Market Integration of Virtual Power Plants

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***Abstract*— We consider a direct control Virtual Power Plant, which is given the task of maximizing the profit of a portfolio of flexible consumers by trading flexibility in Energy and Power Markets. Spot price optimization has been quite intensively researched in Smart Grid literature lately. In this work, however, we develop a three stage market model, which includes Day-Ahead (Spot), Intra-Day and Regulating Power Markets. This allows us to test the hypothesis that the Virtual Power Plant can generate additional profit by trading across several markets.**

We find that even though profits do increase as more markets are penetrated, the size of the profit is strongly dependent on the type of flexibility considered. We also find that penetrating several markets makes profits surprisingly robust to spot price prediction errors.

I. Introduction

The introduction of renewable energy production into the existing power system is complicated by the inherent variability of production technologies, which harvest energy mainly from renewable sources like wind and sun. This means that it becomes increasingly challenging to maintain the real-time balance between production and consumption as the ratio of renewable energy production increases. In a Smart Grid system the inherent flexibility of consumers, such as electric vehicles, heat pumps and HVAC-systems, may be mobilized to play an active part in solving the balancing task.

To achieve this goal, however, we believe that the load control schemes must be fully responsive and non-disruptive, [1]. Consequently we investigate a setup where the actual coordinated operation of the flexible consumers is facilitated by a third party aggregator. This commercial aggregator has implemented a Virtual Power Plant, which is assumed to have direct control of a portfolio of flexible resources.

In a deregulated power market the balance between supply and demand is maintained though a series of markets operating closer and closer to the time of delivery. To make competition fair the Virtual Power Plant must enter these markets and compete on equal terms with other players such as wind farm operators and traditional power plants. The Virtual Power Plant will then help the overall goal of load balancing simply by increasing the capacity in the markets. Market mechanisms will then generate a utilization of the total available capacity, which is cheaper and more efficient.

In this paper we investigate how the Virtual Power Plant operator can potentially make a profit by trading the flexibility of electric vehicles, heat pumps and HVAC-systems in Energy and Power Markets. We first examine how the concepts of fixed and marginal costs (well known for traditional power plants, see e.g. [2] and [3]) applies to Virtual Power Plant operation. We do this in order to investigate how different comfort demands (constraint) determine how flexibility should be traded and also how much profit can be earned.

Other references, such as [4], [5], [6] have also investigated price optimized consumption scheduling for Smart Grid technologies. However, these references investigate a single-stage model where only spot price optimization is performed. In this paper we develop a three stage market model, where Day-Ahead Market, Intra-Day Market and Regulating Power Market are included. The main objective of this paper is consequently to test the hypothesis that a Virtual Power Plant under reasonable assumptions can generate additional profit by participating in several markets.

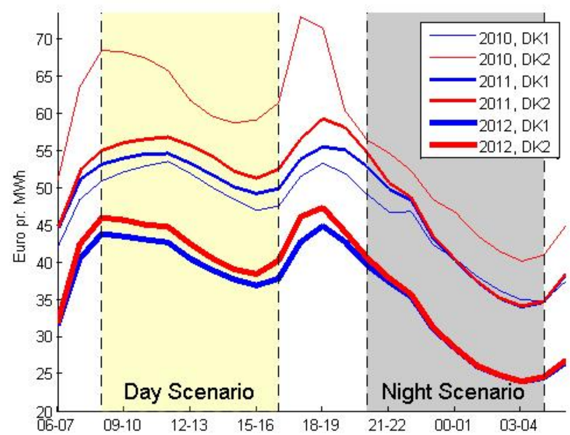

Fig. 1: Average Spot price at each hour of the day in DK1 and DK2.

II. Fixed and Marginal Costs

In economics, fixed costs are necessary expenses, which must be covered in order to enable the production of a given product, but which are not related to the quantity or quality of product produced. In power systems, fixed costs are therefore also referred to as ”overnight” cost, because it is the present day cost of constructing a production facility ”overnight” [2]. While a Virtual Power Plant does not require the construction of a facility as such, there are a number of fixed costs, which must be covered in order for the Virtual Power Plant to be in operation. Examples of such expenses

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For more information on the PhD-project see Mettematics.com.

include marketing, installation, reading and maintenance of communication and metering equipment, development of IT-platform plus customer billing and accounting.

In power systems, marginal costs are defined as the costs/savings associated with producing one more/less kilowatt-hour [2]. For a power production facility, variable costs can therefore be computed as fuel cost per produced power unit plus costs of maintenance and wear. For a Virtual Power Plant, however, the calculations are more complicated. This is because most Smart Grid technologies actually do not consume one more/less kilowatt-hour, but rather advances/postpones the consumption of that kilowatt-hour.

