Optimal Power Consumption in a Central Heating System with Geothermal Heat Pump

F. Tahersima, J. Stoustrup, and H. Rasmussen

Department of Electronic Systems, Aalborg University, Denmark
emails: {fts, jakob, hr@es.aau.dk}

Abstract: A ground source heat pump connected to a domestic hydronic heating network is studied to be driven with the minimum electric power. The hypothesis is to decrease the forward temperature to the extent that one of the hydronic heaters work at full capacity. A less forward temperature would result in a dramatic temperature drop in the room with saturated actuator. The optimization hypothesis is inspired by the fact that, the consumed electric power by the heat pump has a strong positive correlation with the generated forward temperature. A model predictive control scheme is proposed in the current study to achieve the optimal forward temperature. At the lower hierarchy level, local PI controllers seek the corresponding room temperature setpoint. Simulation results for a multi-room house case study show considerable energy savings compared to the heat pump's traditional control scheme.

Keywords: Hydronic heating system; Heat pump; Building energy efficiency; Hierarchical model predictive control.

1. INTRODUCTION

In recent years, there has been a growing interest in energy saving concepts and thermal comfort analysis within the building sector. Efficient control of heating, ventilation and air conditioning (HVAC) systems has a great influence on the thermal comfort sensation of the residents. The other important objective of a well designed control strategy is energy savings, mainly due to the growth of energy consumption, costs and also correlated environmental impacts.

1.1 Motivations and background

Heat pumps are drawing more attentions nowadays due to a surge for energy savings and the quest for mitigation of global warming. The most important benefit of utilizing heat pump systems is that they use 25% to 50% less electrical power than conventional heating or cooling systems. According to EPA, emissions of the ground-source heat pumps (GHP) are up to 44% less than air-source heat pumps (AHP) and up to 72% less than electric resistance heating with standard air-conditioning equipments (Rakhesh et al. (2003)). The other advantage of GHP compared to AHP is the fact that, at depth, the earth has a relatively constant temperature, warmer than the air in winter and cooler than the air in summer.

Here, we are specifically interested in geothermal heat pumps. However the achieved controller scheme can be generalized to AHPs as well. Heat pumps act like refrigerators in reverse and can generate up to 3-4 kWh of heat from 1 kWh of electricity. They transfer heat energy from the underground soil to residential buildings via a network of pipes. See Fig. 1. There are typically two hydronic and one refrigerant circuits interconnected through two heat exchangers. These are: 1) the underground buried brine-filled – mixture of water and anti-freeze – pipes with a small circulating pump; 2) the refrigerant-filled circuit, equipped with an expansion valve and driven by a compressor which is called heat pump; and 3) the indoor under-surface grid of pipes with another small circulating pump which distributes heat to the concrete floor of the building or to the hydronic radiators through a different network of pipes.

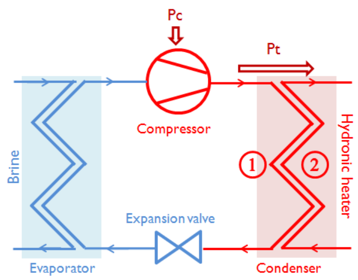

Fig. 1. Fluid circuits of a heat pump.

Traditional heat pump control scheme relies on the direct feedback from outdoor temperature. The objective of the heat pump controller is to seek the water temperature setpoint which is specified based on a prescribed curve, see Fig.2. This curve is suggested by the producer company and is adjusted manually by the heat pump installer. The installer changes the standard slope and offset according to dimensions of the building. Off peak loads easily might happen as a result of a coarse adjustment of such curve. The other inefficiency in power usage occurs because of a bypass stream of the return cooler water.

