Optimal Dispatch Strategy for the Agile Virtual Power Plant

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Abstract— The introduction of large ratios of renewable energy into the existing power system is complicated by the inherent variability of production technologies, which harvest energy from wind, sun and waves. Fluctuations of renewable power production can be predicted to some extent, but the assumption of perfect prediction is unrealistic. This paper therefore introduces the Agile Virtual Power Plant. The Agile Virtual Power Plant assumes that the base load production planning based on best available knowledge is already given, so imbalances cannot be predicted. Consequently the Agile Virtual Power Plant attempts to preserve maneuverability (stay agile) rather than optimize performance according to predictions.

In this paper the imbalance compensation problem for an Agile Virtual Power Plant is formulated. It is proved formally, that when local units are power and energy constrained integrators a dispatch strategy exists, which is optimal regardless of future load/imbalances. The optimal dispatch is obtained at each sample by solving a quadratic program. Finally a simulation example illustrates the optimal dispatch strategy and compares the performance with a (non-optimal) MPC-strategy.

I. INTRODUCTION

Electricity is a so-called just-in-time product, which means that it is instantly consumed at production. This means, that electricity production and consumption must be closely balanced at all times. Production technologies, which harvest energy from natural sources such as wind, sun and waves, are unpredictable and fluctuating by nature. This means that the introduction of large ratios of renewable energy into the existing power system poses a significant challenge in terms of maintaining the real-time balance between production and consumption.

In Smart Grid systems the flexibility of consumers, such as electric vehicles, heat pumps and HVAC-systems, should be mobilized and play an active part in solving the balancing task. With this vision the discrepancies between supply and demand should be evened out via (short-term) storage of energy [5] or by voluntarily displacing consumption in time, so-called *demand-side management* [6].

A Virtual Power Plant is a collection of flexible consumers, which are grouped together and controlled centrally, see [3]. In this paper the flexible consumers are denoted local units and are modeled simply as power and energy constrained integrators.

The considered Virtual Power Plant is part of a larger portfolio of production units, such as wind turbines, solar panels, power plants etc. The entire portfolio is managed

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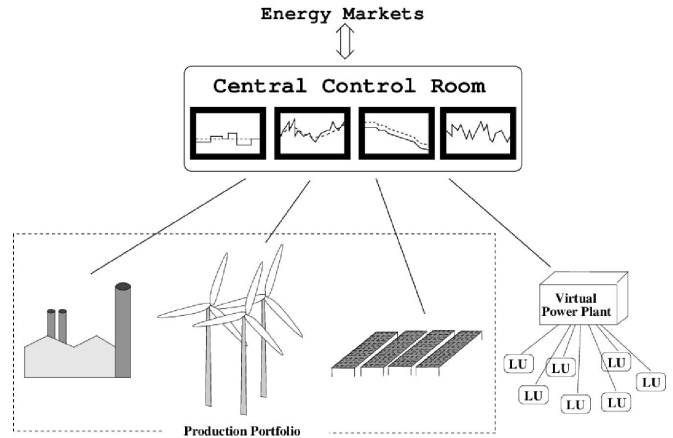
Fig. 1: The Virtual Power Plant is part of a larger portfolio of production units, such as wind turbines, solar panels, power plants etc. The portfolio is controlled centrally by a master controller, which utilizes the capacity of the Virtual Power Plant to compensate imbalances in production, such that the portfolio as a whole is following an agreed schedule.

by a master controller, which trades the production capacity on the energy markets, see Figure 1. Based on the market trading a production schedule for the portfolio is obtained (for more on energy markets see [7] and [8]).

Electricity production and consumption in the (near) future can be estimated based on weather forecast and 24-hour power consumption traces. However, the assumption of perfect prediction is unrealistic, so this paper explores a dispatch strategy for an Agile Virtual Power Plant. Because base load production is already given the master controller utilizes the Virtual Power Plant to compensate unforeseen errors and imbalances, such that the portfolio as a whole is following the agreed schedule, see Figure 2. The Agile Virtual Power Plant therefore attempts to "stay agile", rather than optimize performance according to predictions. The Agile Virtual Power Plant thus plays a role similar to that of traditional power plant reserves in a typical European power system (see [9]).

