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An analytical solution for stability-performance dilemma of hydronic radiators

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ABSTRACT

Thermostatic radiator valves (TRV) have proved their significant contribution in energy savings for several years. However, at low heat demand conditions, an unstable oscillatory behavior is usually observed and well known for these devices. This closed-loop instability is due to the nonlinear dynamics of the radiator which result in a large time constant and a large gain for the radiator at small flow rates. In order to improve stability of radiators under the low demand circumstance, one way is to replace the fixed-parameter controller of TRV with an adaptive controller. This paper presents a gain scheduling controller based on a proposed linear parameter varying model of radiator dynamics. The model is parameterized based on the operating flow rate, room temperature and radiator specifications. Parameters of the model are derived based on the proposed analytic solution that describes dissipated heat by a radiator to ambient air. It is shown via simulations that the designed controller based on the proposed linear parameter varying (LPV) model performs excellent and remains stable in the whole operating conditions.

1. Introduction

Efficient control of heating, ventilation and air conditioning (HVAC)[1] systems has a great influence on the thermal comfort of residents. The other important objective is energy savings, mainly because of the growth of energy consumption, costs and also correlated environmental impacts. A thermostatic radiator valve (TRV)[2] mounted on a hydronic radiator is an excellent example of such energy efficient controller. It cuts down the heating energy consumption up to 20% while improving comfort [1].

Hydronic radiators controlled by TRVs provide good comfort under normal operating conditions. Thermal analysis of the experimental results of a renovated villa in Denmark, built before 1950, has demonstrated that energy savings near 50% were achieved by mounting TRVs on all radiators and fortifying thermal envelopee insulation [2]. Also, various studies are conducted worldwide to conclude that radiant heating consumes less energy compared to that used by a forced air heating system [3–5]. However, well designed room temperature controller is the most important factor and a prerequisite for an energy efficient operation of building system services. Based on the German standard [6], radiators efficiency can be improved from 0.8 to 0.99 by employing adaptive PI controllers instead of centrally controlled supply temperature (see Table 6 in [6]).

However, less studies have been conducted around a very well-know inefficient radiator operation condition. TRVs are usually tuned with a high controller gain in order to maintain the room temperature set point in a high load situation.The inefficiency appears in the seasons with low heat demand especially when the water pump or radiator are over dimensioned [7]. In this situation, due to a small flow rate, loop gain increases; and as a result oscillations in room temperature may occur. These oscillations decrease the life time of the actuators. This problem is addressed in [8] for a central heating system with gas-expansion based TRVs. It is proposed to control the differential pressure across the TRV to keep it in a suitable operating area by estimating the valve position.

In this study, however, we dealt this problem as a dilemma between stability and performance. It has been investigated via simulations that a fixed proportional integral (PI)[3] controller would also fail to guarantee both performance and stability for a radiator in the whole operating conditions. We investigated In this study, pressure drop across the radiator valve is maintained constant unlike what is taken as the control strategy in [8]. Instead, we assumed that estimation of flow rate and thus its control is feasible by accurate adjustment of the valve opening. Automatic adjustment of the valve opening in the case of a gas-based TRV is not possible however with an electric TRV, driven by a batterized stepper motor

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[1] HVAC stands for heating, ventilation and air conditioning system.
[2] TRV represents for thermostatic radiator valve.
[3] PI stands for proportional integral.

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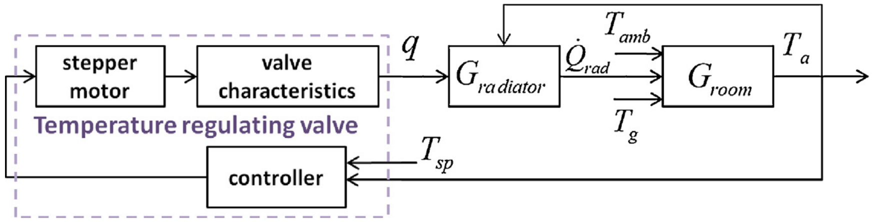

Fig. 1. Closed loop control system of room and radiator

it is an option. In this case, the valve opening is regulated by a stepper motor which allows for precise adjustments. This paper presents a gain scheduling controller for flow adjustment based on a proposed LPV[4] model of radiator dynamics. The model is parameterized based on the operating flow rate, room temperature and radiator specifications. Parameters of the model are derived using the proposed analytic solution that describes dissipated heat by the radiator to ambient air.

It is, also, worth mentioning that the same problem was investigated via simulation based studies in [9] and also via approximation analysis in [10].

In order to validate the controller performance, we utilized simulation models of the HVAC components in Matlab/Simulink. Two approaches for HVAC systems modeling are the forward and the data-driven methods [11]. The first one is based on known physical characteristics and energy balance equations of the air, structural mass and other components of the system, addressed widely in the literature [12–14]. The alternative modeling approach is to use building measurement data with inferential and statistical methods for system identification which is addressed in [15–17]. The latter method requires a significant amount of data and may not always reflect the physical behavior of the system [18].

