A Set-Valued Approach to FDI and FTC: Theory and Implementation Issues

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Abstract: A complete methodology to design robust Fault Detection and Isolation (FDI) filters and Fault Tolerant Control (FTC) schemes for Linear Time-Varying (LTV) systems is proposed. The paper takes advantage of the recent advances in model invalidation using Set-Valued Observers (SVOs) that led to the development of FDI methods for uncertain linear time-varying systems, with promising results in terms of the time required to diagnose faults. An integration of such SVO-based FDI methods with robust control synthesis is described, in order to deploy new FTC algorithms that are able to stabilize the plant under faulty environments. The FDI algorithm is assessed within a wind turbine benchmark model, using Monte-Carlo simulation runs.

Keywords: Fault diagnosis; Fault tolerant control; Linear Time-Varying Systems; Set-Valued Observers; Uncertain Systems

1. INTRODUCTION

The field of Fault Detection and Isolation (FDI) has been studied since the early 70's Willsky [1976], and several techniques have, since then, been applied to different types of systems. An FDI device is key in several applications and, in particular, in those that are *safety critical*. Common examples of systems equipped with FDI devices include aircrafts and a wide range of industrial processes such as the ones described in the following references – Blanke et al. [2001], Patton and Chen [1997], Frank and Ding [1997], Esteban [2004], Collins and Tinglun [2001], Longhi and Moteriù [2009]. An FDI system must be able to bear with different types of faults in sensors and/or actuators, which can occur abruptly or slowly in time. Moreover, model uncertainty (such as unmodeled dynamics) and disturbances must never be interpreted as faults.

A deterministic model-based Fault Detection (FD) system is usually composed of two parts: a filter – see Fig. 1 – that generates residuals, which should be *large* under faulty environments; and a decision threshold, which is used to decide whether a fault is present or not – see Willsky [1976], Patton and Chen [1997], Esteban [2004], Frank and Ding [1994], Massoumnia [1986], Bokor and Balas [2004], Meskin and Khorasani [2009], Wang et al. [2009], Narasimhan et al. [2008] and references therein. The isolation of the fault can, in some cases, be done using a similar approach, *i.e.*, by designing filters for families of faults, and identifying the most likely fault as that associated to the filter with the smallest residuals.

The main idea in such architectures stems from the design of filters that are more sensitive to faults than to disturbances and model uncertainty. This can be achieved, for instance, by using geometric considerations regarding the plant, as in Massoumnia [1986], Longhi and Moteriù [2009], Bokor and Balas [2004], or by optimizing a particular norm minimization objective, such as the $\mathcal{H}_\infty$- or l_1-norm – see Edelmayer et al. [1994], Frank and Ding [1997], Niemann and Stoustrup [2001], Marcos et al. [2005], Collins and Tinglun [2001]. The latter approach provides, in general, important robustness properties, as stressed in Edelmayer et al. [1994], Mangoubi et al. [1995], Patton and Chen [1997], Esteban [2004], by explicitly accounting for model uncertainty. In Savkin and Petersen [1996], integral quadratic constraints for uncertain systems are used for model validation.

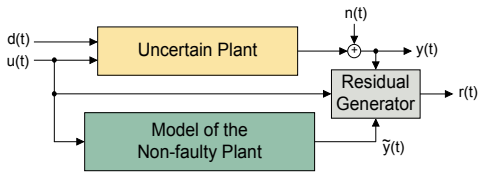

Fig. 1. Residual generation in a classical fault detection (FD) architecture.

As a caveat, these methodologies are, in general, conservative or can only be applied to a restrict class of systems. Moreover, the thresholds used to declare a fault are typically time-varying and highly dependent on the model uncertainty and on the amplitude of the exogenous disturbances and measurement noise.