A similar balancing setup is considered in [1], which investigates energy storage in power system operations. In this work the flexible units are denoted power nodes. These power nodes are essentially energy and power constrained integrators, but ramp constraints and storage loss are also included in the modeling. A Model Predictive Control (MPC) approach is taken, with the assumption of perfect prediction of imbalances. Values for the dispatch parameters are obtained by manual tuning in order to obtain the desired system behavior.
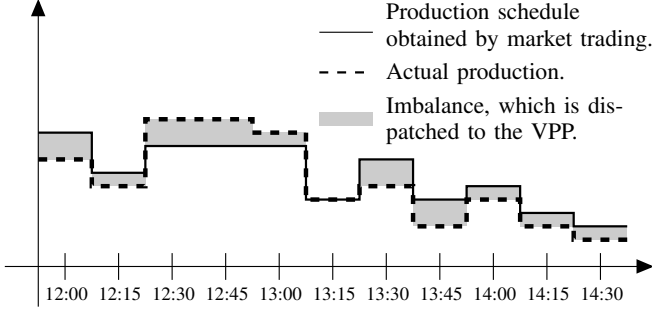
Fig. 2: A production schedule for the production portfolio is obtained by market trading. Any imbalance is then dispatched to the Virtual Power Plant, such that the portfolio as a whole is following the production schedule.

Also [2] considers the operation of storage devices in power systems. In this paper the storage units are also modeled as power and energy constrained integrators, but individual charging and decharging costs are include for each unit. Like [1] the paper [2] also takes an MPC approach to the balancing problem and imbalances are modeled as stochastic processes with diurnal components. It is not explained how values for the dispatch parameters are obtained, but it is indicated that they reflect the monetary costs of charging/decharging and the cost of load shedding.

In the work described above values for the dispatch parameters are fixed based on heuristics or monetary costs. This paper proposes that the dispatch parameters be chosen based on the individual local units' ability to compensate imbalances. Dispatch parameters are therefore calculated based on the state and characteristics (constraints) of each local unit itself.

The contribution of this paper is to formulate the imbalance compensation problem for an Agile Virtual Power Plant. It is proved formally that when the local units have a specific, simple form an optimal dispatch strategy can be obtained at each sample by solving a quadratic optimization problem. Finally, simulation studies show, that for the considered optimization problem the assumption of perfect prediction over a certain horizon does not guarantee optimality.

The remainder of this paper is structured as follows: In Section II, we formulate the imbalance compensation problem for an Agile Virtual Power Plant. Section III presents the main contribution of this paper, namely an optimal dispatch strategy for the imbalance compensation problem. In Section IV, a simulation example illustrates the optimal dispatch strategy and compares the performance with a (non-optimal) MPC-strategy. Finally, Section V gives concluding remarks and suggestions for further work.

II. PROBLEM FORMULATION

A. General Form of the Agile Virtual Power Plant Imbalance Compensation Problem

As explained earlier we consider a Virtual Power Plant, which is part of a larger portfolio of production units. A master controller has direct control of the entire portfolio

and trades the production capacity on the energy markets. Based on the market trading, a base load schedule for the production units is obtained. The Virtual Power Plant is then utilized to compensate for unforeseen errors and imbalances in production, such that the portfolio as a whole is following the agreed schedule.

The Virtual Power Plant has control of a set of local units $\{LU_i\}_{i=1,2,\ldots,N}$, which are governed by individual dynamics and constraints. The Virtual Power Plant offers the capacity of the local units to the master controller and we let $P_{Reserve,i}(k)$ denote the capacity of local unit i, which can be offered to the master controller at sample k. The Virtual Power Plant must offer its full available capacity to the master controller, so the offered capacity at sample k is

$$P_{Reserve}(k) = \sum_{i=1}^{N} P_{Reserve,i}(k).$$

At each sample some volume, $P_{Dispatch}(k)$, is received from the master controller. It is assumed that $0 \leq P_{Dispatch}(k) \leq P_{Reserve}(k)$, such that it is always possible to dispatch $P_{Dispatch}$ to the portfolio. The Virtual Power Plant must dispatch the full volume $P_{Dispatch}(k)$ to the local units and we let $P_i(k)$ denote the quantity dispatched to unit i, so

$$\sum_{i=1}^{N} P_i(k) = P_{Dispatch}(k).$$

The goal of the Virtual Power Plant is to service the master controller as well as possible. The objective is therefore to dispatch $P_{Dispatch}(k)$ to the local units such that $P_{Reserve}(k), k = 0, 1, \ldots, K$, is maximized. This can be formulated as

$$\max_{P_i(\cdot)} \sum_{k=0}^{K} \sum_{i=1}^{N} P_{Reserve,i}(k)$$

$$s.t.$$

$$0 \leq P_{Dispatch}(k) \leq \sum_{i=1}^{N} P_{Reserve,i}(k)$$

$$\sum_{i=1}^{N} P_i(k) = P_{Dispatch}(k)$$

and also subject to the dynamics and constraints of $\{LU_i\}_{i=1,2,\ldots,N}$. This is the general form of the Agile Virtual Power Plant imbalance compensation problem.