In this paper, we adopted heat balance equations of the room dynamics according to the analogous electric circuit, described formerly by the authors in [19]. Radiator dynamics are formulated in two ways. Once, it has been treated as a distributed system in order to analyze the radiator transferred heat. Th e same precise model is used for simulation of radiator dynamics. Secondly, it is approximated by a lumped system for the purpose of controller design.

The remainder of the paper is organized as follows: Section 2 defines the problem. In Section 3, the radiator dissipated heat is derived analytically. Based on the result, control oriented models are developed in Section 4. Utilizing the models, the control structure based on flow adaptation is proposed in the same section. A simulation-based test is conducted in Section 5, exploiting the ODEs of the room and radiator. Discussion and conclusions are given finally in Section 6.

2. Stability-performance dilemma

The case study is composed of a room, a radiator with thermostatic valve and a room temperature sensor. Ambient temperature is the only disturbance which excites the system. It is assumed that heat transfer to the ground is negligible having thick layers of insulation beneath the concrete floor. A block diagram of the system is shown in Fig. 1. All symbols, subscripts and parameter values are listed in Tables 1 and 2. It is, though, worth stating that the chosen values of all parameters are in accordance with the typical experimental and standard values. As mentioned before, the case study is adopted to the one previously studied in [9].

[4] LPV represents linear parameter varying.

Table 1
Symbols and subscripts

Nomenclature	
A	surface area (m^2)
C	thermal capacitance (J/°C)
C_r	thermal capacitance of water and the radiator material (J/°C)
c_w	specific heat capacitance of water (J/kg°C)
d	depth of radiator (m)
h	height of radiator (m)
K	DC gain
K_c	controller gain
K_r	equivalent heat transfer coefficient of radiator (J/s°C)
L	time delay (s)
N	total number of radiator distributed elements
Q	dissipated heat by radiator (W)
q	flow rate of hot water through radiator (kg/s)
T	temperature (°C)
T_i	integration time (s)
T_{in}	inlet water temperature (°C)
T_n	temperature of radiator nth element (°C)
T_{out}	outlet water temperature (°C)
T_{out}^i	steady state T_{out} corresponding to flow rate q_i
U	thermal transmittance (W/m^2 °C)
V_w	water capacity of radiator (m^3)
ρ	density (kg/m^3)
τ	time constant (s)
δ	valve opening
ℓ	length of radiator (m)
Subscripts	
a	room air
amb	ambient temperature (outdoor)
e	envelope
f	floor
hd	high demand
n_1	radiator exponent
rad	radiator
ref	reference temperature of room

Table 2
System parameters

Room parameters		Radiator parameters	
A_e	56 m^2	A_r	1.5 m^2
A_f	20 m^2	C_r	3.1x10^4 J/kg °C
C_a	5.93×10^4 J/°C	c_w	4186.8 J/kg °C
C_e	4×10^4 J/°C	N	45
C_f	1.1×10^4 J/°C	n_1	1.3
U_e	1.2 W/m^2 °C 4×10^4 J/°C	q_{max}	0.015 kg/s
U_f	1.1 W/m^2 °C 4×10^4 J/°C	T_s	70 °C
		V_w	5 L

The dilemma between stability and performance arises when the TRV is controlled by a fixed linear controller. Designing a TRV controller for high demand seasonal condition, usually leads to instability in low demand weather condition. A large loop gain and time constant are the main reasons of this phenomenon. In contrast, selecting a smaller controller gain to handle the instability situation, will result in a poor radiator reaction while the heat demand is high.

An example of overestimated required flow often happens in apartment buildings. Heat demand of the blocks can be very

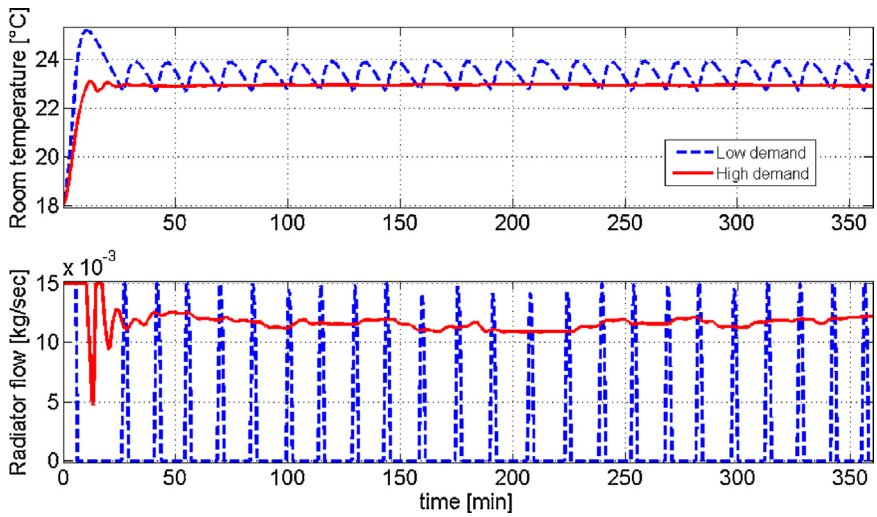

Fig. 2. Undamped oscillations in room temperature and radiator low which occur in low demand situation while the controller is designed for a high demand condition.

different that one block with the largest energy demand takes up as double of the average energy usage by all blocks. In this situation, mass flow is usually adjusted much higher than what is required by majority of apartments which results in over dimensioning and thus instability problem.

