

Reduced-Order LPV Model of Flexible Wind Turbines from High Fidelity Aeroelastic Codes

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Abstract—Linear aeroelastic models used for stability analysis of wind turbines are commonly of very high order. These high-order models are generally not suitable for control analysis and synthesis. This paper presents a methodology to obtain a reduced-order linear parameter varying (LPV) model from a set of high-order linear time invariant (LTI) models. Firstly, the high-order LTI models are locally approximated using modal and balanced truncation and residualization. Then, an appropriate coordinate transformation is applied to allow interpolation of the model matrices between points on the parameter space. The obtained LPV model is of suitable size for designing modern gain-scheduling controllers based on recently developed LPV control design techniques. Results are thoroughly assessed on a set of industrial wind turbine models generated by the recently developed aeroelastic code HAWCStab2.

I. INTRODUCTION

Linear aeroelastic models used for stability analysis of wind turbines are commonly of very high order. Multibody dynamics coupled with unsteady aerodynamics (e.g. dynamic stall) are among the recently developments in wind turbine aeroelasticity [1]. The resulting models contains hundreds or even thousands of flexible modes and aerodynamic delays. In order to synthesize wind turbine controllers, a common practice is to obtain linear time-invariant (LTI) models from a nonlinear model for different operating points. Modern control analysis and synthesis tools are inefficient for such high-order dynamical systems; reducing the model size is crucial to analyze and synthesize model-based controllers.

Model-based control of wind turbines has been extensively researched during the last decade [2]. The linear parameter varying (LPV) framework shown to be suitable to cope, in a systematic manner, with the inherent varying dynamics of a wind turbine over the operating envelope [3], [4], [5]. Wind turbine LPV models are usually simple, first-principles based, often neglecting dynamics related to aerodynamic phenomena and some structural modes. This in turn restricted LPV control of wind turbines to the academic environment only. A procedure to encapsulate high-fidelity dynamics of wind turbines as an LPV system would be beneficial to facilitate industrial use of LPV control.

This paper presents a procedure to obtain a reduced-order LPV wind turbine model from a set of high-order LTI models. Firstly, the high-order LTI models are locally

approximated using modal and balanced truncation and residualization. Then, an appropriate manipulation of the coordinate system is applied to allow interpolation of the model matrices between points of the parameter space. The obtained LPV model is of suitable size for synthesizing modern gain-scheduling controllers based on the recent advances on LPV control design. Time propagation of the varying parameter is not explicitly utilized. Therefore, the procedure assumes that the varying parameter do not vary excessively fast in time, in line with common practices in gain-scheduling control [6]. Results are thoroughly assessed on a set of industrial wind turbine models derived by the recently developed aeroelastic code HAWCStab2 [7].

This paper is organized as follows. The modeling principles of the high-order LTI wind turbine models are exposed in Section II. Section III is devoted to present the proposed method. Section IV brings a numerical example along with results. Conclusions and future work are discussed in Section V.

II. WIND TURBINE MODEL

A nonlinear high-fidelity aeroelastic model is the starting point of the modeling procedure. The wind turbine structure is modeled with nonlinear kinematics based on corotational Timoshenko elements. Aerodynamics are modeled with Blade Element Momentum (BEM) coupled with unsteady aerodynamics based of shed-vorticity and dynamic stall. Linearization is performed analytically around a steady operational state for a given mean wind speed, rotor speed and collective pitch angle. Hansen [7] gives a more complete description of the linear aeroelastic model for an isolated blade. Two main equations of motion, one related to structural dynamics and another related to aerodynamics constitutes the LTI model

$$\begin{aligned} M\ddot{q}_s(t) + (C + G + C_a)\dot{q}_s(t) + (K + K_{sf} + K_a)u_s(t) \\ + A_f x_a(t) = F_s(t) \quad (1) \\ \dot{x}_a(t) + A_d x_a(t) + C_{sa}\dot{q}_s(t) + K_{sa}q_s(t) = F_a(t) \end{aligned}$$

where q_s are the elastic and bearing degrees of freedom, x_a are aerodynamic state variables, M is the structural mass matrix, C the structural damping matrix (Rayleigh), G the gyroscopic matrix, C_a is the aerodynamic damping matrix, K the elastic stiffness matrix, K_{sf} the geometric stiffness matrix, K_a the aerodynamic stiffness matrix, A_f is the coupling of the structure to aerodynamic states, A_d represents aerodynamic time lags, C_{sa} and K_{sa} are coupling matrices to