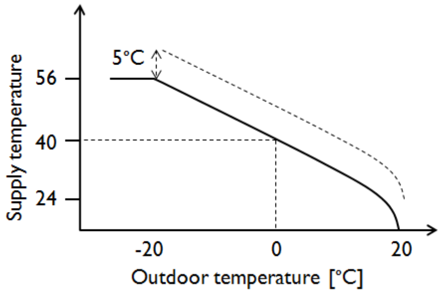

Fig. 2. Graph showing the supply water temperature set-point against ambient temperature in a conventional heat pump control. The dash shows an overhead above the standard curve due to the more heat demand in a specific construction

There has been little attention in the literature to GHPs optimal control in the sense of electric power consumption while it is connected to the system. Some control methods, P, PI and PID with pre-filtering have been tested and compared in Yang et al. (2007) for heat pump control integrated with a floor heating system. In that paper, the water flow rate is fixed to the maximum value (full valve opening with constant differential pressure) and the heat pump is controlled directly based on the feedback from room temperature. This control scheme for a single room is simple and requires little information (room temperature only) to track the room temperature setpoint profile. Undoubtedly, this control method can not be applied to the multiple rooms case, due to the specific heat demand of each single room. Therefore, having local controllers combined with a master controller is inevitable.

In the similar area of chillers liquid-loop control, a new principle called *chilled water temperature reset* (CWTR) has been advocated in recent years, see Piper (1999). In this method, the chilled water set-point is adjusted during the course of the day based on the net energy requirements of the building. Model predictive control in both centralized and distributed schemes is proposed in Chandan et al. (2010) to find the optimal outlet water temperature of chiller.

1.2 Main Contribution

In this study, we employed a simple idea – new in the field of concern – to optimally control the GHP integrated with a domestic hydronic heating network. Suppose we have several rooms in a building, each of which equipped with floor heating (FH) or hydronic radiator (HR) with GHP as hot water supplier. If the forward temperature is lowered to the extent that one of the hydronic valves works at high capacity, the heat pump is absorbing the minimum electric power. The inspiration behind this hypothesis is: the less the forward temperature is the less electric power would be consumed by the heat pump's compressor. The intuition behind the hypothesis is simple: if all the hydronic valves work at partial loads, then the forward temperature is still allowed to be lower, hence the consumed power can be lowered. The optimal point will be attained at the point where at lease one of the heaters goes to saturation.

The main objective of the current work is to present the above-stated principle as the unique optimal solution for driving the GHP integrated into the central heating system. Although, the idea is similar to the one proposed in Chandan et al. (2010), we have proposed a different scheme for designing the distributed model predictive control.

To facilitate the understanding, models of the system components are chosen deliberately simple. A central controller in collaboration with several local controllers is employed to achieve the optimal operating point of all subsystems. Simulation based test compares the new control system efficiency against the traditional one.

1.3 Paper Structure

The paper is organized as follows: Section II introduces the case study which is further investigated through the paper. Section III comes with the models of the system components. Section IV presents a hierarchical control structure which consists of local PI controllers and an MPC as the central controller. The developed control framework is tested by simulations and evaluated in Section V. Final conclusions are given in section VI. All the symbols and subscripts are listed in table 1.

Table 1. Symbols and Subscripts

Nomenclature	
A	surface area (m^2)
C	thermal capacitance ($J/kg\,^\circ$C)
K_r	equivalent heat transfer coefficient of HR($J/sec\,^\circ$C)
K_{fh}	equivalent heat transfer coefficient of FH($J/sec\,^\circ$C)
N, M	total number of HR and FH distributed elements
n_1	radiator exponent
P_c	consumed power by compressor
P_t	transferred power to the secondary side
Q	heat (W)
q	water flow in hydronic heater (kg/sec)
R_i	room number i
T	temperature ($^\circ$C)
T_i, T_j	temperature of the i, j^{th} element (HR,FH) ($^\circ$C)
U	thermal transmittance ($kW/m^2\,^\circ$C)
V	volume (m^3)
τ	time constant
τ_d	time delay of floor heating
Subscripts	
a	air
e	envelop
f	floor
fh	floor heating
hp	heat pump
s	supply water
out	outlet (water)
amb	ambient (temperature)
r	radiator
Ref	reference
w	water

2. CENTRAL HEATING SYSTEM

2.1 Case Study

A single-family detached house is considered as the case study, see Fig. 3. The two small rooms are equipped with hydronic radiators (HR) controlled by thermostatic radiator valves, one in each room. The bigger room has a serpentine floor heating (FH) system. The GHP provides

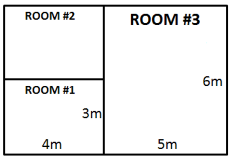

Fig. 3. Sketch of the apartment with three separate heat zones

hot water for the hydronic heaters in the building. Rooms number 1 and 3 have south faced glazings; hence they receive more sun.