Remark 1: (Optimization Target)
In the formulation given above only the upper bound on the available capacity is considered as an optimization target. With this setup we obtain a clear objective, namely maximizing $P_{Reserve}$. If both the upper and lower bounds on the available capacity are included, that is introducing both $\overline{P}_{Reserve}$ and $\underline{P}_{Reserve}$, then the objective becomes less clear. This is because a gain in $\overline{P}_{Reserve}$ will introduce an equivalent loss in $\underline{P}_{Reserve}$. The problem could be handled by considering a less intuitive objective function than the one

presented above, but the penalty for the trade off between positive and negative reserve will invariably be based on heuristics. For now we will therefore only consider the upper bound on the available reserve and as a result it is assumed that the imbalance, $P_{Dispatch}$, is also positive, though this is obviously not a realistic assumption.

B. Agile Virtual Power Plant Imbalance Compensation Problem for Power and Energy Constrained Local Units

In this paper the local units are modeled simply as power and energy constrained integrators and we let $E_i(k)$ denote the energy level in local unit i at sample k.

Definition 1 (Power and Energy Constrained Local Unit): The dynamics and constraints of a power and energy constrained local unit are

$$LU_i(k): \quad E_i(k+1) = E_i(k) + T_s P_i(k)$$
$$0 \leq P_i(k) \leq \overline{P}_i$$
$$0 \leq E_i(k+1) \leq \overline{E}_i$$
$$E_i(0) = E_{i,0},$$

where $k = 0, 1, \ldots, K$, $i \in \mathbb{N}$, $0 \leq \overline{P}_i$, $0 \leq \overline{E}_i$ and $0 \leq E_{i,0} \leq \overline{E}_i$.

For ease of notation we assume that $T_s = 1$ in the following and let $\boldsymbol{LU_N}(k)$ denote a set of $N \in \mathbb{N}$ local units, that is $\{LU_i(k)\}_{i=1,2,\ldots,N}$.

With the choice of power and energy constrained local units we obtain that

$$P_{Reserve,i}(k) = \min(\overline{P}_i, \overline{E}_i - E_i(k)),$$

so the Agile Virtual Power Plant imbalance compensation problem for power and energy constrained local units is

$$\max_{P_i(\cdot)} \sum_{k=0}^{K} \sum_{i=1}^{N} P_{Reserve,i}(k) \tag{1}$$
$$s.t.$$
$$P_{Reserve,i}(k) = \min(\overline{P}_i, \overline{E}_i - E_i(k)) \tag{2}$$
$$0 \leq P_{Dispatch}(k) \leq \sum_{i=1}^{N} P_{Reserve,i}(k) \tag{3}$$
$$\sum_{i=1}^{N} P_i(k) = P_{Dispatch}(k) \tag{4}$$
$$E_i(k+1) = E_i(k) + P_i(k) \tag{5}$$
$$0 \leq P_i(k) \leq \overline{P}_i \tag{6}$$
$$0 \leq E_i(k+1) \leq \overline{E}_i \tag{7}$$
$$E_i(0) = E_{i,0}, \tag{8}$$

where $E_{i,0}$ is the initial energy level of unit i.

To simplify the setup it has been assumed that the Virtual Power Plant is offering $P_{Reserve}$ to the master controller at every sample. When power and energy constrained units are considered, however, $P_{Reserve}$ could be offered for more than one sample without any loss of information using Resource Polytopes as described in [4]. This would give the master controller the benefit of knowledge of future balancing capacity.

III. Optimal Dispatch Strategy

This section presents the main contribution of the article, namely the result, that the optimal dispatch at each sample is independent of future load/imbalances; And the optimal dispatch can be obtained at each sample by solving a quadratic program.

Definition 2 (Agility Factor): Let $LU_i(k)$ denote a power and energy constrained local unit. The *Agility Factor* of local unit i at sample k is defined as

$$\mathcal{K}_i(k) = \frac{\overline{E}_i - E_i(k)}{\overline{P}_i}.$$

Lemma 1: At sample k let $\boldsymbol{LU_N}(k)$ denote a finite set of power and energy constrained local units. A dispatch strategy for problem (1) - (8) can be obtained by solving the program

$$\max_{P_i(k)} \sum_{i=1}^{N} \frac{(\overline{E}_i - E_i(k) - P_i(k))^2}{-2\overline{P}_i} \tag{9}$$
$$s.t.$$
$$P_{Reserve,i}(k) = \min(\overline{P}_i, \overline{E}_i - E_i(k)) \tag{10}$$
$$0 \leq P_{Dispatch}(k) \leq \sum_{i=1}^{N} P_{Reserve,i}(k) \tag{11}$$
$$\sum_{i=1}^{N} P_i(k) = P_{Dispatch}(k) \tag{12}$$
$$E_i(k+1) = E_i(k) + P_i(k) \tag{13}$$
$$0 \leq P_i(k) \leq \overline{P}_i \tag{14}$$
$$0 \leq E_i(k+1) \leq \overline{E}_i \tag{15}$$
$$E_i(0) = E_{i,0}. \tag{16}$$

and for this dispatch strategy the marginal cost/gain of dispatching to local unit i is $\mathcal{K}_i(k+1)$.

