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An interaction measure for control configuration selection for multivariable bilinear systems

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Abstract Control configuration selection is the procedure of choosing the appropriate input and output pairs for the design of decoupled (SISO or block) controllers for multivariable systems. This step is an important prerequisite for a successful industrial control strategy. In industrial practice it is often the case that systems which need to be controlled are non-linear, and linear models are insufficient to describe the behavior of the processes. The focus of this paper is on the problem of control configuration selection for a class of non-linear systems which is known as bilinear systems. A gramian-based interaction measure for control configuration selection of MIMO bilinear processes is described. In general, most of the results on the control configuration selection, which have been proposed so far, can only support linear systems. The proposed gramian-based interaction measure not only supports bilinear processes but also can be used to propose a richer sparse or block diagonal controller structure. The method is illustrated further with the help of some illustrative examples.

Keywords Process control · Bilinear systems · Interaction measure

1 Introduction

The technological world of today has been witnessing the increased complexity due to the rapid development of the process plants and the manufacturing processes. The computational complexity, the reliability problems and the restrictions in communication make the centralized control of such large-scale complex systems expensive and difficult.

To cope with these problems, several decentralized control structures have been introduced and implemented over the last few decades [1]. The decentralized controllers have several advantages, which make them popular in industry. The decentralized controllers are easy to understand for operators, easy to implement and to re-tune [1, 2].

The decentralized control systems design is a two-step procedure. The controller structure selection and input–output pairing is the first main step and the controller synthesis for each channel is the second step of the decentralized control. The focus of this paper is on pairing and the controller structure selection of the decentralized control systems. This issue is a key problem in the design of decentralized and distributed control systems, which directly affects the stability and the performance of the control systems. The interaction measures play an important role in the

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suitable pairing and the controller structure selection for the decentralized and the distributed control. Interaction measures make it possible to study input–output interactions and to partition a process into subsystems in order to reduce the coupling, to facilitate the control and to achieve a satisfactory performance. The interaction measures have received a lot of attention over the last few decades [2, 3]. There are two broad categories of interaction measures in the literature. The first category is the relative gain array (RGA) and its related indices [4–9] and the second category is the family of the gramian-based interaction measures [10–13].

The most well-known and commonly used interaction measure is the relative gain array (RGA), which was first proposed in [4]. In the RGA, d.c. gain of the process is used for the construction of the channel interaction measure. The RGA is not sensitive to delays and more importantly it considers the process just at a particular frequency.

The RGA has been studied by several other researchers (see, e.g. [5, 6]). There are also other similar measures of interaction, which use dc gain of the process e.g. the NI (the Niederlinski index) [7].

The NI does not provide more information for pairing compared to RGA. The RGA and the NI have been extended for input–output pairing of unstable MIMO systems in [2]. The relative interaction array (RIA) is an interaction measure, which is similar to RGA and it is based on considering the interaction as an unmodeled term at d.c.

RIA does not provide more information than the RGA about the channel interactions of the process. These indices use the model of the processes at zero frequency. In [6, 8], the relative dynamic gain array (RDGA) was proposed for the first time. The RDGA shows how the interaction varies over the frequency. The idea is further generalized in [9] by the generalized relative dynamic gains (GRDG). This method was mainly proposed for 2×2 systems.

The second category of interaction measures is the family of gramian-based methods. A method from this category was first proposed in [10] and further in [11]. In this category, the observability and the controllability gramians are used to form the Participation Matrix (PM). The elements of the PM encode the information of the channel interactions. PM is used for pairing and the controller structure selection. The Hankel Interaction Index Array (HIIA) is a similar interaction measure, which was proposed in [12]. The gramian-based interaction measures have several advantages over the interaction measures in the RGA category. The gramian-based interaction measures take the whole frequency range into account rather than a single frequency. This family of the interaction measures suggests more suitable pairing and allows more complicated controller structures. For more details regarding the applications and the differences between two main categories of the interaction measures, see [11–14].

The gramian-based interaction measures have proven to be very useful measures, which encode more information on channel interactions compared to the family of RGA interaction measures. The results on the gramian-based interaction measures, which have been proposed so far, only support linear systems. However, in many applications, linear models are often insufficient to describe the behavior of the processes. On the other hand, due to the complexity of non-linear systems, methods for analyzing non-linear systems or synthesizing their controllers are not as well developed as their linear counterparts. This is the main reason that the interaction measures and controller structure selection issue are not studied extensively for non-linear systems compared to linear systems.

Bilinear systems constitute an important class of non-linear systems which comprise perhaps the simplest class of non-linear models with a lot of practical applications. Bilinear systems enjoy well-established theories and find applications in the variety of fields to describe the practical processes ranging from electrical networks, hydraulic systems to heat transfer, and chemical processes [20]. Moreover, many highly non-linear systems may be modeled as bilinear systems with appropriate state feedback or can be approximated as bilinear systems in the so-called bilinearization process see e.g. [21, 22, 26] and [29].