2.2 Hierarchical Control Structure

Schematic of the hierarchical model predictive controller is depicted in Fig. 4. Local proportional integral (PI) controllers are designed for each hydronic heater based on Ziegler Nichols step response method, (Astrom and Hagglund (1995)). PI controllers seek the respective room temperature setpoint, adjusting the valve opening of HRs and duty cycle of FH's on/off valve. Heat pump's PI controller adjusts the compressor duty cycle to seek the specified temperature setpoint provided by the central controller.

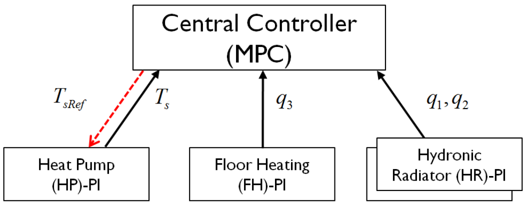

Fig. 4. Schematic of the hierarchical control structure: setponit signal (dashed) and measurements (continuous). Signals' indices correspond to the respective room number.

Model predictive controller receives all the operating points i.e. all flow rates. Based on that, it specifies the supply water temperature for GHP. The minimum supply temperature occurs at a point where at least one of the valves is almost entirely open.

Some of the main assumptions are: 1) An circulating pump in the water circuit is seeking a constant head gain; 2) Maximum valve opening corresponds to the highest capacity in both FH and HRs; 3) Flow rate of HRs are estimated, having TRVs driven by a stepper motor.

3. SYSTEM MODELING

This section is devoted to modeling details of the components and subsystems which are employed in the simulations.

3.1 Simulation Models

Energy balance equations of a single room based on the analogy between thermal systems and electrical circuits are as following:

$$\begin{aligned} C_e\dot{T}_e &= UA_e(T_{amb} - T_e) + UA_e(T_a - T_e) \\ C_f\dot{T}_f &= UA_f(T_a - T_f) + Q_{fh}(t - \tau_d) \\ C_a\dot{T}_a &= UA_e(T_e - T_a) + UA_f(T_f - T_a) + Q_r \end{aligned} \tag{1}$$

The above equations are developed mainly based on Hudson and Underwood (1999). More details regarding the parameters can be found in table 1 and furthermore in Tahersima et al. (2010). Envelope, room air and concrete floor are assumed to be at uniform temperature, i.e. no temperature gradient is considered in any of them. Heat flux via partition walls between the rooms is neglected, provided that temperature differences among the rooms are not noticeable.

Radiator is modeled as a lumped system with N elements in series. The ith section temperature is given by (Hansen (1997)):

$$\frac{C_r}{N}\dot{T}_i = c_w q_r (T_{i-1} - T_i) - \frac{K_r}{N}(T_i - T_a)^{n_1} \tag{2}$$

in which T_i is the radiator's ith element temperature and $i = 1, 2, ..., N$. For details of K_r and the assumptions made in the above formulation, the reader is directed to section 6.3.1 of Hansen (1997).

Assuming a constant pressure drop across the valve, a specific thermostatic valve is modeled with a static polynomial function mapping the valve opening δ to the flow through valve.

$$q = -3.4^{-4}\delta^2 + 0.75\delta \tag{3}$$

The specific TRV has a stepper motor to adjust the valve opening. This is a new type of TRV of which the valve position can be estimated and consequently the flow rate.

