Model Predictive Control for Integration of Industrial Consumers to the Smart Grid under a Direct Control Policy

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***Abstract*— In this paper, we study a three-level hierarchical control framework for integration of industrial consumers to the future smart grid. With this structure, a balance responsible party (BRP) at the top level will be able to provide regulating power from the consumer side to help the power grid in case there is imbalance between demand and supply. We consider heterogeneous consumers at the bottom level, namely a cold storage in supermarkets and a chiller equipped with ice storage. These consumers are under the direct control of a mid-level controller, a so-called aggregator. The aggregator receives a certain amount of power reference within a certain activation time from BRP and applies a model predictive controller (MPC) approach to split up the power reference between different consumers in an optimal way. Each consumer has its own characteristics and constraints that should be honored by the aggregator.**

I. INTRODUCTION

DECREASING fossil fuel energy resources together with increasing environmental concerns, motivate countries to plan on using more renewables such as wind and solar in power generation. Unlike traditional resources which are easy to control and predict, renewable energy resources are intermittent and unpredictable unless for a short horizon. Therefore, keeping balance between production and consumption is becoming more challenging in the future power grid with a high share of fluctuating renewables. To alleviate this issue, we should move toward the smarter grid, so-called "smart grid". The emerging smart grid has been studied from different point of views [1]. In brief, utilization of modern sensors, communication protocol and information exchange to enhance the reliability, flexibility and stability of the power grid will be the vision of the future smart grid [2].

Participation of the consumer side in balancing effort will be one of the main features of the smart grid [3]. Rather than using just generation units to maintain balance and stabilize frequency of the power grid, all parts connected to the grid could play an active role. In this context, active demand refers to the consumers, which can change their consumption behavior in a way that is beneficial to the grid regarding balancing problems. This may cause the consumers to operate outside of their optimal region. Therefore, consumers should have enough incentives to attend in balancing task and to be flexible in power consumption. In other words, flexibility could be traded as a commodity in the future market.

Different control policies can be applied to exploit the flexibility of the consumers. In the literature, two main control policies have been studied which are entitled direct control and indirect control [4] & [5]. In direct control, a grid operator or a third party player has direct access to the local controller located at each consumer site and sends command to change their consumption. This scheme needs a two-way communication between the consumers and the third party. On the contrary, indirect control is a one-way communication scheme where some incentive signals such as price are used to motivate the consumers to change their consumption. Literature review regarding flexible demands reveals that most references are focused on household appliances and indirect control whereas the works on direct control are rather limited. For instance, in [6] an incentive-based consumption scheduling algorithm has been presented where several residential units are connected to a common energy source. Each user is equipped with an automatic energy consumption scheduler which minimizes the energy cost in a game theoretic approach based on a price signal, or the work in [7] which presents an energy management algorithm to optimize energy consumption in a neighborhood level based on price signals. Household appliances are low energy consumers and a lot of them should be aggregated in order to bid in the market. Therefore applying direct control for households could be complicated or even infeasible. Moreover, house owners are not always satisfied to give permission to another entity to have direct access to their appliances. However industrial enterprises are large energy consumers. In practice we can imagine a centralized control architecture with a controller that has the limited number of consumers under its direct jurisdiction based on a contract agreement.

In this paper, we propose a hierarchical control framework, based on direct control policy for integration of industrial consumers to the smart grid. This is similar to the work [8]. However in this paper, each consumer is modeled as a simple storage and the discrepancies between them are not taken into account. Here we consider heterogeneous consumers, namely a supermarket refrigeration system and a chiller equipped with an ice tank for air conditioning in commercial buildings. In [9], a supermarket refrigeration system has been assessed as a fast reserve. Durations of up to 15 minutes have been considered when the supermarket

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could contribute with ancillary services to provide upward and downward regulating power. During an up-regulating period, the supermarket is asked to reduce power consumption whereas, in down-regulating, it consumes more power by lowering the temperature of the cold room. What is gained by combining the flexibilities of different consumers is the main contribution of this paper.