The FDI strategy proposed in this paper uses a different philosophy. Rather than identifying the most likely model of the faulty plant, models that are not compatible with the current input/output data are invalidated, thus avoiding the computation of decision thresholds. To this end, this paper adopts the model falsification technique using Set-Valued Observers (SVOs), described in what follows. In addition, another advantage of the SVO-based

methodology presented herein stems from the fact that it is able to deal with linear time-varying uncertain plants. Alternative set-membership approaches to FDI can be found in Combastel and Raka [2009], Ingimundarson et al. [2009] and references therein, and will be briefly discussed in Section 4.

The use of FDI strategies, however, may not completely void the possibility of having severe failures that, due to delay in the corresponding isolation process, lead to the damage of the diagnosed system beyond repair. Indeed, the time required to detect and isolate certain types of faults is sufficient to lead to the accelerated deterioration of these systems. Therefore, control design methodologies have been developed in the recent years, that take into account these considerations, by deliberately increasing the effects of certain faults, so that they are detected faster. As an example, nested controller and FDI design strategies Marcos et al. [2005], Stoustrup and Niemann [2010] have been proposed that allow faster detection of the faults due to a poorer rejection of the controller with respect to disturbances aligned with these faults. As a shortcoming, the lack of attenuation of the effects of the faults can put into jeopardy the entire controlled system. Once a fault is isolated, the controller can be reconfigured in order to minimize its impact on the performance of the closed-loop system. Such architectures are typically referred to as Fault Tolerant Control (FTC) schemes.

The remainder of this paper is organized as follows. Section 2 introduces the main notation used throughout the article, while Section 3 describes the main concepts regarding model falsification. SVOs for LTV systems are presented in Section 4. These SVOs are used in Section 5 for FDI and FTC, while in Section 6 a brief description of the application of this methodology to a wind turbine is shown. Finally, Section 7 is devoted to the discussion of the proposed approach.

2. PRELIMINARIES AND NOTATION

We assume that the available input/output dataset can be obtained through a Linear Parameter-Varying (LPV) system, described by

$$\begin{cases} x(k+1) = A\big(\rho(k)\big)x(k) + B\big(\rho(k)\big)u(k) + L\big(\rho(k)\big)d(k), \\ y(k) = C\big(\rho(k)\big)x(k) + N\big(\rho(k)\big)n(k), \end{cases} \tag{1}$$

with bounded exogenous disturbances, $d(\cdot)$, uncertain initial state, $x(0) \in X(0)$, control input, $u(\cdot)$, and measurement output, $y(\cdot)$, corrupted by additive noise, $n(\cdot)$. The system is assumed observable and controllable from the exogenous disturbances and control inputs, for all admissible $\rho(\cdot)$. The matrices of the system may be uncertain and are assumed to depend upon a (partially uncertain) time-varying vector of parameters, $\rho(\cdot)$. It is also assumed that

$$|d(k)| := \max_i |d_i(k)| \leq 1,$$

and $|n(k)| \leq \bar{n}$. At each time, k, let $x(k)$ denote the states vector and

$$X(0) := \mathrm{Set}(M_0, m_0),$$

where

$$\mathrm{Set}(M, m) := \{q \in \mathbb{R}^{n_m} : Mq \leq m\} \tag{2}$$

represents a convex polytope, with $M \in \mathbb{R}^{n_m \times n}$, $m \in \mathbb{R}^{n_m}$, and with the inequality taken element-wise. Moreover, let $x(k) \in \mathbb{R}^n$, $d(k) \in \mathbb{R}^{n_d}$, $u(k) \in \mathbb{R}^{n_u}$, and $y(k) \in \mathbb{R}^{n_y}$, for $k \geq 0$. For the sake of simplicity of notation, we redefine $A(k) := A\big(\rho(k)\big), B(k) := B\big(\rho(k)\big), L(k) := L\big(\rho(k)\big), C(k) := C\big(\rho(k)\big), N(k) := N\big(\rho(k)\big)$.