Figs. 2 and 3 show the results of a simulation where oscillations and low performance occur. In the shown simulation results, the forward water temperature is at 50 °C. The proportional integral (PI) controller of TRV is tuned based on Ziegler-Nichols step response method [20].

A remedy to this dilemma is choosing an adaptive controller instead of the current fixed PI controller.

3. System modelling

3.1. Heat balance equations

Radiator is modeled as a lumped system with N elements in series. The nth section temperature is given by, [21]:

$$\frac{C_r}{N}\dot{T}_n = c_w q(T_{n-1} - T_n) - \frac{K_r}{N}(T_n - T_a) \tag{1}$$

in which C_r is the heat capacity of water and radiator material, T_n is the temperature of the radiator's nth section area and $n = 1, 2, \ldots, N$. The temperature of the radiator ending points are inlet temperature: $T_0 = T_{in}$, and return temperature: $T_N = T_{out}$. In this formulation, we assumed that temperature of the radiator surface is the same as influent water through radiator. We have also assumed that heat is transferred between two sections only by mass transport, implying that convective heat transfer is neglected. K_r represents the radiator equivalent heat transfer coefficient which is defined based on one exponent formula, [22] in the following:

$$K_r = \frac{\Phi_0}{\Delta T_{m,0}{}^{n_1}}(T_n - T_a)^{n_1 - 1} \tag{2}$$

in which Φ_0 is the radiator nominal power in nominal condition which is $T_{in,0} = 75\,°\mathrm{C}$, $T_{out,0} = 65\,°\mathrm{C}$ and $T_a = 20\,°\mathrm{C}$. $\Delta T_{m,0}$ expresses the mean temperature difference which is defined as follows:

$$\Delta T_m = \frac{T_{in} - T_{out}}{2} - T_a \tag{3}$$

in nominal condition. The exponent n_1 is usually around 1.3, [22]. In such a case, we can approximate the non-fixed, nonlinear term in K_r with a constant between 3 and 3.4 for a wide enough

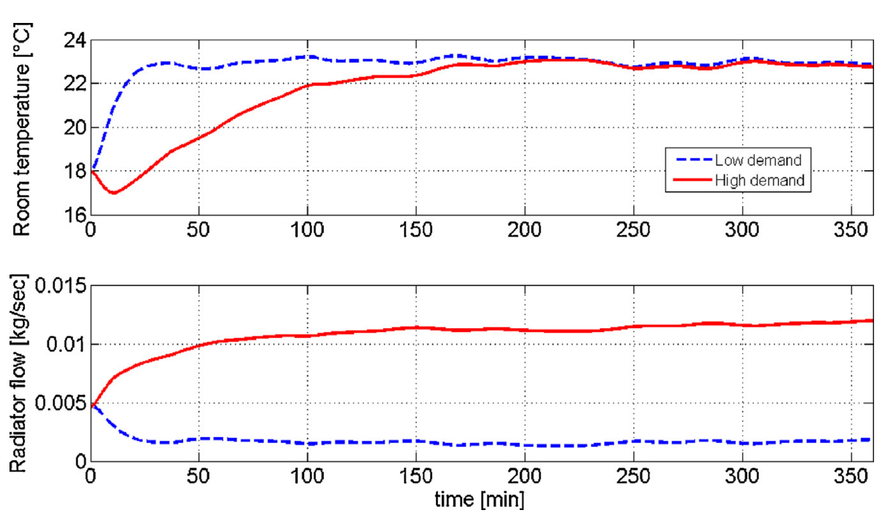

Fig. 3. Poor performance in the cold weather condition, applying the controller designed for the low demand situation

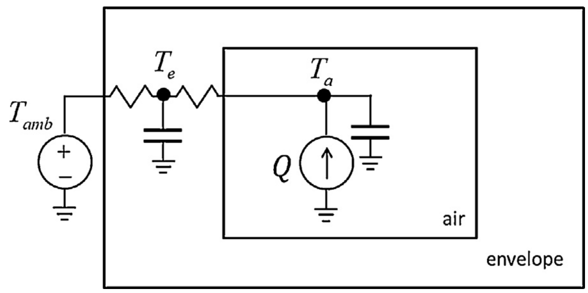

Fig. 4. Analogous electrical circuit to the room thermal model

range of temperature values. Picking 3.2 as the approximation value would result in:

$$K_r = 3.2 \times \frac{\Phi_0}{\Delta T_{m,0}{}^{1.3}} \tag{4}$$

The heat dissipated to the room by the radiator can be described as:

$$Q = \sum_{n=1}^{N} K_r(T_n - T_a) \tag{5}$$

Heat balance equations of the room is governed by the following lumped model [14], that is analogous to the electric circuit shown in Fig. 4:

$$C_e\dot{T}_e = U_eA_e(T_{amb} - T_e) + U_eA_e(T_a - T_e) \tag{6}$$

$$C_a\dot{T}_a = U_eA_e(T_e - T_a) + Q$$

in which T_e represents the building's envelope temperature and T_a represents the room air temperature. Q is the heat dissipated to the room by the radiator. Each of the envelope and room air subsystems are considered as a single lump with uniform temperature distribution.