Proof: First observe that the constraints (2) - (8) are the same as (10) - (16), so at sample k a feasible dispatch strategy for problem (1) - (8) can be obtained by solving (9) - (16).

Next define

$$f(P_i(k)) = \sum_{i=1}^{N} \frac{(\overline{E}_i - E_i(k) - P_i(k))^2}{-2\overline{P}_i},$$

so

$$\nabla f(P_i(k)) = \left[\frac{\overline{E}_1 - E_1(k) - P_1(k)}{\overline{P}_1}, \ldots, \right.$$
$$\left. \frac{\overline{E} - E_N(k) - P_N(k)}{\overline{P}_N} \right]$$
$$= \left[\frac{\overline{E}_1 - E_1(k+1)}{\overline{P}_1}, \ldots, \frac{\overline{E} - E_N(k+1)}{\overline{P}_N} \right]$$
$$= [\mathcal{K}_1(k+1), \ldots, \mathcal{K}_N(k+1)].$$

∎

Definition 3 (Feasible Dispatch Sequence):
Let $\boldsymbol{LU_N}(k)$ denote a finite set of power and energy constrained local units. The sequence $\{\boldsymbol{P_{Dispatch}(k)}\}_{k=0,1,\ldots,K}$ is a *Feasible Dispatch Sequence* associated with $\boldsymbol{LU_N}(k)$ if problem (1) - (8) is feasible for $P_{Dispatch}(k) = \boldsymbol{P_{Dispatch}(k)}, k = 0, 1, \ldots, K$.

Definition 4 (Set of Feasible Dispatch Sequences):
Let $\boldsymbol{LU_N}(k)$ denote a finite set of power and energy constrained local units. The *Set of Feasible Dispatch Sequences* over horizon K for $\boldsymbol{LU_N}(k)$ is denoted $\boldsymbol{\Omega_K}\big(\boldsymbol{LU_N}(k)\big)$.

Definition 5 (Integer Agility Factor System):
A set of $\mathsf{K_{Max}}$ power and energy constrained local units, for which

$$\overline{\mathsf{E}}_1 = \overline{\mathsf{P}}_1, \overline{\mathsf{E}}_2 = 2 \cdot \overline{\mathsf{P}}_2, \ldots, \overline{\mathsf{E}}_{\mathsf{K_{Max}}} = \mathsf{K_{Max}} \cdot \overline{\mathsf{P}}_{\mathsf{K_{Max}}}$$

is denoted an *Integer Agility Factor System*. Observe that for an *Integer Agility Factor System* the index number, i, equals $\mathcal{K}_i$.

The set $\{\mathsf{LU}_j(k)\}_{j=1,2,\ldots,\mathsf{K_{Max}}}$ is denoted $\boldsymbol{\mathsf{LU_{K_{Max}}}}(k)$.

Lemma 2: For any finite set of power and energy constrained local units, $\boldsymbol{LU_N}(k)$, there exists an integer $\mathsf{K_{Max}}$ and an *Integer Agility Factor System*, denoted $\boldsymbol{\mathsf{LU_{K_{Max}}}}(k)$, such that

$$\boldsymbol{\Omega_K}\big(\boldsymbol{LU_N}(k)\big) = \boldsymbol{\Omega_K}\big(\boldsymbol{\mathsf{LU_{K_{Max}}}}(k)\big).$$

Proof: For each local unit in $\boldsymbol{LU_N}(k)$ define $LU_{\lceil \mathcal{K}_i(k) \rceil}$ by

$$\overline{P}_{\lceil \mathcal{K}_i(k) \rceil} = \overline{E}_i - E_i(k) - \lfloor \mathcal{K}_i(k) \rfloor \cdot \overline{P}_i,$$
$$\overline{E}_{\lceil \mathcal{K}_i(k) \rceil} = \left(\overline{E}_i - E_i(k) - \lfloor \mathcal{K}_i(k) \rfloor \cdot \overline{P}_i \right) \cdot \lceil \mathcal{K}_i(k) \rceil$$

and $LU_{\lfloor \mathcal{K}_i(k) \rfloor}$ by

$$\overline{P}_{\lfloor \mathcal{K}_i(k) \rfloor} = \overline{P}_i - \overline{P}_{\lceil \mathcal{K}_i(k) \rceil},$$
$$\overline{E}_{\lfloor \mathcal{K}_i(k) \rfloor} = \left(\overline{P}_i - \overline{P}_{\lceil \mathcal{K}_i(k) \rceil} \right) \cdot \lfloor \mathcal{K}_i(k) \rfloor,$$

so $\boldsymbol{\Omega_K}\big(LU_i(k)\big) = \boldsymbol{\Omega_K}\big(\{LU_{\lceil \mathcal{K}_i(k) \rceil}, LU_{\lfloor \mathcal{K}_i(k) \rfloor}\}\big)$, see Figure 3. Next group these units according to equal $\mathcal{K}$-value to obtain $\boldsymbol{\mathsf{LU_{K_{Max}}}}(k)$, where $\mathsf{K_{Max}} = \max_{i=1,2,\ldots,N} \lceil \mathcal{K}_i(k) \rceil$. ∎

