

Chapter 12

Structured Linear Parameter Varying Control of Wind Turbines

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Abstract High performance and reliability are required for wind turbines to be competitive within the energy market. To capture their nonlinear behavior, wind turbines are often modeled using parameter-varying models. In this chapter, a framework for modelling and controller design of wind turbines is presented. We specifically consider variable-speed, variable-pitch wind turbines with faults on actuators and sensors. Linear parameter-varying (LPV) controllers can be designed by a proposed method that allows the inclusion of faults in the LPV controller design. Moreover, the controller structure can be arbitrarily chosen: static output feedback, dynamic (reduced order) output feedback, decentralized, among others. The controllers are scheduled on an estimated wind speed to manage the parameter-varying nature of the model and on information from a fault diagnosis system. The optimization problems involved in the controller synthesis are solved by an iterative LMI-based algorithm. The resulting controllers can also be easily implemented in practice due to low data storage and simple math operations. The performance of the LPV controllers is assessed by nonlinear simulations results.

12.1 Introduction

Motivated by environmental concerns and the depletion of fossil fuels, as well its mature technological status, wind energy consolidate as a viable sustainable energy source for the decades to come. Over the past 20 years, the global installed capacity

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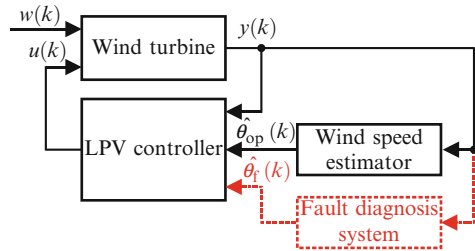
of wind power increased at an average annual growth of more than 25% from around 2.5 GW in 1992 to just under 200 GW at the end of 2010 [14]. Due to ongoing improvements in the wind turbine efficiency and reliability, and higher fuel prices, the cost of electricity produced (COE), which, roughly speaking, takes into account the annual energy production, lifetime of wind turbines, and Operation and Maintenance costs, is becoming economically competitive with conventional power production.

Automatic control is one of the engineering areas that significantly contributed to reduce the cost of wind-generated electricity. In order to reduce COE, a modern wind turbine is not only controlled to maximize energy production but also to minimize mechanical loads. The controlled system also has to comply with external requirements, such as acoustic noise emissions and power quality grid codes. Since many wind turbines are installed at remote locations, the introduction of fault-tolerant control is considered a suitable way of improving reliability/availability and lowering costs of repairs. Finally, the lack of accurate models must be alleviated by robust control strategies capable of securing stability and satisfactory performance despite model uncertainties [19].

From a control point of view, a wind turbine is a challenging system since the wind, which is the energy source driving the machine, is a poorly known stochastic disturbance. Add to that wind turbines inherently exhibit time-varying nonlinear dynamics along their nominal operating trajectory, motivating the use of advanced control techniques such as gain-scheduling, to counteract performance degradation or even instability problems by continuously adapting to the dynamics of the plant. Wind turbine controllers typically consist of multiple gain-scheduled controllers, which are designed to operate in the proximity of a certain operating point. The gain-scheduling approach for industry-standard classical controllers can be either based on switching or interpolation of controller gains [7, 8]. Controller structure may also change by either switching [7] or bumpless transfer [17, 25] according to the wind speed experienced by the wind turbine. The underlying assumption for such control schemes is that parameters only change slowly compared to the system dynamics, which is generally not satisfied in turbulent winds. Additionally, classical gain-scheduling controllers only ensure performance guarantees and stability at the operating points where the linear controllers are designed [22].

A systematic way of designing controllers for systems with linearized dynamics that vary significantly with the operating point is within the framework of linear parameter-varying (LPV) control. An LPV controller can be synthesized after solving an optimization problem subject to linear matrix inequality (LMI) constraints. In control systems for wind turbines, robustness and fault-tolerance capabilities are important properties, which should be considered in the design process, calling for a generic and powerful tool to manage parameter variations and model uncertainties. In this chapter, design procedures for nominal controllers for parameter-varying models as well as active/passive fault-tolerance, are provided. The framework can be trivially extended to design controllers robust to uncertainties in the model [1], e.g., aerodynamic uncertainties [26]. Indeed, handling known parameter dependencies, unknown parameter variations, and faults, constitute the main challenges for the application of wind turbine control.

Fig. 12.1 Block diagram of the controller structures. The black boxes are common to the LPV controllers, while the red dashed box illustrates the fault diagnosis system required by the AFTC



An overview of the proposed control structure is illustrated by the block diagram depicted in Fig. 12.1, where $u(k)$ is the control signal and $w(k)$ is the disturbance. The LPV controllers depend on the measurements $y(k)$ and an estimate of the current operating point, $\hat{\theta}_{op}(k)$, which is used as scheduling parameter. Additionally, a fault diagnosis system provides the scheduling parameter $\hat{\theta}_f(k)$ for the active fault-tolerant controller (AFTC). The extra degree of freedom added by allowing the AFTC to adapt in case of a fault may introduce less conservatism than for the passive fault-tolerant controller. The AFTC is a conventional LPV controller scheduled on $\theta_{op}(t)$ and $\theta_f(t)$; the reason for denoting it an active fault-tolerant controller arises from the origin of the scheduling parameters.

The list of faults occurring in wind turbines is extensive, reflecting the complexity of the machines. On a system level, faults occur in sensors, actuators, and system components, ranging from slow gradual faults to abrupt component failures. The occurrence of faults may change the system behavior dramatically. This motivates us to develop methods for fault diagnosis and fault-tolerant control, offering several benefits:

- Prevent catastrophic failures and faults from deteriorating other parts of the wind turbine, by early fault detection and accommodation.
- Reduce maintenance costs by providing remote diagnostic details and avoiding replacement of functional parts, by applying condition-based maintenance instead of time-based maintenance.
- Increase energy production when a fault has occurred by means of fault-tolerant control.

This chapter gives an overview of the most common faults that can be modelled as varying parameters. For a clear exposure, the fault-tolerant controller is designed to cope with the simple case of a single fault: altered dynamics of the hydraulic pitch system due to low hydraulic pressure. The fault is a gradual fault affecting the control actions of the turbine. The method used also applies to fast parameter variations, i.e., abrupt faults in the extreme case [12].

Realizing advanced gain-scheduled controllers can in practice be difficult and may lead to numerical challenges [19, 21]. Several plant and controller matrices must be stored on the controller memory. Moreover, matrix factorizations and inversions are among the operations that must be done online by the controller at each sampling time [4, 5].

The synthesis procedures presented in this chapter are serious candidates for solving a majority of practical wind turbine control problems, provided a sufficiently good model of the wind turbine is available. We believe that the resulting controller can also be easily implemented in practice due to the following reasons:

- (A.1) *Structured controller*: the controller structure can be chosen arbitrarily. Decentralized of any order, dynamic (full or reduced-order) output feedback, static output, and full state feedback are among the possible structures. This is in line with the current control philosophy within wind industry.
- (A.2) *Low data storage*: the required data to be stored in the control computer memory is only the controller matrices, and scalar functions of the scheduling variables representing plant nonlinearities (basis functions).
- (A.3) *Simple math operations*: the mathematical operations needed to compute the controller gains at each sampling time are look-up tables with interpolation, products between a scalar and a matrix, and sums of matrices.

The versatile controller structure and facilitated implementation comes with a price. Due to the (possible) nonconvex characteristics of the synthesis problem, the controller design is solved by an iterative LMI optimization algorithm that may be demanding from a computational point of view. However, the authors consider that it is worth to transfer the computational burden from the controller implementation to the controller design.

The chapter is organized as follows. Section 12.2 describes the LPV wind turbine plant modeling including typical faults and uncertainties. The LPV controller design procedure, based on an iterative LMI optimization algorithm, is presented in Sect. 12.3. Section 12.4 contains a design example on how state of the art industrial controllers can be designed within the LPV framework. A fault-tolerant gain-scheduled PI pitch controller for the full load region is designed and compared to a gain-scheduled controller without fault accommodation capabilities. Simulation results presented in the same section compares the performance of both LPV controllers to show that pitch actuator faults due to low pressure can be accommodated by the fault-tolerant LPV controller, avoiding the shutdown of the wind turbine. Section 12.5 concludes the paper.

12.2 Wind Turbine LPV model

In this section, an LPV model is derived from a nonlinear time-varying wind turbine model. The nonlinear model consists of several subsystems, namely aerodynamics, the tower, the drive train, the generator, the pitch system, and the converter actuator. The interconnection of the wind turbine submodels is illustrated in Fig. 12.2. For simulation purposes, the wind disturbance input, $V(t)$, is provided by a wind model which includes both tower shadow and wind shear [11] together with a turbulence model [13]. The detailed description of the model is provided in [12]. The submodels are individually described in the sequel.

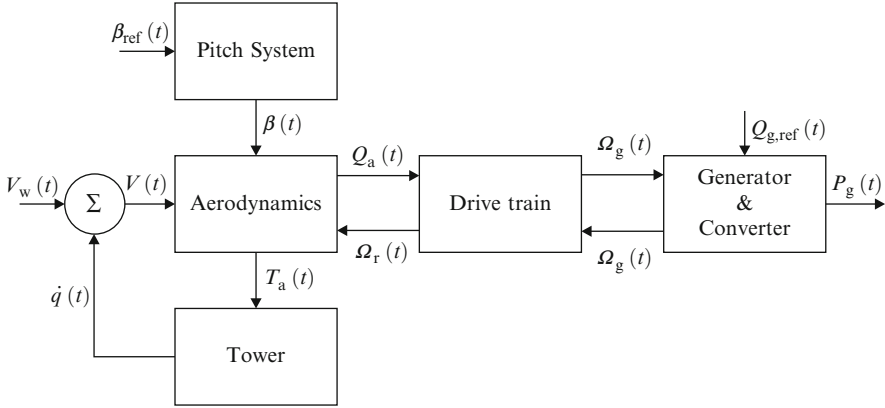


Fig. 12.2 Sub-model-level block diagram of a variable-speed variable-pitch WT

12.2.1 Wind Model

The driving force of a wind turbine is generated by the wind. Therefore, a model of this external input to the wind turbine, $V_w(t)$, has to be provided.

Generally, the wind speed is influenced by several components, which depend on the environment where the wind turbine is located; however, we restrict our model to include only three effects: wind shear, tower shadow, and turbulence. A more thorough wind model can be found in [12]. We will not provide a detailed description of the wind model, but only explain its three components briefly.

Wind shear is caused by the ground and other obstacles in the path of the wind, which cause frictional forces to act on the wind. The frictional forces imply that the mean wind speed becomes dependent on the height above ground level. Therefore, the mean wind speed depends on the location of the three blades. When a blade is located in front of the tower, the lift on that blade decreases because the tower reduces the effective wind speed. This phenomenon is called tower shadow and implies that the force acting on each blade decreases every time a blade is located in front of the tower. Finally, the variations in the wind speed, which are not included in the mean wind speed, are called turbulence and are caused by multiple factors. The utilized turbulence model is based on the Kaimal spectrum that describes the turbulence of a point wind. Since the wind model describes the wind speed averaged over the entire rotor plane, a low-pass filter is applied to smooth the wind speed signal. Figure 12.3 shows an output of the wind model $V_w(t)$. Note that a detailed description of the wind model can be found in [12].