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structural states. F_s and F_a represent forces due to actuators and wind disturbance. The equations in first order form are

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t)\end{aligned}\quad (2a)$$

$$x(t) = [x_a(t) \quad q_s(t) \quad \dot{q}_s(t)]^T \quad u(t) = [Q_g(t) \quad \beta(t) \quad V(t)]^T \quad (2b)$$

where the controllable inputs are the generator torque Q_g and collective pitch angle β , and V is the uniform wind speed disturbance input. The model outputs considered here are the generator angular velocity Ω and tower top lateral displacement q . The first output is usually measured and feed to a speed controller that manipulates the pitch angle β . The second output can be utilized for lateral tower load mitigation by generator torque control [8]. The aeroelastic tool offers the possibility to select other inputs and outputs, but we limit to the ones just mentioned to clearly expose the results.

III. REDUCED ORDER LPV MODEL

Consider N_s stable multiple-input multiple-output (MIMO) LTI dynamical systems (2) of order n corresponding to parameter values $\theta^{(i)}$, $i = 1, 2, \dots, N_s$,

$$S_i : \begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y(t) = C_i x_i(t) + D_i u(t) \end{cases}, \quad i = 1, \dots, N_s. \quad (3)$$

where $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times n_u}$, $C_i \in \mathbb{R}^{n_y \times n}$, $D_i \in \mathbb{R}^{n_y \times n_u}$. We seek a reduced-order parameterized model $S(\theta)$ of order $r < n$ which approximates S_i ,

$$S(\theta) : \begin{cases} \dot{x} = A(\theta)x(t) + B(\theta)u(t) \\ y(t) = C(\theta)x(t) + D(\theta)u(t) \end{cases} \quad (4)$$

where $A(\theta) \in \mathbb{R}^{r \times r}$, $B(\theta) \in \mathbb{R}^{r \times n_u}$, $C(\theta) \in \mathbb{R}^{n_y \times r}$, $D(\theta) \in \mathbb{R}^{n_y \times n_u}$ are continuous functions of a vector of varying parameters $\theta := [\theta_1, \theta_2, \dots, \theta_{N_\theta}]^T$. The dynamics of the original system S_i and the approximated system $S(\theta)$ are assumed to evolve smoothly with respect to $\theta^{(i)}$ and θ , respectively. The parameter θ may represent the current operating point. It also may describe deviations on aerodynamics and structural properties for the sake of parametric model uncertainties. Plant parameters to be designed under an integrated plant-controller synthesis scheme could also be parameterized.

Variation in aerodynamic forces under structural vibration contributes significantly to changes in natural frequencies and damping of some structural modes. A specific procedure that takes these particularities into account is proposed here. A flowchart containing the required steps is depicted in Fig. 1.

Known methods for model reduction constitutes the proposed scheme and are briefly explained in the sequel, in the context of our application. Consult the survey of [9] for a more comprehensive exposure on model reduction.

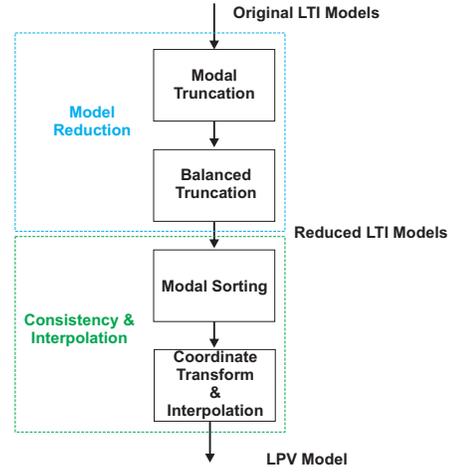


Fig. 1. Scheme overview.