Assuming a constant pressure drop across the valve, the thermostatic valve is modeled with a static polynomial function mapping the valve opening δ to the flow rate q [l/h]:

$$q = -3.4 \times 10^{-4}\delta^2 + 0.75\delta \tag{7}$$

The above equation which is used in simulations, is derived based on experimental data of Danfoss valve body type RA-N 15 [23].

The above presented radiator model is highly nonlinear and not suitable for design of a controller; thus a simplified control oriented LPV model is developed in the next section.

3.2. Control oriented models

For our purposes, the relationship between room air temperature and radiator output heat can be well approximated by a 1st order transfer function as follows:

$$\frac{T_a}{Q}(s) = \frac{K_a}{1 + \tau_a s} \tag{8}$$

Parameters τ_a and K_a can be identified simply via a step response test.

Step response simulations and experiments confirm a first order transfer function between the radiator output heat and input flow rate at a specific operating point as comes in the following:

$$\frac{Q}{q}(s) = \frac{K_{rad}}{1 + \tau_{rad}s} \tag{9}$$

Parameters of the above model are formulated in the next section based on the closed-form solution of the radiator dissipated heat, $Q(t, q, T_a)$.

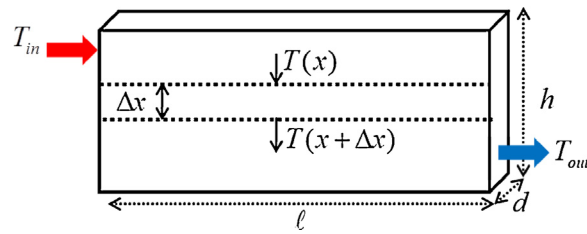

Fig. 5. A radiator section area with the heat transfer equation governed by (9). Entering low to the section is at the temperature $T(x)$ and the leaving low is at $T(x + \Delta x)$.

3.3. Radiator dynamical analysis

In this paper, unlike [9] and [10], we found the precise closed-form map from the operating point to the radiator dissipated heat i.e. $Q(t, q, T_a)$. q and T_a are respectively the hot water flow rate through the radiator and the room temperature. We, previously, derived this map via a simulation study in the form of two profile curves in [9].

To develop the map, $Q(t, q, T_a)$, a step flow is applied to the radiator, i.e. changing the flow rate from q_0 to q_1, at a constant differential pressure across the valve. Propagating with the speed of sound, the flow shift is seen in a fraction of a second all along the radiator. Hence, flow is regarded as a static parameter for $t > 0$, rather than a temperature distribution along the radiator.

Consider a small radiator section Δx with depth d and height h as shown in Fig. 5. The temperature of incoming flow to this section is $T(x)$, while the outgoing flow is at $T(x + \Delta x)$ °C. The temperature is considered to be the same as $T(x)$ throughout a single partition.

The corresponding heat balance equation of this section is given as follows:

$$qc_w(T(x) - T(x + \Delta x)) + K_r\frac{\Delta x}{h}(T_a - T(x)) = C_r\frac{\Delta x}{h}\frac{\partial T}{\partial t} \tag{10}$$

in which flow rate is q_0 at $t = 0$ and q_1 for $t > 0$. C_r is the heat capacity of water and the radiator material defined as: $C_r = c_w\rho_wV_w$. Dividing both sides of (10) by Δx and letting $\Delta x \to 0$, we have:

$$-qc_w\frac{\partial T(x,t)}{\partial x} + \frac{K_r}{h}(T_a - T(x,t)) = \frac{C_r}{h}\frac{\partial T(x,t)}{\partial t} \tag{11}$$

with boundary conditions:

$$T(0,t) = T_{in} \tag{12a}$$

$$T(h,0^-) = T_{out,0} \tag{12b}$$

$$T(h,\infty) = T_{out,1} \tag{12c}$$

in which T_{in} is the constant temperature of supply water, $T_{out,0}$ and $T_{out,1}$ are return water temperatures corresponding to q_0 and q_1 respectively. The first solution candidate would be a separable solution like $T(x, t) = T(t) \times X(x)$. Substituting it into (11), gives:

$$T(0,t) = c_1e^{k_1t} + c_2 \tag{13}$$

which implies a contradiction according to 12a.

Before proceeding to solve the full PDE (11), we need to find the two boundary conditions $T_{out,0}$ and $T_{out,1}$. For this purpose, take the steady state form of (11):

$$-qc_w\frac{dT}{dx} + \frac{K_r}{h}(T_a - T(x)) = 0 \tag{14}$$

which can be written as:

$$\frac{dT}{dx} + \frac{\beta}{\gamma}T(x) = T_a \tag{15}$$

with constants $\beta = \frac{K_r}{C_r}$ and $\gamma = \frac{qc_wh}{C_r}$.