The rest of the paper is structured as follows. In section II, we describe our proposed control set up for offering flexibility. In section III, we then present a brief overview of our case studies and the dynamic models of them. In section IV, we formulate the optimization problem. Simulation results are provided in section V. Finally we conclude the paper in section VI.

II. PROPOSED CONTROL SET-UP

To stabilize the frequency of power grid, balance between production and consumption of electricity should be kept all the times. In power system, transmission system operators (TSO) are responsible to sustain the reliability of the power grid. In order to accomplish this, TSOs purchase ancillary services from generation or consumption. Ancillary services include a variety of services such as regulating power or voltage control. In [10], a comprehensive description of ancillary services which are provided in Denmark is presented. In this paper, we introduce a three level hierarchical structure to provide regulating power from industrial consumers (Fig. 1). In this structure, balance responsible party (BRP) is at the top level. BRPs are trading companies which are economical responsible for supplying power of a number of consumers. They could sell and buy power at different markets. For instance, in a day-ahead market, power is traded a day before it is used by the consumer. By utilizing the flexibility of consumers, BRPs will be able to minimize the deviation between the power bought and the actual power consumed. Moreover, BRPs can trade with TSO in other markets like regulating power market. In case of power deficit in the grid, upward regulating power could be provided by either increase in production or decrease in consumption. When there is power surplus, downward regulating power could be obtained by decrease in production or increase in consumption. In the proposed setup, consumers are committed to follow a time-varying power within a certain period of time which we call it an activation time. This setup is based on a contract agreement where both sides agree on the necessary terms. For instance, the duration of activation time, maximum capacity of the consumers, consumers 'constraints and the penalties in case of deviation from commitment should be specified in the contract. On the other hand, handling a large number of consumers would be a hard task from a computational complexity point of view for a BRP. Therefore we define a mid-level controller, which is located at the aggregator. In this direct control approach, the aggregator plays an important role. Aggregator distributes the power reference, which it receives from BRP, between the consumers within the activation time.

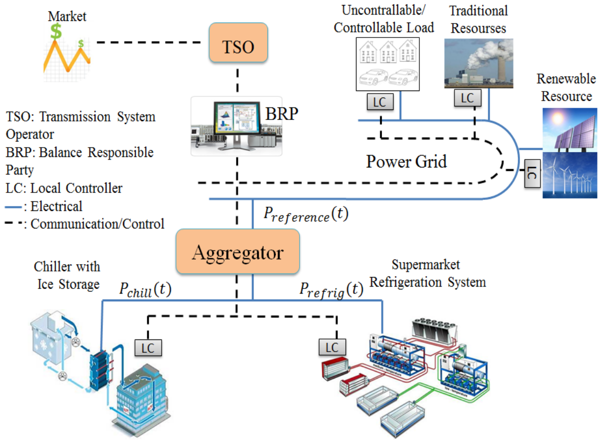

Fig. 1. Hierarchical Control Structure

Here we consider heterogeneous consumers at the bottom level to investigate the benefits coming from the aggregation of flexibilities of different consumers. Our case studies are a supermarket refrigeration system and a chiller equipped with ice storage. Each consumer has an optimal operation, which is defined by the local controller. After they are activated by the aggregator, they will be deviated from optimal operation. Deviation from optimal operation could be interpreted as a cost of flexibility. The aggregator needs to minimize the cost of flexibility while satisfying the power reference requirement from BRP. To this end, it requires a dynamic model of the consumers and the constraints of them. At each time instant, the below optimization problem should be solved at the aggregator:

Minimize {cost of flexibility}
Subject to:
Consumers' dynamics
Consumers' constraints
$P_{reference}(t) = P_{refrig}(t) + P_{chill}(t)$

where $P_{refrig}(t)$ and $P_{chill}(t)$ are the power consumption of refrigerator and chiller respectively. $P_{reference}(t)$ is the regulating power that the aggregator promises to provide during the activation time. We just consider the down regulating scenario in this paper, when the consumers are asked to consume more than they need. In the following section, each case study is explained briefly.