The considered floor heating has a serpentine piping embedded into a heavy concrete. Heat flux from pipes exterior is considered only upward. Employing a similar modeling as radiator, the distributed lump model is governed by:

$$\frac{C_{fh}}{M}\dot{T}_j = c_w q_{fh}(T_{j-1} - T_j) - \frac{K_{fh}}{M}(T_j - T_a) \tag{4}$$

in which T_j represents the jth element temperature with $j = 1, 2, ..., M$. Distribution of lumped elements are considered to be along the pipe. We have also assumed that heat is transferred between two sections only by mass transfer, implying that convective heat transfer is neglected. Constants C_{fh} and K_{fh} depend on the floor and pipes material. For more details, please see Hu et al. (1995).

Floor heating valve has an on-off thermal wax actuator. This actuator is controlled by pulse width modulation signal in practice. However, without loss of generality, we designed FH controller in continuous time.

3.2 Control Oriented Models

We have presented low-order models for control design purposes based on the relatively sophisticated simulation models of previous section.

Each room temperature pertains to the heat of radiator or floor heating via a 3rd order transfer function which can be approximated with a first order transfer function. The model parameters are derived for each room separately.

The relationship between radiator output heat and influent water flow around a specific operating point can be approximated by a first order transfer function. The approximation precision suffices for the control purposes.

$$\frac{Q_r}{q_r}(s) = \frac{k_r}{1+\tau_r s} \tag{5}$$

Parameters can be found via linearizion around an operating point, via simulation or experiment. These parameters are found previously in Tahersima et al. (2011) composing a linear parameter varying (LPV) model. In that paper, parameters were found by linearizion around operating points and were presented as some profile curves.

The transfer function between output heat and flow through FH:

$$\frac{Q_{fh}}{q_{fh}}(s) = \frac{k_{fh}}{1+\tau_{fh} s} e^{-\tau_d s} \tag{6}$$

Constants k_{fh} and τ_{fh} depend on the floor heating operating point, i.e. the flow and inlet water temperature. These parameters are estimated by linearization around specific operating points.

Closed loop transfer function of the heat pump system is approximated by its dominant dynamic between the supply water temperature and its setpoint as following:

$$\frac{T_s}{T_{sRef}}(s) = \frac{1}{1+\tau_{hp} s} \tag{7}$$

4. HIERARCHICAL CONTROL DESIGN

Local control units in cooperation with a central controller is considered as shown in Fig. 4. A local unit is a FH or HR system controlled by a PI controller. With a single unit in each room, PI is tuned based on the specific room's dynamic. Both flow rate and influent water temperature are manipulated variables of a single heating unit. While flow rate is controlled in the local unit, the forward temperature is adjusted in the central controller. Central controller receives valve opening as the status signal from all other units. Connection between the valve opening and the flow rate is via the fixed polynomial (3), independent of the pressure drop across the valve. A circulating pump seeks a constant differential pressure across all units. Henceforth, we use flow rate instead of valve opening in the central controller unit.

4.1 PI based Local Controllers

We presented LPV models of the system local units in the modeling section. In spite of the variable model parameters, we designed fixed PI controllers for each unit. It means satisfying performance measures and stability margins coarsely. Although, a gain schedule controller could handle variable parameters to maintain high performance measures, a simple PI controller is granted to simplify the proof of concept. Such gain schedule controller is designed in Tahersima et al. (2011).

PI controllers are designed for each FH and HR unit integrated with the corresponding room. The integrated models are:

$$\begin{aligned} \frac{T_a}{q_r} &= \frac{k_1}{(1+\tau_1 s)(1+\tau_r s)} \\ \frac{T_a}{q_{fh}} &= \frac{k_2}{(1+\tau_2 s)(1+\tau_{fh} s)} e^{-\tau_d s} \end{aligned} \tag{8}$$

PI controller is designed based on Ziegler Nichols step response method. The time delay of floor heating is not taken into consideration for local controller design. However, it is considered in the model predictive control design.

4.2 MPC based Central Controller

Central controller (CC) determines the reference forward temperature of heat pump based on the operating point of other local units. CC decreases forward water temperature until one of the heating units works at full capacity. The resulted minimum forward temperature based on the house heat demand corresponds to the minimum electric power consumption by the heat pump.