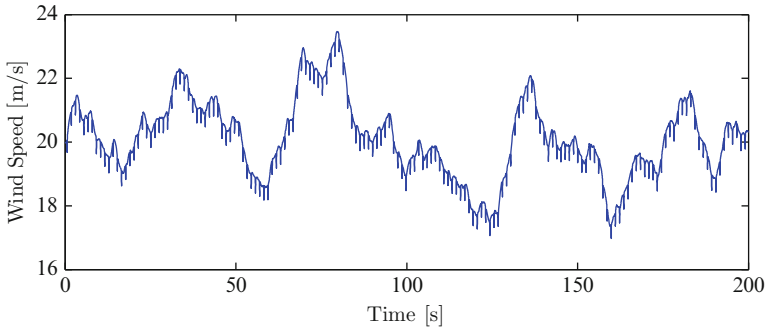


Fig. 12.3 Output of the wind model at a constant rotor speed. The periodic decrease of the wind speed is caused by tower shadow

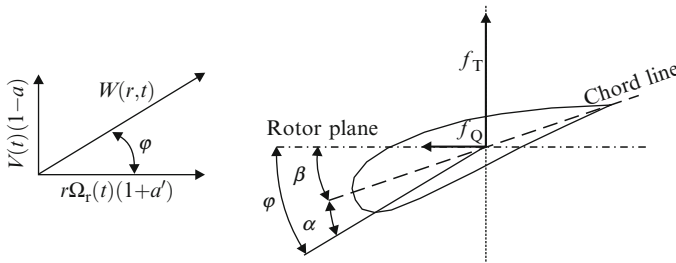


Fig. 12.4 Forces on a blade element

12.2.2 Nonlinear Model

The rotor of a wind turbine converts kinetic energy of the wind into rotational energy of the rotor blades and shaft. Aerodynamic forces over the rotor blades are often determined with the assumptions of blade element momentum (BEM) theory [15]. Figure 12.4 illustrates the forces and velocity vectors on a blade element.

Assuming a symmetric aerodynamic rotor driven by a uniform inflow, and neglecting unsteady aerodynamic effects, the local tangential f_Q and axial f_T forces along the local blade radius r are given by

$$f_Q = \frac{1}{2} \rho c(r) W^2(r, t) \left(C_L(r, \alpha(r, t)) \sin \varphi(r, t) - C_D(r, \alpha(r, t)) \cos \varphi(r, t) \right) \quad [\text{N}],$$

$$f_T = \frac{1}{2} \rho c(r) W^2(r, t) \left(C_L(r, \alpha(r, t)) \sin \varphi(r, t) + C_D(r, \alpha(r, t)) \cos \varphi(r, t) \right) \quad [\text{N}]$$

with the squared local inflow velocity $W^2(r, t)$, local angle of attack $\alpha(r, t)$ and local inflow angle $\varphi(r, t)$ described as

$$\begin{aligned} W^2(r, t) &= (V(t)(1 - a(r)))^2 + (r\Omega_r(t)(1 + a'(r)))^2 \quad [\text{m}^2/\text{s}^2], \\ \alpha(r, t) &= \varphi(r, t) - \phi(r) - \beta(t) \quad [^\circ], \\ \varphi(r, t) &= \tan^{-1} \left(V(t)(1 - a(r)) (r\Omega_r(t)(1 + a'(r)))^{-1} \right) \quad [^\circ]. \end{aligned}$$

In the above expressions, ρ is the air density, $c(r)$ is the local chord length, $C_L(r, \alpha)$ and $C_D(r, \alpha)$ are the local steady-state lift and drag coefficients, $V(t)$ is a mean wind speed over the rotor disk, $\Omega_r(t)$ is the rotor speed, $a(r)$ and $a'(r)$ are the axial and tangential flow induction factors, respectively, $\phi(r)$ is the local chord twist angle along the blade, and $\beta(t)$ is the blade pitch angle.

In the aerodynamic model, we assume that a yawing system exists, which always keeps the rotor plane perpendicular to the direction of the wind; hence, $V(t)$ is always perpendicular to the rotor plane. However, as the rotor rotates the resulting wind speed at a blade, called the inflow velocity $W(r, t)$, has an angle φ with respect to the rotor plane. The drag force given by $1/2\rho cW^2C_D$ is defined to point in the opposite direction as $W(r, t)$ and the lift force given by $1/2\rho cW^2C_L$ is perpendicular to drag force. Via projections of these forces, we obtain f_Q and f_T .

The aerodynamic torque Q_a and thrust force T_a produced by the rotor can be expressed as the summation of the forces over the B number of rotor blades

$$Q_a(V, \Omega_r, \beta, a, a') = B \int_0^R f_Q(r, V, \Omega_r, \beta, a(r), a'(r)) r \, dr \quad [\text{Nm}], \quad (12.1a)$$

$$T_a(V, \Omega_r, \beta, a, a') = B \int_0^R f_T(r, V, \Omega_r, \beta, a(r), a'(r)) \, dr \quad [\text{N}]. \quad (12.1b)$$

After integration, the aerodynamic torque and thrust are represented as

$$Q_a(t) = \frac{1}{2\Omega_r(t)} \rho A V^3(t) C_P(\lambda(t), \beta(t)) \quad [\text{Nm}], \quad (12.2a)$$

$$T_a(t) = \frac{1}{2} \rho A V^2(t) C_T(\lambda(t), \beta(t)) \quad [\text{N}] \quad (12.2b)$$

with the tip-speed ratio denoting the ratio between the blade tip and the mean wind speed

$$\lambda(t) = \frac{\Omega_r(t) R}{V(t)} \quad [·],$$

where R is the rotor radius and $A = \pi R^2$ is the rotor swept area. The power coefficient $C_P(\lambda, \beta)$ and thrust coefficient $C_T(\lambda, \beta)$ are smooth surfaces usually given in tabular form. Figure 12.5 depicts C_P and C_T surfaces of a typical 2 MW wind turbine.

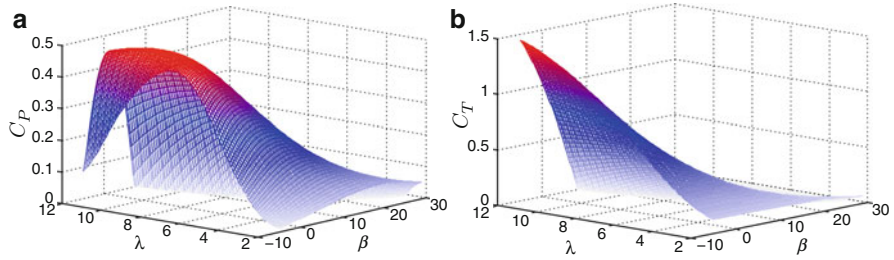


Fig. 12.5 Power and thrust coefficients of a typical utility-scale wind turbine. (a) Power coefficient (b) Thrust coefficient

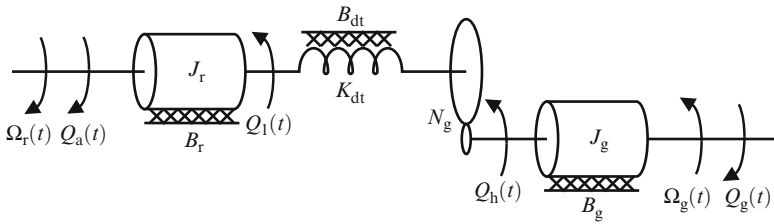


Fig. 12.6 Diagram of the drive train of the wind turbine

Aerodynamic torque Q_a drives a drive train model consisting of a low-speed shaft and a high-speed shaft having inertias J_r and J_g , and friction coefficients B_r and B_g . The shafts are interconnected by a transmission having gear ratio N_g , combined with torsion stiffness K_{dt} , and torsion damping B_{dt} . This results in a torsion angle, $\theta_\Delta(t)$, and a torque applied to the generator, $Q_g(t)$, at a speed $\Omega_g(t)$. The model of the drive train is shown in Fig. 12.6 and given by

$$J_r \dot{\Omega}_r(t) = Q_a(t) + \frac{B_{dt}}{N_g} \Omega_g(t) - K_{dt} \theta_\Delta(t) - (B_{dt} + B_r) \Omega_r(t) \quad [\text{Nm}], \quad (12.3a)$$

$$J_g \dot{\Omega}_g(t) = \frac{K_{dt}}{N_g} \theta_\Delta(t) + \frac{B_{dt}}{N_g} \Omega_r(t) - \left(\frac{B_{dt}}{N_g^2} + B_g \right) \Omega_g(t) - Q_g(t) \quad [\text{Nm}], \quad (12.3b)$$

$$\dot{\theta}_\Delta(t) = \Omega_r(t) - \frac{1}{N_g} \Omega_g(t) \quad [\text{rad/s}]. \quad (12.3c)$$

The thrust T_a acting on the rotor introduces fore–aft tower bending described by the axial nacelle linear translation $q(t)$. Sideward movements are ignored by neglecting yawing and drive train reaction torque on the tower. The tower translates in the same direction as the wind; therefore, aerodynamic torque and thrust are in

fact driven by the relative wind speed $V(t) = V_w(t) - \dot{q}(t)$. The tower dynamics is modeled as a mass-spring-damper system

$$M_t \ddot{q}_t(t) = T_a(t) - B_t \dot{q}_t(t) - K_t q_t(t), \quad (12.4)$$

where M_t is the modal mass of the first fore-aft tower bending mode, B_t is structural damping coefficient, and K_t is the modal stiffness for axial nacelle motion due to fore-aft tower bending.

Hydraulic pitch systems are satisfactorily modeled as a second order system with a time delay, t_d , and reference angle $\beta_{\text{ref}}(t)$

$$\ddot{\beta}(t) = -2\zeta \omega_n \dot{\beta}(t) - \omega_n^2 \beta(t) + \omega_n^2 \beta_{\text{ref}}(t - t_d), \quad (12.5)$$

where the natural frequency, ω_n , and damping ratio, ζ , specify the dynamics of the model. To represent the limitations of the pitch actuators, for simulation purposes the model includes constraints on the slew rate and the range of the pitch angle.

Electric power is generated by the generator, while a power converter interfaces the wind turbine generator output with the utility grid and controls the currents in the generator. The generator torque in (12.6) is controlled by the reference $Q_{g,\text{ref}}(t)$. The converter dynamics are approximated by a first order system with time constant τ_g and time delay $t_{g,d}$

$$\dot{Q}_g(t) = -\frac{1}{\tau_g} Q_g(t) + \frac{1}{\tau_g} Q_{g,\text{ref}}(t - t_{g,d}). \quad (12.6)$$

Just as for the model of the pitch system, the slew rate and the operating range of the generator torque are both bounded to match the limitations of the real system. The power produced by the generator can be approximated from the mechanical power calculated in (12.7), where η_g denotes the efficiency of the generator, which is assumed constant

$$P_g(t) = \eta_g \Omega_g(t) Q_g(t). \quad (12.7)$$

12.2.3 Linear Varying Parameters

From the model description, is clear that a wind turbine is a nonlinear, time-varying system. What is not apparent is how to find an LPV description that captures this dynamic behavior. Wind turbines can be represented as Quasi-LPV models [6, 19] and Linear Fractional Transformation models [19], but the most common approach relies on the classical linearization around equilibrium or operating points resulting in a linearized LPV model [5, 19, 21]. The latter approach is adopted in this chapter.

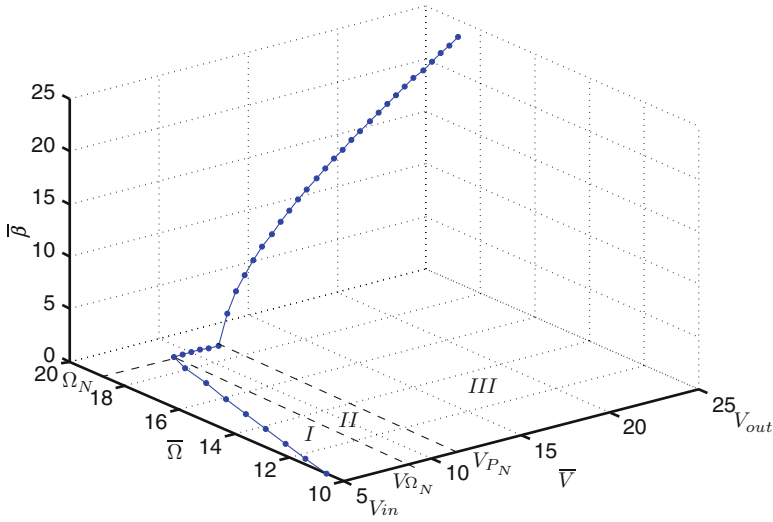


Fig. 12.7 Operating locus of a typical utility-scale wind turbine

12.2.3.1 Aerodynamics

The underlying assumption of a wind turbine LPV model based on linearization is that wind speed, rotor speed, and pitch angle can be described by slow and fast components

$$V(t) = \bar{V}(t) + \hat{V}(t), \quad \Omega_r(t) = \bar{\Omega}_r(t) + \hat{\Omega}_r(t), \quad \beta(t) = \bar{\beta}(t) + \hat{\beta}(t),$$

The collection of operating points $(\bar{V}, \bar{\Omega}, \bar{\beta})$ is what defines the control strategy of a wind turbine, selected to match steady-state requirements such as maximize energy capture, minimize static loads, and limit noise emissions.