III. CASE STUDIES

A. *Supermarket Refrigeration*

In supermarkets, goods that need to be kept cold are placed in display cases or cold storages. A vapor-compression cycle is used to remove heat from the goods. The liquid refrigerant enters the evaporator where it absorbs heat from the goods and turns to vapor. The vapor then flows through the compressor and condenser to be compressed and expel the heat and again returns to the evaporator in the liquid form to close the cycle. The compressor is the main power consumer in this cycle.

The temperature of the cold room, T_{cr}, could be simply described by the following equations:

$$m_{food} c_{p,food} \frac{dT_{cr}}{dt} = \dot{Q}_{load-cr} - \dot{Q}_e \qquad (1)$$
$$\dot{Q}_{load-cr} = (UA)_{amb-cr} (T_{amb} - T_{cr})$$

where Q_e represents the heat removed from the cold room by the evaporator and $Q_{load-cr}$ is the heat load from the surrounding. m_{food} and $c_{p,food}$ are the mass and specific heat capacity of the refrigerated goods and $(UA)_{amb-cr}$ is the overall heat transfer coefficient between ambient and cold room. T_{amb} is the ambient temperature. Power consumed by the compressor is obtained from:

$$\dot{Q}_e = COP_{refrig} P_{refrig} \qquad (2)$$

COP_{refrig} is the coefficient of performance of the refrigerator that is assumed to be constant in this paper. Temperature of the cold room could vary within a certain limits: $T_{cr,min} \leq T_{cr} \leq T_{cr,max}$ and this opens a space for the system to offer flexibility. We assume that the local controller at supermarket tries to run a system with its minimum power consumption to keep the temperature just below the upper limit in order to consume less energy. Hence, it is always possible for the system to provide downward regulating power by increase in power consumption. Lowering the temperature of the cold room leads to store energy in the refrigerated goods, which could be regained by turning off the compressor afterwards. However, heat load from the surrounding increases as the cold room temperature decreases. This means that some part of the energy we put in to the system could not be stored and therefore will be lost. Extra energy stored in the cold room after activation is given by:

$$Q_{refrig} = m_{food} c_{p,food} (T_{cr,\max} - T_{cr}) \qquad (3)$$

By defining $x_r =: Q_{refrig}$ as a state, the following linear state space model describes the energy changing after activation:

$$\dot{x}_r(t) = A_r x_r(t) + B_r u_r(t) + D_r$$
$$A_r = \frac{-(UA)_{amb-cr}}{m_{food} c_{p,food}} \qquad (4)$$
$$B_r = COP_{refrig}$$
$$D_r = (UA)_{amb-cr} (T_{cr,\max} - T_{amb})$$

where $u_r(t) =: P_{refrig}(t)$ is the input power to the refrigeration system.

B. Chiller with Ice Storage

A chiller is part of the air conditioning system that is used to cool down a building and provide a satisfactory comfort level. Same as refrigeration system, chiller utilizes the vapor-compression cycle to remove heat from a liquid, typically brine. Cooled brine then circulates through the pipes to cool down the air via a heat exchanger. Air conditioning systems in commercial buildings usually consume a significant amount of power, which often coincides with the high-peak hours of electricity consumption in the power grid. Adding ice storage to this system could help the power grid and at the same time reduce the cost of energy for the building. The basic idea is to store cold energy by charging the storage during off-peak hours e.g. at night and use the ice storage to cool the building during the day. The flexibility of the ice storage is rather large due to the large latent heat of water. Compared to a refrigerator, more energy could be saved and the system could provide downward regulating power for a long term.

The dynamics of the ice tank is difficult to describe as the phase change from water to ice occurs during freezing. At different times, there could be water, ice or both of them in the tank. In this paper, we assume that the tank is always in two-phase mode. In two-phase mode, the temperature of the tank can be assumed to be equal to zero all the time. Since the energy stored by lowering the temperature of water or frozen ice is much smaller than what is stored during freezing, this is a reasonable assumption. Heat transfer between brine and water during freezing, Q_{ice}, could be described by the following equation:

$$L \frac{dm_{water}}{dt} = \dot{Q}_{ice} \qquad (5)$$
$$\dot{Q}_{ice} = (UA)_{brine-water} (T_b - 0)$$

where $(UA)_{brine-water}$ is the heat transfer coefficient between brine and water and L is the specific latent heat of water. T_b and m_{water} represent the temperature of brine and mass of water respectively. For a constant power consumption, as the ice builds up in the tank, the rate of heat removed from the tank decreases. This is because of the lower heat transfer coefficient between brine and ice compared to the water and brine. We model this effect by defining a time varying UA value as follows:

$$(UA)_{brine-water} = \frac{1}{R} \qquad (6)$$
$$R = R_0 + R_1 \left(\frac{m_{ice}}{m_{water,\max}} \right)$$