Therefore, the steady state temperature, $T(x, t)|_{t\to\infty}$ will be achieved as:

$$T(x) = c_1 e^{-\frac{\beta}{\gamma}x} + c_0 \tag{16}$$

at a specific flow rate q. Substituting the above equation in (15) gives $c_0 = T_a$. Knowing $T(0) = T_{in}$, c_1 is also found. Finally $T(x)$ looks like:

$$T(x) = (T_{in} - T_a)e^{-\frac{\beta}{\gamma}x} + T_a \tag{17}$$

Therefor the two boundary conditions are:

$$T_{out,0} = (T_{in} - T_a)e^{-\frac{\beta}{\gamma_0}x} + T_a \tag{18a}$$

$$T_{out,1} = (T_{in} - T_a)e^{-\frac{\beta}{\gamma_1}x} + T_a \tag{18b}$$

corresponding to the flow rates q_0 and q_1.

Next, we solve (11) for $T(x, t)$ in frequency domain. Taking Laplace transform of this equation will give:

$$-qc_w \frac{\partial \tilde{T}(x,s)}{\partial x} + \frac{K_r}{h}\left(\frac{T_a}{s} - \tilde{T}(x,s)\right) = \frac{C_r}{h}\left(s\tilde{T}(x,s) - T(x,0)\right) \tag{19}$$

which is simplified to:

$$\frac{\partial \tilde{T}(x,t)}{\partial x} + \frac{s+\beta}{\gamma}\tilde{T}(x,s) = \frac{\beta}{\gamma}\frac{T_a}{s} + \frac{1}{\gamma}T(x,0) \tag{20a}$$

$$T(x,0) = (T_{in} - T_a)e^{-\frac{\beta}{\gamma_0}x} + T_a \tag{20b}$$

$$Boundary\quad Condition : \tilde{T}(0,s) = \frac{T_{in}}{s} \tag{20c}$$

with $\beta = \frac{K_r}{C_r}$, $\gamma_0 = \frac{q_0 c_w h}{C_r}$ and $\gamma_1 = \frac{q_1 c_w h}{C_r}$. The initial condition, $T(x, 0)$ is obtained using (17). The solution to the above differential equation comes out of inspection as follows:

$$\tilde{T}(x,s) = c_1 e^{-\frac{\beta}{\gamma_0}x} + c_2 e^{-\frac{s+\beta}{\gamma_1}x} + c_0 \tag{21}$$

$$c_0 = \frac{T_a}{s} \tag{22}$$

$$c_1 = \frac{T_{in} - T_a}{s + \beta(1 - \frac{\gamma_1}{\gamma_0})}$$

$$c_2 = \frac{T_{in}}{s} - c_0 - c_1$$

The time response is obtained via taking inverse Laplace transform of the above frequency response. It is shown in the following:

$$T(x,t) = (T_{in} - T_a)e^{-\beta t - \frac{\beta}{\gamma_0}(x - \gamma_1 t)}\left(u(t) - u(t - \frac{x}{\gamma_1})\right) + (T_{in} - T_a)e^{-\frac{\beta}{\gamma_1}x}u(t - \frac{x}{\gamma_1}) \tag{23}$$

in which $u(t)$ is a *unit step function*.

We, however, are interested in the radiator output heat $Q(t)$ to find K_{rad} and τ_{rad} in (9). It is defined as:

$$Q(t) = \int_0^h \frac{K_r}{h}(T(x,t) - T_a)\,dx \tag{24}$$

Taking time derivative of the above equation gives:

$$\frac{dQ}{dt} = \int_0^h \frac{K_r}{h}\frac{\partial T(x,t)}{\partial t}\,dx \tag{25}$$

and rewriting the result using (11):

$$\begin{aligned}\frac{dQ}{dt} &= \int_0^h \beta\left(-qc_w\frac{\partial T}{\partial x} + \frac{K_r}{h}(T_a - T(x,t))\right)dx \\ &= \beta q c_w (T_{in} - T_{out}(t)) - \beta Q\end{aligned} \tag{26}$$

which turns into the following differential equation:

$$\frac{dQ}{dt} + \beta Q = \beta q c_w (T_{in} - T_{out}(t)) \tag{27a}$$

$$T_{out}(t) = T(h,t) \tag{27b}$$

in which $T(h, t)$ is obtained using (23). The solution to the above first order differential equation via inspection is:

$$\begin{aligned}Q(t) \;&= Q(0)e^{-\beta t} + c_w q(T_{in} - T_a)(1 - e^{-\beta t}) + \\ &+ \frac{c_w q(T_{in} - T_a)\gamma_0}{\gamma_1}e^{-\beta t}\left(e^{-\frac{\beta}{\gamma_0}h} - e^{-\frac{\beta}{\gamma_0}(h - \gamma_1 t)}\right) - \\ &- c_w q(T_{in} - T_a)e^{-\beta t}\left(\frac{\gamma_0}{\gamma_1}(1 - e^{-\frac{\beta}{\gamma_0}(h-\gamma_1 t)}) - (1 - e^{-\frac{\beta}{\gamma_1}(h - \gamma_1 t)})\right)u(t - \frac{h}{\gamma_1})\end{aligned} \tag{28}$$

In the next section, we utilize the derived formula to extract the radiator's gain and time constant for the approximation LPV model.