A typical control strategy of a generic 2 MW wind turbine is depicted in Fig. 12.7. A more detailed treatment of different operating strategies for wind turbines [5, 7] is outside the scope of this chapter. Three subareas on a typical control strategy can be distinguished:

- (A.1) On Region I (V_{in} to V_{Ω_N}) the energy capture is maximized by keeping the aerodynamic efficiency as high as possible. This can be accomplished by tracking a rotational speed set point using generator torque as the control input variable. Pitch actuation is not utilized for tracking purposes; the pitch angle remains at the value of maximum aerodynamic efficiency. With only one input and one output to be controlled, a multivariable controller is not necessary on this region. Notice that $\bar{\Omega}$ is proportional to $\bar{V}$ as a consequence of optimal aerodynamic efficiency.

- (A.2) On Region II (V_{Ω_N} to V_{P_N}), the wind turbine maintains constant rotational speed at a nominal value Ω_N , by acting on the generator torque. The rotational speed is limited due to acoustic noise emission limits. Pitch actuation remains unused. A multivariable controller is still not needed.
- (A.3) On Region III (V_{P_N} to V_{out}), rated power P_N is reached and the main goal is to minimize power fluctuations. Small fluctuations on the generator torque around rated value add damping to the drive train torsional mode and fine control the electrical power. Therefore, pitch angle should be gradually increased as wind speed rises to limit generated power by lowering the rotor aerodynamic efficiency. In some wind turbines, active tower damping is also implemented on this region.

A linearization-based LPV model is obtained by classical linearization around the operating points given by the control strategy. The aerodynamic model is exclusively the source of time-varying nonlinearities. A first order Taylor series expansion of (12.2) leads to the following linearized representations of torque and thrust:

$$Q_a \approx \bar{Q}_{a(\bar{V}, \bar{\Omega}, \bar{\beta})} + \left. \frac{\partial Q_a}{\partial V} \right|_{(\bar{V}, \bar{\Omega}, \bar{\beta})} \hat{V} + \left. \frac{\partial Q_a}{\partial \Omega_r} \right|_{(\bar{V}, \bar{\Omega}, \bar{\beta})} \hat{\Omega}_r + \left. \frac{\partial Q_a}{\partial \beta} \right|_{(\bar{V}, \bar{\Omega}, \bar{\beta})} \hat{\beta}, \quad (12.8a)$$

$$T_a \approx \bar{T}_{a(\bar{V}, \bar{\Omega}, \bar{\beta})} + \left. \frac{\partial T_a}{\partial V} \right|_{(\bar{V}, \bar{\Omega}, \bar{\beta})} \hat{V} + \left. \frac{\partial T_a}{\partial \Omega_r} \right|_{(\bar{V}, \bar{\Omega}, \bar{\beta})} \hat{\Omega}_r + \left. \frac{\partial T_a}{\partial \beta} \right|_{(\bar{V}, \bar{\Omega}, \bar{\beta})} \hat{\beta}, \quad (12.8b)$$

where $\bar{Q}_{a(\bar{V}, \bar{\Omega}, \bar{\beta})}$ and $\bar{T}_{a(\bar{V}, \bar{\Omega}, \bar{\beta})}$ are equilibrium components of the aerodynamic torque and thrust, respectively. The partial derivatives of Q_a and T_a are given by

$$\begin{aligned} \frac{\partial Q_a}{\partial V} &= \frac{\rho AV^2}{2\Omega_r} \left(3C_P + V \frac{\partial C_P}{\partial \lambda} \frac{\partial \lambda}{\partial V} \right), & \frac{\partial T_a}{\partial V} &= \frac{\rho AV}{2} \left(2C_T + V \frac{\partial C_T}{\partial \lambda} \frac{\partial \lambda}{\partial V} \right), \\ \frac{\partial Q_a}{\partial \Omega_r} &= \frac{\rho AV^3}{2\Omega_r} \left(\frac{\partial C_P}{\partial \lambda} \frac{\partial \lambda}{\partial \Omega_r} - \frac{C_P}{\Omega_r} \right), & \frac{\partial T_a}{\partial \Omega_r} &= \frac{\rho AV^2}{2} \frac{\partial C_T}{\partial \lambda} \frac{\partial \lambda}{\partial \Omega_r}, \\ \frac{\partial Q_a}{\partial \beta} &= \frac{\rho AV^3}{2\Omega_r} \frac{\partial C_P}{\partial \beta}, & \frac{\partial T_a}{\partial \beta} &= \frac{\rho AV^2}{2} \frac{\partial C_T}{\partial \beta}, \end{aligned} \quad (12.9)$$

and must be evaluated at the time-varying equilibrium point $(\bar{V}, \bar{\Omega}, \bar{\beta})$. The partial derivatives of a typical 2 MW wind turbine for the whole operational envelope are depicted in Fig. 12.8. The aerodynamic partial derivatives given by (12.9), hereafter also referred to as aerodynamic gains, are varying parameters in an LPV wind turbine model.

With the assumption that the wind turbine is operating on the nominal trajectory specified in Fig. 12.7, the equilibrium values for pitch angle and rotor/generator speed can be described uniquely by the wind speed, e.g., $\bar{\Omega}(\bar{V})$, $\bar{\beta}(\bar{V})$. This means that the wind turbine can be described by an LPV model scheduled on wind speed only

$$\theta_{op}(t) := \bar{V}(t). \quad (12.10)$$

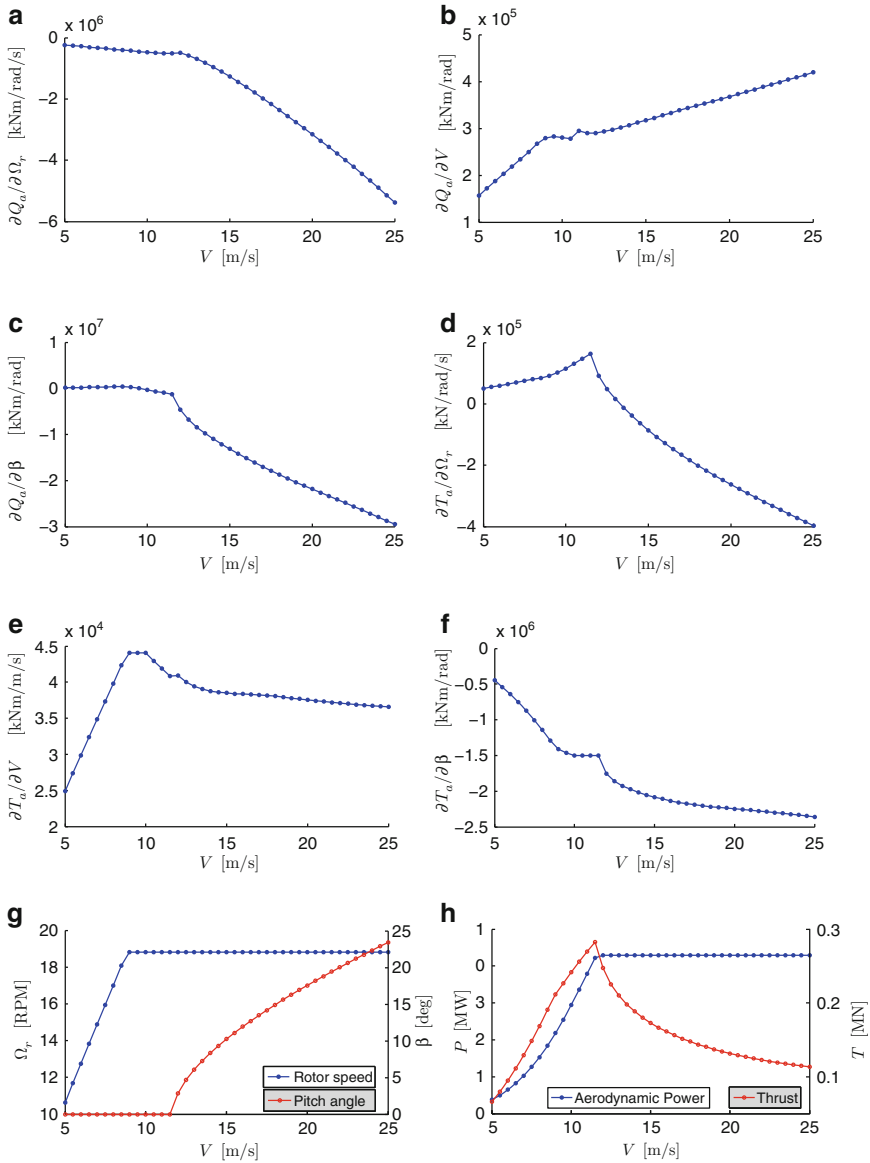


Fig. 12.8 Aerodynamic parameters of a typical 2 MW wind turbine. (a) From rotor speed to torque (b) From wind speed to torque (c) From pitch angle to torque (d) From rotor speed to thrust (e) From wind speed to thrust (f) From pitch angle to thrust (g) Rotor speed and pitch angle (h) Power and thrust

Depending on the region of interest in the control strategy and the model complexity, the varying parameters can be approximated as an explicit function of the scheduling variable. An affine representation is always preferable to diminish the computational cost of solving an LMI-constrained optimization. If tower dynamics are omitted and the aim is to design a controller for Region III, the aerodynamic torque gains can be fairly well approximated by a linear function of the wind speed. In this case, the parameter variations in the nominal LPV plant model are approximated using an affine description in the wind speed [26]. If tower dynamics are taken into account, the aerodynamic gains can be fairly approximated by polynomial functions in Region III. For the most general case, which is the design of a single LPV controller covering the full control strategy locus, representing the aerodynamic gains by polynomials is difficult and one has to resort to grid-based methods at high computational cost [5, 21].

On most wind turbines, the wind speed is measured by an anemometer on the nacelle, which only measures the wind speed at a single point in space and is affected by the presence of the rotor. Therefore, this measurement is not a good estimate of (12.10). To obtain the wind speed for scheduling purposes, an effective wind speed estimator must be designed [20]. The effective wind speed is defined as the spatial average of the wind field over the rotor plane with the wind stream being unaffected by the wind turbine. By inspecting the output of wind models and real field measurements, we determine the rate bounds on the effective wind speed $\hat{\theta}_{\text{op}}(t)$ to be -2 m/s^2 and 2 m/s^2 .

12.2.3.2 Faults

Faults in a wind turbine have different degrees of severity and accommodation requirements. A safe and fast shutdown of the wind turbine is necessary to some of them, while to others the system can be reconfigured to continue power generation. Linear parameter varying control can be applied in the case of failures that gradually change system's dynamics. The most common faults along with their magnitude and the rate at which they can be introduced are summarized in Table 12.1.

Pitch position and generator speed measurements are the sensors most affected by failures. Originated by either electrical or mechanical anomalies, they can result in either a bias or a gain factor on the measurements. A biased pitch sensor measurement affects both the pitch system model and the pitch angle measurement. When the bias is introduced, the pitch actuator model and measurement equation are modified as

$$\ddot{\beta}(t) = -2\zeta\omega_n\dot{\beta}(t) - \omega_n^2(\beta(t) + \beta_{\text{bias}}(t)) + \omega_n^2\beta_{\text{ref}}(t - t_d), \quad (12.11a)$$

$$\beta_{\text{mes}}(k) = \beta(k) + \beta_{\text{bias}}(k) + v_{\beta}(k), \quad (12.11b)$$

where $v_{\beta}(k)$ is a measurement noise. A bias can either be a result of inaccurate calibration of the pitch system or be an gradual fault.