3. MODEL FALSIFICATION

The problem of *model falsification* appears in several areas where we are interested in distinguishing among an eligible set of dynamic systems. The simplest model falsification problem one can think of is that of stating whether or not a given dynamic model is *compatible* with the current observed input/output data. However, it is important to notice that a model can never be validated in practice. Indeed, if the model is compatible with the input/output data up to time t, it need not be compatible at time $t + \delta$, where $\delta > 0$. Therefore, one can only say that *a given model is not falsified (or invalidated) by the current input/output data*. On the other hand, a model is obviously *invalidated* or *falsified* once it is not compatible with the observations. Hence, we usually refer to *model falsification* rather than *model validation*, since the latter is not achievable in practice.

As an example, suppose that there are four possible models, M_1, M_2, M_3, and M_4, for a given plant. We are interested in deciding which model (if any) is able to explain the input/output data sequence that we are obtaining from the sensors and actuators' commands. Therefore, assume that, at a given initial time, t_o, all the four models are plausible, as depicted in Fig. 2. Further suppose that, at time t_1, model M_4 is invalidated, *i.e.*, the sensors readings cannot be explained by model M_4. Moreover, consider that, at time t_2, model M_2 is invalidated and that, finally, model M_1 is invalidated at time t_3. Then, at time t_3, we conclude that the only model capable of explaining the input/output time-series generated by the plant is model M_3.

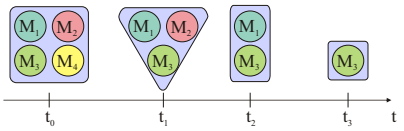

Fig. 2. Example of the time-evolution of a set of models that are able to describe the input/output behavior of a given plant.

3.1 Model Falsification in the Literature

Unmodeled dynamics (present in virtually every physical system) and adverse exogenous disturbances, can result in erroneous model falsification. Therefore, worst-case approaches, rather than stochastic approaches, are more suitable to address this type of problems. In fact, the solution proposed in Poolla et al. [1994] for uncertain LTI systems, and later on extended to LTV systems in Bianchi and Sánchez-Peña [2010], assumes that the system is described by an LTI nominal model interconnected with an LTI or LTV unknown system, denoted by Δ. This uncertain system Δ can be used, for instance, to describe unmodeled dynamics and parametric uncertainty. However, the methods provided in Poolla et al. [1994], Bianchi and Sánchez-Peña [2010] are not recursive, which means that, after a given amount of input/output data is obtained, we check whether or not the data is compatible with our model of the system. Hence, the complexity of the algorithms grows with the number of iterations.

The model falsification strategy presented in this paper uses a philosophy similar to that of Poolla et al. [1994], Bianchi and Sánchez-Peña [2010], but proposes a recursive algorithm which can be used to run in real-time. As shown in the sequel, this method guarantees that valid models of the plant are never falsified. Moreover, under certain

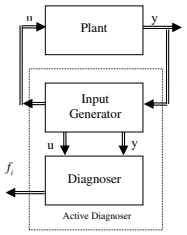

Fig. 3. Structure of an active fault diagnoser.

distinguishability conditions discussed herein, it is also shown that the *correct* model of the plant is selected.

3.2 The (In)Distinguishability Problem

Due to noise and uncertainty on the model of the system, it is possible that an input/output data is consistent with more than one model and, therefore, we cannot distinguish the correct one among a set of valid models of the dynamic system. A remedy to this is to use active diagnosis methods to improve the distinguishability between valid models by exciting the system using an auxiliary input signal. In active diagnosis, the *diagnoser* generates an input that excites the system, to decide whether the output represents a normal or a faulty behavior and, if possible, decide which fault has occurred. The generated input must perturb the system from the operation point but, at the same time, not lead the system to instability or to an unacceptable performance. The area of active diagnosis has attracted a considerable attentions in recent years – see Nikoukhah et al. [2002], Nikoukhah and Campbell [2006], Niemann and Poulsen [2005], Niemann [2006], Tabatabaeipour [2010] and references therein.

The structure of an active diagnoser in depicted in Fig. 3. It consists of a generator and a diagnoser. The generator generates an input sequence $U = [u(0), \dots, u(T_d - 1)]$ which is applied to the system and then occurrence of fault f is determined by the diagnoser by observing the applied input sequence and the output sequence $Y = [y(0), \dots, y(T_d)]$.

