Stability Performance Dilemma in Hydronic Radiators with TRV

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***Abstract*— Thermostatic Radiator Valves (TRV) have proved their significant contribution in energy savings for several years. However, at low heat demands, an unstable oscillatory behavior is usually observed and well known for these devices. It happens due to the nonlinear dynamics of the radiator itself which results in a high gain and a large time constant for the radiator at low flows. If the TRV is tuned in order to dampen the oscillations at low heat loads, it will suffer from poor performance and lack of comfort, i.e. late settling, when full heating capacity is needed. Based on the newly designed TRVs, which are capable of accurate flow control, this paper investigates achievable control enhancements by incorporating a gain schedulling control scheme applied to TRVs. A suitable linear parameter varying model is derived for the radiator which governs the gain scheduler. The results are verified by computer simulations.**

I. INTRODUCTION

Efficient control of heating, ventilation and air conditioning (HVAC) systems has a great influence on the thermal comfort of residents. The other important objective is energy savings, mainly because of the growth of energy consumption, costs and also correlated environmental impacts.

Hydronic radiators controlled by thermostatic radiator valves (TRV) provide good comfort under normal operating conditions. Thermal analysis of the experimental results of a renovated villa in Denmark, built before 1950, has demonstrated that energy savings near 50% were achieved by mounting TRVs on all radiators and fortifying thermal envelope insulation [1].

To maintain the temperature set point in a high load situation, TRVs are usually tuned with a high controller gain.The inefficiency appears in the seasons with low heat demand especially when the water pump or radiator are over dimensioned [2]. In this situation, due to a low flow rate, loop gain increases; and as a result oscillations in room temperature may occur. Besides discomfort, oscillations decrease the life time of the actuators. This problem is addressed in [3] for a central heating system with gas-expansion based TRVs. It is proposed to control the differential pressure across the TRV to keep it in a suitable operating area using an estimate of the valve position.

In this study, we investigated the problem as a dilemma between stability and performance. The case study of the paper is a HVAC system including a room and a hydronic radiator controlled by a TRV. In this study, pressure drop across the radiator valve is maintained constant unlike what is taken as the control strategy in [3]. Instead, flow control is assumed to be feasible by the accurate adjustment of the valve opening. The valve opening is regulated by a stepper motor which allows the concrete adjustment. We have proposed control oriented models for the system components as functions of operating conditions. In this way, the nonlinear radiator model is replaced by a linear parameter varying model. Based on the proposed modeling, gain scheduling is chosen among various possible control structures to design the TRV controller.

Control oriented models are derived based on energy balance equations of the system components. Generally, there are two approaches for HVAC systems modeling, the forward approach and the data-driven (inverse) approach [4]. The first one is based on known physical characteristics and energy balance equations of the air, structural mass and other components of the system. In this approach, three methods of heat balance, weighting factor and thermal network are addressed widely in the literature [5], [6], [7]. The alternative modeling approach is to use building measurement data with inferential and statistical methods for system identification which is addressed in [8], [9], [10]. The main drawback of this method is that it requires a significant amount of training data and may not always reflect the physical behavior of the system [11].

In this study, the control oriented model of the room is employed based on the lumped capacitance model described formerly in [12]. An extension of the *one exponent model*, addressed in [13] is proposed for describing the radiator dynamics.

The rest of the paper is organized as follows: Section II describes the system components. Control oriented model of the HVAC system is developed using the simulation models in Section III. Section IV proposes the control structure based on flow adaptation. Simulation results are illustrated in the same section. Discussion and conclusions are given finally in Sections V and VI.

II. SYSTEM DESCRIPTION

The HVAC system is composed of a room, a radiator with thermostatic valve and a temperature sensor. Disturbances which excite the system are ambient temperature, heat from the radiator and ground temperature. The latter input affects the room temperature through thick layers of insulation and a heavy concrete. The block diagram of the system is shown in Fig. 1. Symbols, subscripts whcih are used in the paper and their corresponding amount are shown in table I and table II respectively. The parameters' value are calculated mainly based on [4].