Lemma 3: Let there be given an *Integer Agility Factor System*, $\boldsymbol{\mathsf{LU_{K_{Max}}}}(k)$, and a sequence $\{\boldsymbol{P_{Dispatch}(k)}\}_{k=0,1,\ldots,K}$ and define

$$\ell_n = \left\{ k = 0, 1, \ldots, K | \boldsymbol{P_{Dispatch}(k)} > \sum_{j=n}^{\mathsf{K_{Max}}} \overline{\mathsf{P}}_j \right\}$$
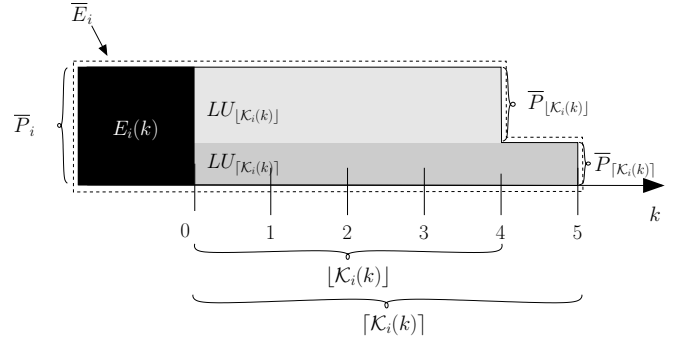

Fig. 3: Lemma 2: Any power and energy constrained local unit can be expressed as two equivalent units, which have integer *Agility Factors*.

for $n = 1, 2, \ldots, \mathsf{K_{Max}}$. Then

$$\{\boldsymbol{P_{Dispatch}(k)}\}_{k=0,1,\ldots,K} \in \boldsymbol{\Omega_K}\big(\boldsymbol{\mathsf{LU_{K_{Max}}}}(k)\big),$$

if and only if

$$P_{Dispatch}(k) \geq 0, \ k = 0, 1, \ldots, K \tag{17}$$
$$\sum_{k=0}^{K} P_{Dispatch}(k) \leq \sum_{j=1}^{\mathsf{K_{Max}}} \overline{\mathsf{E}}_j \tag{18}$$
$$\ell_1 = \emptyset \tag{19}$$

and

$$\sum_{k \in \ell_n} \left(\boldsymbol{P_{Dispatch}(k)} - \sum_{j=n}^{\mathsf{K_{Max}}} \overline{\mathsf{P}}_j \right) \leq \sum_{j=1}^{n-1} \overline{\mathsf{E}}_j \tag{20}$$

for $n = 2, 3, \ldots, \mathsf{K_{Max}}$, see Figure 4.

Proof: Consider an *Integer Agility Factor System* consisting of just one local unit $\{\mathsf{LU}_1(k)\}$, where

$$\overline{\mathsf{E}}_1 = \overline{\mathsf{P}}_1.$$

Then

$$\{\boldsymbol{P_{Dispatch}(k)}\}_{k=0,1,\ldots,K} \in \boldsymbol{\Omega_K}\big(\{\mathsf{LU}_1(k)\}\big),$$

if and only if

$$P_{Dispatch}(k) \geq 0, \ k = 0, 1, \ldots, K$$
$$\sum_{k=0}^{K} P_{Dispatch}(k) \leq \overline{\mathsf{E}}_1$$

and

$$\ell_1 = \emptyset.$$

Next consider an *Integer Agility Factor System* consisting of two local units $\{\mathsf{LU}_1(k), \mathsf{LU}_2(k)\}$, where

$$\overline{\mathsf{E}}_1 = \overline{\mathsf{P}}_1, \overline{\mathsf{E}}_2 = 2 \cdot \overline{\mathsf{P}}_2.$$

Then

$$\{\boldsymbol{P_{Dispatch}(k)}\}_{k=0,1,\ldots,K} \in \boldsymbol{\Omega_K}\big(\{\mathsf{LU}_1(k), \mathsf{LU}_2(k)\}\big),$$
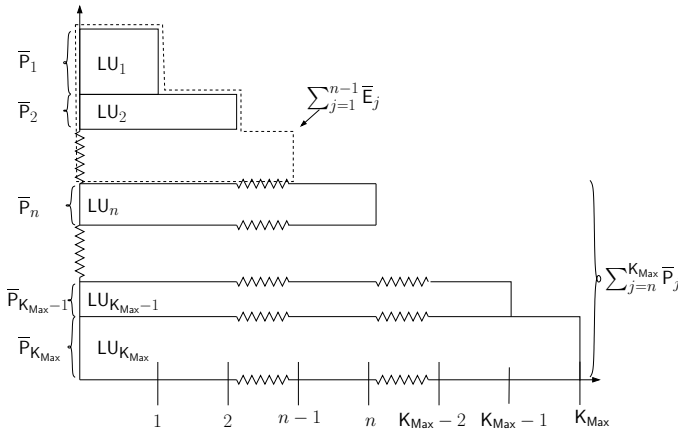
Fig. 4: Lemma 3: Samples where $P_{Dispatch}(k)$ exceeds $\sum_{j=n}^{K_{Max}} \overline{P}_j$ are denoted ℓ_n. For these samples any quantity larger than $\sum_{j=n}^{K_{Max}} \overline{P}_j$ must be dispatched to units $\{LU_j(k)\}_{j=1,2,\dots,n-1}$.