$(UA)_{brine-water}$ is the reciprocal of the thermal resistance; R. R_0 and R_1 are constant values which indicate the overall thermal resistance between brine and water and the rate of change of thermal resistance after ice builds up. $m_{water,max}$ represents the maximum mass of water in the tank. From (6), we can see that as the mass of ice increases, thermal resistance, R, increases which leads to decrease of UA value. We assume a linear relation between T_b and power consumption of chiller as follows:

$$T_b = \alpha P_{chill} + \beta \qquad (7)$$

Chiller in conjunction with ice storage can operate in different modes. Three basic operation modes are charging, direct cooling and passive cooling. In charging mode, the ice storage is charged by the chiller and there is no cooling load from the building. In direct cooling, the ice storage is not utilized and the cooling load is provided by chiller. The chiller is shut off in passive mode and the ice storage is used exclusively to serve the cooling load. Fig. 2 shows a simple diagram of such a system:

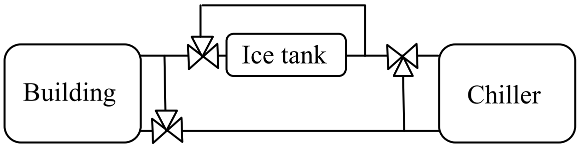

Fig. 2. Simple diagram of chiller + ice storage

In principle, the system could be able to switch between these modes or a combination of them by regulating the three-way valves. For instance, it can provide downward regulating power by charging the ice storage while providing cooling load to the building simultaneously. However, running the system for simultaneous direct cooling and charging is not optimal because of the significant drop in coefficient of performance, COP_{chill} that decreases along with evaporation temperature. In order to make ice and store energy in the tank, brine temperature has to be lower than zero. Otherwise, heat transfer could not occur between the brine and water. This implies a considerable drop in evaporator temperature from a direct cooling situation, which means a significant drop in COP_{chill}. In other words, when the chiller is activated by the aggregator to consume more power than it is needed, no energy is stored in the ice tank unless the power reference could provide the required evaporation temperature. We define this power reference as threshold power, $P_{chill,threshold}$. From (7), $P_{chill,threshold} = -\beta/\alpha$ which correspond to $T_b = 0^{\circ}C$. Same as refrigeration system, extra energy stored in the ice tank is given by:

$$Q_{chill} = L(m_{water,\max} - m_{water}) \qquad (8)$$

By defining $x_{ch} =: Q_{chill}$ and $u_{ch}(t) =: P_{chill}(t)$ as a state and input of the system respectively, we have the following state space model for describing the dynamics of thermal energy stored in the ice tank:

$$\begin{aligned} \dot{x}_{ch}(t) &= \begin{cases} B_{ch}(t)u_{ch}(t) + D_{ch}(t) & u_{ch}(t) > P_{chill,threshold} \\ 0 & u_{ch}(t) \le P_{chill,threshold} \end{cases} \\ B_{ch}(t) &= -\alpha(UA)_{brine-water} \\ D_{ch}(t) &= -\beta(UA)_{brine-water} \end{aligned} \qquad (9)$$

where B_{ch} and D_{ch} are time-varying as they are state-dependent according to (6). Since the ice storage is isolated, the heat loss to the surrounding is rather small compared to the refrigerator. We assume the heat loss to be zero for ice storage.