Model predictive control (MPC) is chosen as CC. Features like handling constraints, disturbances and setpoint profile tracking in a systematic way, have made MPC a very popular tool in many process applications (Maciejowski (2002)). We, specifically, count on the constraint handling specification of MPC in this paper. However, MPC is chosen as to fulfill other future targets in this specific case study, i.e. disturbance rejection.

We did not take the whole integrated state space model (Fig. 4) into consideration as MPC model. Instead we relied on the relationship between flow rates and forward temperature of each unit. Such linear relationship is $\dot{q} = -\alpha q - \beta T_s(t-\tau_d)$. A pure delay term should only be considered in association with floor heating unit. We need to derive the parameters of this dynamical equation first. The corresponding transfer function looks like:

$$\frac{q}{T_s} = \frac{-\beta}{s+\alpha} \tag{9}$$

The DC gain is found by equating the outgoing heat in one situation with the new situation in steady state. Suppose supply water temperature is changed from T_s^1 to T_s^2. Then, the steady state flow would change from q_1 to q_2. The new flow rate can be achieved from:

$$c_w q_1(T_s^1 - T_{out}) = c_w q_2(T_s^2 - T_{out}) \tag{10}$$

In both sides of the above equation, T_{out} is the same, provided that the flow rate is limited by a balancing task initially at installation phase. Therefore, the DC gain is:

$$\frac{\beta}{\alpha} = -q_1 \times \frac{T_s^1 - T_{out}}{T_s^2 - T_{out}} \tag{11}$$

To find α, we take a look to the channel through which q is influenced by T_s. The dominant pole in this channel belongs to the room air dynamic together with a pure time delay connected to the concrete floor. Therefore, $\alpha = \frac{1}{\tau_a}$ and $\beta = \alpha \times DCgain$.

The applied model to predict the influence of forward temperature on flow rate is as following:

$$
\begin{aligned}
R_i:&\ \dot{q_i} = -\alpha q_i + \beta T_s \quad i = 1,2 \qquad (12)\\
R_3:&\ \dot{q_3} = -\alpha q_3 + \beta T_s(t - \tau_d)\\
HP:&\ \dot{T_s} = -\frac{1}{\tau_{hp}} T_s + \frac{1}{\tau_{hp}} T_{sRef}
\end{aligned}
$$

The presented MPC minimizes the following cost functional:

$$
\begin{aligned}
J:&\ \min_{T_{sRef}} \theta T_s^2 + \phi \Delta T_{sRef}^2 \qquad (13)\\
&\ s.t.\ 0 \le q_i \le q_{iMax}
\end{aligned}
$$

The objective function is a summation of two terms with weights θ and ϕ that can be tuned. The first term seeks minimization of electric power consumption. The second term prevents abrupt changes in actuation signal. This online optimization problem can be solved using standard solvers e.g. MATLAB. We selected the prediction horizon such that it includes all significant dynamics i.e. room air plus time delay of concrete floor. Control interval is chosen based on the operation time of the slowest actuators which are on-off thermal wax actuators. We chose the control horizon not to be less than the fastest control loop settling time.

5. SIMULATION RESULTS

The potential energy saving with the proposed control scheme is investigated via a simulation test. The case study shown in Fig. 3 is simulated employing the accurate nonlinear models described in Section 3.1.

5.1 Simulation

We demonstrated a situation where the house heat demand varies during a day. While heat demand is more in the first room initially, the demand peak is shifted to the second room when solar radiation heats up the first and third rooms.

Ambient temperature is an unmeasured disturbance input for the system. A sinusoid with the period of 24 hours models the ambient temperature. In this simulation, behavior of the system in a period of two days is simulated.

The maximum flow through FH is $0.1 Lit/sec$ and through each HR is $0.015 Lit/sec$. In Fig. 6 the flow of FH is scaled.

As shown in Fig. 6, the temperature of the forward water decreases from 34°C at the day-time of the first day to 32°C in the day-time during the second day due to the solar radiation. This decrease in the demanded forward temperature is translated to a shorter compressor operation time and consequently to a lower power consumption.