Fixed costs should obviously be recovered over time in order for the Virtual Power Plant to prove a profitable concept. Fixed costs, however, do not affect the prices at which the Virtual Power Plant should bid into the market. For any production facility it is true that if the *market price* is higher than the *marginal cost* of production then the facility will earn (*market price* - *marginal cost*) per unit produced. Production facilities should therefore always bid at marginal cost, since making a small profit is better than making no profit at all.

Taking this to the consumption side it is found that if the *market price* is lower than *marginal cost* of consumption then the Virtual Power Plant will earn (*marginal cost* - *market price*) per unit consumed. Consequently, the Virtual Power Plant should also bid on the market at marginal costs.

As mentioned earlier, however, a Virtual Power Plant does not simply increase or decrease its consumption, but rather advances or postpones consumption. Therefore, marginal costs of a Virtual Power Plant can only be determined if market prices are known. However, this assumption is hardly ever satisfied at the time of bidding, so the Virtual Power Plant must use a best estimate of prices to determine its own marginal costs and thus appropriate bidding price. Marginal costs for the Virtual Power Plant will be discussed and exemplified much further in Section IV-B and V-B.

Notice that throughout the paper, up- and down-regulation is defined in accordance with classical conventions for the consumption side. This means the up-regulation for a flexible consumer corresponds to a decrease in consumption and down-regulation corresponds to an increase in consumption.

III. Flexibility Modeling

In the present paper, flexibility is defined based on the *Buckets*, *Batteries and Bakeries*-taxonomy presented in [7]. The first model, denoted the *Bucket*, is a power and energy constrained integrator with a drain. The *Battery* is also a power and energy constrained integrator, but without the drain and with the added restriction that the unit must be fully charged at a specific deadline. Finally the *Bakery* extends the *Battery* with the additional constraint that the process must run as a batch process at constant power consumption. We let $P_i(k)$ denote the power consumption of unit i at sample k and let $E_i(k)$ denote the energy level in unit i at sample k. Note also that unless otherwise stated variables are real positive scalars.

Formal definitions of a *Bucket*, a *Battery* and a *Bakery* are given in Definition 1, 2 and 3 respectively. In the following T_s denotes the size of the time step, $\underline{P}_i$ and $\overline{P}_i$ denote limits on consumption rate, $\underline{E}_i$ and $\overline{E}_i$ denote limits on energy storage levels and $v_i(k)$ is a boolean-valued variable, which state whether a *Bakery* is running at sample k.

Definition 1 (Bucket): The dynamics and constraints of a *Bucket* with drain α are

$\mathit{Bucket}_i(k)$:

$$E_i(k+1) = \alpha E_i(k) + T_s\big(P_i(k) + P_{i,Plan}(k)\big) \quad \text{(A.1)}$$
$$\underline{P}_i - P_{i,Plan}(k) \le P_i(k) \le \overline{P}_i - P_{i,Plan}(k) \quad \text{(A.2)}$$
$$\underline{E}_i \le E_i(k) \le \overline{E}_i \quad \text{(A.3)}$$
$$E_i(0) = E_{i,0}, \quad \text{(A.4)}$$

where $k = 0,1,\dots,\infty$, $i = 1,2,\dots,N^{\mathit{Buckets}}$, $0 \le \alpha \le 1$, $\underline{P}_i \le 0 \le \overline{P}_i$, $\underline{P}_i \le P_{i,Plan} \le \overline{P}_i$ and $\underline{E}_i \le E_{i,0} \le \overline{E}_i$.

Definition 2 (Battery): The dynamics and constraints of a *Battery* are

$\mathit{Battery}_i(k)$:

$$E_i(k+1) = E_i(k) + T_s\big(P_i(k) + P_{i,Plan}(k)\big) \quad \text{(B.1)}$$
$$0 - P_{i,Plan}(k) \le P_i(k) \le \overline{P}_i - P_{i,Plan}(k) \quad \text{(B.2)}$$
$$0 \le E_i(k) \le \overline{E}_i \quad \text{(B.3)}$$
$$E_i(0) = E_{i,0}, \quad \text{(B.4)}$$
$$E_i(T_{end,i}) = \overline{E}_i, \quad \text{(B.5)}$$

where $k = 0,1,\dots,\infty$, $i = 1,2,\dots,N^{\mathit{Batteries}}$, $T_{end,i} \in \mathbb{N}$, $0 \le \overline{P}_i$, $0 \le P_{i,Plan} \le \overline{P}_i$ and $0 \le \overline{E}_i$.