In this paper, the gramian-based interaction measure is extended to support bilinear processes. The proposed interaction measure is used for pairing and the controller structure selection.

The paper is organized as follows. In the next section, we review the concept of the gramians for bilinear systems. The interpretation of the controllability and observability gramians is also discussed in this section. Section 3 presents how gramians can be used to quantify the channel interactions for bilinear systems. The application of the proposed interaction measure in pairing and the controller structure selection is explained in this section. Section 4 presents the extension of the proposed interaction measure for possible non-square bilinear systems. In Sect. 5, the proposed method is illustrated further with the help of some illustrative examples including a bilinear model of a hydraulic rotary multi-motor system. Section 6 concludes the paper.

The notation used in this paper is as follows: M^* denotes the transpose of matrix if $M \in \mathbb{R}^{n\times m}$ and complex conjugate transpose if $M \in \mathbb{C}^{n\times m}$. The standard notation $>, \geq$ $(<, \leq)$ is used to denote the positive (negative) definite and semidefinite ordering of matrices. $Struc(\Pi)$ denotes the structure of a MIMO system Π. For a $m \times p$ MIMO system Π with input $u(t) \in \mathbb{R}^m$ and output $y(t) \in \mathbb{R}^p$, $Struc(\Pi) = [\pi_{ij}]_{p\times m}$ is a symbolic array where $\pi_{ij} = *$, if there exists a subsystem in Π with input u_j and output y_i. Otherwise: $\pi_{ij} = 0$.

2 Controllability and observability gramians

The controllability and the observability gramians are well-known matrices, which are widely used to check the controllability and the observability of dynamical systems. The gramians are also widely used in the process of model order reduction [15, 16]. The controllability and observability gramians show how difficult a system is to control and to observe, respectively. This is an interesting feature which makes gramians very useful for pairing and for the control structure selection.

For a discrete-time bilinear dynamical system which is described by

$$\Pi : \begin{cases} x(k+1) = Ax(k) + \sum_{j=1}^{m} N_j x(k) u_j(k) \\ \qquad\qquad + Bu(k), \\ y(k) = Cx(k). \end{cases} \tag{1}$$

Here $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^p$, the controllability gramian P is defined as [17–19]:

$$P := \sum_{i=1}^{\infty} \sum_{k_i=0}^{\infty} \cdots \sum_{k_1=0}^{\infty} P_i P_i^*, \tag{2}$$

where

$$\begin{aligned} &P_1(k_1) = A^{k1} B, \\ &P_i(k_1, \ldots, k_i) \\ &\quad = A^{k_i} \begin{bmatrix} N_1 P_{i-1} & N_2 P_{i-1} & \cdots & N_m P_{i-1} \end{bmatrix}, \end{aligned} \tag{3}$$

and the observability gramian Q is defined as

$$Q := \sum_{i=1}^{\infty} \sum_{k_i=0}^{\infty} \cdots \sum_{k_1=0}^{\infty} Q_i^* Q_i, \tag{4}$$

where

$$\begin{aligned} &Q_1(k_1) = CA^{k1}, \\ &Q_i(k_1, \ldots, k_i) = \begin{bmatrix} Q_{i-1}N_1 \\ Q_{i-1}N_2 \\ \vdots \\ Q_{i-1}N_m \end{bmatrix} A^{k_i}. \end{aligned} \tag{5}$$

The gramians are given by the solutions of the generalized Lyapunov equations [19]:

$$\begin{aligned} &APA^* - P + \sum_{j=1}^{m} N_j P N_j^* + BB^* = 0, \\ &A^*QA - Q + \sum_{j=1}^{m} N_j^* Q N_j + C^*C = 0. \end{aligned} \tag{6}$$

The generalized Lyapunov equations can be solved iteratively. The controllability gramian P can be obtained from [19]:

$$P = \lim_{i\to\infty} \hat{P}_i \tag{7}$$

where

$$\begin{aligned} &A\hat{P}_1 A^* - \hat{P}_1 + BB^* = 0, \\ &A\hat{P}_i A^* - \hat{P}_i + \sum_{j=1}^{m} N_j \hat{P}_{i-1} N_j^* + BB^* = 0, \\ &\quad i = 2, 3, \ldots. \end{aligned} \tag{8}$$