A. Model Reduction

A reduced order model is commonly obtained by truncation of appropriate states. Let the state vector x_i be partitioned into $x_i := [x_{r,i} \quad x_{t,i}]^T$ where $x_{r,i}$ is the vector of retained states and $x_{t,i}$ is the vector of truncated states. The original system is partitioned accordingly

$$\begin{aligned}\begin{bmatrix} \dot{x}_{r,i}(t) \\ \dot{x}_{t,i}(t) \end{bmatrix} &= \begin{bmatrix} A_{rr,i} & A_{rt,i} \\ A_{tr,i} & A_{tt,i} \end{bmatrix} \begin{bmatrix} x_{r,i}(t) \\ x_{t,i}(t) \end{bmatrix} + \begin{bmatrix} B_{r,i} \\ B_{t,i} \end{bmatrix} u(t) \\ y &= [C_{r,i} \quad C_{t,i}] \begin{bmatrix} x_{r,i}(t) \\ x_{t,i}(t) \end{bmatrix} + Du(t)\end{aligned}\quad (5)$$

and the reduced model is simply given by the state-space equation of the retained states

$$\begin{aligned}\dot{x}_{r,i} &= A_{rr,i}x_{r,i}(t) + B_{r,i}u(t), \\ y &= C_{r,i}x_{r,i} + Du(t)\end{aligned}\quad (6)$$

If the original model is a stable system so is its truncated counterpart. While truncation tends to produce a good approximation in the frequency domain, the zero frequency gains (DC gains) are not guaranteed to match. This can be of particular importance in a wind turbine model because some aerodynamic states may not influence the transient behavior but can contribute significantly to low frequency gains. Matching DC gains can be enforced by a model residualization method by setting the derivative of $x_{t,i}$ to zero in (5) and solving the resulting equation for $x_{r,i}$. After trivial manipulations, the reduced model is given by

$$\begin{aligned}\dot{x}_{r,i} &= [A_{rr} - A_{rt}A_{tt}^{-1}A_{tr}]_i x_{r,i} + [B_r - A_{rt}A_{tt}^{-1}B_t]_i u(t) \\ y &= [C_r - C_tA_{tt}^{-1}A_{tr}]_i x_{r,i} + [D - C_tA_{tt}^{-1}B_t]_i u(t)\end{aligned}\quad (7)$$

Note that $A_{tt,i}$ is assumed invertible for (7) to hold. Residualization is performed in both modal and balanced reduction steps.

1) *Modal Truncation*: Due to size and numerical properties associated with large size systems and low damped dynamics, model reduction algorithms based on Hankel singular values can fail to produce a good reduced model.

In order to start the reduction process, the original model is truncated to an intermediate size for subsequent reduction in a more accurate way. In modal form the system is put into a modal realization before states are truncated [10]. The modal form realization has the state matrix A in block diagonal form with either 1×1 or 2×2 blocks when the eigenvalue is real or complex, respectively. Let system S_i be represented in modal form,

$$S_{m,i} : \begin{cases} \dot{x}_i(t) = A_{m,i}x_{m,i}(t) + B_{m,i}u(t) \\ y(t) = C_{m,i}x_{m,i}(t) + D_{m,i}u(t) \end{cases} \quad (8a)$$

$$\begin{aligned} A_{m,i} &= \text{diag}(A_{m,k,i}), \\ A_{m,k,i} &= -e_{k,i} \quad \text{for real eigenvalues,} \\ A_{m,k,i} &= \begin{bmatrix} -\xi_{k,i}\omega_{k,i} & \omega_{k,i}\sqrt{1-\xi^2} \\ \omega_{k,i}\sqrt{1-\xi^2} & -\xi_{k,i}\omega_{k,i} \end{bmatrix} \quad \text{for complex eigen.} \\ B_{m,i} &= \begin{bmatrix} B_{m,1,i} \\ B_{m,2,i} \\ \vdots \\ B_{m,k,i} \end{bmatrix}, \quad C_{m,i} = [C_{m,1,i} \quad C_{m,2,i} \quad \dots \quad C_{m,k,i}] \\ i &= 1, \dots, N_s, \quad k = 1, \dots, N_m. \end{aligned} \quad (8b)$$

where N_m is the number of modes, $\xi_{k,i}$ and $\omega_{k,i}$ are the damping ratio and natural frequency of mode k and model i . The diagonal blocks are usually arranged in ascending order according to their eigenvalue magnitudes. The magnitude of a complex eigenvalue is $\omega_{k,i}$ while for a purely real eigenvalue is $e_{k,i}$. The retained states are then the ones with magnitudes smaller than a chosen threshold $\bar{\omega}$. The intermediate model must contain all modes within the frequencies of interest for control design. A large number of states (300 to 450) is expected at this stage since many modes are of low frequency.