IV. OPTIMIZATION PROBLEM

As we stated in Section II, the aggregator wants to minimize the cost of flexibility between heterogeneous consumers while satisfying the power reference from the BRP. The consumers could be seen as thermal energy storages with different characteristics. Fig. 3 illustrates a symbolic representation of the consumers:

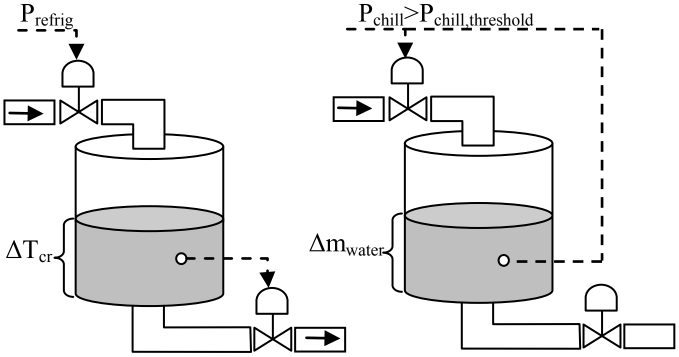

Fig. 3. Symbolic representation of cold storage in supermarket (right) and ice storage (left)

A cold room in a supermarket could be seen as a storage that has a time varying drainage which depends on the state of the charge of the tank. An ice tank is like a storage without any drainage, however the opening degree of the inlet valve depends on the state of the charge. The inlet valve is totally closed for $P_{chill}(t) < P_{chill,threshold}$. Minimizing the cost of flexibility is equivalent to maximizing the total amount of energy stored in both thermal storages. We consider the situation when we want to store some amount of extra energy in two storages. The more energy can be stored during the activation, the more it can be regained after the activation time. In other words, the aggregator will benefit more by keeping the devices off and using the stored energy after the activation. Therefore, the optimization problem at aggregator can be formulated as below. Power consumption of the chiller and supermarket refrigeration system, $u_{ch}(t), u_r(t)$, are the manipulated variables.

$$\begin{aligned} &\max_{u_r^d, u_{ch}^d} \quad \left(x_r^d(N) + x_{ch}^d(N)\right) \\ &subject \quad to \\ &x_r^d(t+1) = A_r^d x_r^d(t) + B_r^d u_r^d(t) + D_r^d \\ &x_{ch}^d(t+1) = x_{ch}^d(t) + (B_{ch}^d(t)u_{ch}^d(t) + D_{ch}^d(t))\delta(t) \qquad (10) \\ &0 \le u_r^d(t) \le u_{r,\max}, \quad 0 \le u_{ch}^d(t) \le u_{ch,\max} \\ &0 \le x_r^d(t) \le m_{food} c_{p,food}(T_{cr,\max} - T_{cr,\min}) \\ &0 \le x_{ch}^d(t) \le L m_{water,\max} \\ &\alpha u_{ch}^d(t) + \beta \le T_{b,\max}, \quad u_{ch}^d(t) + u_r^d(t) = P_{reference}^d(t) \\ &\delta(t) = \{0,1\} \end{aligned}$$

The superscript d indicates the correspondent discrete value of system states and parameters. To satisfy the cooling load from the building for chiller system, we define a constraint on maximum brine temperature, $T_{b,max}$. N is the prediction horizon for MPC controller which is time-varying in this case. Initial value for N is equal to the duration of activation time. This value decreases by one at each time instant. As we can see, the optimization problem is a mixed integer problem because of the integer value, δ. To convert the problem to a linear convex problem, we apply a method proposed by Bemporad and Morari in [11]. Then the convex optimization problem has the following form, where z(t) is the auxiliary variable and ε is a small tolerance.