Definition 3 (Bakery): The dynamics and constraints of a *Bakery* are

$\mathit{Bakery}_i(k)$:

$$E_i(k+1) = E_i(k) + T_s\big(P_i(k) + P_{i,Plan}(k)\big), \quad \text{(C.1)}$$
$$\overline{P}_i v_i = P_i(k) + P_{i,Plan} \quad \text{(C.2)}$$
$$0 \le E_i(k) \le \overline{E}_i, \quad \text{(C.3)}$$
$$E_i(0) = E_{i,0}, \quad \text{(C.4)}$$
$$E_i(T_{end,i}) = \overline{E}_i, \quad \text{(C.5)}$$
$$0 \le \sum_{l=k}^{k+T_{run,i}-1} v_i(l) - T_{run,i}\big(v_i(k) - v_i(k-1)\big), \quad \text{(C.6)}$$

where $k = 0,1,\dots,K$, $0 \le \overline{P}_i$, $0 \le P_i \le \overline{P}_i$, $\overline{E}_i = \overline{P}_i T_{run,i} - E_{i,0}$, $v_i(k) \in \{0,1\}$, $i = 1,2,\dots,N^{\mathit{Bakeries}}$, $T_{end,i} \in \mathbb{N}$ and $T_{run,i} \in \mathbb{N}$.

IV. Market Theory and Model

This section gives a short introduction to the Nordic Power Markets and next extends this introduction to a market model.

A. The Nordic Power Markets

In the Nordic countries the balance between production and consumption at the market level is maintained by means of Day-Ahead Markets, Intra-Day Markets, Regulating Power Markets and Balancing Power Markets (after-day settlement). This section gives a general description of the setup.

As the name suggests, the Day-Ahead Market (the Spot Market) operates before the actual time of delivery. Producers and wholesalers make bids for production and consumption in future time slots and prices are settled based on a double auction. Once prices on the Day-Ahead Market are settled (Market Clearing), the market is closed.

On the Day-Ahead Market producers and wholesalers have made bids based on the best available knowledge at the time of bidding. As time progresses, however, better forecasts become available. The Day-Ahead Market is therefore followed by the Intra-Day Market (the Elbas Market), where players have the option of adjusting their initial production and consumption schedules in future time slots. The Intra-Day Market is a continuous market where trading takes place up until one hour before the hour of delivery. The Intra-Day Market consists of two lists, which are continuously updated: One list for power purchases and one for power sales. Whenever there is a match within these lists (meaning that a player is willing to purchase power at a price which is higher than another players sales price), these two bids are activated and removed from the lists. This means that the Intra-Day market is more bilateral in nature than the other markets.

If players do not follow the schedule generated on the Day-Ahead and Intra-Day markets, they generate a need for balancing, i.e. up- or down-regulation. Up- and down-regulation are performed by spare capacity denoted *reserves*, which are in place because ”the price mechanism cannot work fast enough to balance consumption and production in real time” [3]. Traditionally, reserves are provided by specific power plants, which are operating at less than full capacity, so they can ramp up or down as needed. In the Nordic markets reserve services are traded on the Regulating Power Market. Having a designated power market insures that a competitive price is paid for Regulating Power. In the Regulating Power Market, bids can be made up to 15 minutes before the hour of delivery. If a need for regulation arises during the hour of operation, then bids are activated in accordance with the highest price of the block of most inexpensive bids until the requested regulation is accumulated.

After the actual time of delivery, metered data of actual production/consumption is evaluated. In the after-day settlement (or Balancing Power Market), producers and wholesalers are invoiced according to their trades across the Day-Ahead, Intra-Day and Regulating Power markets. In the Balancing Power Market the cost of Regulating Power is also transferred to any player that deviated from the contracted production/consumption.

B. Market Model

After the introduction above, we now develop a market model based on historic data. The model consists of a series of optimization problems, which are solved one by one, each time using the latest and most updated information. Since the model is based on historic data there is no feedback in the formation of prices, meaning that prices is not generated dynamically. Consequently, the model is only valid if we assume that the amount of flexibility bid into the system is small enough not to affect the formation of the price cross significantly. On the other hand, since calculations are based only on historic data, results are not blurred by assumptions or estimated correlations.

We denote by $(\cdot)^*_{\{t_1-t_2\}}$ a list of elements associated with each hour of the interval from t_1 to t_2, e.g. $€^*_{\{01:00-04:00\}} = [€(01:00-02:00), €(02:00-03:00), €(03:00-04:00)]$. Also $(\cdot)^*_{\{12:00-12:00\}}$ denotes values associated with a 24 hour period from 12:00 noon till 12:00 noon of the following day.

The market model has four main stages as depicted in Table I and the trading algorithm is also summarized in **Algorithm 1**.

The first stage is the Day-Ahead Market, which the Virtual Power Plant can bid into based on predictions of market prices. Day-Ahead prices are denoted $€_{\text{Day-Ahead},\{12:00\text{-}12:00\}}$, so if the Virtual Power Plant wants to maximize its profit it should bid according to the solution of

$$\min_{P(k)} \sum_{k=\{12:00-12:00\}} €_{\text{Predictions of Day-Ahead}}(k) P(k) \tag{1}$$

$$s.t.$$

$$(A.1)-(A.4), (B.1)-(B.5) \text{ and } (C.1)-(C.6), \tag{2}$$

where P_{Plan} is zero for all units and all time slots.