3.4. Radiator LPV model

Parameters K_{rad} and τ_{rad} of the radiator LPV model (9), are derived based on the best first order fit to the step response of the radiator dissipated heat (28). Using the tangent to $Q(t)$ at $t = 0^+$, we obtain the time constant. The slope of the tangent at $t = 0^+$ is made equal to the first derivative of

$$Q_{final} + (Q_0 - Q_{final})e^{-\frac{t}{\tau_{rad}}} \tag{29}$$

i.e. the first order approximation of $Q(t)$. It gives:

$$\tau_{rad} = \frac{Q_{final} - Q_0}{q_1 c_w \beta (T_{in} - T_a)(\frac{\gamma_0}{\gamma_1} - 1)} \tag{30}$$

Steady state gain is also obtained as follows:

$$K_{rad} = \frac{Q_{final} - Q_0}{q_1}$$

in which Q_{final} and Q_0 are the dissipated heat by the radiator in steady state corresponding to flow rates q_1 and q_0 respectively.

These two parameters depend also on room temperature and supply water temperature. However, we have assumed a constant feed water temperature for the heating system. Therefore, variations of K_{rad} and τ_{rad} against a number of flow rates and room temperatures are shown in the following figure. Room temperature varies between 5 and 25°C and flow rate changes between the minimum and the maximum flow rates i.e. 0 and 360 $\frac{kg}{h}$.

It is apparent from Fig. 6 that radiator's time constant does not vary with room temperature alterations. However, the small signal gain decreases with an increase in the room temperature which seems rational. There is also a slight difference between the simulation and the analytic results with regard to the time constant. This is due to employing different methods in τ_{rad} calculations. In simulation, the time constant is taken as the time when 0.63 of the final value is met while in calculations, this is derived based on the tangent to $Q(t)$ and its first order approximation.

Fig. 7 shows how the LPV model (9) with the derived parameters matches the simulation model (1). In the next section, we will design a gain scheduling controller based on the developed radiator LPV model.

4. Gain scheduling controller design based on flow adaptation

In the previous section, we developed a linear parameter varying model of the radiator instead of the high-order nonlinear model

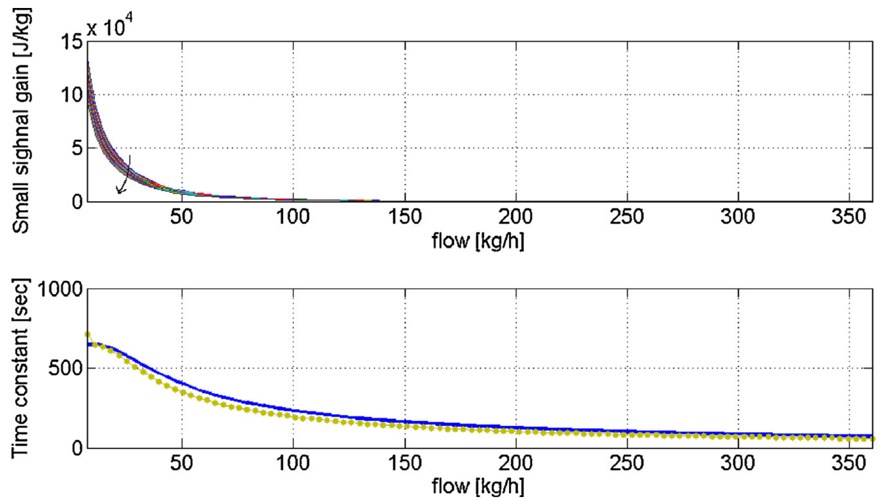

Fig. 6. K_{rad} and τ_{rad} deviations against low rate and room temperature variations. The direction of air temperature increase is shown via an arrow. The results are shown for both simulation (dotted) and analytic (solid) results. Comparing the result of analytic solution with simulation, the system gain is the same throughout all operating points, however there is marginal variations in the system time constant. This is due to employing the best 1st order fit of $Q(t)$.

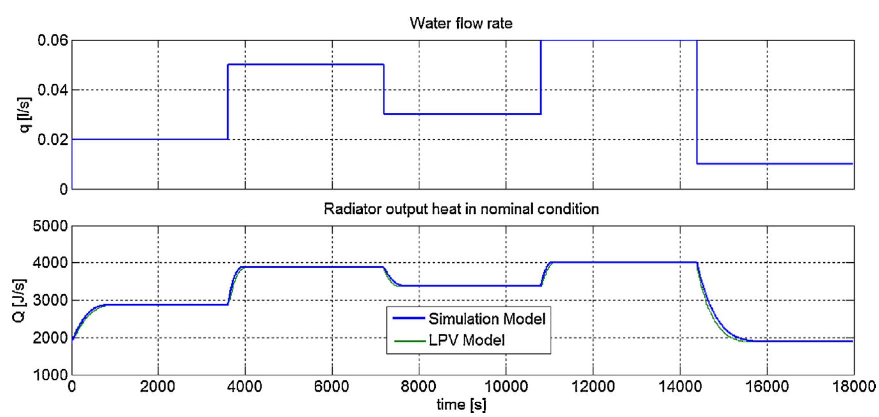

Fig. 7. Comparison of the developed LPV model and the precise simulation model. Flow rate is altered in an stepwise form. Dissipated heat from the radiator with the two models are simulated and compared.