The active diagnosis problem can be stated as follows:

Problem 1. **Active diagnosis problem:** Given the set $\mathcal{M} = \{M_0, \dots, M_n\}$ describing behaviors of the system with no fault and subject to faults $\{f_1, \dots, f_n\}$, respectively, find a sequence of inputs U such that (U, Y) can only be described by a unique M_i.

In other words, the set $\mathcal{M}$ must be distinguishable – see Rosa and Silvestre [2011]. If such an input sequence exists, *i.e.* if the system is diagnosable, then we can look for the optimal solution, where optimality can be interpreted in different senses. The problem can be formulated as a feasibility test problem as follows:

$$\text{Find } T_d, u_{T_d} \text{ s.t.} \tag{3}$$

$$\begin{cases} x_i(k) \in X_i(0) \\ x_i(k+1) = A_i(\rho(k))x_i(k) + B_i(\rho(k))u(k) + L_i(\rho(k))d(k) \\ y_i(k) = C_i(\rho(k))x_i(k) + N_i(\rho(k))n(k) \\ i = 0, \cdots, n \\ y_i(T_d) - y_j(T_d) \neq 0, \ i, j \in \{0, \dots, n\}, i \neq j \\ |n(k)| \leq \bar{n} \\ |d(k)| \leq \bar{d} \end{cases}$$

This problem is in general nonconvex. In this work, we assume that the general form of the auxiliary input signal is given as a periodic signal of the form $u(k) = Asin(wk)$, with parameters A and w – the companion paper Casau et al. [2011] illustrates the applicability of this tool. The problem is to find the appropriate amplitude A and the frequency w of the input signal that guarantees distinguishability of the corresponding outputs despite noise and disturbance. For a given A_0 and w_0, T_{d_0}, if there exist a noise and disturbance sequences and a initial condition such that the following problem is feasible, then we can not guarantee that the models are distinguishable:

$$\begin{cases} x_i(k) \in X_i(0) \\ u(k) = A_0 sin(w_0 k) \\ x_i(k+1) = A_i(\rho(k))x_i(k) + B_i(\rho(k))u(k) + L_i(\rho(k))d(k) \\ y_i(k) = C_i(\rho(k))x_i(k) + N_i(\rho(k))n(k) \\ i = 0, \cdots, n \\ y_i(T_{d_0}) - y_j(T_{d_0}) = 0, \ i, j \in \{0, \dots, n\}, i \neq j \\ |n(k)| \leq \bar{n} \\ |d(k)| \leq \bar{d} \end{cases} \tag{4}$$

Now, to solve (3) we look for A_0, w_0, T_{d_0} that render (4) infeasible. Therefore, we parameterize (4) over A_0, w_0, and T_{d_0} and use an appropriate gridding of the parameter range and check feasibility of (4) at each grid point. The optimal signal can be found by choosing the optimal value of the parameter vector that makes (4) infeasible. The proposed method yields solving a finite number of linear programming problems that, for a reasonable grid density, is computationally efficient.

4. SET-VALUED OBSERVERS

4.1 Introduction

If a dynamic model is not able to explain the output of the actual system, given the applied control inputs and bounds on the exogenous disturbances, it is straightforward to conclude that such a model is not compatible with the actual dynamics of the plant. Hence, this section is devoted to the description of a technique that allows one to systematically design filters, which, in turn, are going to be used for model falsification. These filters are referred to as Set-Valued Observers (SVOs) – see Witsenhausen [1968], Schweppe [1968, 1973], Milanese and Vicino [1991] and references therein for an overview on SVOs –, as they are able to provide set-valued estimates of the state of the plant, based upon a) the dynamic model of the system (which may be uncertain); b) the output measurements; c) the control inputs; d) and the bounds on the exogenous disturbances and measurement noise.

This type of observers, jointly with the model falsification paradigm described in the previous section, naturally arises as a solution to distinguish among models of dynamic systems.