2) *Balanced Truncation*: The order of the intermediate system is further reduced by balanced truncation. In balanced truncation [11] the system is transformed to a balanced realization. A MIMO LTI system of the form (3) is said to be balanced if, and only if, its controllability and observability grammians are equal and diagonal, i.e. $P_i = Q_i = \text{diag}(\sigma_1, \dots, \sigma_n)$, where $\sigma_1, \dots, \sigma_n$ denotes the Hankel singular values sorted in decreasing order and matrices P_i, Q_i are the controllability and the observability Gramians. The gramians are solutions of the following Lyapunov equations

$$\begin{aligned} A_i P_i + P_i A_i^T + B_i B_i^T &= 0 \\ A_i^T Q_i + Q_i A_i + C_i^T C_i &= 0 \end{aligned} \quad (9)$$

If this holds, the balanced system is given by

$$S_{b,i} : \begin{cases} \dot{x}_{b,i} = W_i^T A_i V_i x_{b,i}(t) + W_i^T B_i u_i(t) \\ y(t) = C_i V_i x_{b,i}(t) + D_i u_i(t) \end{cases} \quad (10)$$

$i = 1, \dots, N_s.$

where $x_b \in \mathbb{R}^n$, $V = UZ\Sigma^{-1/2}$ and $W = LY\Sigma^{-1/2}$, together with the factorizations $P = UU^T$, $Q = LL^T$ and the singular

value decomposition $U^T L = Z\Sigma Y^T$ [12]. This state coordinate equalizes the input-to-state and state-to-output energy transfers, making the Hankel singular values a measure of the contribution of each state to the input/output behavior.

Denote $V_{i(r)}$ and $W_{i(r)}$ the first r columns of V_i and W_i . The reduced-order systems \hat{S}_i

$$\hat{S}_i : \begin{cases} \dot{\hat{x}}_i = \hat{A}_i \hat{x}_i(t) + \hat{B}_i u_i(t) \\ \hat{y}(t) = \hat{C}_i \hat{x}_i(t) + \hat{D}_i u_i(t) \end{cases} \quad i = 1, \dots, N_s. \quad (11)$$

are obtained by truncation when the projectors $V_{i(r)}$ and $W_{i(r)}$ are applied to the intermediate sized model. In words, the balanced truncation removes the states with low Hankel singular values, thus not much information about the system will be lost. When applied to a stable system, balanced truncation preserves stability and guarantees an upper bound on the approximation error in an H_∞ sense [13]. Expected order of the final reduced system is 7 to 20 states. The choice of the final order depends on the required model complexity and admissible error between the full and reduced model.

B. State-Space Consistency & Interpolation

Consider the balanced reduced models \hat{S}_i and put them in modal form. The first step towards a consistent state-space representation is to assure that all modes keep their positions in the state matrix throughout the parameter space. The second step to a consistent state-space is to ensure that values of the entries of the system matrices change smoothly between each LTI system. At this point, the system matrices cannot be readily interpolated because the modal and balanced similarity transformations applied to the original system are not unique. One could think of interpolating the system in modal form. Indeed, the state matrix A is unique up to a permutation of the location of the modes and could easily be interpolated, but the similarity transformation that puts the system in modal form is not unique. Therefore, matrices \hat{B} and \hat{C} may have entries with abrupt value changes. The balanced realization is unique up to a sign change and consequently abrupt sign changes in the system matrices may occur from one LTI system to another. As suggested by [14], these issues can be corrected by properly changing the sign of the correspondent eigenvectors.

Instead of correcting the eigenvectors before similarity transformations, we propose to transform the reduced order LTI systems into a representation based on the companion canonical form. No unique canonical form for multivariable systems is known to exist [15]. However, there exist algorithms which, for a system under arbitrary similarity transformation, find a unique companion form [16]. One algorithm with such properties is implemented in the function *canon* of MATLAB. The companion form is poorly conditioned for most state-space computations [17]. In order to avoid numerical issues, each mode k of the reduced system in modal coordinates is transformed into a companion realization. The system matrices of this particular realization