(1). To control the system, among various possible control structures, a gain scheduling approach is selected which is a very useful technique for reducing the effects of parameter variations [24].

The term of flow adaptation, here, is chosen to further emphasize on the operating point-dependent controller. The main idea of designing an adaptive controller is to transform the system (9) to one which is independent of the operating point. The controller, subsequently, would be designed for the new transformed system which is a linear time invariant (LTI) system. The block diagram of this controller is shown in Fig. 8.

The function g is chosen in a way to cancel out the variable dynamics of the radiator and to place a pole instead in a desired position. The desired position corresponds to a high flow rate or a high demand condition. In this situation, the radiator has the fastest dynamic. Therefore, the simplest candidate for the linear transfer function g is a phase-lead structure, as follows:

$$g(K_{rad}, \tau_{rad}) = \frac{K_{hd}}{K_{rad}} \frac{\tau_{rad}s + 1}{\tau_{hd}s + 1} \tag{31}$$

in which K_{hd} and τ_{hd} correspond to the gain and time constant of radiator in the high demand situation when the flow rate is maximum. Consequently, the transformed system would behave always similar to the high demand situation. By choosing high demand as the desired situation, we give the closed loop system the possibility to place the dominant poles as far as possible from the imaginary axis, and as a result as fast as possible.

The controller of the transformed linear time invariant (LTI)[5] system is, therefore, a fixed PI controller. The rationale for choosing a PI controller is to track a step reference with zero steady state error. Parameters of this controller is calculated based on the Ziegler–Nichols step response method [20]. To this end, the transformed second order control-oriented model i.e.

$$\frac{T_a}{q} = \frac{K_a K_r}{(1 + \tau_a s)(1 + \tau_r s)} \tag{32}$$

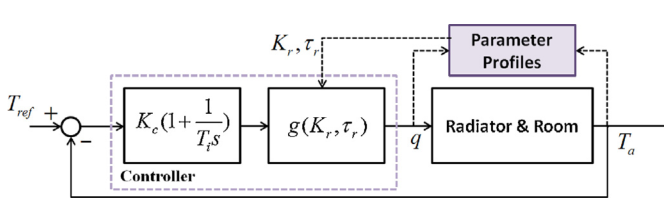

Fig. 8. Block diagram of the closed loop system with linear transformation

[5] LTI stands for linear time invariant.

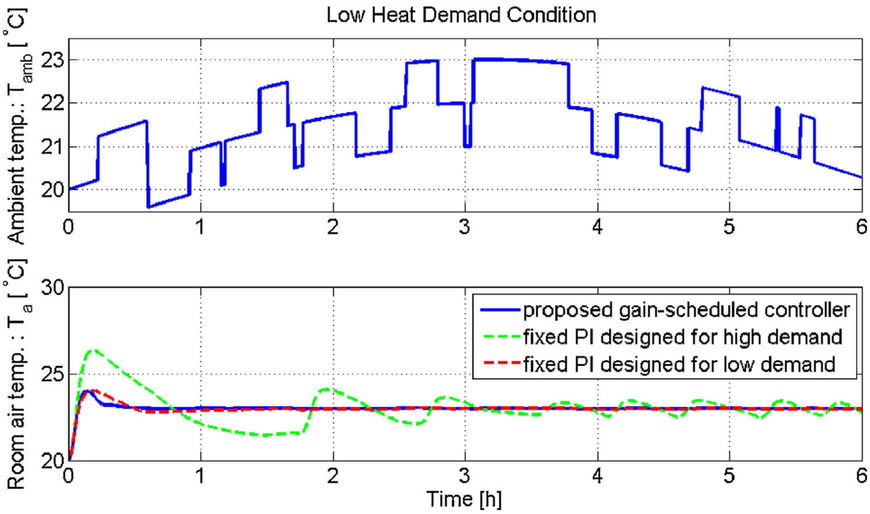

Fig. 9. (Top) ambient temperature, (bottom) room temperature for three controllers. The results of simulation with low adaptive controller together with two fixed PI controllers are shown. The PI controller designed for the high demand situation encounters instability in the low heat demand condition.