The maximum flow rate is limited here to 90% of maximum flow in order not to push the valves into fully-open saturated status. Otherwise, no actuation capacity is left for compensating exogenous disturbances.

5.2 Evaluation of the Results

In this section, we have compared the energy consumption by the heat pump against the conventional heat pump control. Currently, the dominant method of heat pump

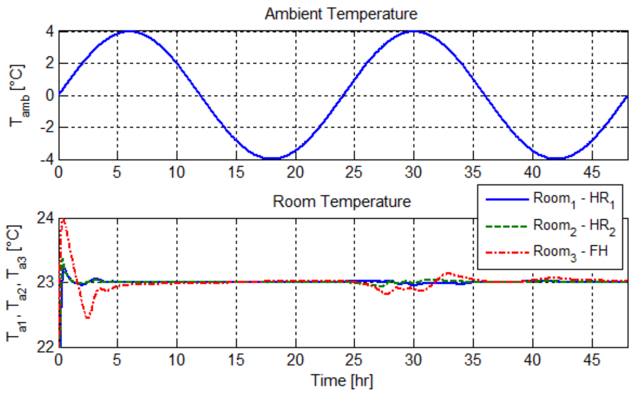

Fig. 5. Top: Ambient temperature variations during 48 hours. Bottom: Temperature variations of the three rooms. At the earlier times of the second day, solar radiation through glazing causes a temperature increase in the southern rooms.

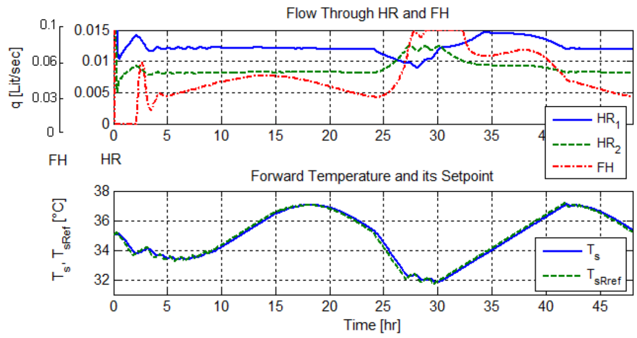

Fig. 6. Top: Flow through radiators and floor heating. FH's flow is scaled. Bottom: Forward temperature of GHP and the reference of this temperature. Due to the solar radiation at the earlier times of the second day, the flow through the first radiator starts to fall and consequently the forward temperature of GHP. This causes that the other radiator in the northern room and the FH demand for more flow and start to increase and works around 90% capacity. The slow response of the FH system is due to the delay imposed by the heavy floor.

control is based on a feed-forward approach. The supply water temperature is specified via a predefined map as shown in Fig. 2 which has been employed from Danfoss (2008). The offset and the slop of this curve is usually adjusted manually by the installer. If, with the standard settings of heat pump, the demanded heat can not be provided due to the large dimensions of the building or a poor thermal insulation, an overhead would be considered by the installer.

The comparison is performed on the same case study with the same disturbance model and setpoints. The forward temperature is calculated based on two methods. Relation ship between electric power consumption and the house's heat demand is

$$
P_c = \frac{P_t}{COP} \qquad (14)
$$

in which COP is the heat exchanger's coefficient of Performance and P_t is the transferred heat to the house. Since energy loss of the building is the same independent of the forward temperature, P_t is the same in both methods. But, a lower forward temperature resulted from our control scheme means a higher COP and thus a lower P_c. A COP curve is shown in Fig. 7 which is the result of an investigation over 100 models of heat pumps Staffell (2009).