are

$$\begin{aligned}
A_{c,i} &= \text{diag}(A_{c,k,i}), \quad B_{c,i} = \begin{bmatrix} B_{c,1,i} \\ B_{c,2,i} \\ \vdots \\ B_{c,k,i} \end{bmatrix}, \\
C_{c,i} &= [C_{c,1,i} \quad C_{c,2,i} \quad \dots \quad C_{c,k,i}], \\
\left\{ \begin{array}{l} A_{c,k,i} = -a_{k,i} \\ B_{c,k,i} = \begin{bmatrix} 1 & b_{1,k,i} & \dots & b_{n_u-1,k,i} \end{bmatrix} \\ C_{c,k,i} = \begin{bmatrix} c_{1,k,i} \\ \dots \\ c_{n_y-1,k,i} \\ 0 \end{bmatrix} \end{array} \right. & \text{for real eigenvalues,} \\
\left\{ \begin{array}{l} A_{c,k,i} = \begin{bmatrix} 0 & -a_{k,i,1} \\ 1 & -a_{k,i,2} \end{bmatrix}, \\ B_{c,k,i} = \begin{bmatrix} 0 & b_{11,k,i} & \dots & b_{1n_u-1,k,i} \\ 1 & b_{21,k,i} & \dots & b_{2n_u,k,i} \end{bmatrix} \\ C_{c,k,i} = \begin{bmatrix} c_{11,k,i} & \dots & c_{1r,k,i} \\ \vdots & \ddots & \vdots \\ c_{n_y1,k,i} & \dots & c_{n_yr,k,i} \end{bmatrix} \end{array} \right. & \text{for complex eigen.} \\
i = 1, \dots, N_s, \quad k = 1, \dots, N_m. &
\end{aligned} \tag{12}$$

The characteristic polynomial of each mode appears in the rightmost column of the matrix $A_{c,k,i}$. The entries of $A_{c,k,i}$, $B_{c,k,i}$ and $C_{c,k,i}$ may be easily checked for possible inconsistencies of a particular mode, by detecting abrupt value changes between LTI systems. The state-space matrices are now at a realization suitable for interpolation. Let $z(\theta)$ be one matrix entry, function of θ . We focus on the polynomial dependence

$$z(\theta) = \sum_{k=1}^{N_p} \eta_k p_k(\theta) \tag{13}$$

where p_k is a set of multivariate polynomials on the parameters $\theta_1, \dots, \theta_{N_\theta}$ and η_k are coefficients to be determined. Let z_i be the values of a matrix element for $i = 1, \dots, N_s$. Define the following matrices

$$\begin{aligned}
H &= \begin{bmatrix} p_1(\theta^{(1)}) & \dots & p_{N_p}(\theta^{(1)}) \\ \vdots & \ddots & \vdots \\ p_1(\theta^{(N_s)}) & \dots & p_{N_p}(\theta^{(N_s)}) \end{bmatrix} = [P_1 \quad \dots \quad P_{N_p}] \\
Z^T &= [z_1 \quad \dots \quad z_{N_s}], \quad \Gamma^T = [\eta_1 \dots \eta_{N_p}].
\end{aligned} \tag{14}$$

A linear least squares fit minimizes the quadratic error between $z(\theta^{(i)})$ and z_i , $i = 1, \dots, N_s$

$$\Gamma^* = \arg \min_{\Gamma} (Z - H\Gamma)^T (Z - H\Gamma) \tag{15}$$

The optimal Γ^* is determined by initially computing a singular value decomposition of H

$$\Upsilon \Xi \Psi^T = \text{svd}(H) \tag{16}$$

With the decomposition at hand, the solution to the linear least squares problem is given by

$$\Gamma^* = \Psi \Xi^+ \Upsilon^T Z \tag{17}$$

where Ξ^+ stands for the Moore-Penrose pseudoinverse of Ξ . Repeating the above procedure for each matrix entry results in the polynomial approximations of the matrices $A(\theta)$, $B(\theta)$, $C(\theta)$, $D(\theta)$ that can be used for subsequent analysis and design of controllers.

IV. NUMERICAL EXAMPLE

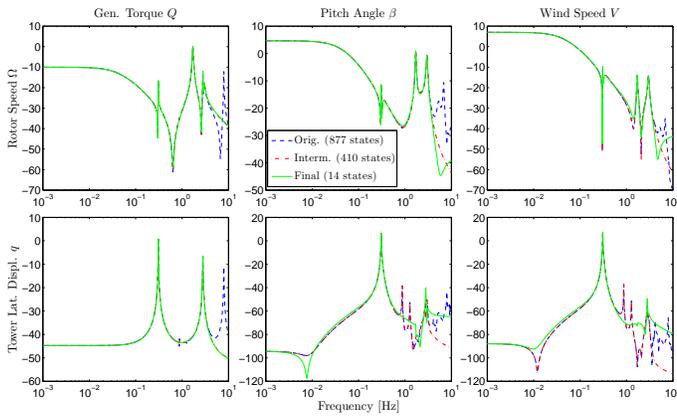
In this section, the proposed procedure is applied to the NREL 5MW reference wind turbine model [18]. The aim is to find an LPV model encapsulating the wind turbine dynamics operating at the full load region. Large scale MIMO LTI models with 877 states are computed by the aeroelastic code HAWCStab2 [7] for wind speeds equidistant 1 m/s ($\theta^{(i)} \in \{12, 13, \dots, 25\}$). The model is parameterized by the mean wind speed $\theta := \bar{V}$. A fifth-order polynomial dependence of the LPV system matrices