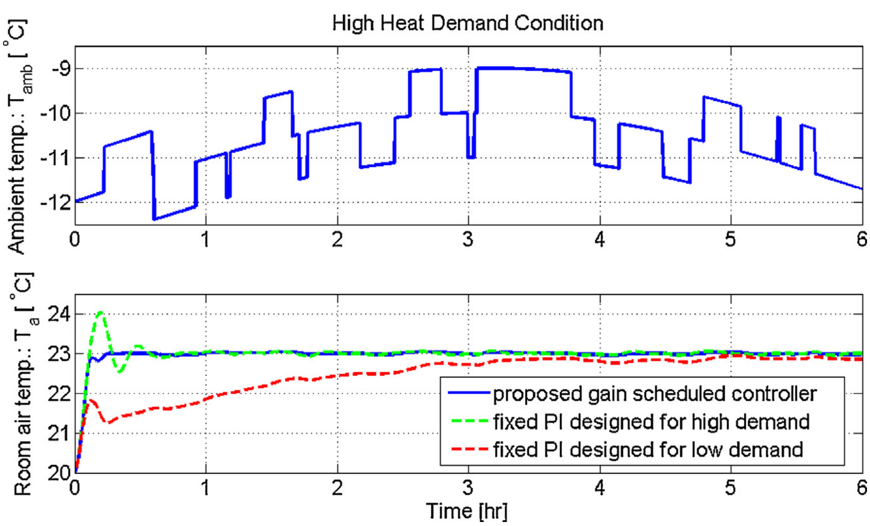

Fig. 10. (Top) ambient temperature, (bottom) room temperature for three controllers. The results of the simulation with low adaptive controller together with two fixed PI controllers are shown. The PI controller designed for the low demand condition is very slow for the high demand situation.

is approximated by a first-order system with a time delay as follows:

$$\frac{T_a}{q}(s) = \frac{k}{1+\tau s}e^{-Ls} \tag{33}$$

The time delay and time constant of the above model can be found easily by looking into the time response of the second-order model (32) to a unit step input q. The step response is derived and shown in the following:

$$T_a(t) = K_{hd}K_a\left(1+\frac{\tau_{hd}}{\tau_a-\tau_{hd}}e^{\frac{-t}{\tau_{hd}}}+\frac{\tau_a}{\tau_{hd}-\tau_a}e^{\frac{-t}{\tau_a}}\right)q(t) \tag{34}$$

in which $q(t) = u(t)$ is the unit step input.

The apparent time constant and time delay are calculated based on the time when 0.63 and 0.05 of the final value of T_a is achieved, respectively. The positive solution of the following equation gives the time delay when $\chi = 0.05$ and the time constant when $\chi = 0.63$.

$$(2-\chi)t^2 - 2\chi(\tau_{hd}+\tau_a)t^2 - 4\chi\tau_{hd}\tau_a = 0 \tag{35}$$

Solving the above equation for τ and L, the PI parameters comes out of the Ziegler–Nichols step response method. The parameters are the integration time $T_i = 3L$ and proportional gain $K_c = \frac{0.9}{a}$ with $a = k\frac{L}{T}$ and DC gain $k = K_{hd} \times K_a$.

5. Simulation results

We designed the proposed flow adaptive controller for the case study which was described earlier in Section 2. The controller is applied to simulation models of room and radiator. Parameter values used in simulation are listed in Table (2). PI controller parameters are obtained as $K_c = 0.01$ and $T_i = 400$. Ambient temperature is considered as the only source of disturbance for the system. In a partly cloudy weather condition, the effect of intermittent sunshine is modeled by a fluctuating outdoor temperature. To this end, a random binary signal is added to a sinusoid with the period of 12 hours to model the ambient temperature.

Simulation results with the designed controller and the corresponding ambient temperature are depicted in Figs. 9 and 10. The results are compared to the case with fixed PI controllers designed for both high and low heat demand conditions.

The simulation results of the proposed control structure show significant improvement in the system performance and stability compared to the fixed PI controller.

6. Discussions and conclusions

In this study, we investigated an inefficient operating conditions of TRV controlled radiators. The condition is discussed as a dilemma between stability and performance which we dealt with using a new generation of thermostatic radiator valves. Using the new TRV, flow estimation and control becomes possible. Based on the estimated flow, we have developed a gain scheduled controller which guarantees both performance and stability for the radiator system. To this end, we derived analytically, low-order models of the room-radiator system parameterized based on the estimated operating point.

All gain scheduling control approaches operate based on the basic assumption that all system states can be measured or estimated and a generalized observability holds [24]. In this study, however, we should clarify the validity of this assumption. The parameters that we need to measure or estimate are room temperatures and radiator flow rates. Measuring the first state is mandatory when the goal is seeking a reference for this temperature. However, radiator flow rate is not easily measurable.

To have an estimation of the radiator flow rate, one possibility is to use a new generation of TRVs in which the valve is driven by a stepper motor. Experiments show that this TRV can give a rather precise estimation of the valve opening. Knowing this fact and assuming a constant pressure drop across the radiator valve, we would be able to estimate the flow rate.

We have shown throughout the paper that using the new generation of TRVs, a gain scheduling controller would guarantee the performance of the radiator operation. In the current study, however we did not discuss the robustness of the proposed controller with respect to the model uncertainties. This task is postponed to be studied in future studies.

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