Based on the trading in the Day-Ahead Market a 24-hour base plan denoted $\boldsymbol{P}^*_{Plan,\{12:00-12:00\}}$ is generated.

In the next stage of the model, the Intra-Day market is opened. If there is activity on the Intra-Day market, the Virtual Power Plant can do additional trading here to further increase its profit. Often, however, there is not activity on the Intra-Day market in all hours of the day (See Figure 2). In an hour where there is no activity, some estimate must be used by the Virtual Power Plant to make decisions and do trading. In an hour where there is no activity on the Intra-Day market it is assumed that the Virtual Power Plant uses Day-Ahead prices as best estimation of regulating prices during that hour. These prices are denoted $€_{\text{Intra-Day with Day-Ahead Padding}}$. The Virtual Power Plant therefore bids into the Intra-Day Market according to the solution of

$$\min_{P(k)} \sum_{k=\{12:00-12:00\}} €_{\text{Intra-Day with Day-Ahead Padding}}(k) P(k) \tag{3}$$

$$s.t.$$

$$(A.1)-(A.4), (B.1)-(B.5) \text{ and } (C.1)-(C.6), \tag{4}$$

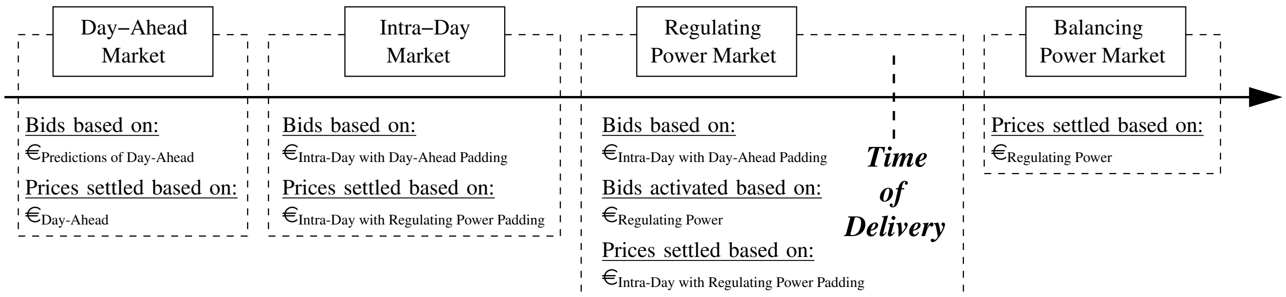

TABLE I: Market Model.

where P_{Plan} is now set according to the Day-Ahead trading.

At this time, however, the Day-Ahead Market is closed so the Virtual Power Plant cannot actually trade at spot price. Instead it must go into imbalances for which it obviously has to pay $€_{\text{Regulating Power}}$. Bidding on the Intra-Day market is therefore done based on $€_{\text{Intra-Day with Day-Ahead Padding}}$, but costs/profits are settled based on $€_{\text{Intra-Day with Regulating Power Padding}}$.

In the final stage of the market model, the Virtual Power Plant must make up- and down-regulation bids into the Regulating Power Market. In order to determine the appropriate bidding price for e.g. time slot 12:00 to 13:00 the Virtual Power Plant must solve

$$\min_{P(k)} \sum_{k=\{13:00-12:00\}} €_{\text{Intra-Day with Day-Ahead Padding}}(k)P(k) \tag{5}$$

$$s.t.$$

$$(A.1)-(A.4), (B.1)-(B.5) \text{ and } (C.1)-(C.6) \tag{6}$$

$$P_{Bucket}(12:00-13:00) = P_{limit,\,Bucket} \tag{7}$$

$$P_{Battery}(12:00-13:00) = P_{limit,\,Battery} \tag{8}$$

$$P_{Bakery}(12:00-13:00) = P_{limit,\,Bakery} \tag{9}$$

where P_{Plan} is now the base load plan after both Day-Ahead and Intra-Day trading. Problem (5) to (9) must be solved for two values of P_{limit} in order to find both the up- and down-regulation bid price. The problem must therefore first be solved for P_{limit} equal to the maximum up-regulation adjustment that the unit can make to its consumption within the restrictions of its constraints, dynamics and base load plan between 12:00 and 13:00. Next the problem is solved for the maximum down-regulation adjustment. For each unit in the portfolio the Virtual Power Plant will then place an up- and a down-regulation bid of

$$\sum_{k=\{13:00-12:00\}} -€_{\text{Intra-Day with Day-Ahead Padding}}(k)\frac{P_{(5)-(9)}(k)}{P_{limit}} \tag{10}$$

where $P_{(5)-(9)}$ is the solution of (5)-(9) for each of the two values of P_{limit}.