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} + \sum_{d=1}^5 \begin{bmatrix} A_d & B_d \\ C_d & D_d \end{bmatrix} \theta^d \tag{18}$$

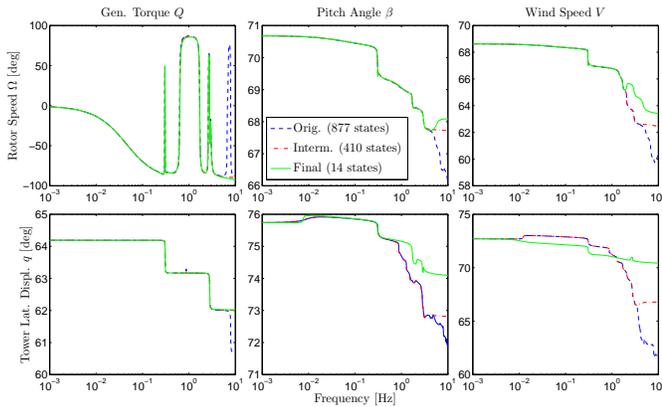
gives a fair trade-off between interpolation accuracy and polynomial order. Due to the different units of inputs and outputs, the LTI systems should be suitably normalized before the order is reduced. Parameter-independent scales are applied to all LTI models such that expected signal excursions are normalized to 1. The generator torque input was scaled to 5% of the rated torque. The pitch angle input and wind speed input remained unscaled. The rotor angular velocity is scaled to the maximum excursion desired in closed-loop, 5 % of its nominal value. The lateral tower top displacement was scaled with the H_∞ -norm of the inputs to this output channel.

Bode plots of the original, intermediate and final reduced order models for an operating point $\theta = 15$ m/s are depicted in Fig. 2. The intermediate model with 410 states resulted from modal truncation and residualization of the original system. A balanced truncation with residualization further reduced the size to 14 states. The magnitude plots have an excellent agreement in the frequencies of interest. The residualization in both steps assisted to a better fit of the low frequency content of the tower displacement output. Attempts to use balanced truncation directly on the original system failed to produce a good approximation due to numerical ill-conditioning.

The balanced realization "misses" a low frequency anti-resonance related to the transfer function from wind speed input to tower displacement output. Notice the differences in magnitude and more pronouncedly phase around 10^{-2} Hz. However, this anti-resonance does not contribute significantly



(a) Magnitude



(b) Phase

Fig. 2. Bode plots of the original, intermediate and final reduced order models for a mean wind speed of 15 m/s.

to the input-output behavior of the MIMO system. A comparison of the minimum and maximum singular values is depicted in Fig. 3 and shows an excellent agreement.

Step responses of the original, intermediate and final reduced order models for a mean wind speed of 15 m/s are depicted in Fig. 4. Except for some high frequency content in the signal from generator torque to tower position, the responses are identical.

The location of the poles of the LPV system for a $2N_s$ grid of equidistant operating points is illustrated in Fig. 5. The arrows indicate how the poles move for increasing mean wind speeds. A smooth evolution of the poles along the full load region is noticeable.

The relative difference of the Hankel singular values of the interpolated LPV system and the reduced order system defined as

$$\sigma_{rel,r,i} = \frac{\sigma_{int,r,i} - \sigma_{r,i}}{\sigma_{r,i}} \times 100, \quad i = 1, \dots, N_s \quad (19)$$

serves as a measure of the quality of the interpolation. A good fit can be corroborated by some metrics of $\sigma_{rel,r,i}$ given in Tab. I. The mean difference in the Hankel singular values is only 0.27% and the maximum difference just 2.75%.

