Model Predictive Control for Power Flows in Networks with Limited Capacity

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***Abstract*— We consider an interconnected network of consumers powered through an electrical grid of limited capacity. A subset of the consumers are *intelligent consumers* and have the ability to store energy in a controllable fashion; they can be filled and emptied as desired under power and capacity limitations. We address the problem of maintaining power balance between production and consumption using the intelligent consumers to ensure smooth power consumption from the grid. Further, certain capacity limitations to the links interconnecting the consumers must be honored. In this paper, we show how this problem can be formulated as an optimization problem, leading directly to the design of a model predictive controller. Using this scheme, we are able to incorporate predictions of future consumption and exploit knowledge of link limitations such that the intelligent consumers are utilized ahead of time ensuring high performance.**

I. Introduction

With an increasing focus on climate-related issues and rising fossil fuel prices, the penetration of renewable energy sources is likely to increase in the foreseeable future throughout the developed world [1]. Since many of these renewable sources of energy are difficult to control, base load units (*e.g.*, fossil fuel-fired co-generation plants) must be kept in reserve to compensate for temporary shortages. The higher the percentage of renewable sources and the more fluctuating the power production, the harder the regulation task becomes for the base load units (see *e.g.* [2]). This *balancing problem* is typically solved centrally by a Balance Responsible Entity for a given power grid region, by activating or de-activating controllable reserves via an Automatic Generation Control system (see *e.g.* [3]).

Traditionally, control of large, networked systems is achieved by designing local, subsystem-based controllers that ignore the interactions between the different subsystems [4]. However, it is well known that such designs can lead to poor performance and coordinated solutions have thus been pursued in recent years. [3] and [5] present distributed model-based predictive control (MPC) schemes to solve the Automated Generation Control problem, albeit without taking uncontrollable energy sources into account. [6] uses distributed MPC to solve the balancing problem by actively controlling a portfolio of fossil fuel fired power plants in order to counteract fluctuations induced by renewable sources such as wind farms. However, most existing solutions have so far only considered the production side.

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A *smart grid* is an electric power system, where both producers and consumers are equipped with control capabilities that allow them to participate in these balancing efforts, for instance by allowing local devices with large time constants to store more or less energy at convenient times and thereby adjusting the momentary consumption, see *e.g.* [7] and [8]. One obvious method to do so is by exploiting large thermal time constants in deep freezers, refrigerators, local heat pumps etc.; extra energy can be stored during off-peak hours, and the accumulated extra cooling or heating can then be used by turning compressors and similar devices on less frequently during peak hours, see *e.g.* [9]. Implementing such capabilities requires local measurement and feedback of current energy and power demands [10]. Consumers equipped with such measurement and feedback capabilities will be referred to as *intelligent consumers* in the following. Such an intelligent consumer could also represent a large number of units aggregated into one consumer.

Recently, [11] introduced a hierarchical MPC design to distribute resources to intelligent consumers that makes active use of the consumers to counteract quickly fluctuating imbalances. Since the consumers do require a certain amount of energy over time in order to satisfy local performance requirements, *e.g.* quality of foodstuff kept in cold storage, constraints on both instantaneous power and energy consumed over a specific time horizon had to be considered for each consumer. However, the setup considered in [11] was idealized in many ways; for example, the grid topology was completely ignored. That is, it was not taken into account that the power grid itself has limits to how much power it can convey at any given point in time from one node to another and that these constraints may be different from one part of the grid to another.

In this paper, we extend the design in [11]. We consider a number of both intelligent consumers and uncontrollable consumers interconnected in a network. The uncontrollable consumers are characterized by power consumptions that cannot be controlled but that we have good predictions of due to the very competitive energy market, where such predictions are most valuable. The intelligent consumers, on the other hand, are characterized by the ability to store energy in a controllable fashion.

A controller is responsible for ensuring balance between power consumption and production. The controller can balance the uncontrollable consumption by assigning power directly from the supplier, but at a significant cost; it is therefore advantageous for the controller to utilize the storage possibilities in the intelligent consumers. Further, the controller must ensure that the grid capacity limitations are honored.

