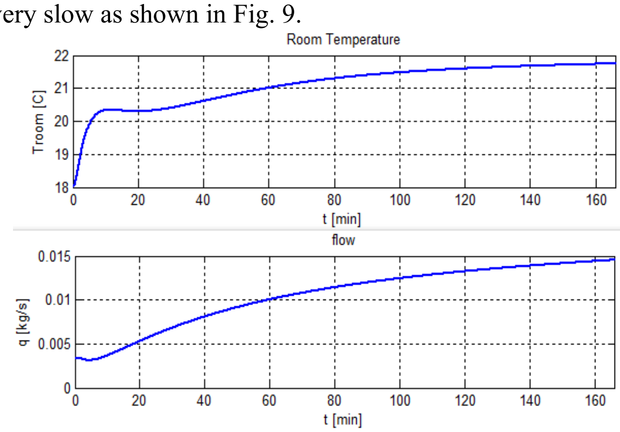

Fig. 9. Response of the system in high demand conditions controlled with the PI tuned for low demand situations

The oscillatory behavior is addressed previously in the literature [9] where the TRV is a gas expansion based valve. The solution proposed in that paper was based on controlling the pump speed to give a fixed valve position. This way, the controller makes the small signal gain independent of the heat demand.

In our case, we have direct control on the valve opening using the TRV derived by a step motor. A gain scheduling control can be designed for different operational regions of radiator. These operational regions can be realized based on the small signal gain curve shown in Fig.5. Adaptive approaches will be studied in future research.

IV. Conclusion

A precise simulation model for radiator based on discrete-element lumped model is proposed and optimized. The optimization is in term of best approximation of the radiator small signal gain for all operating points. The oscillatory behavior of radiator in low demand situations as a drawback of thermostatic radiator valves is addressed in this paper. The results of simulations approve that oscillations occur partly due to the high gain of radiator in low demand weather conditions and subsequently low flows. The large time constant at low demand situations compared to the high demand conditions will add to the stability problems.

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