if and only if

$$P_{Dispatch}(k) \geq 0, \ k = 0, 1, \dots, K$$
$$\sum_{k=0}^{K} P_{Dispatch}(k) \leq \sum_{j=1}^{2} \overline{E}_j$$
$$\ell_1 = \emptyset.$$

and

$$\sum_{k \in \ell_2} \left(P_{Dispatch}(k) - \overline{P}_2 \right) \leq \overline{E}_1$$

Using equivalent reasoning, in the general case, that is when considering $LU_{K_{Max}}(k)$, we obtain (17) to (20). ∎

Lemma 4: Let $LU_N(k)$ denote a finite set of power and energy constrained local units and let some quantity $P_{Dispatch,0}$ satisfying $0 \leq P_{Dispatch,0} \leq \sum_{i=1}^{N} P_{Reserve,i}(k)$ be given.

Also let $LU_N^R(k+1)$ be a set of local units obtained by a feasible dispatch of $P_{Dispatch,0}$ to $LU_N(k)$ and let $LU_N^Q(k+1)$ be the set of local units obtained by dispatching according to the solution of problem (9) - (16). Then

$$\Omega_K\big(LU_N^R(k+1)\big) \subseteq \Omega_K\big(LU_N^Q(k+1)\big).$$

Proof: First let $LU_{K_{Max}}(k)$ be the *Integer Agility Factor System* associated with $LU_N(k)$ as given by Lemma 2. Then by Lemma 3 the set of feasible dispatch sequences is given by (17) to (20).

Next for each $n = 2, 3, \dots, K_{Max}$ let α_n denote the ratio of $P_{Dispatch,0}$, which is dispatched to units $j = 1, 2, \dots, n-1$. After dispatch of $P_{Dispatch,0}$ we have that $\{P_{Dispatch}(k)\}_{k=0,1,\dots,K}$ is a feasible dispatch sequence

of the system if and only if

$$P_{Dispatch}(k) \geq 0, \ k = 0, 1, \dots, K$$
$$\sum_{k=0}^{K} P_{Dispatch}(k) \leq \sum_{j=1}^{K_{Max}} \overline{E}_j - P_{Dispatch,0}$$
$$\ell_1 = \emptyset$$

and

$$\sum_{k \in \ell_n} \left(P_{Dispatch}(k) - \sum_{j=n}^{K_{Max}} \overline{P}_j \right)$$
$$\leq \sum_{j=1}^{n-1} \overline{E}_j - \alpha_n \cdot P_{Dispatch,0} \quad (21)$$

for $n = 2, 3, \dots, K_{Max}$.

It follows from (21), that the maximum set of feasible dispatch sequences after dispatch of $P_{Dispatch,0}$ is obtained by minimizing α_n for all n, that is

$$\min_{P_j(k)} \alpha_n, \quad n = 2, 3, \dots, K_{Max},$$

subject to (12) to (16). This also means for each n dispatch as much as possible to the local units $n+1, n+2, \dots, K_{Max}$ and it follows by Lemma 1, that this is exactly what is obtained by the dispatch strategy (9) to (16).

Finally $\Omega_K\big(LU_N^R(k+1)\big) \subseteq \Omega_K\big(LU_N^Q(k+1)\big)$, since no other dispatch can generate higher upper bounds on (21) than what is obtained by (9) to (16). ∎

Theorem 1: Dispatching according to the solution of (9) to (16) at each sample, yields an optimal dispatch strategy for (1) to (8).