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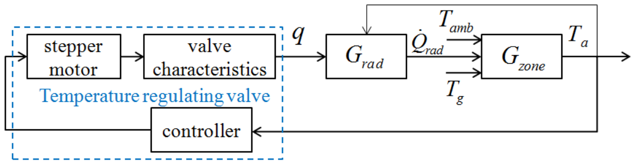

Fig. 1. Block diagram of the room temperature control system

Fig. 2 and Fig. 3 show a test where oscillations and low performance occur respectively. In this test the forward water temperature is at 50°C. The proportional integral (PI) controller of TRV is tuned based on Ziegler-Nichols step response method employed from [14].

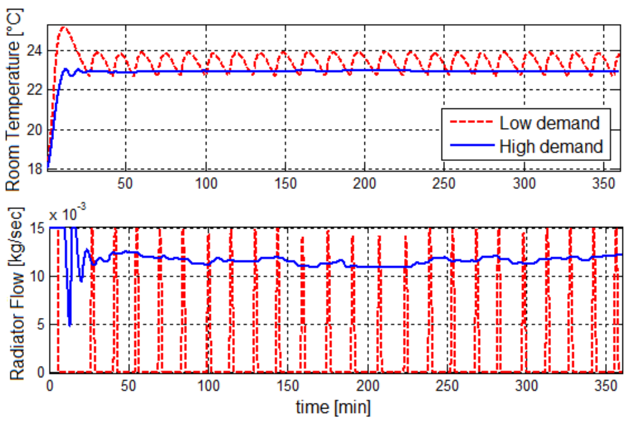

Fig. 2. Undamped oscillations in room temperature and radiator flow which occur in low demand situation while the controller is designed for high demand condition.

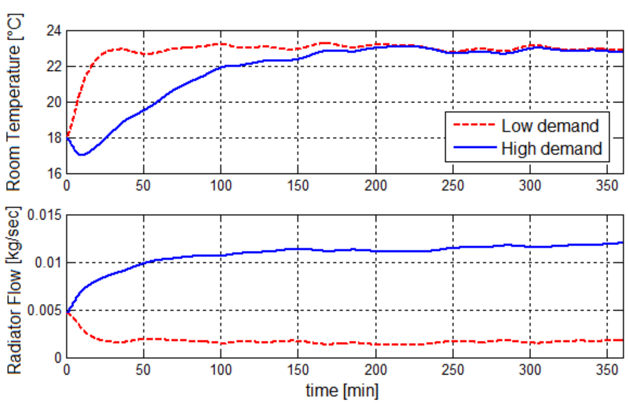

Fig. 3. Poor performance in the cold weather condition, applying the controller designed for the low demand situation

III. System Modeling

A. Simulation Models

A radiator is a distributed system which can be considered as N pieces in series. Using one exponent method for modeling the radiator output heat, the n^{th} section is given by, [13]:

$$\frac{C_{rad}}{N}\dot{T}_n = H_q(T_{n-1} - T_n) - \frac{\Phi_0}{N}\left(\frac{T_n - T_a}{\Delta T_{m,0}}\right)^{n_1} \quad (1)$$

in which C_{rad} is heat capacity of the water and the radiator material, T_n is temperature of the radiator's nth element with $n = 1, 2, \ldots, N$. Temperature of the end points are water inlet temperature $T_0 = T_{in}$ and outlet temperature $T_N = T_{out}$. $H_q = c_w q$ and Φ_0 is the nominal power of the radiator in nominal condition which is $T_{in,0} = 90\,°\text{C}$, $T_{out,0} = 70\,°\text{C}$, and $T_a = 20\,°\text{C}$. $\Delta T_{m,0}$ represents the mean temperature difference defined as:

$$\Delta T_m = \frac{T_{in} - T_{out}}{2} - T_a \quad (2)$$

in nominal condition. n_1 in (1) is an exponent which varies between 1.2 and 1.4 for different radiators.