Based on the structure of the problem, it follows naturally to design a model predictive controller. Based on two simulation examples, we show that the developed MPC controller indeed is able to utilize the intelligent consumers such that high performance is achieved. We use the examples to show that the MPC controller uses the predictive abilities to ensure balance without stressing the supplier; based on consumption predictions, the controller is able to fill or empty the energy storages ahead of time, to compensate for future known events. Further, the examples show that the MPC controller is able to exploit knowledge of grid capacities and thereby reduce congestion problems by preemptive action.

The outline of the rest of the paper is as follows. First, in Section II we describe the system setup under consideration. Next, in Section III we present the main result of the work: a predictive control strategy that takes simple grid constraints into account in the distribution of power to intelligent consumers. Section IV presents simulation examples that illustrate the feasibility of the design, and finally Section V describes future work, while VI sums up the work.

II. Modeling

We consider a setup as depicted in Figure 1. The figure illustrates two types of consumers; a set of m uncontrollable consumers and a set of n intelligent consumers.

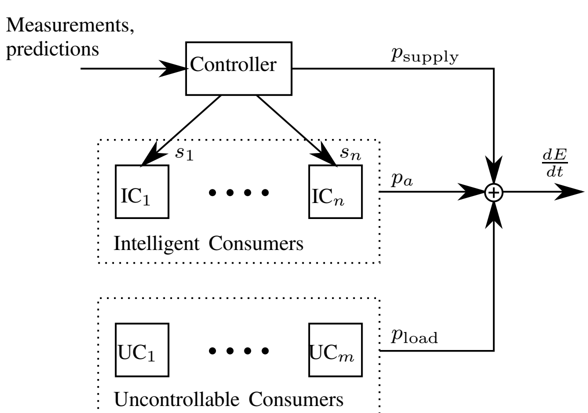

Fig. 1. The signal flow in the network. The controller must reduce the power imbalance E by appropriate utilization of the ICs without stressing the power supplier.

The n intelligent consumers are characterized by power consumptions $p = (p_1, \ldots, p_n) \in \mathbf{R}^n$, and a total consumption $p_a = \mathbf{1}^T p$, where $\mathbf{1}$ is a vector of ones, *i.e.* $\mathbf{1} = (1, \ldots, 1) \in \mathbf{R}^n$. The consumption p_i of an intelligent consumer consists of a drain rate and a storage rate

$$p_i(t) = s_i(t) + r_i(t) \tag{1}$$

$$\frac{dE_i(t)}{dt} = s_i(t) \tag{2}$$

where r_i is the drain rate while s_i is the storage rate and $E_i(t)$ is the stored energy, as illustrated in Figure 2. As an example, consider a house with electrical heating as an intelligent consumer. Some energy is lost due to heat exchange with the outside world corresponding to the drain rate r_i. If the supplied power is larger than this drain rate, *i.e.* $s_i(t) > 0$, thermal energy is stored in the house and E_i increases. This allows us to supply little or zero power to the house at a later time such that $s_i(t) < 0$ whereby we will use the stored energy and E_i will decrease. With this understanding we note, that a negative s_i does not necessarily mean that we supply electrical power to the grid, but simply that we use less than the natural drain rate r_i. Finally note, that for simplicity we assume that the drain rate is independent on the amount of stored energy.

The stored energy E can take various forms; if the intelligent consumer i is a house with electrical heating, E_i would be thermal energy, while E_i would be electrical energy if consumer i was an electric vehicle. The amount of energy

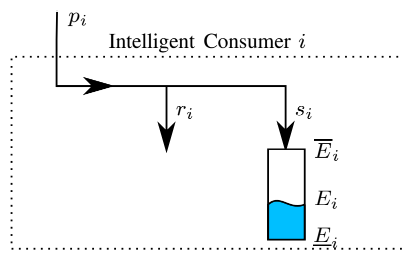

Fig. 2. Model of an intelligent consumer consisting of a drain rate r_i and a storage rate s_i, thus with a total consumption p_i. The stored energy is denoted E_i.

stored in intelligent consumer i can be freely regulated via storage rate s_i under limitations regarding rate and capacity:

$$\underline{s}_i \leq s_i(t) \leq \overline{s}_i \tag{3}$$

$$\underline{E}_i \leq E_i(t) \leq \overline{E}_i, \tag{4}$$

where the constants $\underline{s}_i$, $\overline{s}_i$, $\underline{E}_i$, $\overline{E}_i \in \mathbf{R}$ describe these limits. For a house with electrical heating, the energy levels $\underline{E}_i$, $\overline{E}_i$ would describe the lowest and highest allowed temperature in the house (comfort limits). The rate limits $\underline{s}_i$, $\overline{s}_i$ would describe lower and upper bounds on the power we can put into or avoid putting into the house.