Proof: Let $LU_N(k)$ denote a finite set of power and energy constrained local units. Observe, that at any sample $n \geq k$

$$P_{Reserve}(n) =$$
$$\max_{P_{Dispatch}(\cdot) \in \Omega_K\big(LU_N(n)\big)} P_{Dispatch}(n). \quad (22)$$

Next let $\{P_{Dispatch}(k)\}_{k=0,1,\dots,K}$ be any sequence in $\Omega_K\big(LU_N(k)\big)$. Also let $\{LU_N^{Opt}(k)\}_{k=0,1,\dots,K}$ denote the optimal sequence of sets of local units, that is the sequence of sets of local units obtained by dispatching according to the solution of (1) to (8). Finally let $\{LU_N^Q(k)\}_{k=0,1,\dots,K}$ denote the sequence of sets of local units obtained by dispatching according to the solution of (9) to (16) at each sample. By Lemma 4

$$\Omega_K\big(LU_N^{Opt}(k)\big) \subseteq \Omega_K\big(LU_N^Q(k)\big), \quad k = 0, 1, \dots, K.$$

It now follows from (22) that dispatching according to the solution of (9) to (16) at each sample, yields an optimal dispatch strategy for (1) to (8). ∎

Remark 2: (Agile, Linear Dispatch Strategy)
By using the *Agility Factors* of the local units *before* dispatch as weights in the objective function, we can formulate a linear problem, which also generates a "$\mathcal{K}$-greedy" dispatch strategy. This strategy is denoted the linear strategy and at sample k it is obtained by

$$\max_{P_i(k)} \sum_{i=1}^{N} \mathcal{K}_i(k) P_i(k) \qquad (23)$$

subject to (2) - (8).

The linear strategy, however, is not optimal, which is illustrated by the following example: Consider a system of the two local units given in Table I and let $P_{Dispatch}(k) = 10$. The solution of problem (23) subject to (2) - (8) is then $P_1(k) = 10$ and $P_2(k) = 0$. This means that $P_{Reserve}(k+1) = 15$, since $E_1(k+1) = 15$ and $E_2(k+1) = 6$. A higher value of $P_{Reserve}(k+1)$, however, is obtained by setting $P_1(k) = 5.5$ and $P_2(k) = 4.5$, since this makes $E_1(k+1) = 10.5$ and $E_2(k+1) = 10.5$, so $P_{Reserve}(k+1) = 19$. This shows that the linear strategy is not optimal.

The problem is that the linear strategy distributes according to $\mathcal{K}_i(k)$, that is, the state before dispatch, and does not consider the dynamic effects of the current dispatch. Since the quadratic optimization problem has marginal costs/gain of $\mathcal{K}_i(k+1)$ it exactly considers the dynamic effects of the current dispatch.

	LU_1	LU_2
$\overline{P}$	10	10
$\overline{E}$	20	20
$E(k)$	5	6
$\mathcal{K}_i(k)$	1.5	1.4

TABLE I: System of local units for which the agile, linear strategy is not optimal for $P_{Dispatch}(k) = 10$.

IV. SIMULATION EXAMPLE

To illustrate the different dispatch strategies a simulation example has been constructed and implemented. The performance of the optimal and linear dispatch strategies are compared to a predictive dispatch strategy in which perfect prediction of $P_{Dispatch}$ is assumed over a certain prediction horizon.

At each sample k the optimal dispatch strategy solves the optimization problem (9) - (16) and the linear strategy solves problem (23) subject to (2) - (8). At each sample k the predictive dispatch strategy solves the optimization problem (1) - (8) by assuming perfect prediction of $P_{Dispatch}$ over

the horizon $N_{Predict}$:

$$\max_{P_i(\cdot)} \sum_{n=0}^{N_{Predict}} \sum_{i=1}^{N} P_{Reserve,i}(k+n) \qquad (24)$$

$s.t.$

$$P_{Reserve,i}(k+n) = \min(\overline{P}_i, \overline{E}_i - E_i(k+n)) \quad (25)$$

$$0 \le P_{Dispatch}(k+n) \le \sum_{i=1}^{N} P_{Reserve,i}(k+n) \quad (26)$$

$$\sum_{i=1}^{N} P_i(k+n) = P_{Dispatch}(k+n) \qquad (27)$$

$$E_i(k+1+n) = E_i(k+n) + P_i(k+n) \qquad (28)$$

$$-\overline{P}_i \le P_i(k+n) \le \overline{P}_i \qquad (29)$$

$$0 \le E_i(k+1+n) \le \overline{E}_i \qquad (30)$$

$$E_i(0) = E_{i,0}, \qquad (31)$$

where $N_{Predict}$ is less than K.

At each sample k in the simulations we choose $P_{Dispatch}(k)$ randomly from a uniform distribution subject to the constraint

$$P_{Dispatch}(k) \le \min \big[P_{Reserve,Optimal}(k),$$
$$P_{Reserve,Linear}(k),$$
$$P_{Reserve,Predictive}(k) \big],$$

which insures that problem (9) - (16), problem (23) subject to (2) - (8) and problem (24) - (31) are all feasible.

Nine local units are included in the simulations and parameters for these are given in Table II. Additional simulation parameters are $K = 90$, $T_s = 1$, and $N_{Predict} = 3$.

i	$\overline{P}_i$	$\overline{E}_i$	$E_{i,0}$	$\mathcal{K}_i(0)$
1	1	40	0	40
2	2	50	0	25
3	3	45	0	15
4	4	120	0	30
5	5	175	0	35
6	6	270	0	45
7	7	35	0	5
8	8	160	0	20
9	9	90	0	10

TABLE II: Parameters for the local units.