At each remaining hour of the day the Virtual Power Plant should repeat this approach and adjust its base load plan according to $P_{(5)-(9)}$ whenever a bid is activated. Notice that up-regulation bids should be as low as possible to get activated and down-regulation bids should be as high as possible to get activated.

Finally, since Intra-Day trading and Regulating Power trading are settled at $€_{\text{Intra-Day with Regulating Power Padding}}$ we do not need an independent stage for the balancing market, since the appropriate imbalances have already been paid/compensated at the price of Regulating Power.

Algorithm 1: Market Trading

from January 1^{st} **to** December 31^{st}
  Generate $€_{\text{Predictions of Day-Ahead}}$ and solve (1) to (2).
  Purchase $\boldsymbol{P}^*_{Plan}$ in the Day-Ahead Market according to solution of (1) to (2).
  Retrieve $€_{\text{Day-Ahead}}$ and $€_{\text{Intra-Day}}$ and generate $€_{\text{Intra-Day with Day-Ahead Padding}}$.
  Solve (3) to (4) and update $\boldsymbol{P}^*_{Plan}$ according to purchase/sales in the Intra-Day market.
  from 12:00 **to** 11:00
    Solve (5) to (9) and bid into the Regulating Power
    Market according to (10).
    If bid activated **then** update $\boldsymbol{P}^*_{Plan}$.
  end from
end from

V. Analysis and Results

A. Market Data

The present analysis focuses on the Danish Power Market, so Day-Ahead prices, average Intra-Day prices and Regulating Power prices for DK1 (Western Denmark) and DK2 (Eastern Denmark) in 2010, 2011 and 2012 form the basis of the main analysis. The data set can be downloaded from [8]. Figure 1 shows the average Day-Ahead price for each hour of the day. In later simulations we will consider a day and a night scenario (inspired by an electric vehicle in frequent use). These scenarios are also depicted in Figure 1.

B. Single-Day Illustration

Based on the taxonomy presented in Section III and the market model developed in Section IV we now illustrate how the Virtual Power Plant should bid each of the flexibility types in the taxonomy into the markets. The algorithm is based on the assumption that the Virtual Power Plant is continuously trying to maximize its profit based on the best available knowledge. Prices from February 3^{rd}, 2012 in DK1 from 08:00 to 17:00 are randomly chosen as illustrating (see Figure 2). We consider a portfolio consisting of one unit of each type in the taxonomy, with parameters values $\alpha = 0.9, \underline{P} = -1MW, \overline{P} = 1MW, \underline{E} = 0MWh, \overline{E} = 3MWh$ for all units, which corresponds to $T_{run} = 3\ hours$ for the *Bakery*.

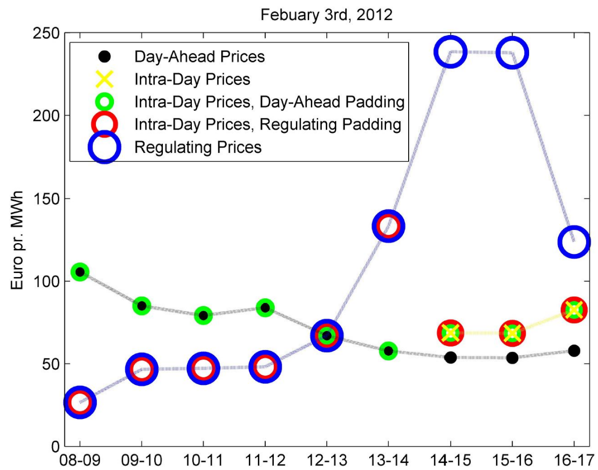

Fig. 2: Electricity Prices in DK1 on February 3^{rd}, 2012.

The first prices that are settled are the Day-Ahead Prices. If we assume that the Virtual Power Plant is capable of predicting these prices exactly (the implications of this assumption will be investigated further in Section V-E), then the Day-Ahead base load plan for each of the units will be as in Figure 3. Here it can be seen that the *Battery* and *Bakery* have paid 165 € to satisfy their base load requirements and that the *Bucket* has not yet made any profit.