Defining the constant term $\frac{\Phi_0}{N \Delta T_{m,0}^{n_1}}$ as equivalent heat transfer coefficient, K_{rad}, (1) can be rewritten as:

$$\frac{C_{rad}}{N}\dot{T}_n = H_q(T_{n-1} - T_n) - K_{rad}(T_n - T_a)^{n_1} \quad (3)$$

The power transferred by the radiator to the room air can be calculated as:

$$\dot{Q}_{rad} = \sum_{n=1}^{N} K_{rad}(T_n - T_a)^{n_1} \quad (4)$$

Heat balance equations of the room is governed by the following lumped model [7]:

$$\begin{aligned} C_e\dot{T}_e &= U_eA_e(T_{amb} - T_e) + U_eA_e(T_a - T_e) \quad (5)\\ C_f\dot{T}_f &= U_fA_f(T_a - T_f)\\ C_a\dot{T}_a &= U_eA_e(T_e - T_a) + U_fA_f(T_f - T_a) + \dot{Q}_{rad} \end{aligned}$$

in which T_e represents the envelop temperature, T_f the temperature of the concrete floor and T_a the room air temperature. Q_{rad} is the heat power transferred to the room by radiator. Each of the envelop, floor and room air are considered as a single lump with uniform temperature distribution.

Assuming a constant pressure drop across the valve, the thermostatic valve is modeled as a static polynomial function:

$$q = -3.4 \times 10^{-4}\delta^2 + 0.75\delta \quad (6)$$

The above function is mapping the valve opening δ to the water flow through the valve.

B. Control Oriented Models

Step response simulations and experiments confirm a first order relationship between the radiator output heat and the input flow around a specific operating point:

$$\frac{\dot{Q}_{rad}}{q}(s) = \frac{K_r}{1 + \tau_r s} \quad (7)$$

The static gain K_r and the time constant τ_r depend on the operating point of the system i.e. corresponding flow and room temperature. In order to develop the low order model thoroughly, relationship between these parameters and the operating point will be derived based on simulation tests. Fig. 4 shows these relationships for a specific radiator.

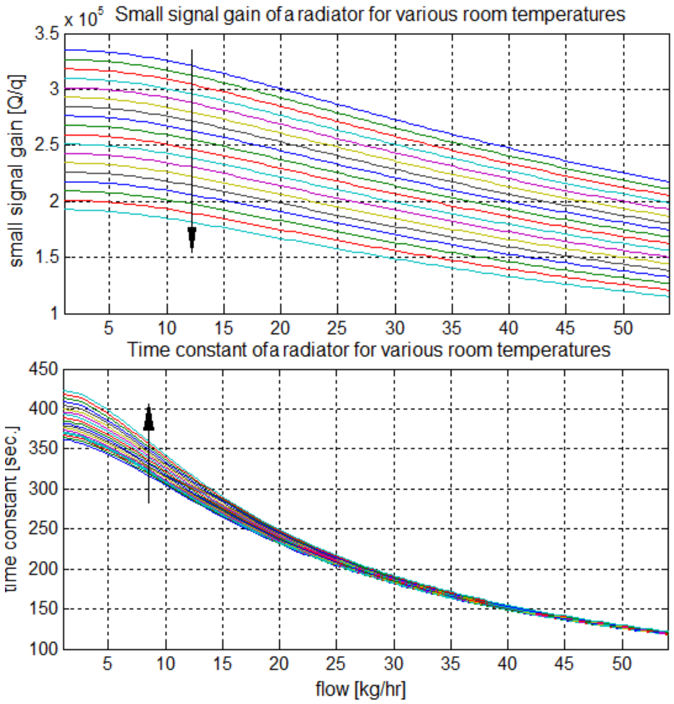

Fig. 4. Static gain and time constant variations for various values of the radiator flow and room temperature. The arrows show the direction of room temperature increase. Room temperature is changed between $-10\,^{\circ}$C and $24\,^{\circ}$C and flow is changed between the minimum and the maximum flow

To derive the curves, radiator is simulated with small steps as the input flow. Static gain and time constant are achieved for a specific room temperature. The room temperature is then changed by 2°C and the procedure is repeated.