The m uncontrollable consumers are characterized by power consumptions $q = (q_1, \ldots, q_m) \in \mathbf{R}^m$, yielding a total consumption $p_{\text{load}} = \mathbf{1}^T q$.

We define the system imbalance E as the integrated mismatch between production and consumption

$$\frac{dE(t)}{dt} = p_{\text{supply}}(t) - p_{\text{load}}(t) - p_a(t), \tag{5}$$

where p_{supply} denotes the power requested from the power supplier, see Figure 1. The interpretation of this imbalance depends on the system under consideration, but could *e.g.* represent deviation from planned operation. In this case, the imbalance would be penalized economically according to up- and down regulation prices.

The requested power p_{supply} is subject to power limits

$$\underline{p}_{\text{sup}} \leq p_{\text{supply}}(t) \leq \overline{p}_{\text{sup}}, \tag{6}$$

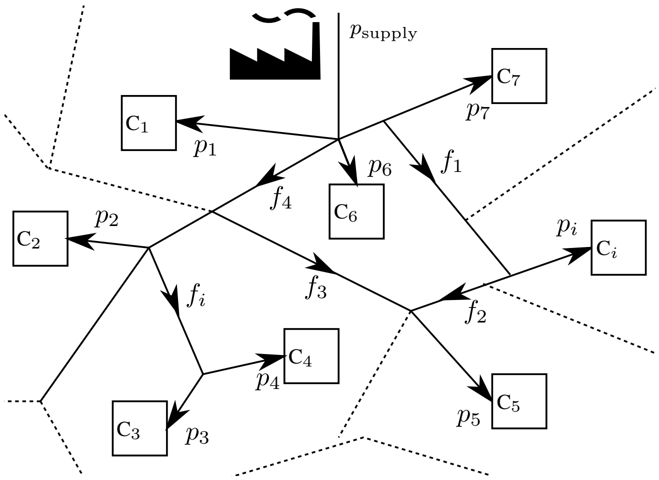

Fig. 3. A number of intelligent consumers (ICs) and uncontrollable consumers (UCs) powered through a network of links.

due to physical constraints of the power supplier. Further, it is desired to keep p_{supply} smooth to avoid stressing the power plant.

Next, we consider the power flows in the network. The $n+m$ consumers are connected to the grid through a network of links, as illustrated in Figure 3. Let l and v denote the number of links and nodes, respectively, and let $f = (f_1, \dots, f_l) \in \mathbf{R}^l$ denote the flows through the links. We can then represent the coupling between flows and power consumptions as

$$Ff(t) = Pp(t) + Qq(t), \tag{7}$$

where $F \in \mathbf{R}^{v\times l},\ P \in \mathbf{R}^{v\times n},\ Q \in \mathbf{R}^{v\times m}$. The entries in F, P, Q describe the network interconnections:

$$(F)_{ij} = \begin{cases} 1 & \text{if flow } j \text{ enters node } i \\ -1 & \text{if flow } j \text{ leaves node } i \\ 0 & \text{if flow } j \text{ is not connected to node } i \end{cases}$$

$$(P)_{ij} = \begin{cases} 1 & \text{if IC}_j \text{ is connected to node } i \\ 0 & \text{if IC}_j \text{ is not connected to node } i \end{cases}$$

$$(Q)_{ij} = \begin{cases} 1 & \text{if UC}_j \text{ is connected to node } i \\ 0 & \text{if UC}_j \text{ is not connected to node } i, \end{cases}$$

where $(X)_{ij}$ denotes the (i, j)th entry in X.

As each link in the network is limited in capacity, the flows are constrained by

$$-\overline{f}_j \le f_j(t) \le \overline{f}_j, \tag{8}$$

where $\overline{f}_j$ represents the capacity limitation of link j.

III. Controller Synthesis

The objective of the controller is twofold. The controller must

- maintain system balance (between consumption and production),
- avoid stressing the power supplier.