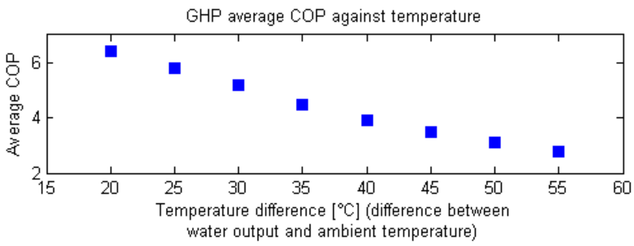

Fig. 7. Average COP of around 100 heat pump models against the temperature rise across the heat pump

Relying on the curve in Fig. 7, the COP corresponding to the forward temperatures are calculated for both methods which is shown in Fig. 8. Integrating the inverse of COP multiplied by P_t over the period of two days, we calculated the electric power consumption. The percent of energy saving with the proposed MPC method and the conventional control scheme is shown in table 2.

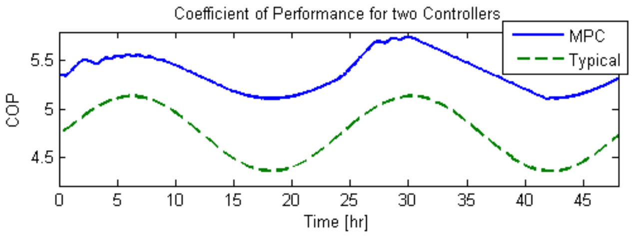

Fig. 8. Heat pump coefficient of performance for two methods, the proposed MPC controller and a typical heat pump controller.

Table 2. Comparison of average electric power consumption $[KW]$

	MPC	Typical	Energy saving (%)
Well insulated	32	37	13.5
Weakly insulated	33	42	21.4

6. CONCLUSION

A hypothesis for heat pump energy optimization is proposed. Heat pump consumes the least electric power when at least one of the hydronic heaters in the house work at full load. The proposition relies on the fact that the power consumed by heat pump has a strong positive correlation with the water forward temperature. Employing simplified low order models and simple local controllers, this paper serves as the proof of concept. Nevertheless, the proposed hypothesis for optimization is general and could really contribute to reduced power consumption in almost any type of heat pump based building. More detailed simulations and real life experiments are subjects of future works. Besides, energy measurements at the compressor end would be presented to evaluate the efficiency improvement.

REFERENCES

Astrom, K. and Hagglund, T. (1995). *PID Controllers*. ISA, North Carolina.

Chandan, V., Mishra, S., and G. Alleyne, A. (2010). Predictive Control of Complex Hydronic Systems. *American Control Conference*.

Danfoss (2008). Maintenance instructions dhp-h opti pro. Maintenance instruction.

Hansen, L.H. (1997). *Stochastic modeling of central heating systems*. Ph.D. thesis, DTU.

Hu, S.Y., Hayes, R.E., and Wood, R.K. (1995). Simulation of the dynamic behavior of a hydronic floor heating system. *Heat Recovery & CHP*, 15(6).

Hudson, G. and Underwood, C.P. (1999). A simple building modeling procedure for matlab/simulink. 99(2), 777–783.

Maciejowski, J.M. (2002). *Predictive Control with Constraints*. Prentice Hall, Harlow, England.

Piper, J. (1999). *Operations and maintenance manual for energy management*. ME Sharpe.

Rakhesh, B., Venkatarathnam, G., and Srinivasa Murthy, S. (2003). Experimental studies on a heat pump operating with r22, r407c and r407a: Comparison from an exergy point of view. *Journal of Energy Resources Technology, Transactions of the ASME*, 125(2), 101–112.

Staffell, I. (2009). A review of domestic heat pump coefficient of performance. Technical report.

Tahersima, F., Stoustrup, J., and Rasmussen, H. (2011). Stability-performance dilemma in TRV-based hydronic radiators. *Submitted for publication*.

Tahersima, F., Stoustrup, J., Rasmussen, H., and Gammeljord Nielsen, P. (2010). Thermal analysis of an hvac system with trv controlled hydronic radiator. *2010 IEEE International Conference on Automation Science and Engineering*, 756–761.

Yang, Z., Pedersen, G., Larsen, L., and Thybo, H. (2007). Modeling and control of indoor climate using a heat pump based floor heating system. *IECON 2007 - 33rd Annual Conference of the IEEE Industrial Electronics Society*, 2985–2990.