Fig. 4 shows that the static gain and time constant of the heat-flow transfer function are extremely dependent on the flow rate. The high gain and the long time constant in the low heat demand conditions mainly contribute to the oscillatory behavior. The model of room-radiator can be written as:

$$\frac{T_a}{q}(s) = \frac{K_r K_a}{(1+\tau_r s)(1+\tau_a s)} \qquad (8)$$

Room parameters, K_a and τ_a can be estimated easily by preforming a simple step response experiment. We obtained these parameters based on [4] assuming specific materials for the components.

Estimating the flow rate and measuring the room temperature, corresponding radiator parameters can be achieved using the curves in Fig. 4. Consequently, the model (8) will be determined. These curves are approximated by two sets of polynomials using Matlab curve fitting toolbox, cftool.

IV. Gain Scheduling Control Design based on Flow Adaptation

In the previous section, we developed a linear parameter varying model which approximates the radiator's nonlinearities of (3). In order to alleviate the effects of parameter variations, gain scheduling control is selected among the various possible control structures which is adapted based on [15]. Therefore, the title of flow adaptation is reflecting the dependence of controller parameters to the radiator flow.

The main idea of the designed controller is to transform the primary parameter varying system model i.e. (8) to a system independent of the operating point. A controller would be designed based on the transformed linear time invarient (LTI) system. Block diagram of this controller is shown in Fig. 5.

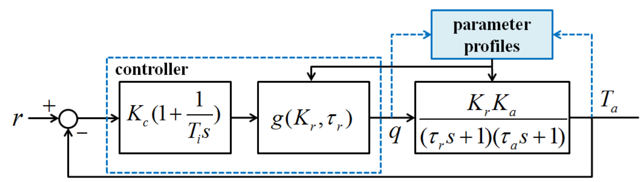

Fig. 5. Block diagram of the controller based on linear transformation

The function g is chosen such that it cancels out the varying pole of the radiator and places a pole instead in the desired position. This position corresponds to the radiator's farthest pole in left half plane associated to the high flow rates. Therefore, the simplest candidate for the linear transfer function g is a phase-lead structure, (9).

$$g(K_r,\tau_r) = \frac{K_{r,hd}}{K_r}\frac{\tau_r s+1}{\tau_{r,hd}s+1} \qquad (9)$$

in which $K_{r,hd}$ and $\tau_{r,hd}$ correspond to the gain and time constant of radiator in the highest demand situation when the flow rate is maximum. Consequently, the transformed system is equivalent to (8) at the high load operating point which corresponds to the system parameters $K_{r,hd}$ and $\tau_{r,hd}$. By choosing the high demand as the desired situation, we give the closed loop system the prospect to have the dominant poles as far as possible from the origin, and as a result as fast as possible.

The controller for the transformed LTI system is a fixed PI controller then. The parameters of this controller is calculated based on Ziegler-Nichols step response method [14]. To this end, the transformed second order system is approximated by a first-order system with a time delay, (10). The choice of PI controller is to track a step reference with zero steady state error.

$$\frac{T_a}{q}(s) = \frac{k}{1+\tau s}e^{-Ls} \qquad (10)$$

The time delay and time constant of the above model can be found by a simple step response time analysis:

$$\begin{aligned} T_a(t) = K_{r,hd}K_a(1 + &\frac{\tau_{r,hd}}{\tau_a-\tau_{r,hd}}e^{\frac{-t}{\tau_{r,hd}}} \\ &+\frac{\tau_a}{\tau_{r,hd}-\tau_a}e^{\frac{-t}{\tau_a}})q(t) \end{aligned} \qquad (11)$$

in which $q(t) = u(t)$ is the unit step input. The apparent time constant and time delay are calculated based on the time when 0.63 and 0.05 of the final value is achieved respectively. In the following, χ is exploited as an auxiliary parameter. The positive solution of the following equation gives the time delay when $\chi = 0.95$ and the time constant when $\chi = 0.37$.