Table 12.1 Specification of ranges and rate limits of gradual faults

Fault	Specification
Pitch sensor	
Bias	$\dot{\beta}_{\text{bias}}(t) \in [-1^\circ/\text{month}, 1^\circ/\text{month}]$ $\beta_{\text{bias}}(t) \in [-7^\circ, 7^\circ]$
Pitch actuator	
High air content	$\dot{\theta}_{\text{ha}}(t) \in [-1/\text{month}, 1/\text{month}]$ $\theta_{\text{ha}}(t) \in [0, 1]$
Pump wear	$\dot{\theta}_{\text{pw}}(t) \in [0, 1/(20 \text{ years})]$ $\theta_{\text{pw}}(t) \in [0, 1]$
Hydraulic leakage	$\dot{\theta}_{\text{hl}}(t) \in [0, 1/(100 \text{ s})]$ $\theta_{\text{hl}}(t) \in [0, 1]$
Pressure drop	$\dot{\theta}_{\text{pd}}(t) \in [-0.033/\text{s}, 0.033/\text{s}]$ $\theta_{\text{pd}}(t) \in [0, 1]$
Generator speed sensor	
Proportional error	$\dot{\theta}_{\text{pe}}(t) \in [-1/\text{month}, 1/\text{month}]$ $\theta_{\text{pe}}(t) \in [-0.1, 0.1]$

A proportional error on the generator speed sensor changes the sensor gain. The measurement equation

$$\Omega_{\text{g,mes}}(k) = (1 + \theta_{\text{pe}}(k)) \Omega_{\text{g}}(k) + v_{\Omega_{\text{g}}}(k) \quad (12.12)$$

is a linear function of the gain deviation θ_{pe} , where $v_{\Omega_{\text{g}}}(k)$ is a measurement noise.

The power converter and pitch systems are the actuators most likely to fail. Power converter faults can result in an offset of the generated torque due to an offset in the internal converter control loops. An offset in the internal converter control loops modifies the generator and converter model as follows:

$$\dot{T}_{\text{g}}(t) = -\frac{1}{\tau_{\text{g}}}(Q_{\text{g}}(t) + Q_{\text{g,bias}}(t)) + \frac{1}{\tau_{\text{g}}}T_{\text{g,ref}}(t - t_{\text{g,d}}), \quad (12.13)$$

where $Q_{\text{g,bias}}(t)$ is an offset of the generated torque.

A fault changes the dynamics of the pitch system by varying the damping ratio and natural frequency from their nominal values ζ_0 and $\omega_{\text{n},0}$ to their faulty values ζ_{f} and $\omega_{\text{n},\text{f}}$. The dynamics of the pitch system can then be represented as

$$\ddot{\beta}(t) = -2\zeta(\theta_{\text{f}})\omega_{\text{n}}(\theta_{\text{f}})\dot{\beta}(t) - \omega_{\text{n}}^2(\theta_{\text{f}})\beta(t) + \omega_{\text{n}}^2(\theta_{\text{f}})\beta_{\text{ref}}(t - t_{\text{d}}) \quad [^\circ/\text{s}^2] \quad (12.14)$$

with the parameters changing according to a convex combination of the vertices of the parameter sets [18]

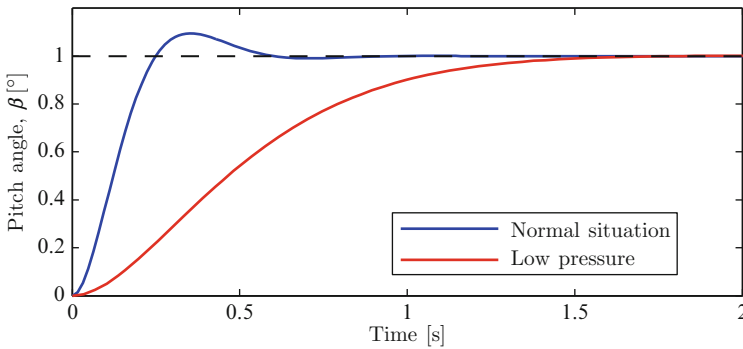
$$\omega_{\text{n}}^2(\theta_{\text{f}}) = (1 - \theta_{\text{f}})\omega_{\text{n},0}^2 + \theta_{\text{f}}\omega_{\text{n,lp}}^2, \quad (12.15\text{a})$$

$$-2\zeta(\theta_{\text{f}})\omega_{\text{n}}(\theta_{\text{f}}) = -2(1 - \theta_{\text{f}})\zeta_0\omega_{\text{n},0} - 2\theta_{\text{f}}\zeta_{\text{lp}}\omega_{\text{n,lp}}, \quad (12.15\text{b})$$

Table 12.2 Parameters for the pitch system under different conditions

Fault	Parameters
No fault	$\omega_n = 11.11 \text{ rad/s}$, $\zeta = 0.6$
High air content in the oil	$\omega_{n,ha} = 5.73 \text{ rad/s}$, $\zeta_{ha} = 0.45$
Pump wear	$\omega_{n,pw} = 7.27 \text{ rad/s}$, $\zeta_{pw} = 0.75$
Hydraulic leakage	$\omega_{n,hl} = 3.42 \text{ rad/s}$, $\zeta_{hl} = 0.9$
Pressure drop	$\omega_{n,hl} = 3.42 \text{ rad/s}$, $\zeta_{hl} = 0.9$

The normal air content in the hydraulic oil is 7%, whereas high air content in the oil corresponds to 15%. Pump wear represents the situation of 75% pressure in the pitch system while the parameters stated for hydraulic leakage corresponds to a pressure of only 50%

**Fig. 12.9** Step responses of hydraulic pitch model under normal (*blue*) and fault (*red*) conditions

where $\theta_f \in [0, 1]$ is an indicator function for the fault with $\theta_f = 0$ and $\theta_f = 1$ corresponding to nominal and faulty conditions, respectively. Pitch system failures are usually occasioned by the following reasons:

- Pump Wear is introduced very slowly and results in low pump pressure. When $\theta_f(t) = 0$ the pump delivers the nominal pressure, but as $\theta_f(t)$ goes to one the pressure drops. Notice that $\dot{\theta}_f(t) \geq 0$ for all t , since the wear is irreversible without replacing the pump. The fault described by $\theta_f = 1$, corresponding to a pressure level of 75%, can emerge after approximately 20 years of operation.
- Hydraulic leakage is introduced considerably faster than pump wear. Again $\dot{\theta}_f(t) \geq 0$ for all t , since a leakage cannot be reversed without repair of the system. Notice that the pressure for $\theta_f = 1$ corresponds to 50% of the nominal pressure.
- High air content in the oil is a fault that, in contrast to pump wear and hydraulic leakage, may disappear; hence, $\dot{\theta}_f(t)$ can be both positive and negative. The extreme values caused by $\theta_f = 0$ and $\theta_f = 1$ correspond to air contents of 7% and 15% in the hydraulic oil.

Values for the natural frequency and damping ratio of the pitch system under faults are described in Table 12.2. Step responses for the normal and fault conditions in the case of high air content in the oil are illustrated in Fig. 12.9.

If a number n_{θ_f} of faults are considered on the modeling, θ_f denotes a vector of scheduling parameters

$$\theta_f = [\theta_{f,1}, \dots, \theta_{f,m}], \quad m = 1, \dots, n_{\theta_f}.$$

12.2.3.3 System Description

The synthesis of LPV controllers are posed similarly to the $\mathcal{H}_\infty$ control of linear systems. The first step is to identify the input variable w known as disturbance or exogenous perturbation, and the fictitious output variable z called performance output or error. Next, weighting functions for these inputs and outputs are chosen, usually rational functions of the Laplace operator s stressing the frequencies of interest. The standard state-space interconnections of the LPV model of the plant and the weighting functions are called augmented plant, given by the general continuous-time LPV system description shown in (12.16)

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B_w(\theta(t))w(t) + B_u(\theta(t))u(t), \\ z(t) &= C_z(\theta(t))x(t) + D_{zw}(\theta(t))w(t) + D_{zu}(\theta(t))u(t), \\ y(t) &= C_y(\theta(t))x(t) + D_{yw}(\theta(t))w(t), \end{aligned} \quad (12.16)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}^{n_w}$ is the vector of exogenous perturbation, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $z(t) \in \mathbb{R}^{n_z}$ is the controlled output, and $y(t) \in \mathbb{R}^{n_y}$ is the measured output. $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, $D(\cdot)$ are continuous functions of the time-varying parameter vector $\theta = [\theta_{op} \quad \theta_f]$.

Possible types of dependency of the aerodynamic gains on the scheduling parameters have already been discussed in the Aerodynamics subsection. The general case where no restrictions are imposed on the parameter dependence is treated here [4, 5]. It is necessary to choose scalar functions of the varying parameters such that the LPV model of the augmented plant (12.16) is affine in these functions. That is,

$$\begin{aligned} \begin{bmatrix} A(\theta) & B_w(\theta) & B_u(\theta) \\ C_z(\theta) & D_{zw}(\theta) & D_{zu}(\theta) \\ C_y(\theta) & D_{yw}(\theta) & D_{yu}(\theta) \end{bmatrix} &= \begin{bmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{bmatrix}_0 + \sum_i \begin{bmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{bmatrix}_i \rho_i(\theta), \\ &+ \sum_m \begin{bmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{bmatrix}_m \theta_{f,m}, \quad i = 1, \dots, n_\rho, \quad m = 1, \dots, n_{\theta_f}, \end{aligned} \quad (12.17)$$

where $\rho_i(\theta)$ are scalar functions known as basis functions. The aerodynamic partial derivatives are natural candidates for basis functions related to plant nonlinearities [5]

$$\begin{aligned}\rho_1(\theta) &:= \frac{1}{J_r} \frac{\partial Q_a}{\partial \Omega} \Big|_{\bar{V}}, & \rho_2(\theta) &:= \frac{1}{J_r} \frac{\partial Q_a}{\partial V} \Big|_{\bar{V}}, & \rho_3(\theta) &:= \frac{1}{J_r} \frac{\partial Q_a}{\partial \beta} \Big|_{\bar{V}}, \\ \rho_4(\theta) &:= \frac{1}{M_t} \frac{\partial T_a}{\partial \Omega} \Big|_{\bar{V}}, & \rho_5(\theta) &:= \frac{1}{M_t} \frac{\partial T_a}{\partial V} \Big|_{\bar{V}}, & \rho_6(\theta) &:= \frac{1}{M_t} \frac{\partial T_a}{\partial \beta} \Big|_{\bar{V}},\end{aligned}$$

where the division by J_r and M_t is adopted to improve numerical conditioning.

12.3 LPV Controller Design Method

In this section, an LMI-based optimization procedure for designing structured discrete-time LPV controllers is presented. Decentralized controllers of any order, fixed-order, and static output feedback (SOF) are among the possible control structures. Stability is assessed via a parameter-dependent Lyapunov function with varying parameters having their rates of variation contained in a compact closed convex set. A parameter-varying nonconvex condition for an upper bound on the induced $\mathcal{L}_2$ -norm performance is solved via an iterative LMI-based algorithm [1, 2].

An open-loop, discrete-time augmented LPV system with state-space realization of the form

$$\begin{aligned}x(k+1) &= A(\theta)x(k) + B_w(\theta)w(k) + B_u(\theta)u(k), \\ z(k) &= C_z(\theta)x(k) + D_{zw}(\theta)w(k) + D_{zu}(\theta)u(k), \\ y(k) &= C_y(\theta)x(k) + D_{yw}(\theta)w(k)\end{aligned}\tag{12.18}$$

is considered for the purpose of synthesis, where $x(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{R}^{n_w}$ is the vector of disturbance, $u(k) \in \mathbb{R}^{n_u}$ is the control input, $z(k) \in \mathbb{R}^{n_z}$ is the controlled output, and $y(k) \in \mathbb{R}^{n_y}$ is the measured output. $A(\theta)$, $B(\theta)$, $C(\theta)$, $D(\theta)$ are continuous functions of some time-varying parameter vector $\theta = [\theta_1, \dots, \theta_{n_\theta}]$. The same matrix notation to both the continuous-time augmented plant (12.16) and the discrete-time counterpart (12.18) have been adopted. Throughout the text, the context makes it clear when a continuous or discrete system is being referred to.