This means that the imbalance E must be driven to zero by the controller. As it is costly to assign power from the power supplier p_{supply} for fast regulation, it is attractive for the controller to involve the intelligent consumers in the balancing; the intelligent consumers will provide this regulation freely under the given power and capacity limitations.

In the following we formulate the task of the controller as an optimization problem based on the models presented above. As the dynamics of the intelligent consumers are pure integration, we can easily formulate discrete approximations. In the rest of the paper, we use discrete time models where k is used to indicate sample number and a sample time of 1 s is used to ease the notation.

A. Objectives

Based on a finite horizon N, we formulate the following three objectives of the controller at time k.

a) Imbalance Reduction: The main task of the controller is to minimize the imbalance E between production and consumption. We can describe the imbalance to be minimized as

$$J_e(k) = \sum_{\kappa=k+1}^{k+N} \|E(\kappa)\|^2.$$

b) Low Stress on Power Supplier: It is further desired to avoid stressing the power supplier, which is accomplished by assigning power from the power plant smoothly. We formulate this as a minimization of the change in p_{supply}

$$J_p(k) = \sum_{\kappa=k}^{k+N-1} \|p_{\text{supply}}(\kappa) - p_{\text{supply}}(\kappa-1)\|^2.$$

c) Energy Storage Mid-Ranging: Finally, it is desirable to keep the energy storages close to their respective mid-points, hereby allowing large freedom for preemptive action. By using $(\overline{E}_i - \underline{E}_i)/2$ as the energy mid-point, we can formulate this storage mid-ranging as

$$J_m(k) = \sum_{\kappa=k+1}^{k+N} \sum_{i=1}^{n} \|E_i(\kappa) - (\overline{E}_i - \underline{E}_i)/2\|^2.$$

B. Optimization Problem

At time k we look N steps into the future and minimize the cost $J(k) = (J_e(k), J_p(k), J_m(k)) \in \mathbf{R}^3_+$ subject to the dynamics and the given constraints. This can be expressed as the following optimization problem.

$$\begin{array}{ll} \text{minimize} & \lambda^T J(k) \\ \text{subject to} & E(\kappa+1) = \\ & \qquad E(\kappa) + p_{\text{supply}}(\kappa) - \mathbf{1}^T q(\kappa) - \mathbf{1}^T p(\kappa) \\ & E_i(\kappa+1) = E_i(\kappa) + s_i(\kappa) \\ & p_i(\kappa) = s_i(\kappa) + r_i(\kappa) \\ & \underline{s}_i \le s_i(\kappa) \le \overline{s}_i \\ & \underline{E}_i \le E_i(\kappa) \le \overline{E}_i \\ & \underline{p}_{\text{sup}} \le p_{\text{supply}}(\kappa) \le \overline{p}_{\text{sup}} \\ & Ff(\kappa) = Pp(\kappa) + Qq(\kappa) \\ & -\overline{f}_j \le f_j(\kappa) \le \overline{f}_j \end{array}$$

where $\kappa = k, \dots, k+N-1$ and where $i = 1, \dots, n$ and $j = 1, \dots, l$. The variables are $p_i(\kappa),\ E(\kappa+1),\ E_i(\kappa+1),\ s_i(\kappa),\ p_{\text{supply}}(\kappa),\ f_j(\kappa) \in \mathbf{R}$, while $\lambda \in \mathbf{R}^3_+$ is a vector valued parameter providing a weighting between the three objectives. The data to the optimization problem is $r_i(\kappa),\ q_i(\kappa),\ E_i(k),\ E(k) \in \mathbf{R}$. Discrete time equivalents of Equations (1) – (8) are used.

Note that this is a standard MPC problem, see *e.g.* [12].

C. Controller Algorithm

Based on the optimization presented above, we formulate an algorithm for controlling the intelligent consumers as follows. The controller algorithm implements the above optimization in a receding horizon fashion.

1) Gather estimates of the future drain rates of the intelligent consumers $[r_i(k), \ldots, r_i(k+N-1)]$ and power consumptions of the uncontrollable consumers $[q_i(k), \ldots, q_i(k+N-1)]$. Further, gather the current energy levels $E_i(k)$ and the imbalance $E(k)$.
2) Solve the MPC optimization problem presented in Section III-B. Let the solution be denoted $[s_i^\star(k), \ldots, s_i^\star(k+N-1)]$ for the storage rates and $[p_{\text{supply}}^\star(k), \ldots, p_{\text{supply}}^\star(k+N-1)]$ for power of the supplier.
3) Apply the power $s_i^\star(k)$ to intelligent consumer i for $i = 1, \ldots, n$ and assign the power $p_{\text{supply}}^\star(k)$ from the power supplier.
4) Increase k by one and repeat from step 1.