$$(\chi + 1)t^2 + 2(\tau_{r,hd} + \tau_a)(\chi - 1)t^2 + a(\chi - 1)\tau_{r,hd}\tau_a = 0 \quad (12)$$

Having τ and L calculated, the parameters of the regulator obtained by Ziegler-Nichols step response method would be the integration time $T_i = 3L$ and the proportional gain $K_c = \frac{0.9}{a}$ with $a = k\frac{L}{T}$ and $k = K_{r,hd} \times K_a$. k is the static gain.

A. Simulation Results

The proposed controller parameterized based on the radiator parameters is applied to the simulation models of the room and radiator. Parameters of the PI controller are found based on the parameter values in table II as $K_c = 0.01$ and $T_i = 400$. Ambient temperature is considered as the only source of disturbance for the system. In a partly cloudy weather condition, the effect of intermittent sunshine is modeled by a fluctuating outdoor temperature. A random binary signal is added to a sinusoid with the period of two hours to model the ambient temperature.

Simulation results with the designed controller and the corresponding ambient temperature are depicted in Fig. 6 and Fig. 7. The results are compared to the case with fixed PI controllers designed for both high and low heat demand conditions.

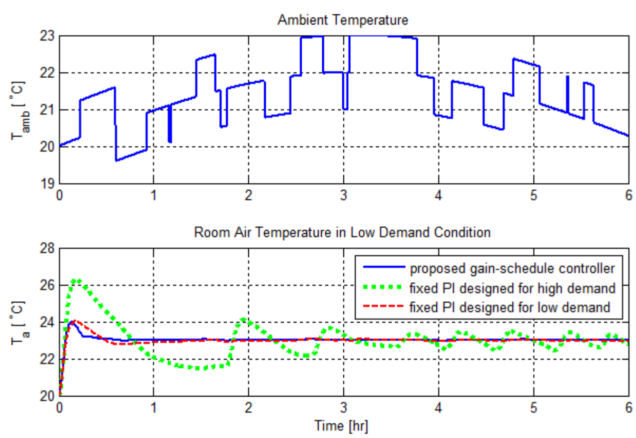

Fig. 6. (Top) ambient temperature, (bottom) room temperature for three controllers. The results of simulation with flow adaptive controller together with two fixed PI controllers are shown. The PI controller designed for the high demand situation encounters instability in the low heat demand condition.

The simulation results of the proposed control structure show significant improvement in the system performance and stability compared to the fixed PI controller.

V. Discussions

All the gain scheduling control approaches are based on this assumption that all states can be measured and a generalized observability holds [15]. In this study, we also need to clarify if this assumption is valid. The parameters that we need to measure or estimate are room temperature

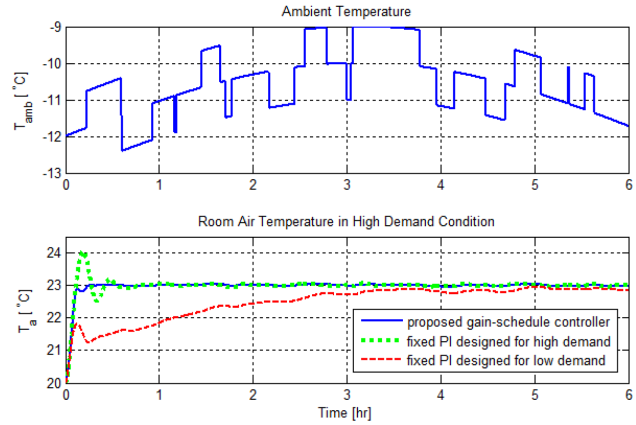

Fig. 7. (Top) ambient temperature, (bottom) room temperature for three controllers. The results of the simulation with flow adaptive controller together with two fixed PI controllers are shown. The PI controller designed for the low demand condition is very slow for the high demand situation.

and radiator flow rate. Measuring the first state is mandatory when the goal is seeking a reference for this temperature. However, radiator flow is not easily measurable.