Assume θ ranges over a hyperrectangle denoted Θ

$$\Theta = \{\theta : \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i, i = 1, \dots, n_\theta\}.$$

The rate of variation $\Delta\theta = \theta(k+1) - \theta(k)$ belongs to a hypercube denoted $\mathcal{V}$

$$\mathcal{V} = \{\Delta\theta : |\Delta\theta_i| \leq v_i, i = 1, \dots, n_\theta\}.$$

The LPV controller has the form

$$\begin{aligned}x_c(k+1) &= A_c(\theta)x_c(k) + B_c(\theta)y(k), \\u(k) &= C_c(\theta)x_c(k) + D_c(\theta)y(k),\end{aligned}\tag{12.19}$$

where $x_c(k) \in \mathbb{R}^{n_c}$ and the controller matrices are continuous functions of θ . Note that depending on the controller structure, some of the matrices may be zero. The controller matrices can be represented in a compact way

$$K(\theta) := \begin{bmatrix} D_c(\theta) & C_c(\theta) \\ B_c(\theta) & A_c(\theta) \end{bmatrix}.\tag{12.20}$$

The interconnection of system (12.18) and controller (12.19) leads to the following closed-loop LPV system denoted S_{cl} :

$$S_{cl} : \begin{cases} x(k+1) = \mathcal{A}(\theta, K(\theta))x_{cl}(k) + \mathcal{B}(\theta, K(\theta))w(k), \\ z(k) = \mathcal{C}(\theta, K(\theta))x_{cl}(k) + \mathcal{D}(\theta, K(\theta))w(k), \end{cases}\tag{12.21}$$

where the closed-loop matrices are [24]

$$\begin{aligned}\mathcal{A}(\theta, K(\theta)) &= \mathbf{A}(\theta) + \mathbf{B}(\theta)K(\theta)\mathbf{M}(\theta), \quad \mathcal{B}(\theta, K(\theta)) = \mathbf{D}(\theta) + \mathbf{B}(\theta)K(\theta)\mathbf{E}(\theta), \\ \mathcal{C}(\theta, K(\theta)) &= \mathbf{C}(\theta) + \mathbf{H}(\theta)K(\theta)\mathbf{M}(\theta), \quad \mathcal{D}(\theta, K(\theta)) = \mathbf{F}(\theta) + \mathbf{H}(\theta)K(\theta)\mathbf{E}(\theta),\end{aligned}$$

with

$$\begin{aligned}\mathbf{A}(\theta) &= \begin{bmatrix} A(\theta) & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{M}(\theta) = \begin{bmatrix} C_y(\theta) & 0 \\ 0 & I \end{bmatrix}, \quad \mathbf{B}(\theta) = \begin{bmatrix} B_u(\theta) & 0 \\ 0 & I \end{bmatrix}, \\ \mathbf{C}(\theta) &= \begin{bmatrix} C_z(\theta) & 0 \end{bmatrix}, \quad \mathbf{F}(\theta) = D_{zw}(\theta), \\ \mathbf{E}(\theta) &= \begin{bmatrix} D_{yw}(\theta) \\ 0 \end{bmatrix}, \quad \mathbf{D}(\theta) = \begin{bmatrix} B_w(\theta) \\ 0 \end{bmatrix}, \quad \mathbf{H}(\theta) = \begin{bmatrix} D_{zu}(\theta) & 0 \end{bmatrix}.\end{aligned}$$

This general system structure can be particularized to some usual control topologies. If $K(\theta)$ is an *unconstrained* matrix and $n_c = 0$, the problem becomes a SOF. The static state feedback (SSF) is a particular case of SOF, when the system output is a full rank linear transformation of the state vector $\forall \theta$. If $n = n_c$, the full-order dynamic output feedback arises. In a structured control context, more elaborate control systems can be designed by constraining $K(\theta)$. A fixed-order dynamic output feedback has $n_c < n$. For decentralized controllers of arbitrary order, the structure of $K(\theta)$ is constrained to be

$$K(\theta) := \begin{bmatrix} \text{diag}(D_c(\theta)) & \text{diag}(C_c(\theta)) \\ \text{diag}(B_c(\theta)) & \text{diag}(A_c(\theta)) \end{bmatrix},$$

where $\text{diag}(\cdot)$ stands that $(\cdot)$ has a block-diagonal structure.

The design of a closed-loop system usually consider performance specifications that can be characterized in different ways. Define $T_{zw}(\theta)$ as the input–output operator that represents the forced response of (12.21) to an input signal $w(k) \in \mathcal{L}_2$ for zero initial conditions. The induced $\mathcal{L}_2$ -norm of a given input–output operator

$$\|T_{zw}\|_2 := \sup_{(\theta, \Delta) \in \Theta \times \mathcal{V}} \sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2}$$

is commonly utilized as a measure of performance of LPV systems and allows formulating the control specification as in $\mathcal{H}_\infty$ control theory. It is of interest to note that an upper bound $\gamma > 0$ on the induced $\mathcal{L}_2$ -norm $\|T_{zw}\|_2$ can be interpreted in terms of the upper bound on the system's energy gain

$$\lim_{h \rightarrow \infty} \sum_{k=0}^{h-1} z(k)^T z(k) < \gamma^2 \lim_{h \rightarrow \infty} \sum_{k=0}^{h-1} w(k)^T w(k).$$

The LPV system (12.21) is said to have performance level γ when it is exponentially stable and $\|T_{zw}\|_2 < \gamma$ holds. An extension of the bounded real lemma (BRL) for parameter-varying systems provides sufficient conditions to analyze the performance level, by solving a constrained LMI optimization problem [10, 27]. For a given scalar γ and a given LPV controller $K(\theta)$, if there exists a θ -dependent matrix function $\mathcal{P}(\theta) = \mathcal{P}(\theta)^T$ satisfying

$$\begin{bmatrix} \mathcal{P}(\theta + \Delta\theta) & \mathcal{A}(\theta, K(\theta))\mathcal{P}(\theta) & \mathcal{B}(\theta, K(\theta)) & 0 \\ * & \mathcal{P}(\theta) & 0 & \mathcal{P}(\theta)\mathcal{C}(\theta, K(\theta))^T \\ * & * & \gamma I & \mathcal{D}(\theta, K(\theta))^T \\ * & * & * & \gamma I \end{bmatrix} > 0 \quad (12.22)$$

$\forall(\theta, \Delta\theta) \in \Theta \times \mathcal{V}$, then the system S_{cl} is exponentially stable and $\|T_{zw}(\theta)\|_2 < \gamma$. The symbol $*$ means inferred by symmetry.

The parameter-varying BRL just shown can be also applied to the case where $w(k)$ is not an energy signal ($\|w(k)\|_2$ not finite) but has a nonzero root mean-square (RMS) value

$$w_{\text{RMS}} := \left[\lim_{h \rightarrow \infty} \frac{1}{h} \sum_{k=0}^{h-1} w(k)^T w(k) \right]^{1/2} \neq 0.$$

In this context, $\mathcal{L}_2$ -norm of a system is given in terms of the RMS values of the signals of interest, instead of $\|\cdot\|_2$. Such a situation is more appropriate to interpret control performance of a wind turbine, since the turbulent wind is a stochastic disturbance that persists for long periods of time, thus $\|w(k)\|_2$ is not a good measure of the signal.

When an LPV controller with performance level γ is not given but should be found (synthesized), the inequality (12.22) is no longer an LMI in the unknown variables due to the product between the variables $K(\theta)$ and $\mathcal{P}(\theta)$. Thus, convex

optimization algorithms cannot be applied to the condition *as it is*. Reformulations into sufficient (possibly conservative) LMI constraints are readily available for particular controller structures and type of parameter dependencies [9, 10].

We propose to design the controllers via an iterative algorithm, instead of attempting to reduce the problem to LMIs. The iterative algorithm relies on the following equivalent non-LMI parametrization that is suitable for iterative design [2]. If there exist $K(\theta)$, $\mathcal{P}(\theta) = \mathcal{P}(\theta)^T$, and $\mathcal{G}(\theta)$ satisfying:

$$\begin{bmatrix} \mathcal{P}(\theta + \Delta\theta) & \mathcal{A}(\theta, K(\theta)) & \mathcal{B}(\theta, K(\theta)) & 0 \\ * & -\mathcal{G}(\theta)^T \mathcal{P}(\theta) \mathcal{G}(\theta) + \mathcal{G}(\theta)^T + \mathcal{G}(\theta) & 0 & \mathcal{C}(\theta, K(\theta))^T \\ * & * & \gamma I & \mathcal{D}(\theta, K(\theta))^T \\ * & * & * & \gamma I \end{bmatrix} > 0, \quad (12.23)$$

$\forall (\theta, \Delta\theta) \in \Theta \times \mathcal{V}$, then the system S_{cl} is exponentially stable and $\|T_{zw}(\theta)\|_2 < \gamma$.

The affine dependence of the reformulated condition on $K(\theta)$ allows the controller matrices to be variables, irrespective of the chosen controller structure. The inequality remains nonconvex due to the product between $\mathcal{P}(\theta)$ and the introduced slack variable $\mathcal{G}(\theta)$. Furthermore, it involves the satisfaction of infinitely many inequalities, since (12.23) should hold for all $(\theta, \Delta\theta) \in \Theta \times \mathcal{V}$.

In order to make the problem computationally tractable, the iterative algorithm solves LMI optimization problems with the slack matrix $\mathcal{G}(\theta)$ constant during an iteration. An *iteration* should be understood to be an LMI-constrained optimization. The use of $\mathcal{G}(\theta)$ as a parameter-dependent slack variable is facilitated by updating its value at each iteration according to some predefined rule. In particular, the update rule is

$$\mathcal{G}(\theta)^{\{j+1\}} = \left(\mathcal{P}(\theta)^{\{j\}} \right)^{-1}, \quad (12.24)$$

where $\{\cdot\}$ is the iteration index and j is the current iteration number.

The iterative algorithm for the design of a structured LPV controller with minimum performance level γ is formulated next.

Algorithm 0: Set $j = 0$, a convergence tolerance ε , an initial $\mathcal{G}(\theta)^{\{0\}}$ and start to iterate:

(A.1) For fixed $\mathcal{G}(\theta)^{\{j\}}$, find $\mathcal{P}(\theta)^{\{j\}}$, $\mathcal{P}(\theta + \Delta\theta)^{\{j\}}$, $K(\theta)^{\{j\}}$, and $\gamma^{\{j\}}$ satisfying the LMI-constrained problem

Minimize γ subject to (12.23).

(A.2) If $|\gamma^{\{j\}} - \gamma^{\{j-1\}}| \leq \varepsilon$, stop. Otherwise, $\mathcal{G}(\theta)^{\{j+1\}} = \left(\mathcal{P}(\theta)^{\{j\}} \right)^{-1}$, set $j = j + 1$ and go to step 1.

12.3.1 Initial Slack Matrix $\mathcal{G}(\theta)^{\{0\}}$

The initial value of $\mathcal{G}(\theta)^{\{0\}}$ required to initialize Algorithm 0 can be obtained in different ways. If a given initial controller $K(\theta)$ satisfies the following optimization problem:

$$\text{Minimize } \gamma \text{ subject to (12.22), } \forall(\theta, \Delta\theta) \in \Theta \times \mathcal{V},$$

then the resulting $\mathcal{P}(\theta)$ can be utilized to derive $\mathcal{G}(\theta)^{\{0\}} = \mathcal{P}(\theta)^{-1}$. The example section shows the usage of this approach.