Hereby we have a controller that is able to react preemptive to future known events, while taking given physical constraints of the system into account.

IV. Simulation Example

The examples presented in this section show the benefits of utilizing the storage capacities of intelligent consumers, and illustrate that MPC is an attractive control scheme to accomplish this task.

In order to keep the methods of this paper generic to both the transmission level and the distribution level, we do not include any units on the consumers. Hereby we do not specify whether the intelligent consumers represent a single electrical unit or a large number of aggregated units. Further, the simulation example is kept at a conceptual level with only $n = 4$ intelligent consumers and $l = 4$ links, such that the behavior of the controller is clear (see Figure 4). We impose capacity and power constraints for the storages

$$\underline{s}_i \leq s_i(k) \leq \overline{s}_i$$
$$0 \leq E_i(k) \leq \overline{E}_i,$$

i.e. we let $\underline{E}_i = 0$ for simplicity. Further we have constraints on the link capacities and power limits on the production

$$-\overline{f}_j \leq f_j(k) \leq \overline{f}_j$$
$$\underline{p}_{\text{sup}} \leq p_{\text{supply}}(k) \leq \overline{p}_{\text{sup}}.$$

The limits on energy storages and on link capacities are presented in Table I while $\underline{p}_{\text{sup}}$, $\overline{p}_{\text{sup}}$ are chosen to be -10.0 and 10.0, respectively. We assume that the drain rates are constant $r_i(k) = r_i$ and use the values presented in Table I.

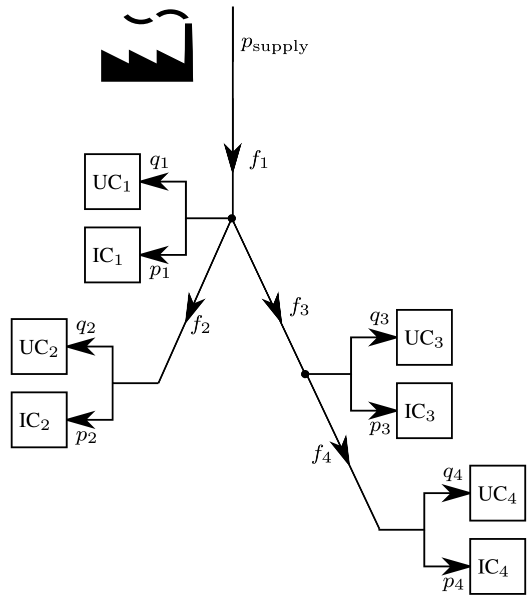

Fig. 4. Simulation example setup. Four intelligent consumers and four uncontrollable consumers are interconnected by four links.

$\overline{E}_1 = 4.0$	$\underline{s}_i = -4.7$	$\overline{s}_i = 3.4$	$r_1 = 4.0$	$\overline{f}_1 = 40.0$
$\overline{E}_2 = 5.0$	$\underline{s}_i = -3.3$	$\overline{s}_i = 3.5$	$r_2 = 1.0$	$\overline{f}_2 = 10.0$
$\overline{E}_3 = 4.0$	$\underline{s}_i = -4.2$	$\overline{s}_i = 4.0$	$r_3 = 2.0$	$\overline{f}_3 = 25.0$
$\overline{E}_4 = 5.0$	$\underline{s}_i = -2.8$	$\overline{s}_i = 5.5$	$r_4 = 3.0$	$\overline{f}_4 = 15.0$

TABLE I
Key parameters used in the simulation example.

Due to the network structure, as presented in Figure 4, the coupling between flows and power consumptions can be described as

$$Ff(k) = p(k) + q(k)$$

where

$$F = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In the following we present simulation results using a prediction horizon of $N = 10$ and an appropriate weight vector λ.

A. Overall Performance

The behavior of the controller is illustrated in the following. We compare two cases, one where the controller is allowed to utilize the storage facilities in the intelligent consumers and one where this is not allowed. In both cases we observe the imbalance E and utilization of the power p_{supply}. In the case where it is not allowed to utilize the intelligent consumers, the controller will simply choose p_{supply} such that the optimal trade-off between imbalance and power supply stress is found.