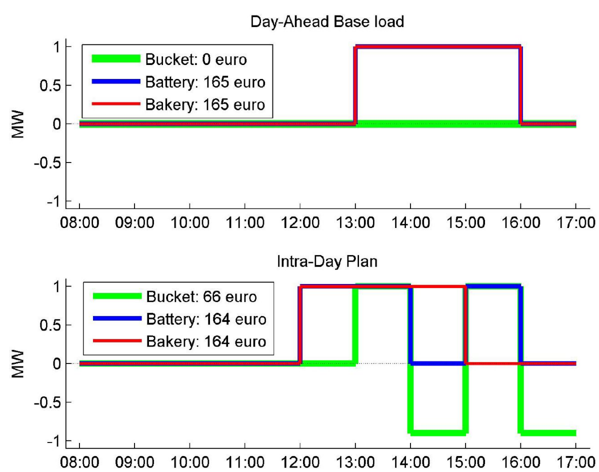

Fig. 3: Base load plan based on perfect prediction of Day-Ahead prices and Intra-Hour plan based on trading in open Day-Ahead time slots and the assumption that Regulating Power prices will equal Spot prices in DK1 on February 3^{rd}, 2012 for each type of flexibility in the taxonomy.

Next the Intra-Day Market is opened. It can be seen from Figure 2 that in DK1 there was only trading in the Intra-Day Market between 14:00 and 17:00 on February 3^{rd}, 2012 (yellow crosses). Since there is some activity on the Intra-Day market, the Virtual Power Plant can try to make an additional profit on new trading. The results of Intra-Day trading are shown Figure 3. The *Battery* and *Bakery* sell power between 14:00 and 15:00 and between 15:00 and 16:00 respectively. They are then scheduled to over consume between 12:00 and 13:00 instead, because spot prices were low here. This is a gamble, because the Day-Ahead Market is no longer open and there is no activity on the Intra-Day market in the time slot between 12:00 and 13:00, so the Virtual Power Plant-Operator cannot buy the power anywhere. However, since the regulating price between 12:00 and 13:00 ends up equal to the spot price the gamble earns 1 € per unit.

In the Intra-Day market the *Bucket* is scheduled to over-consume 1 MWh between 15:00 and 16:00 in order to be able to sell 0.9 MWh between 16:00 and 17:00. This earns the *Bucket* a profit of 6 €. Unfortunately it is scheduled to do the same between 13:00 and 15:00. If the regulating power price at time 13:00-14:00 had equaled the spot price, then this would have earned the *Bucket* a profit of 4 €. As it turns out, however, the regulating price at 13:00-14:00 is 133 €, so the trade would actually costs the *Bucket* 72 €, if it had done no further trading on the considered day. Later in the day, however, the *Bucket* bids into the Regulating Power Market during that critical hour and therefore the trading will not be as costly to the *Bucket* as it first looked. This is exactly the advantage of trading in several markets.

Next the Virtual Power Plant must bid into the Regulating Power Market for the time slot 08:00 to 09:00. Since no power have been purchased for any units to consume during this time slot, the Virtual Power Plant cannot make any up-regulation bids (recall that up and down regulation is defined based on the production conventions, so up-regulation corresponds to a decrease in consumption). It can, however, make three down-regulation bids:

- The *Bucket* is bid at 77 € for 1 MW down-regulation. If the *Bucket* gets activated for down regulation (increase in consumption), it will be able to under-consume 0.9 MWh between 09:00 and 10:00. Given the current expected prices, the Virtual Power Plant assumes that this will save 77 €, so if up-regulation power can be purchased for less than 77 € between 08:00 to 09:00, a profit will be made.
- The *Battery* is bid at 68 € for 1 MW down-regulation. If the *Battery* gets activated for down-regulation between 08:00 to 09:00, then it will be able to under consume between 15:00 and 16:00, which is expected to save 68 €. So again, if up-regulation power can be purchased for the *Battery* at less than 68 €, then a profit will be made.
- The *Bakery* is bid at 29 € for 1 MW down-regulation. If the *Bakery* should start early, then it must sell all power between 12:00 and 15:00 and over-consume between 09:00 and 11:00. This change to the consumption schedule is expected to cost 29 €, so down-regulation