Alternatively, an iterative feasibility algorithm can be created by relaxing the inequality (12.23). Instead of requiring the inequality to be positive definite (> 0), a variable term is included to the right hand side ($> \text{diag}(\tau I, \tau G^T G, \tau I, \tau I)$), where τ is a scalar variable. The algorithm maximizes τ until the value reaches a certain chosen $v > 0$.

Algorithm 1: Set $j = 0$, a convergence tolerance ε , a $v > 0$, an initial $\mathcal{G}(\theta)^{\{0\}} = I$ and start to iterate:

(A.1) For fixed $\mathcal{G}(\theta)^{\{j\}}$, find $\mathcal{P}(\theta)^{\{j\}}$, $\mathcal{P}(\theta + \Delta\theta)^{\{j\}}$, $K(\theta)^{\{j\}}$, $\gamma^{\{j\}}$, and scalar τ satisfying the LMI-constrained problem

$$\text{Maximize } \tau \text{ subject to (12.23) with the right hand side changed from } > 0 \text{ to } > \text{diag}(\tau I, \tau G^T G, \tau I, \tau I), \text{ and } \tau < v.$$

(A.2) If $|\tau^{\{j\}} - \tau^{\{j-1\}}| \leq \varepsilon$, stop. Otherwise, $\mathcal{G}(\theta)^{\{j+1\}} = (\mathcal{P}(\theta)^{\{j\}})^{-1}$, set $j = j + 1$ and go to step 1.

The resulting $\mathcal{G}(\theta)^{\{0\}}$ can subsequently be used to initialize Algorithm 0.

12.3.2 From Infinite to Finite Dimensional

The LMI problems of Algorithm 0 involve infinitely many LMIs, as θ and $\Delta\theta$ are defined in a continuous space. When LMIs depend affinely on θ and $\Delta\theta$, the synthesis problem at each iteration is reduced to an optimization problem with a finite number of LMIs checked at $(\theta, \Delta\theta) \in \text{Vert } \Theta \times \text{Vert } \mathcal{V}$. Note that $\text{Vert } \Theta$ is the set of all vertices of Θ . For LMIs polynomially θ -dependent, relaxations based on multiconvexity arguments also reduce the problem to check LMIs at the vertices of the parameter space [1, 2]. This procedure, based on sufficient conditions, may lead to extra conservatism. In the general case, where no restrictions on the parameter dependence are imposed, one has to resort to ad-hoc gridding methods [4]. The gridding procedure consists of defining a gridded parameter subset denoted $\Theta_g \subset \Theta$, designing a controller that satisfies the LMIs $\forall \theta \in \Theta_g$, and checking the LMI-constraints in a denser grid. If the last step fails, the process is repeated with a finer grid.

Due to the assumption of general parameter dependence of the open-loop plant on the scheduling variables (12.17), the gridding approach is used in the controller design. The Lyapunov and the LPV controller matrices are affine in the basis functions

$$\mathcal{P}(\theta) = P_0 + \sum_{i=1}^{n_p} \rho_i(\theta_k) P_i + \sum_{i=1}^{n_{\theta_f}} \theta_{f,i} P_{n_p+i}, \quad (12.25a)$$

$$K(\theta) = K_0 + \sum_{i=1}^{n_p} \rho_i(\theta) K_i + \sum_{i=1}^{n_{\theta_f}} \theta_{f,i} K_{n_p+i}. \quad (12.25b)$$

Due to the bounded parameter rate set $\mathcal{V}$ assumed known, the Lyapunov function at sample $k+1$ can be described as

$$\mathcal{P}(\theta + \Delta\theta) = P_0 + \sum_{i=1}^{n_p} \rho_i(\theta + \Delta\theta) P_i + \sum_{i=1}^{n_{\theta_f}} (\theta_{f,i} + \Delta\theta_{f,i}) P_{n_p+i}. \quad (12.26)$$

Note the general parameter dependence of (12.26) on $\Delta\theta$ occasioned by $\rho_i(\theta + \Delta\theta)$. Conveniently, the basis functions at sample $k+1$ are represented as a linear function of $\rho(\theta)$ and $\Delta\theta$

$$\rho_i(\theta + \Delta\theta) := \rho_i(\theta) + \frac{\partial \rho_i(\theta)}{\partial \theta} \Delta\theta, \quad (12.27)$$

thereby turning inequality (12.23) affine dependent on the rate of variation $\Delta\theta$. Thus, it is sufficient to verify (12.23) with (12.26) and (12.27) only at Vert $\mathcal{V}$.

The iterative algorithm for a chosen grid $\Theta_g \subset \Theta$ is presented in the sequel.

Algorithm 2: Set $j = 0$, a convergence tolerance ε , initialize $\mathcal{G}(\theta)^{\{0\}} \forall \theta \in \Theta_g$, and start to iterate:

(A.1) For fixed $\mathcal{G}(\theta)^{\{j\}}$, and $i = 0, 1, \dots, n_p + n_{\theta_f}$, find $P_i^{\{j\}} > 0$, $K_i^{\{j\}}$, and $\gamma^{\{j\}}$ satisfying the LMI-constrained problem

$$\text{Minimize } \gamma \text{ subject to (12.23), } \forall (\theta, \Delta\theta) \in \Theta_g \times \text{Vert}\mathcal{V}.$$

(A.2) If $|\gamma^{\{j\}} - \gamma^{\{j-1\}}| \leq \varepsilon$, stop. Otherwise, $\mathcal{G}(\theta)^{\{j+1\}} = \mathcal{P}(\theta)^{\{j\}-1}$, $\forall \theta \in \Theta_g$. Set $j = j+1$ and go to step 1.

The Lyapunov variable $\mathcal{P}(\theta)^{\{j\}} > 0$ may be close to singular at each iteration, making the inversion required to compute $\mathcal{G}(\theta)$ possibly ill conditioned. To alleviate this issue, an additional LMI constraint

$$\mathcal{P}(\theta)^{\{j\}} > \mu I,$$

improves numerical condition of the inversion by imposing a lower bound on the eigenvalues of $\mathcal{P}(\theta)^{\{j\}}$, where $\mu > 0$ is a chosen scalar. There exists a tradeoff between the value of μ and the attained value of γ . Higher values of μ may lead to more conservative controllers, although from our experience, the small value of μ required to better condition the inversion does not influence significantly on the performance level γ .

The gridding procedure for controller synthesis can be summarized by the following steps:

- (A.1) Define a grid Θ_g for the compact set Θ .
- (A.2) Find initials $\mathcal{G}(\theta)^{\{0\}}$, $\forall \theta \in \Theta_g$.
- (A.3) Solve Algorithm 2.
- (A.4) Define a denser grid.
- (A.5) Verify the feasibility of the LMI (12.22) with the computed controller $K(\theta)$, in each point of the new grid. If it is infeasible, choose a denser grid and go to step 2.

12.3.3 Controller Implementation

The iterative LMI optimization algorithm provides the controller matrices $A_{c,i}$, $B_{c,i}$, $C_{c,i}$, $D_{c,i}$, for $i = 0, 1, \dots, n_p + n_{\theta_f}$. These matrices, the basis functions, and the value of the scheduling variables are the only required information to determine the control signal u . At each sample time k , the scheduling variable θ is measured (or estimated) and a control signal is obtained as follows:

- (A.1) Compute the value of the basis functions $\rho_i(\theta)$, for $i = 0, 1, \dots, n_p$. The basis functions may be stored in a lookup table that takes θ as an input and outputs an interpolated value of $\rho(\theta)$.
- (A.2) With the value of the basis functions in hand, determine the controller matrices $A_c(\theta)$, $B_c(\theta)$, $C_c(\theta)$, $D_c(\theta)$ according to

$$\begin{aligned}
 A_c(\theta) &= A_{c,0} + \sum_{i=1}^{n_p} \rho_i(\theta) A_{c,i} + \sum_{i=1}^{n_{\theta_f}} \theta_{f,i} A_{c,n_p+i}, \\
 B_c(\theta) &= B_{c,0} + \sum_{i=1}^{n_p} \rho_i(\theta) B_{c,i} + \sum_{i=1}^{n_{\theta_f}} \theta_{f,i} B_{c,n_p+i}, \\
 C_c(\theta) &= C_{c,0} + \sum_{i=1}^{n_p} \rho_i(\theta) C_{c,i} + \sum_{i=1}^{n_{\theta_f}} \theta_{f,i} C_{c,n_p+i}, \\
 D_c(\theta) &= D_{c,0} + \sum_{i=1}^{n_p} \rho_i(\theta) D_{c,i} + \sum_{i=1}^{n_{\theta_f}} \theta_{f,i} D_{c,n_p+i}.
 \end{aligned}$$

- (A.3) Once the controller matrices have been found, the control signal $u(k)$ can be obtained by the dynamic equation (12.19) of the LPV controller, which only involves multiplications and additions.

12.4 Example: LPV PI Controller Tolerant to Pitch Actuator Faults

The proportional and integral (PI) is the most utilized controller by the wind energy industry. At low wind speeds, the PI speed control using generator torque as controlled input can be quite slow, thus tuning is not significantly challenging. However, at high wind speeds, the PI speed control using pitch angle as controlled input strongly couples with the tower dynamics, denoting a multivariable problem, and should be properly designed. Inappropriate gain selection can make rotational speed regulation “loose” around the set point or make the system unstable, as well as excite poorly damped structural modes [7].

The concepts seen throughout this chapter are here applied to the state-of-the-art controller structure of the wind turbine industry [8]. The present example intends to show that theoretical rigorosity on the design of gain-scheduled controllers may bring advantages in terms of performance and reliability of wind turbines in a closed loop.

12.4.1 Controller Design

For a clear and didactic exposure, the adopted control structure depicted in Fig. 12.10 is simpler than an industry-standard Region III controller [8], but includes the most common control loops.

The generator speed is regulated by a PI controller of the form

$$G_{PI} := k_p(\theta) + k_i(\theta) \frac{(s + z_I)}{s},$$

where s denotes the Laplace operator. Instead of a pure integrator, the PI controller is composed by an integrator filter

$$G_I(s) := \frac{s + z_I}{s},$$

for reasons to be explained later, where the filter zero z_I is a design parameter.

It is possible to provide an extra signal by using an accelerometer mounted in the nacelle, allowing the controller to better recognize between the effect of wind speed disturbances and tower motion on the measured power or generator speed. With

A power controller for reducing fast power variations is treated simplistically as a proportional feedback from generator speed to generator torque. Considering a constant power control scheme, the generator torque can be represented as a function of the generator speed. The proportional feedback is nothing but the partial derivative of generator torque with respect to generator speed

$$\frac{\partial Q_g(\Omega_g)}{\partial \Omega_g} = -\frac{P_N}{N_g \Omega_{g,N}^2}.$$

In real implementations, a slow integral component is added to the loop to include asymptotic power tracking.