Figure 5 (top) illustrates the four uncontrollable consumptions q_1 to q_4. The four consumptions constitute p_{load} as

$p_{\text{load}} = \mathbf{1}^T q$. The resulting accumulated imbalance E and utilization of power from the power supplier p_{supply} are also shown in Figure 5. We compare the case where the MPC controller regulates the intelligent consumers (red, dashed) with the case where the intelligent consumers are not utilized (blue, solid). We note a significant reduction of the imbalance E and a smoothing of p_{supply}.

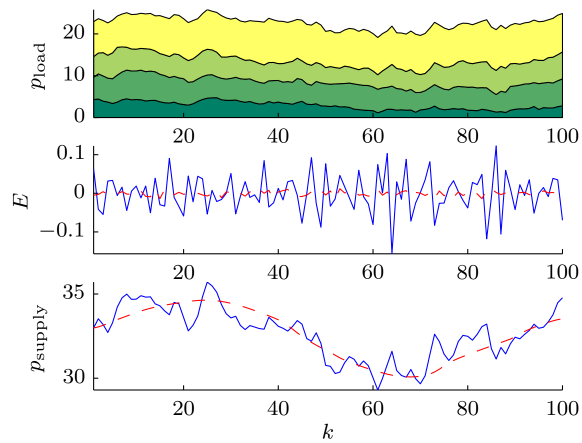

Fig. 5. Top: the uncontrollable consumptions from q_1 (yellow, top) to q_4 (dark green, bottom). Middle and bottom: responses E and p_{supply}, respectively, with MPC control of the intelligent consumers (red, dashed) and with no active intelligent consumers (blue, solid).

B. Performance Improvement due to Prediction

Next, we examine how the MPC controller is able to handle constraints and benefit from consumption predictions. We illustrate this by considering a case where consumer 1 suddenly increases the power consumption, q_1, while the remaining consumers have constant consumptions. This results in a power consumption as illustrated in Figure 6 (top). The controller is assumed to be able to make a good prediction of this step (this could reflect a factory starting production at a known time of the day). Figure 6 illustrates that by utilizing the storage facilities of the intelligent consumers, the MPC is able to keep the imbalance close to zero, while only smoothly utilizing the power supplier p_{supply} (red, dashed curves). For comparison, the response to the same load without prediction results in an undesired abrupt change in p_{supply} and a significant imbalance (blue, solid curves).

Figure 7 shows the corresponding energy levels of the intelligent consumers. In the case of prediction (red, dashed curves), the intelligent consumers contribute to the smooth transition of p_{supply}; all four intelligent consumers use the external power p_{supply} to fill their reservoirs before the step in the load occurs, and start unloading once the step occurs. This action ahead of time, allows the external power p_{supply} to increase smoothly over 40 samples, instead of an undesired rapid change causing congestion. With no prediction (blue, solid curves), the intelligent consumers are not able to fill their reservoirs ahead of time and are therefore incapable of allowing a smooth transition.

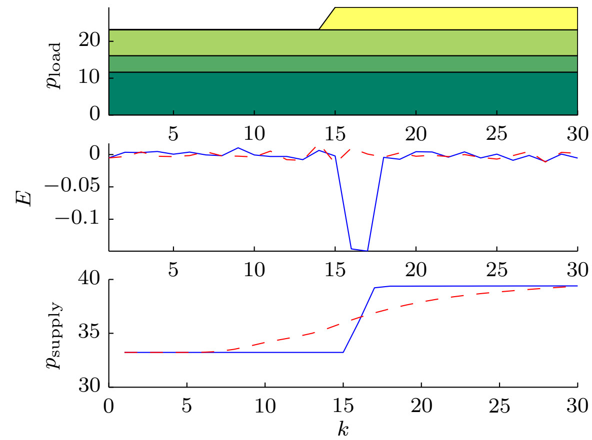

Fig. 6. Response to a step in the load q_1 of consumer 1 (upper plot) where we observe the resulting imbalance E and power from the supplier p_{supply} (bottom two plots). A comparison is presented with a controller utilizing predictions of the step (red, dashed) and a controller not utilizing this prediction (blue, solid).