$$\begin{aligned}
&\max_{u_r^d, u_{ch}^d} \quad \left(x_r^d(N) + x_{ch}^d(N)\right) \\
&\textit{subject} \quad \textit{to} \\
&x_r^d(t+1) = A_r^d x_r^d(t) + B_r^d u_r^d(t) + D_r^d \\
&x_{ch}^d(t+1) = x_{ch}^d(t) + B_{ch}^d(t) z(t) + D_{ch}^d(t)\delta(t) \\
&0 \le u_r^d(t) \le u_{r,\max}, \quad 0 \le u_{ch}^d(t) \le u_{ch,\max} \\
&0 \le x_r^d(t) \le m_{food} c_{p,food}(T_{cr,\max} - T_{cr,\min}) \\
&0 \le x_{ch}^d(t) \le L m_{water,\max}, \quad \alpha u_{ch}^d(t) + \beta \le T_{b,\max} \\
&u_{ch}^d(t) + u_r^d(t) = P_{reference}^d(t) \\
&-(u_{ch,\max} - P_{chill,threshold} + \varepsilon)\delta(t) \le -u_{ch}^d(t) + P_{chill,threshold} - \varepsilon \\
&P_{chill,threshold}\delta(t) \le u_{ch}^d(t), \quad z(t) \le u_{ch}^d(t), \quad z(t) \le u_{ch,\max}\delta(t) \\
&z(t) \ge u_{ch}^d(t) - u_{ch,\max}(1-\delta(t)), \quad z(t) \ge 0, \quad \delta(t) = \{0,1\}
\end{aligned} \tag{11}$$

V. SIMULATION RESULTS

In this section, simulation results for a control architecture consisting of one cold storage which contains of frozen meat in a supermarket and one ice storage under the control of an aggregator are provided. Numerical values of system parameters for simulation are listed below:

supermarket		***chiller***	
m_{food}	*200kg*	$m_{water,max}$	*500kg*
$c_{p,food}$	*2.01kJ/kg°C*	L	*334kJ/kg*
$(UA)_{amb\text{-}cr}$	*0.3kW/°C*	R_0	*1°C/kW*
COP_{refrig}	*3*	R_1	*20°C/kW*
$T_{cr,min}$	*-20°C*	α	*-3°C/kW*
$T_{cr,max}$	*-10°C*	β	*15°C*
$u_{r,max}$	*10kW*	$u_{ch,max}$	*10kW*
T_{amb}	*15°C*	$T_{b,max}$	*8°C*

Table 1. Parameters used for simulation

We assume the initial temperature of cold room and the initial mass of water in the tank are $T_{cr,int}=-10^{\circ}C$ and $m_{water,int}=500kg$ respectively. Activation time is chosen to be 1hour. The following figures show the allocated power and energy stored in thermal storages for three different numbers of power references: $P_{reference}(t)=5.2kW$, $P_{reference}(t)=5.8kW$ and $P_{reference}(t)=13.5kW$.

Before activation, power consumption for supermarket, $P_{refrig}(t)=2.5kW$ which is needed to maintain the temperature of cold room at $T_{cr}=-10^{\circ}C$. Chiller also consumes $P_{chill}=2.3kW$ to provide $Tb=8^{\circ}C$ which is the maximum brine temperature that is needed to meet cooling load from the building. As we can see in Fig. 4, for $P_{reference}(t)=5.2kW$, extra energy is just stored in the cold storage and no energy is stored in the ice tank. Generally, for low power reference, since the heat load to the surrounding in cold storage is rather low and on the other hand, power reference in not large enough to produce ice, aggregator utilizes supermarket exclusively to store energy. When the power reference increases, heat load in the cold storage increases. Hence, the more power dedicated to the chiller, the more energy can be stored as the there is no loss in the ice tank. However for $P_{reference}(t)=5.8kW$, the power is not sufficient to provide the threshold power. Supermarket needs to consume at least *2.5kW* unless the constraint on cold room temperature will be violated. Therefore, in this case, the aggregator first devotes all power (except the power needed to satisfy the cooling load from the building) to the supermarket. This causes the cold room temperature decreases. After that, chiller consumes the whole power while the power consumption of supermarket is equal to zero. As soon as the cold room temperature reaches the maximum value, the aggregator switches to the supermarket again (Fig. 5).

For $P_{reference}(t)=13.5kW$, both storages are utilized simultaneously. In this case, more power is dedicated to chiller system. The aggregator allocates 10kW to chiller which is the maximum power consumption of the chiller system.