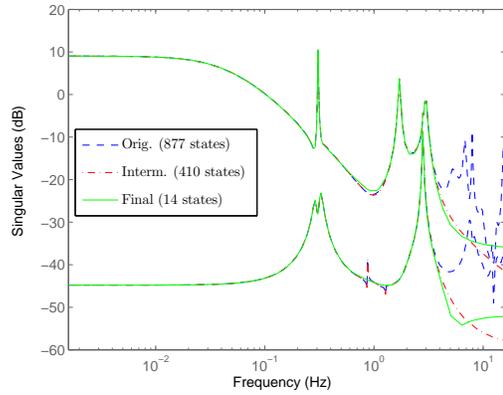


Fig. 3. Singular values of the original, intermediate and final reduced order models for a mean wind speed of 15 m/s.

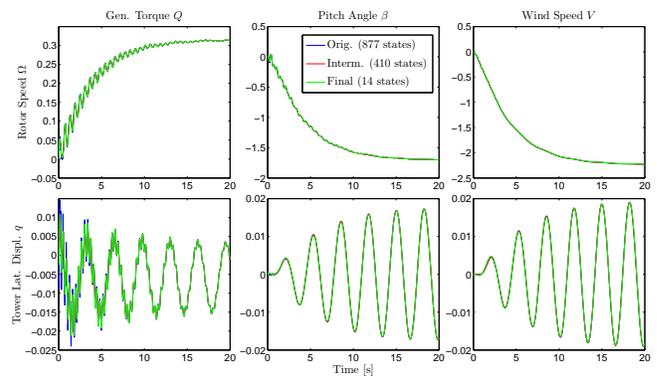
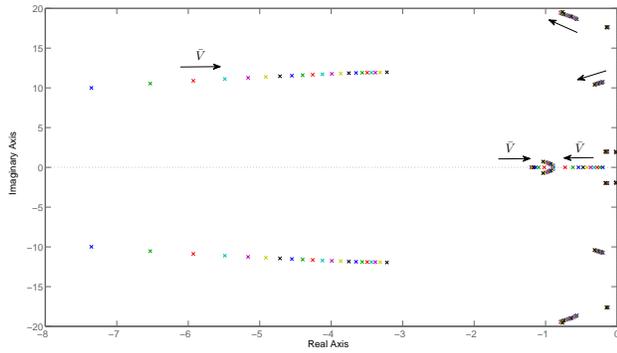


Fig. 4. Step responses of the original, intermediate and final reduced order models for a mean wind speed of 15 m/s.

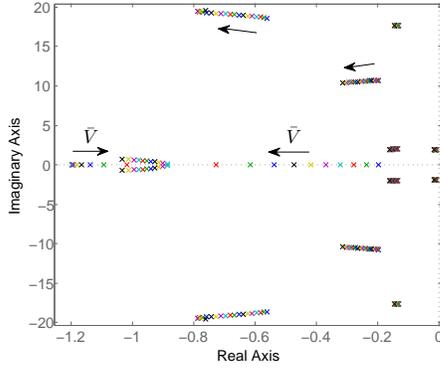
V. CONCLUSION & FUTURE WORK

This paper presents a procedure to obtain a reduced-order LPV model of a wind turbine from a set of high-order LTI models. Finding ways to encapsulate high-fidelity LTI aeroelastic models as an LPV system is an important step to increase the utilization of recent advances in LPV control by the wind turbine industry. The proposed procedure starts by model reduction of the high-order LTI systems at different values of the parameter space. Manipulations of the state-space coordinates follows, in order to arrive at low-order consistent LTI systems for subsequent interpolation. The reduced-order LPV system has a suitable size for analysis and synthesis of controllers and presents smoothly varying dynamics along the scheduling parameter range.

A subject for future work is to initially interpolate the set of high-order LTI models and later apply an appropriate reduction method to realize a reduced order LPV model. Preserving structure reduction methods applied directly in the second order vector equations of motion in an interesting topic to be studied. Model complexity versus required polynomial degree and a comparison with models obtained by first-principles is also subject of future work.



(a) Pole Map



(b) Detail

Fig. 5. Pole location of the LPV model for frozen values of the varying parameter θ .

TABLE I

DIFFERENCE IN THE HANKEL SINGULAR VALUES BETWEEN THE LPV AND REDUCED ORDER SYSTEM FOR FROZEN VALUES OF θ .

Max	Min	Mean	Std. dev
2.75	0.001	0.27	0.57

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