The problem of designing SVOs – also referred to as set-membership filtering design – has been extensively studied in the literature. One of the first algorithms developed to compute (ellipsoidal) set-valued estimates of the state of a system was introduced in Schweppe [1968] and Schweppe [1973]. In Yang and Yongmin [2009], an approach to the synthesis problem of SVOs for LTV plants with nonlinear equality constraints is described. A method for active mode observation of switching systems, based on SVOs, has been recently proposed in Baglietto et al. [2009]. Zonotope-based approaches to fault detection were also recently proposed in Combastel and Raka [2009], Ingimundarson et al. [2009].

The SVO-based methodology adopted in this paper is an extension of the work in Shamma and Tu [1999]. In fact, the results in Rosa et al. [2010] are a generalization of the set-valued state estimation for LTV systems, which is able

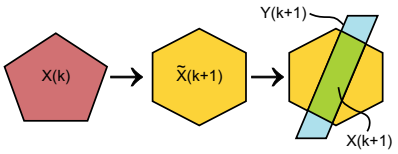

Fig. 4. Prediction and update cycles.

to handle model uncertainty. Indeed, this section briefly describes how to design SVOs which are able to provide set-valued estimates of the state, under different scenarios, namely parametric uncertainty in the input, output or matrices of the dynamics of the state-space representation of the plant. The proposed method is, in general, less computationally demanding when compared to zonotope-based approaches.

As illustrated in Fig. 4, the SVOs prediction cycle consists in estimating the set of possible states, $\tilde{X}(k+1)$, at time $k+1$, based upon the model of the system and the set-valued estimate of the state at time k. The update cycle comprises the computation of the states, $Y(k+1)$, which are compatible with the measured output of the plant, and the intersection of this set with $\tilde{X}(k+1)$.

4.2 SVOs for LTV Dynamic Models

Let $X(k+1)$ represent the set of possible states at time $k+1$, *i.e.*, the state $x(k+1)$ verifies (1) with $x(k) \in X(k)$ if and only if $x(k+1) \in X(k+1)$. The goal of an SVO is to find $X(k+1)$, based upon (1) and with the additional knowledge that $x(k) \in X(k), x(k-1) \in X(k-1), \cdots, x(k-N) \in X(k-N)$ for some finite N. We further require that for all $x \in X(k+1)$, there exists $x^\star \in X(k)$ such that, for $x(k) = x^\star$, the observations are compatible with (1). In other words, we want $X(k+1)$ to be the smallest set containing all the solutions to (1).

The computation of $X(k+1)$ based upon $X(k)$ for systems with no model uncertainty can be performed using the technique described in Shamma and Tu [1999]. Indeed, let the system be described by (1), and assume that the matrices of the dynamics are exactly known. For the sake of simplicity, assume that $N\big(\rho(k)\big) = I$ for all $\rho(k), k \geq 0$. Then, as shown in Shamma and Tu [1999], $x(k+1) \in X(k+1)$ if and only there exist $x(k)$, $n(k)$ and $d(k)$, such that, for the current measurement, $y(k+1)$, we have

$$P(k) \begin{bmatrix} x(k+1) \\ x(k) \\ d(k) \end{bmatrix} \leq \begin{bmatrix} B(k)u(k) \\ -B(k)u(k) \\ \mathbf{1} \\ \mathbf{1} \\ \tilde{m}(k) \\ m(k-1) \end{bmatrix} =: p(k) \qquad (5)$$

where

$$P(k) := \begin{bmatrix} I & -A(k) & -L(k) \\ -I & A(k) & L(k) \\ 0 & 0 & I \\ 0 & 0 & -I \\ \tilde{M}(k) & 0 & 0 \\ 0 & M(k-1) & \end{bmatrix}, \tilde{M}(k) = \begin{bmatrix} C(k+1) \\ -C(k+1) \end{bmatrix},$$

$$\tilde{m}(k) = \begin{bmatrix} \bar{n} + y(k+1) \\ \bar{n} - y(k+1) \end{bmatrix},$$

and where $M(k-1)$ and $m(k-1)$ are defined such that $X(k) = \text{Set}\,(M(k-1), m(k-1))$. The inequality in (5) provides a description of a set in $\mathbb{R}^{2n+n_d}$, denoted by

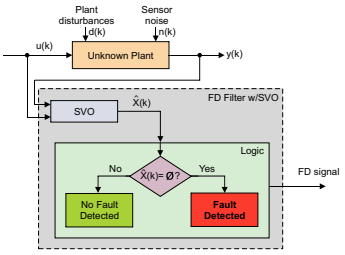

Fig. 5. Fault detection using SVOs.