VI. CONCLUSION

We have presented a hierarchical control architecture to utilize the flexibility of the industrial consumers in the future smart grid. By aggregating heterogeneous consumers under the direct jurisdiction of an MPC-based controller, downward regulating power can be provided for a specific time horizon. Two case studies have been investigated in this paper, namely the supermarket and the chiller with ice storage. Simulation results show that different strategies will be applied in order to distribute power between the consumers in an optimal way.

It should be noted that, in this paper, the down regulating scenario has only been studied. We assume the temperature of the cold room is kept at the maximum level and the ice tank has enough capacity to store more ice. This is a reasonable assumption since it leads to less energy consumption. Considering this assumption, the aggregator is always able to provide downward regulating power within the agreed level. If the aggregator fails to fulfill the agreements, it should pay penalty to the BRP. BRP's action to compensate the resulting mismatch is not the subject of this paper. In case of up regulating, consumers need to change their set points such that they will be able to reduce their consumption whenever they are activated. From optimization point of view, this case is similar to the down regulating except that here, the aggregator needs to minimize the cost of flexibility before the activation time and the power references should be distributed during this time. During the activation, the consumers will turn off their devices. To achieve this, the aggregator requires a good prediction of the time of activation in order to optimize the energy consumption. This will be studied for the future work.

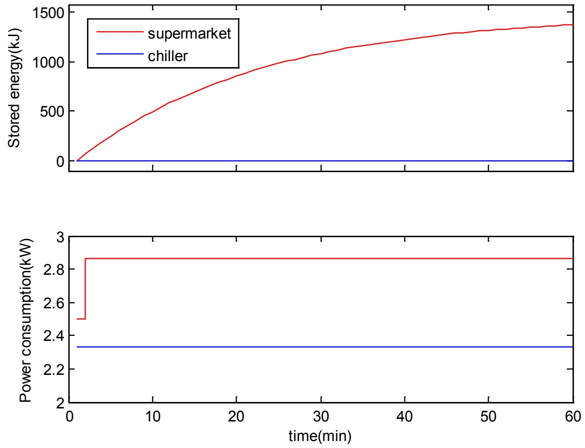

Fig. 4. Lower: power consumption, Upper: stored energy, $P_{reference}(t)=5.2kW$

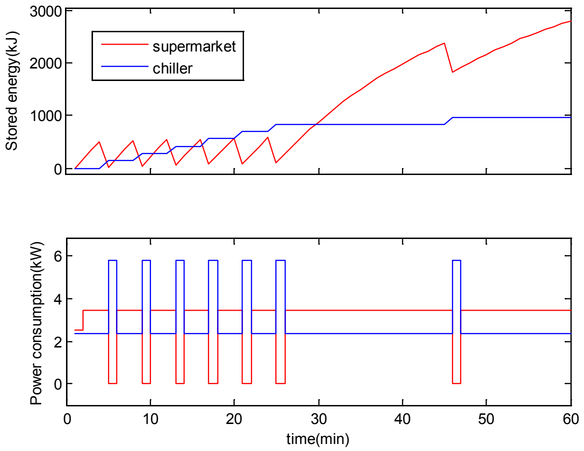

Fig. 5. Lower: power consumption, Upper: stored energy, $P_{reference}(t)=5.8kW$

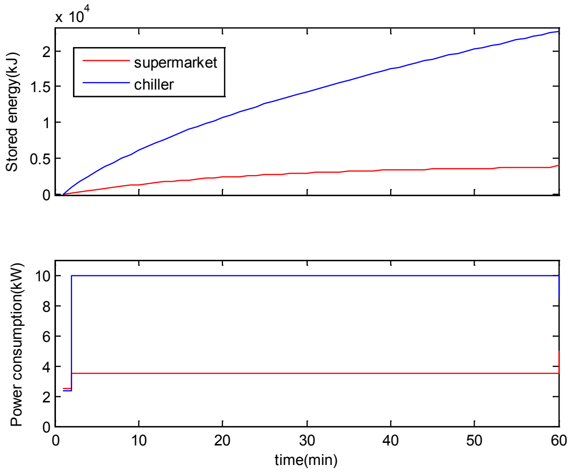

Fig. 6. Lower: power consumption, Upper: stored energy, $P_{reference}(t)=13.5kW$

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