Instead of the classical control techniques, the design of PI speed and tower feedback loops are revisited under the LPV framework. For a didactic and clear exposure, the interconnection of the drive train with the damper is now considered as a first order low pass filter from aerodynamic torque to generator speed, and the rotor speed proportional to the generator speed. The LPV controller can now be designed to trade off the tracking of generator speed and tower oscillations with control effort (wear on pitch actuator). The dynamic model of the variable-speed wind turbine can then be expressed as an LPV model of the form

$$G: \begin{cases} \dot{x} = A(\theta)x + B_w(\theta)\hat{u} + B_u(\theta)\beta_{\text{ref}} \\ y = C_y x \end{cases},$$

where states, controllable input and measurements are

$$x = [\Omega_r \dot{q} q \dot{\beta} \beta x_{\Omega,i}]^T, \quad u = \beta_{\text{ref}}, \quad y = [\Omega_g y_{\Omega,i} \dot{q}]^T.$$

with open-loop system matrices

$$A(\theta) = \begin{bmatrix} \rho_1(\theta) - \frac{1}{J_r + J_g N_g^2} \frac{\partial Q_g}{\partial \Omega} & -\rho_2(\theta) & 0 & 0 & \rho_3(\theta) & 0 \\ \rho_4(\theta) & -\frac{1}{M_t} B_t - \rho_5(\theta) - \frac{K_t}{M_t} & 0 & \rho_6(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44}(\theta_f) - a_{12}(\theta_f) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ N_g & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_u = [\rho_2(\theta) \ \rho_5(\theta) \ 0 \ 0 \ 0 \ 0]^T, \quad B_w = [0 \ 0 \ 0 \ b_{4,1}(\theta_f) \ 0 \ 0]^T, \quad C_y = \begin{bmatrix} N_g & 0 & 0 & 0 & 0 \\ z_1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$a_{12}(\theta_f) = b_{41}(\theta_f) = (1 - \theta_f(t))\omega_{n,0}^2 + \theta_f(t)\omega_{n,lp}^2,$$

$$a_{44}(\theta_f) = -2(1 - \theta_f(t))\zeta_0\omega_{n,0} - 2\theta_f(t)\zeta_{lp}\omega_{n,lp}.$$

The basis functions $\rho_1(\theta), \dots, \rho_6(\theta)$ related to the parameter-varying aerodynamic gains are selected as

$$\begin{aligned} \rho_1 &:= \frac{1}{J_r + J_g N_g^2} \frac{\partial Q}{\partial \Omega} \bigg|_{\hat{V}}, \quad \rho_2 := \frac{1}{J_r + J_g N_g^2} \frac{\partial Q}{\partial V} \bigg|_{\hat{V}}, \quad \rho_3 := \frac{1}{J_r + J_g N_g^2} \frac{\partial Q}{\partial \beta} \bigg|_{\hat{V}}, \\ \rho_4 &:= \frac{1}{M_t} \frac{\partial T}{\partial \Omega} \bigg|_{\hat{V}}, \quad \rho_5 := \frac{1}{M_t} \frac{\partial T}{\partial V} \bigg|_{\hat{V}}, \quad \rho_6 := \frac{1}{M_t} \frac{\partial T}{\partial \beta} \bigg|_{\hat{V}}. \end{aligned}$$

Notice the PI controller integrator filter G_I conveniently augmented into the state-space of G , represented by the state $x_{\Omega,i}$ and the output $y_{\Omega,i}$. The plant G_p is defined as the wind turbine model solely (plant G without the augmentation of G_I).

Considering G as the plant for synthesis purposes, the LPV controller structure reduces to a parameter-dependent SOF of the form

$$\begin{aligned} K(\theta) &= D_{c,0} + \sum_{i=1}^6 \rho_i(\theta) D_{c,i} + \theta_f D_{c,7}, \quad D_{c,n} := [D_{p,n} \ D_{i,n} \ D_{\dot{q},n}], \\ n &= 0, 1, \dots, 7. \end{aligned}$$

Controller tuning follows a procedure similar to the $\mathcal{H}_\infty$ design. Notice that, for fixed values of the varying parameter θ , and initially neglecting the tower velocity feedback, the controller design becomes a mixed sensitivities optimization problem intended to minimize the norm

$$\left\| \begin{bmatrix} W_{z1} G_I S G_v \\ W_u G_{PI} S G_v \end{bmatrix} \right\|_\infty,$$

where S is the sensitivity defined as $S := (I + G_p G_{PI})^{-1}$, G_v is the transfer function from $\hat{V}$ to $\hat{\Omega}_g$, W_{z1} and W_u are weighting functions. The weight W_{z1} applied to the generator speed deviations can be used to shape the closed-loop response of rotational speed in face of wind disturbances, given by $\hat{\Omega}(t) = S G_v \hat{V}(t)$. The desired sensitivity in closed loop is

$$S_\Omega(s) := \frac{s^2 + 2\xi_\Omega \omega_\Omega s}{s^2 + 2\xi_\Omega \omega_\Omega s + \omega_\Omega^2},$$

where the natural frequency ω_Ω and damping ratio ξ_Ω are design parameters that select the desired second-order closed-loop behavior. The desired sensitivity S_Ω can be applied as a loop-shaping weight by defining W_{z1} as

$$W_{z1}(s) := \frac{1}{G_I(s) S_\Omega(s)} = \frac{s^2 + 2\xi_\Omega \omega_\Omega s + \omega_\Omega^2}{(s + z_1)(s + 2\xi_\Omega \omega_\Omega)}.$$

W_u is a first order high-pass filter that penalizes high-frequency content on the pitch angle

$$W_u(s) := k_3 \frac{s + z_3}{s + p_3}.$$

W_{z1} and W_u governs the tradeoff between rotational speed regulation and pitch wear. Due to the resonance characteristics of the transfer function from $\hat{V}$ to $\dot{q}$, the weighting function W_{z2} is chosen as a scalar k_2 that tradeoffs the desired tower damping.

Two LPV controllers are designed, one fault intolerant and another tolerant to pitch actuator faults. The only difference on their synthesis is the inclusion of the fault-dependent terms $P_7\theta_f$ and $D_{c,7}\theta_f$ of the Lyapunov and controller matrices, respectively. The parameters for the loop-shaping weight W_{z1} are selected as $\omega_\Omega = 0.6283$ rad/s (0.1 Hz) and $\xi_\Omega = 0.7$, with the zero of the integrator filter located at $z_1 = 1.0$ rad/s. A special attention must be devoted to the choice of W_u . Due to the fact that the pitch system has slower dynamics in the presence of low oil pressure, the bandwidth of this filter must be made large enough to allow rotational speed and tower damping control in the occurrence of faults. Defining Ω_{3P} as three times the nominal rotational speed $\Omega_{r,N}$, in the present example, $k_3 = 1$, $p_3 = 1.5\Omega_{3P}$ and $z_3 = 15\Omega_{3P}$.

Remember that the iterative LMI algorithm is a synthesis procedure in discrete time. Therefore, the augmented LPV plant in continuous time is discretized using a bilinear (Tustin) approximation [3] with sampling time $T_s = 0.02$ s, at each point $\Theta_g \times \text{Vert} \mathcal{V}$. The rate of variation of the scheduling variables in continuous time must as well be converted to discrete-time by the relation $\Delta\theta(k) = T_s\Delta\theta(t)$.

The initial slack matrices $G(\theta, \Delta\theta)^{\{0\}}$, $\forall(\theta, \Delta\theta) \in \Theta_g \times \text{Vert} \mathcal{V}$ required to initialize the LMI-based algorithm are determined from the solution of the following LMI optimization problem:

$$\text{Minimize } \gamma \text{ subject to } (12.22), (12.26), (12.27), \forall(\theta, \Delta\theta) \in \Theta_g \times \text{Vert} \mathcal{V}$$

with a given initial controller $K(\theta)$. The resulting Lyapunov matrix determines $G(\theta, \Delta\theta)^{\{0\}} = \mathcal{P}(\theta, \Delta\theta)^{-1}$. The proportional and integral gains of the given initial controller can be computed by placement of the poles of the transfer function from $\hat{V}$ to $\hat{\Omega}_g$. Neglecting pitch actuator dynamics, and considering a pure integrator, the k_p and k_i gains can be described analytically as [16]

$$k_p(\theta) = \frac{2\xi_\Omega \omega_\Omega (J_r + N_g^2 J_g) - N_g \frac{\partial Q_g}{\partial \Omega_g} + \rho_1(\theta)}{-N_g \rho_3(\theta)}, \quad k_i(\theta) = \frac{\omega_\Omega^2 (1 + \xi_\Omega^2) (J_r + N_g^2 J_g)}{-N_g \rho_3(\theta)}.$$

The tower feedback gain of the initial controller is $k_{\dot{q}}(\theta) = 0$, meaning no active tower damping.

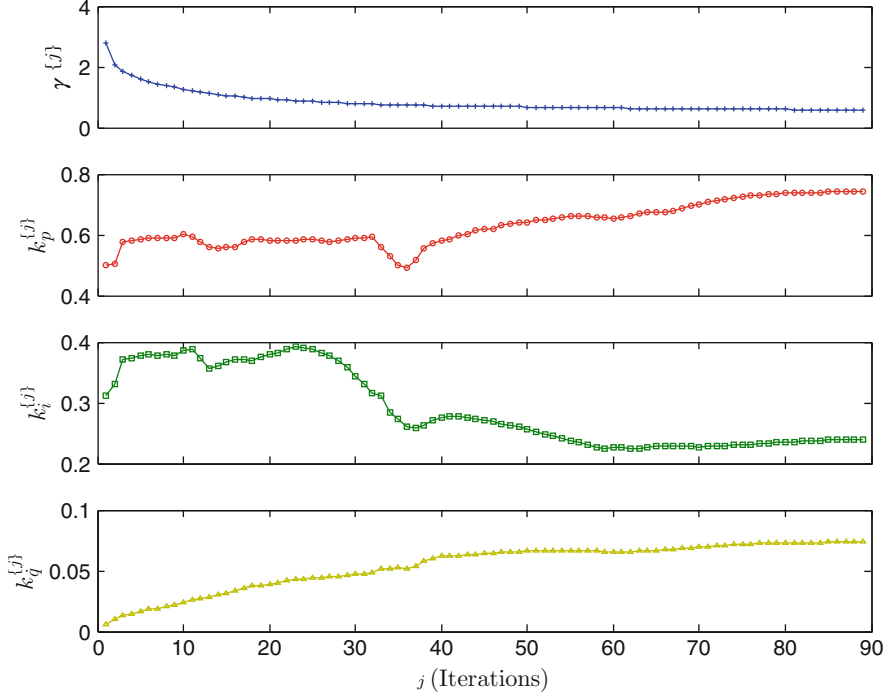


Fig. 12.11 Evolution of performance level γ and controller gains k_p , k_i , k_q during the iterative LMI synthesis. Controller gains computed at $\theta_{op} = 15$ m/s, $\theta_f = 0$

Convergence tolerance of the iterative algorithm is set to $\varepsilon = 10^{-3}$. After 89 iterations, convergence is achieved to a performance level $\gamma = 0.586$. The evolution of $\gamma^{(j)}$ versus the iteration number is depicted in Fig. 12.11, where the monotonically decreasing property of the sequence is noticeable. The proportional and integral gains depicted on the figures are multiplied by the gearbox ratio N_g for better illustration. The controller gains $K(\theta) = [k_p(\theta), k_i(\theta), k_q(\theta)]$ computed at $\theta_{op} = 15$ m/s, $\theta_f = 0$, during the course of the iterative LMI algorithm, are also shown. The synthesis procedure converge to controller gains different than the gains of the initial controller. The tower feedback gain k_q , null in the initial controller, has converged to a nonzero value, meaning active tower damping.

The proportional, integral, and tower feedback gains as three-dimensional surfaces of the scheduling parameters $\bar{V}$ and θ_f are illustrated in Fig. 12.12a–c. The controller gains capture the dependence of the LPV system on the wind speed given by the basis functions. Compare the shape of the surfaces with the aerodynamic gains (Fig. 12.8). Also notice the slight changes in k_p and k_q and the changes in k_i scheduled by θ_f .