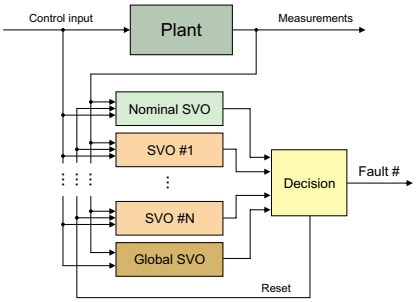

Fig. 6. Fault Detection and Isolation using Set-Valued Observers (FDI-SVO) architecture.

$$\Gamma(k+1) = \text{Set}\,(P(k), p(k))\,.$$

Therefore, it is straightforward to conclude that

$$\hat{x} \in X(k+1) \Leftrightarrow \underset{x \in \mathbb{R}^n, d \in \mathbb{R}^{n_d}}{\exists} : \begin{bmatrix} \hat{x} \\ x \\ d \end{bmatrix} \in \Gamma(k+1)$$

Hence, the set $X(k+1)$ can be obtained by projecting $\Gamma(k+1)$ onto the subspace of the first n coordinates, which, in turn, can be done resorting to the *Fourier-Motzkin elimination method* (see Shamma and Tu [1999], Keerthi and Gilbert [1987]). Therefore, one ends up with a description of all the admissible $x(k+1)$, which does not depend upon specific $x(k)$ nor $d(k)$.

4.3 SVOs for Uncertain Dynamic Models

For plants with uncertainties, the set $X(k+1)$ is, in general, non-convex, even if $X(k)$ is convex. Thus, it cannot be represented by a linear inequality as in (2). The generalization of the previous results to uncertain dynamic systems is omitted here due to space limitations, and can be found in Rosa [2011].

5. FDI AND FTC USING SVOS

In this section, the applicability of the SVOs to Fault Detection and Isolation (FDI) and Fault Tolerant Control (FTC) is going to be discussed. In both cases, we take advantage of the model falsification technique described in Section 3 to identify the model of the plant. In particular, the logic depicted in Fig. 5 is used, in order to detect faults, i.e., a lack of consistency between the measurements obtained from the sensors, and the model of the plant in nominal (non-faulty) operation.

5.1 FDI using SVOs

The FDI-SVO methodology adopted in this paper was introduced in Rosa et al. [2010], and the corresponding general architecture is illustrated in Fig. 6.

This architecture requires two additional SVOs, besides the faults isolation SVOs, namely a) one SVO for the

non-faulty (probably uncertain and time-varying) plant – referred to as *Nominal SVO*; b) another SVO – referred to as *Global SVO* – providing set-valued estimates of the state, which are valid not only for the non-faulty plant, but also for the faulty plant.

The *Nominal SVO* is used for fault detection only. If the state estimate of this SVO is the empty set, a fault has occurred. Hence, the fault isolation SVOs are initialized with the state estimate of the *Global SVO*. A fault is completely isolated whenever a single fault isolation SVO has a non-empty set-valued state estimation. It should be stressed that the FD filters that are designed for specific faults, are only initialized with the set-valued state estimate of the *Global SVO* when they are signaled by the *Nominal FD* filter that a fault has occurred.

5.2 Passive Fault Tolerant Control

After the occurrence of a given fault, the FDI system may require several measurements before such an event is detected and isolated. Thus, in this article, we propose the use of robust controllers that, at the cost of a possible slight decrease in terms of performance under non-faulty scenarios, guarantees stability of the system even under faulty environments. These controllers are designed to take into account only certain types of faults that are typically harder to detect. Hence, such robust controllers provide the FDI system with further time to determine the exact location of the fault and, then, to select a controller which is more adequate to handle the failure, as described in the following subsection. The synthesis of controllers that are robust against different types of uncertainties and time-variations on the dynamics of the plant has, indeed, deserved considerable attention over the last decades. The interested reader is referred, for instance, to Skogestad and Postlethwaite [2005], Zhou et al. [1996].