To estimate the radiator's flow rate, one possibility is using a new generation of TRVs which drive the valve using a stepper motor. It is claimed that this TRV can give an estimation of the valve opening. Provided the valve opening degree, its characteristic and the pressure difference, flow rate would be estimated.

We have shown through the paper that using the new generation of TRVs, gain scheduling control would guarantee efficiency of the radiator system. However, this claim would be defensible when the flow rate estimation is done in practice through an easy, reliable method. This issue, besides robustness of the proposed controller and quantifying energy savings will be studied in the future works.

VI. Conclusion

The dynamical behavior of a TRV controlled radiator is investigated. A dilemma between stability and performance for radiator control is presented. We dealt with the dilemma using a new generation of thermostatic radiator valves. With the new TRV, flow estimation and control based on energy demand would be possible. Based on the estimated flow, we have developed a gain scheduling controller which guarantees both performance and stability for the radiator system. To this end, we derived low-order models of the room-radiator system. The model is parameterized based on the estimated operating point which is radiator flow rate. Gain scheduled controller is designed for the derived time varying model at the end.

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TABLE I

SYMBOLS AND SUBSCRIPTS

Nomenclature	
A	surface area (m^2)
C	thermal capacitance $(J/kg\,^\circ\mathrm{C})$
g	linear transformation function
G	transfer function
K	static gain
K_c	controller gain
K_{rad}	equivalent heat transfer coefficient of radiator $(J/sec\,^\circ\mathrm{C})$
L	time delay $(sec.)$
N	total number of radiator distributed elements
n_1	radiator exponent
$\dot{Q}$	heat (W)
q	water flow through radiator (kg/sec)
T	temperature $(^\circ\mathrm{C})$
T_i	integration time
T_n	temperature of the radiator n^{th} element $(^\circ\mathrm{C})$
U	thermal transmittance $(kW/m^2\,^\circ\mathrm{C})$
V	volume (m^3)
ρ	density (kg/m^3)
τ	time constant $(sec.)$
Φ_0	nominal power of radiator (W)
ΔT_m	mean water temperature $(^\circ\mathrm{C})$
Subscripts	
a	room air
amb	ambient temperature (outdoor)
e	envelope
f	floor
g	ground
in	inlet (water temperature)
o	outlet (water)
out	outlet (water temperature)
p	a fraction of sun radiation heating the floor
rad, r	radiator
s	solar radiation
w	water

TABLE II

SYSTEM PARAMETERS

Room Parameters		Radiator Parameters	
A_e	$56\ m^2$	A_r	$1.5\ m^2$
A_f	$20\ m^2$	C_{rad}	$3.1 \times 10^4\ J/kg\,^\circ\mathrm{C}$
C_a	$5.93 \times 10^4\ J/kg\,^\circ\mathrm{C}$	c_w	$4186.8\ J/kg\,^\circ\mathrm{C}$
C_e	$5 \times 10^4\ J/kg\,^\circ\mathrm{C}$	N	45
C_f	$1.1 \times 10^4\ J/kg\,^\circ\mathrm{C}$	n_1	1.3
U_e	$1.2\,kW/m^2\ ^\circ\mathrm{C}$	q_{max}	$0.015\ kg/sec$
U_f	$1.1\,kW/m^2\ ^\circ\mathrm{C}$	T_s	$70\ ^\circ\mathrm{C}$
p	neglected	V	$5\ Liter$
Q_e	neglected	Φ_0	$1700\ W$
Q_s	neglected	ρ_w	$998\ kg/m^3$

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