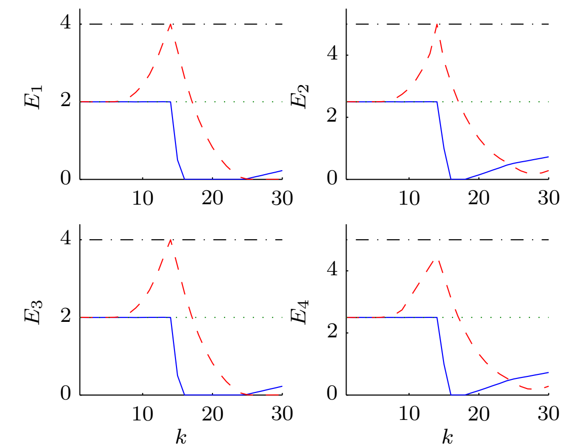

Fig. 7. The four energy levels E_i in the case of predictive control (red, dashed) and no prediction (blue, solid) when applying a load as presented in Figure 6. Further, the upper energy levels $\overline{E}_i$ (black dash-dotted) and the energy mid-points $(\overline{E}_i - \underline{E}_i)/2$ (green dotted) are depicted.

Figure 8 shows the corresponding link flows f along with the link capacities. In the predictive case (red, dashed curves), the four reservoirs start filling up the reservoirs ahead of time, saturating f_4. This is the reason that reservoir 4 is only partially filled prior to the step in load, see Figure 7. This is in contrast to the case with no prediction (blue, solid curves), where the controller does not act ahead of time, and therefore does not use the full capacity of link 4.

We sum up and conclude on the results in Section VI.

V. Discussion

In the presented method, two rough assumptions are used. The first is that the intelligent consumers are seen as ideal storages and the second is that only the predictable noise is considered. A natural extension of this work is therefore to extend the MPC algorithm to handle storages, that are not ideal, as presented in *e.g.* [13], and further to handle

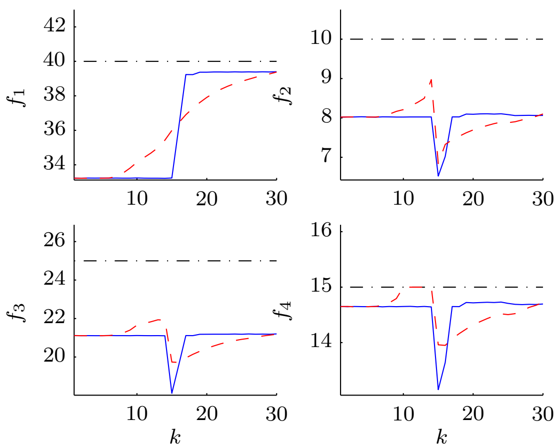

Fig. 8. The four link flows corresponding to the step in p_{load} presented in Figure 6 in the predictive case (red, dashed) and with no prediction (blue, solid). The capacity limits $\overline{f}_j$ are black dashed.

unpredictable noise. We intend to address this in our future research.

Another important issue is that we have assumed that all consumers in the network are under the jurisdiction of the same balancing responsible. In our future research, we will consider the questions that arise when there are several balancing responsible companies in the network, as is the case in a liberalized energy market.

Finally we note that the presented method is only suitable for a relatively small number of consumers, as the computational burden scales poorly with the number of states ($O(n^3)$), see [14]. This calls for alternative methods when the system is large *e.g.* in the case of control on a national level. One approach to remedy this problem is to use a hierarchical approach, where a high-level controller controls a number of so called *aggregators*. Each aggregator then controls a small number of consumers, such that the computational burden is reduced and distributed among the aggregators. This concept is presented in [11] and would be a natural extension of the controller design presented in this work.

VI. Conclusion

In this paper, an MPC approach was proposed for the control of intelligent consumers connected to the power grid through a network of limited capacity. The MPC strategy is well suited for this problem, as it directly incorporates consumption predictions and system limitations; given good predictions within the control horizon, we are able to handle the trade-off between the objectives optimally, while honoring all constraints.

The presented simulation examples illustrate the advantages of using MPC to control the intelligent consumers where we are able to exploit consumption predictions and handle system constraints. The result is that the controller is able to act ahead of time, ensuring balance without stressing the power supplier.

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