The simulation results are depicted in Figure 5. In the first part of the simulations the three methods perform equally well. After sample 15, however, the optimal and linear strategies are able to compensate for a larger imbalance than the predictive strategy, even though the three methods have the exact same local units at their disposal and have to balance the exact same load. As expected, the linear non-predictive strategy performs worse than the optimal strategy, but still much better than the predictive strategy.

The explanation for why the optimal strategy outperforms the predictive strategy can be found in Figure 6, which depicts the energy levels in each of the nine local units. The optimal strategy is able to get a better utilization of e.g. LU_7 and LU_9 early in the simulations, which allows it to stay clear of $\overline{E}$ for all local units until sample 70.
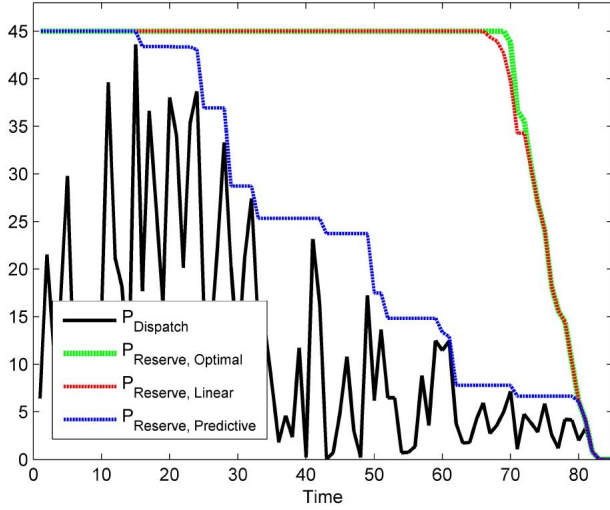
Fig. 5: Simulation Results: $P_{Reserve}$ is an upper bound on the amount of imbalance which could potentially be dispatched to the Virtual Power Plant. A high value of $P_{Reserve}$ thus means a better utilization of the available flexibility. At each sample the predictive dispatch strategy assumes perfect prediction of $P_{Dispatch}$ over the next three samples; An assumption which is *not* made by the optimal and linear strategies.

In the first part of the simulations the three methods perform equally well. After sample 15, however, the optimal and linear dispatch strategies are able to compensate for a larger imbalance than the predictive dispatch strategy. This happens even though the three methods have the exact same local units at their disposal and have to balance the exact same load.
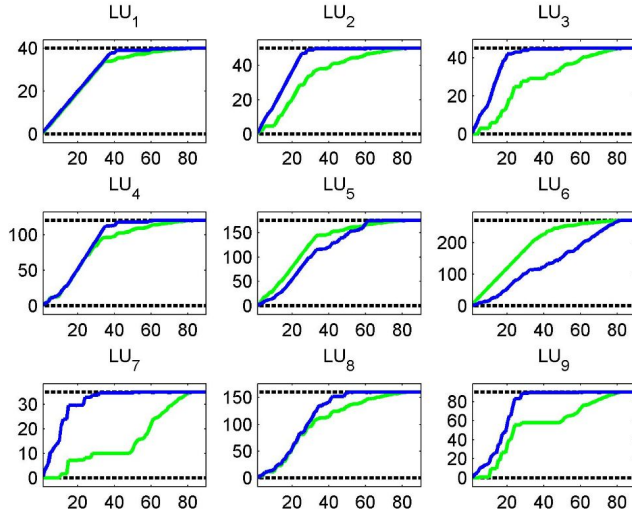

Fig. 6: Simulation Results: Energy levels for each of the nine energy and power constrained local units (LUs) considered in the simulations (Parameters for the local units are given in Table II). Blue depicts the predictive strategy and green depicts the optimal strategy.

Notice that the optimal strategy has a better utilization of e.g. LU_7 and LU_9 early in the simulations. This allows the optimal strategy to stay clear of $\overline{E}$ for all local units until sample 70.

V. DISCUSSION

This paper presented the imbalance compensation problem for an Agile Virtual Power Plant and proved the optimality of an associated dispatch strategy when the local units are power and energy constrained integrators. Furthermore, simulation results indicated, that for the considered optimization problem the assumption of perfect prediction is not enough to insure optimality.

Further research will address an extension of the setup by adding availability and minimum runtime constraints to the local unit models. When such temporal constraints are added it seems unlikely that an optimal strategy can be found for any future trajectory of $P_{Dispatch}$. It might, however, be possible to formulate general strategies or rules-of-thumb for these models even without the assumption of prediction. This would be highly advantageous for large scale problems where computational demands become significant.

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