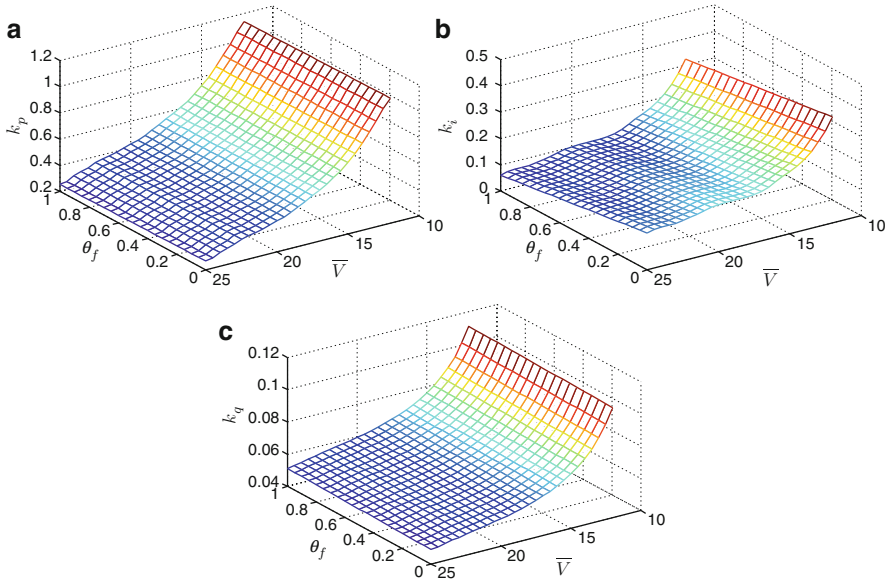


Fig. 12.12 Proportional, integral and tower feedback gains as functions of the operating point and fault scheduling variables

12.4.2 Simulation Results

The performance of the LPV controllers are accessed in a nonlinear wind turbine simulation environment [12]. The effective wind speed is estimated by an unknown input observer that uses measurements of generator speed, generator torque, and pitch angle [20]. Figures 12.13a–12.14d depict time series of the variables of interest resulting from a 600 s simulation. A mean speed of 17 m/s with 12% turbulence intensity and shear exponent of 0.1 characterizes the wind field (Fig. 12.13a). At time $t = 200$ s, the pitch system experiences a fault with θ_f increasing from 0 to 1 (Fig. 12.13b). At $t = 430$ s, the pitch system comes to normality with θ_f decreasing from 1 to 0. Both variations on the fault scheduling variable are made with maximum rate of variation.

Results of LPV controllers intolerant and tolerant to pitch actuator faults are compared to support a discussion of the consequences of the fault on the closed-loop system as well as fault accommodation. When the wind turbine is controlled by the fault intolerant LPV PI controller, the rotational speed (Fig. 12.13c) experiences poor and oscillatory regulation during the occurrence of faults, more pronouncedly while θ_f is varying. The threshold for a shutdown procedure due to overspeed is usually between 10% and 15% over the nominal speed [23]; in this particular case, the overspeed would not cause the wind turbine to shut down. The FT-LPV PI controller successfully accommodates the fault, maintaining rotor speed

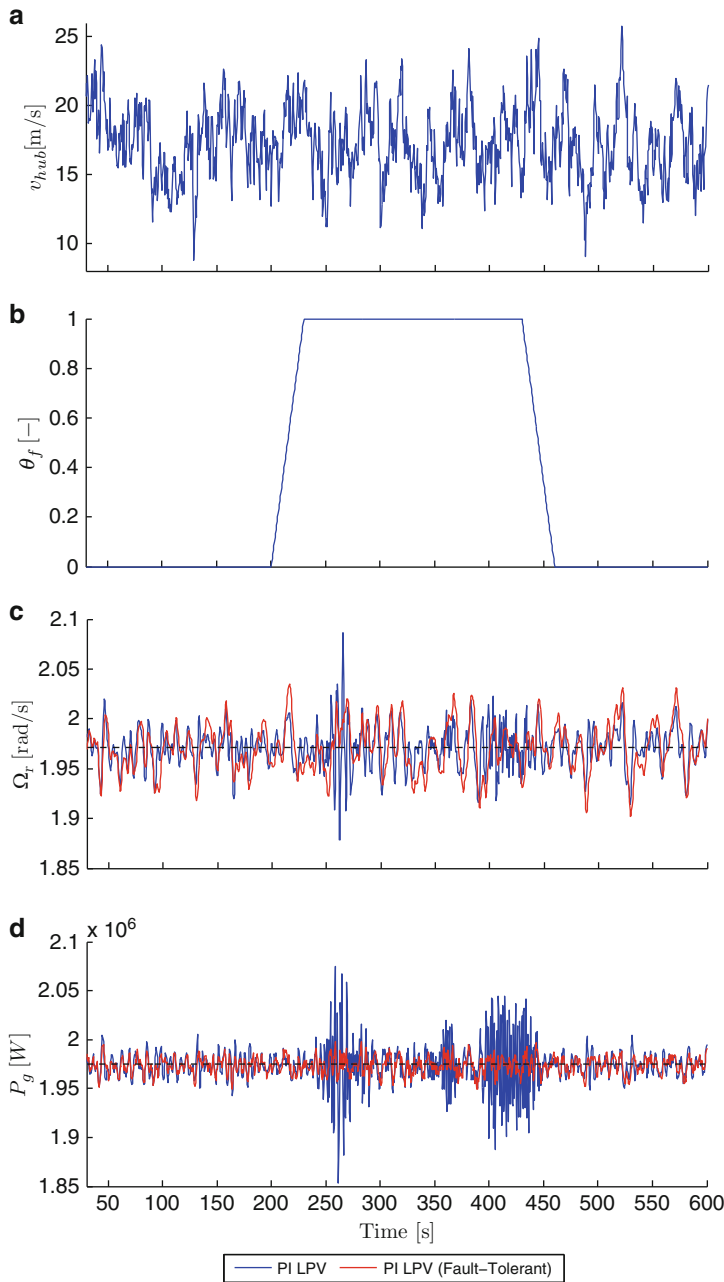


Fig. 12.13 Time series of (a) hub height wind speed (b) fault scheduling variable (c) rotor speed and (d) electrical power. Simulation results of a 2 MW wind turbine controlled by a fault-intolerant and a fault-tolerant LPV PI controller

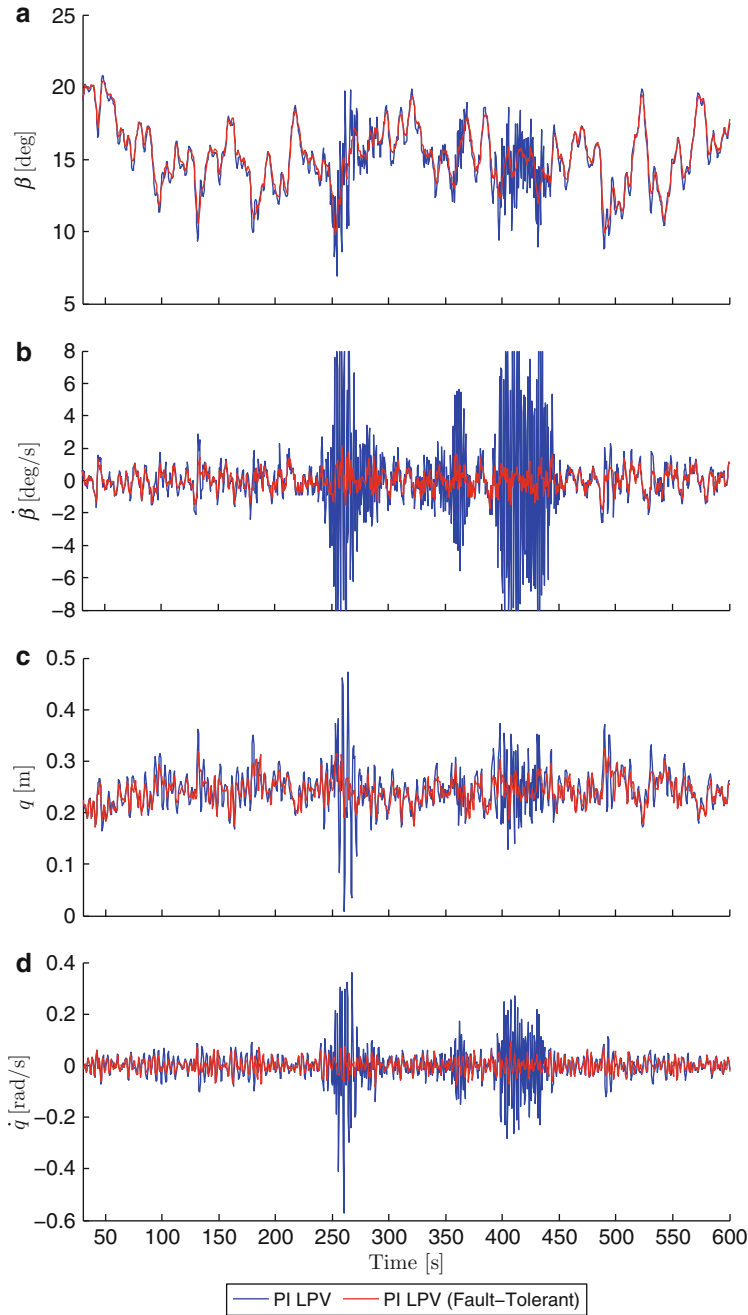


Fig. 12.14 Time series of (a) pitch angle (b) pitch velocity (c) fore–aft tower position and (d) fore–aft tower velocity. Simulation results of a 2 MW wind turbine controlled by a fault-intolerant and a fault-tolerant LPV PI controller

properly regulated. Oscillatory power overshoots of up to 6% of the nominal power (Fig. 12.13d) degrades power quality; the same does not happen to the FT-LPV controlled system.

More serious than the effects on rotational speed and power are the consequences of faults on the pitch system and tower. Excessive pitch angle excursions during faults (Fig. 12.14a) with the limits on velocity of ± 8 deg/s being reached (Fig. 12.14b) may cause severe wear on pitch bearings. The FT-LPV controller maintain pitch excursions and velocities within normal limits. The tower experiences displacements (Fig. 12.14c) of up to 0.48 m, an increase of approximately 60% when compared to the FT-LPV. The displacements comes along with very high tower velocities of almost 0.4 m/s, 260% higher than the fault accommodated case.

In such a situation, the supervisory controller would shut down the wind turbine due to excessive vibrational levels measured by the nacelle accelerometer. The same would not be necessary if the wind turbine is controlled by the FT-LPV. Therefore, fault-tolerance leads to higher energy generation and availability. It also collaborates to a better management of condition-based maintenance; higher priority of maintenance can be given to wind turbines with faults that cannot be accommodated by the control system. These are examples of the benefits that the LPV control design framework presented in this chapter can bring to wind turbines in closed loop with industry-standard as well as more elaborate controllers.

12.5 Conclusions

This chapter initially presents the modeling of a wind turbine model as an LPV system, considering faults on actuators and sensors. Later, an iterative LMI-based algorithm for the design of structured LPV controllers is described. This constitutes a unified LMI-based design framework to address gain-scheduling, fault-tolerance, and robustness on the design of wind turbine controllers.

The method is based on parameter-dependent Lyapunov functions, which reduces conservativeness of control for systems with rate bounds, which is the case in this work. The iterative algorithm may be computationally expensive depending on the number of plant states and scheduling variables, but brings desired flexibility in terms of the controller structure: decentralized of any order, dynamic (reduced-order) output feedback, SOF, and state feedback are among the possible ones. Moreover, the resulting controller can also be easily implemented in practice due to low data storage and simple math operations. In fact, the required data to be stored on the controller memory is only the controller matrices, and scalar functions of the scheduling variables representing plant nonlinearities. The mathematical operations needed to compute the controller at each sampling time are look-up tables with interpolation, products between a scalar and a matrix, and sums of matrices.

A design example of a fault-tolerant controller for the Region III, with a structure similar to the state-of-the-art industrial controllers, intends to show that theoretical rigorousness on the design of gain-scheduled controllers may bring advantages in

terms of performance and reliability of wind turbines in closed loop. The presented framework is not limited to the specific example shown. Due to its flexibility, the framework can be applied to other known wind turbine controller structures or even to explore different control philosophies.

Simulations indeed confirm that the fault-tolerant LPV controllers have superior performance in the occurrence of faults. The LPV controller designed for the nominal system start oscillating when the fault is introduced. In a real situation, the supervisory controller would shut down the wind turbine due to excessive vibrational levels measured by the nacelle accelerometer. The same would not be necessary if the wind turbine is controlled by the FT-LPV. Therefore, higher energy generation and availability is achieved. It also contributes to a better management of condition-based maintenance; priority on maintenance can be given to wind turbines with faults that cannot be accommodated by the control system.

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