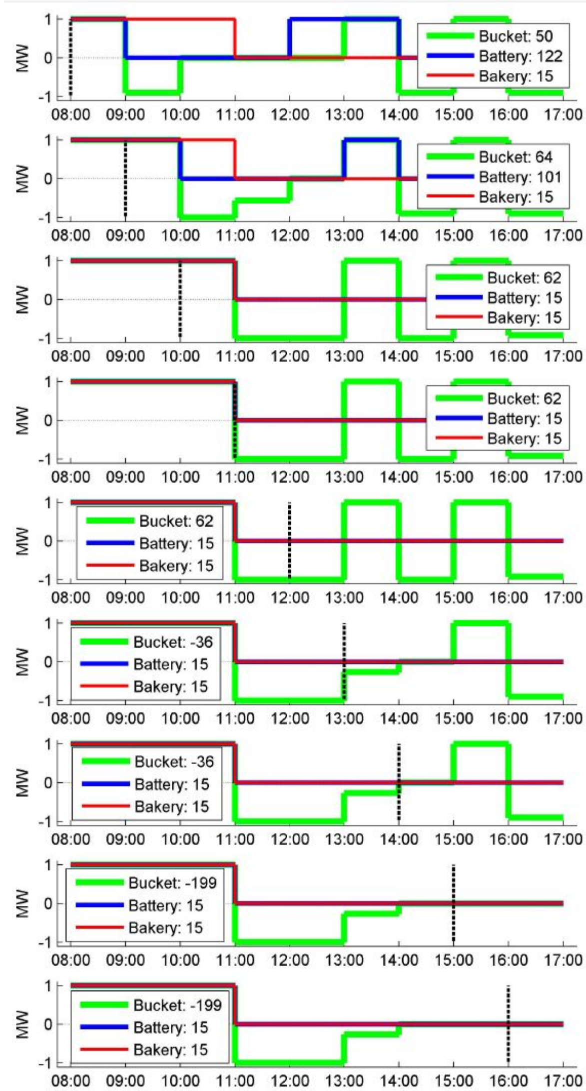

Fig. 4: Consumption plan for each hour of February 3^{rd}, 2012, DK1 as trading in the Regulating Power Market progresses.

power at time 08:00-09:00 must be cheaper than 29 € in order for the *Bakery* to be interested in moving its consumption.

The down-regulation power price between 08:00 and 09:00 comes out at 26 €, so all bids are activated. Now the cost associated with the *Bucket* drops down to 50 € and the *Battery* has paid 122 € for its total power consumption, if it does no more trading today. The *Bakery* is very lucky, as it turns out that by starting earlier it will end up paying just 13 € for its total power consumption that day.

Table II states all the bids that the Virtual Power Plant would make on February 3^{rd}, 2012, if it continuously attempts to maximize its profit based on the best available knowledge. The adjusted consumption plans for the units as the day progresses are depicted in Figure 4. By the end of the day, the *Bucket* has earned 199 € while both the *Battery* and the *Bakery* have paid only 15 € for their total power consumption compared to their initial cost of 165 €. Thus

		Bucket		*Battery*		*Bakery*	
	Activated	**Down**	**Up**	**Down**	**Up**	**Down**	**Up**
08-09	Down	77	N/A	68	N/A	29	N/A
09-10	Down	67	N/A	67	N/A	N/A	N/A
10-11	Down	56	N/A	58	N/A	N/A	N/A
11-12	Down	44	N/A	N/A	N/A	N/A	N/A
12-13		52	N/A	N/A	N/A	N/A	N/A
13-14	Up	N/A	56	N/A	N/A	N/A	N/A
14-15	Up	62	N/A	N/A	N/A	N/A	N/A
15-16	Up	N/A	74	N/A	N/A	N/A	N/A
16-17	Up	0	N/A	N/A	N/A	N/A	N/A

TABLE II: Bids made into the Regulating Power Market. Entries are underlined if bids are available/activated.

offering to be flexible on February 3^{rd}, 2012 could have turned out to be very good business.

Notice especially that when the *Bucket* did trading on the Intra-Day Market, its total cost went up, because it did not know that the Regulating Power prices would be unfavorable later. By bidding into the Regulating Power Market, however, the *Bucket* saves itself from actually consuming between 13:00 and 14:00 where regulating prices are high and therefore recovers its loss.

C. Full Year Simulations

In the next simulation example, we consider one unit of each type in the taxonomy. Parameter values are again $\alpha = 0.9, \underline{P} = -1MW, \overline{P} = 1MW, \underline{E} = 0MWh, \overline{E} = 3MWh$. The *Battery* and the *Bakery* are limited to trading between 08:00 to 17:00 (Day) and 20:00 to 05:00 (Night), whereas the *Bucket* is allowed to trade round-the-clock.

The results of trading the portfolio according to the algorithm given in Section IV-B are given in Table III. It can be seen that profits/savings do indeed increase as more markets are penetrated. The benefit is largest for the *Bucket* and relatively limited for the *Bakery*. However, it is on average always beneficial to offer flexibility to the system. In some scenarios, such as the *Battery*, 2012, DK1, Day-scenario costs actually increase from Day-Ahead to Intra-Day market. This is because Day-Ahead is done with perfect prediction of prices, whereas Intra-Day trading is sometimes settled at Regulating Power price. After the *Battery* has participated in the Regulating Power Market there are still savings to be obtained by trading in several markets, however.

D. Virtual Power Plant Profit

Since the *Battery* and *Bakery* both have base load requirements to satisfy, it is possible for the Virtual Power Plant to achieve savings, but not an actual profit as is the case for the *Bucket*. A sensible agreement between the unit owner and the Virtual Power Plant could thus be that the unit owner should cover the Day-Ahead base load cost. Any additional profit gained in the Intra-Day and Regulating Power Markets should then be shared evenly between the unit owner and the Virtual Power Plant. With this setup, Table IV shows which flexibility type is most profitable for the Virtual Power Plant. Is is found that the *Bucket* is far more profitable than the *Battery*, which again generates more than twice as much profit as the *Bakery*.