5.3 FTC using SVOs

The Fault Tolerant Control using Set-Valued Observers (FTC-SVO) architecture is depicted in Fig. 7. The Decision-block is responsible for selecting the appropriate controller, based upon the set-valued estimates provided by the bank of SVOs. Each controller is designed as in the previous subsection, so that robust-stability is guaranteed while a given fault is not detected and isolated. The FTC-SVO method uses a mixed solution, between an active FDI algorithm and a passive FTC. Therefore, on the one hand, the FDI system applies a persistence of excitation on the plant, whenever the measured signals hinder the distinguishability of the faults – see Section 3.2. On the other hand, the controller synthesized for the nominal system is also robust to mild variations on the dynamics of the plant, so that faults can be accommodated while the FDI subsystem is not able to reconfigure the controller.

If the system is operating normally, the *Nominal SVO* provides non-empty set-valued state estimates for the plant, and thus the *Nominal Controller* is connected to the loop. This controller must also be able to accommodate a fault, should it occur, until the FDI algorithm (see Section 5.1) detects and isolates this fault. After that, if fault $\#i$ is isolated, then controller $\#i$ is connected to the loop, substituting the nominal one.

6. SIMULATION RESULTS

The applicability of the technique presented in this paper to a wind turbine is fully described in the companion paper Casau et al. [2011]. However, for the sake of completeness, some of the main results are also shown here. For details on the dynamic model of the wind turbine and related faulty scenarios, the reader is referred to Odgaard et al. [2009]. The simulation results presented in this section are summarized in Table 1.

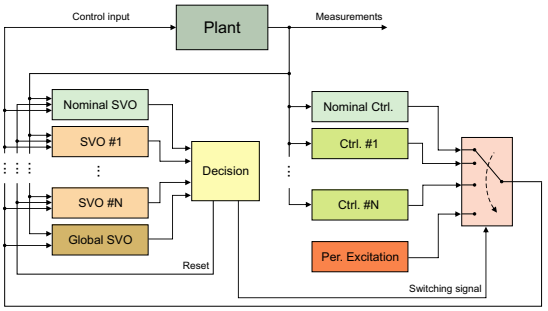

Fig. 7. Fault Tolerant Control using Set-Valued Observers (FTC-SVO) architecture.

Fault no.	Detections	False Detections	Median Time [s]	Min. Time [s]	Max. Time [s]
1	12	0	0.01	0.01	0.01
2	12	0	0.07	0.03	0.09
3	12	0	0.01	0.01	0.01
4	12	0	0.01	0.01	0.01
5	12	0	0.01	0.01	0.01
6	12	0	0.27	0.13	0.28
7	12	0	0.09	0.07	0.11
8	9	3	0.01	0.01	0.01

Table 1. Fault detection simulations results

7. CONCLUSIONS

This paper described Fault Detection and Isolation (FDI) and Fault Tolerant Control (FTC) methodologies, applicable to Linear Time-Varying (LTV) systems, that take advantage of recent advances in the Set-Valued Observers (SVOs) theory to invalidate dynamic models. Contrary to residual-based approaches, the suggested method need not the computation of decision thresholds, which are highly dependent on the exogenous disturbances, measurement noise, and model uncertainty. Some of the computational issues that arise in the implementation of such methods are also briefly discussed. In terms of FTC, a mixed active-passive approach was adopted. In particular, robust controllers were used to accommodate faults during the period the FDI system is trying to isolate them. Once a fault is isolated, the controller is reconfigured so as to minimize the impact on the closed-loop plant. Monte-Carlo simulations were performed on a faulty wind turbine, showing that only a few measurements are necessary, in general, to detect and isolate faults.

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