Year	Area	*Bucket* Day-Ahead	*Bucket* Intra-Day	*Bucket* Regulating	Scenario	*Battery* Day-Ahead	*Battery* Intra-Day	*Battery* Regulating	*Bakery* Day-Ahead	*Bakery* Intra-Day	*Bakery* Regulating
2010	DK1	-6.823	-7.544	-20.792	Day,	50.984	51.281	48.877	51.307	51.432	50.793
					Night	38.154	38.047	35.083	38.233	38.095	35.605
	DK2	-21.409	-21.998	-28.937	Day	62.139	62.367	58.965	62.453	62.872	61.609
					Night	44.067	43.927	42.775	44.170	43.935	43.329
2011	DK1	-8.655	-9.969	-27.880	Day,	53.178	52.488	47.403	53.404	52.440	50.425
					Night	37.376	36.873	34.103	37.463	37.041	35.686
	DK2	-10.156	-11.386	-30.047	Day	55.192	55.001	47.221	55.460	55.786	51.084
					Night	37.539	37.449	35.430	37.621	37.403	36.638
2012	DK1	-11.707	-14.639	-35.981	Day,	39.843	39.345	31.409	40.059	40.058	37.176
					Night	26.638	26.296	24.587	26.682	26.531	25.895
	DK2	-14.541	-17.383	-39.282	Day	41.171	40.186	31.188	41.420	40.757	37.393
					Night	26.724	26.317	24.716	26.752	26.449	26.048
Total		-73.292	-82.919	-182.919		513.005	509.576	461.757	515.025	512.799	491.681

TABLE III: Profit/savings in € obtained by trading the portfolio according the the algorithm given in Section IV-B.

Year	Area	*Bucket*	Scenario	*Battery*	*Bakery*
2010	DK1	-10.396	Day	-1.053	-257
			Night	-1.535	-1.314
	DK2	-14.469	Day	-1.587	-422
			Night	-646	-420
2011	DK1	-13.940	Day	-2.888	-1.489
			Night	-1.636	-889
	DK2	-15.023	Day	-3.986	-2.188
			Night	-1.054	-492
2012	DK1	-17.991	Day	-4.217	-1.442
			Night	-1.025	-393
	DK2	-19.641	Day	-4.991	-2.014
			Night	-1.004	-352
Total		-91.459		-25.624	-11.672
Percentage		100%		14%	6%

TABLE IV: Virtual Power Plant profit in €, when the Day-Ahead base load costs of the *Battery* and *Bakery* are covered by the unit owner.

	Battery	*Bakery*
Perfect Prediction	461.757	491.681
With error	464.064	497.147
Difference	2.308	5.466
Percentage	0.5%	1.1%

TABLE V: Increase in costs when the *Battery* and *Bakery* are not allowed to purchase power during the cheapest hour on the Day-Ahead Market.

E. Sensitivity Analysis

Since we have assumed perfect prediction of Day-Ahead prices and since the *Battery* and *Bakery* savings are relatively limited it is relevant to investigate how sensitive the savings are to prediction errors on Day-Ahead prices. To do this all calculations are repeated, but now units are not allowed to purchase power during the cheapest hour in the Day Ahead market. The results are given in Table V and it is found, that the savings are surprisingly unaffected by the prediction error: Just 0.5% and 1.1% increases in costs for the *Battery* and the *Bakery*, respectively.

It has also been investigated how the *Bucket* profit is affected by the size of the energy drain. Again parameter values are $\alpha = 0.9, \underline{P} = -1MW, \overline{P} = 1MW, \underline{E} = 0MWh, \overline{E} = 3MWh$ and the results are depicted in Figure 5. As expected the profit is heavily influenced by the size of the energy drain, but even with a drain of 20% per hour the total profit is still more than 50.000 €.

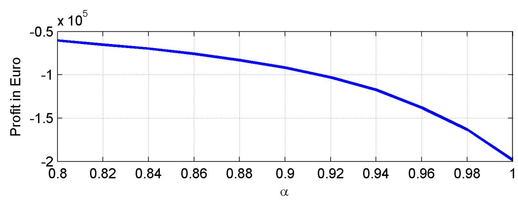

Fig. 5: Profit of the *Bucket* summarized over DK1 and DK2 and 2010, 2011 and 2012 as a function of energy drain.

VI. Conclusion

In this paper we have developed a three stage electric power market model, which include Day-Ahead (Spot), Intra-Day and Regulating Power Markets. By use of this model we have confirmed the hypothesis that a Virtual Power Plant operator can increase its profit by trading in several markets from day to day. We have also found that the profit is highly sensitive to the type of flexibility considered, but surprisingly robust to errors in Day-Ahead price predictions.

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