In other words, all what we need to do is to run (8) and to compute $\hat{P}_i$'s and terminate the computation when $\hat{P}_i$ converges to a constant matrix. Dually the observability gramian can be computed [19]. The controllability and observability gramians are defined analogously for continuous-time systems. Let Π be a continuous-time bilinear system described by

$$\Pi : \begin{cases} \dot{x}(t) = Ax(t) + \sum_{j=1}^{m} N_j x(t) u_j(t) \\ \qquad + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{9}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, the controllability gramian P is defined as [30]

$$P := \sum_{i=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} P_i P_i^* \, dt_1 \cdots dt_i, \tag{10}$$

where

$$\begin{aligned} &P_1(t_1) = e^{At_1} B \\ &P_i(t_1, \ldots, t_i) \\ &\quad = e^{At_i} \begin{bmatrix} N_1 P_{i-1} & N_2 P_{i-1} & \ldots & N_m P_{i-1} \end{bmatrix} \end{aligned} \tag{11}$$

and the observability gramian is defined as

$$Q := \sum_{i=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} Q_i^* Q_i \, dt_1 \cdots dt_i, \tag{12}$$

where

$$\begin{aligned} &Q_1(t_1) = Ce^{At_1}, \\ &Q_i(t_1, \ldots, t_i) = \begin{bmatrix} Q_{i-1} N_1 \\ Q_{i-1} N_2 \\ \vdots \\ Q_{i-1} N_m \end{bmatrix} e^{At_i}. \end{aligned} \tag{13}$$

These gramians are given by the solutions of the generalized Lyapunov equations:

$$AP + PA^* + \sum_{j=1}^{m} N_j P N_j^* + BB^* = 0, \tag{14}$$

$$A^* Q + AQ + \sum_{j=1}^{m} N_j^* Q N_j + C^* C = 0. \tag{15}$$

Analogous to discrete-time case, the generalized Lyapunov equations can be solved iteratively. More details on gramians and its interpretations and computations are discussed in [19] and [30].

3 Interaction measure

In this section, an interaction measure for the square discrete-time and continuous-time bilinear MIMO processes (Fig. 1) is built upon the notion of the gramians. The trace of the cross gramian is used as a convenient basis to present the channel interaction and to select the most appropriate controller structure.

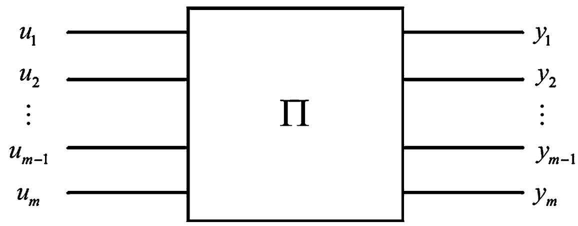

Fig. 1 MIMO system with input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^m$

For a square MIMO bilinear system Π with representation (1), we have

$$\begin{aligned} &B = [b_1 \quad b_2 \quad \ldots \quad b_m], \\ &C^* = [c_1 \quad c_2 \quad \ldots \quad c_m]. \end{aligned} \tag{16}$$

A set of elementary SISO systems can be associated to the MIMO system, such that each SISO system has a single input $u_j(t)$ and single output $y_i(t)$. The state-space representation of each elementary system is given by

$$\Pi_{ij} : \begin{cases} x(k+1) = Ax(k) + N_j x(k) u_j(k) + b_j u_j(k), \\ y_i(k) = c_i^* x(k), \end{cases} \tag{17}$$

with gramians P_j and Q_i. The controllability gramian P_j and the observability gramian Q_i for the elementary systems are the solutions to

$$\begin{aligned} &AP_j A^* - P_j + N_j P_j N_j^* + b_j^* b_j = 0, \\ &A^* Q_i A - Q_i + N_j^* Q_i N_j + c_i^* c_i = 0. \end{aligned} \tag{18}$$

Following a similar procedure for a continuous-time MIMO bilinear system Π with representation (9), a set of elementary SISO systems can be associated to the system:

$$\Pi_{ij} : \begin{cases} \dot{x}(t) = Ax(t) + N_j x(t) u_j(t) + b_j u_j(t), \\ y_i(t) = c_i^* x(t). \end{cases} \tag{19}$$

The controllability gramian P_j and the observability gramian Q_i for the elementary systems are the solutions to

$$
\begin{aligned}
&AP_j + P_jA^* + N_jP_jN_j^* + b_jb_j^* = 0,\\
&A^*Q_i + AQ_i + N_j^*Q_iN_j + c_i^*c_i = 0.
\end{aligned}
\tag{20}
$$

The information of the channel interaction which is obtained from the gramians of the elementary systems is encompassed into the so-called participation matrix (PM):

$$\Psi = [\psi_{ij}] \in \mathbb{R}^{m\times m}, \tag{21}$$

where

$$\psi_{ij} = \frac{trace[P_jQ_i]}{\sum_{i=1}^{m}\sum_{j=1}^{m} trace[P_jQ_i]}. \tag{22}$$

Note that $0 \le \psi_{ij} < 1$ and $\sum_{i=1}^{m}\sum_{j=1}^{m}\psi_{ij} = 1$.

The participation matrix highlights the elementary subsystems, which are more important in the description of MIMO systems, and in this way it shows the suitable pairing and the appropriate controller structure to select.

For pairing and the controller structure selection, the structure of the nominal system Π_n needs to be obtained. The nominal model is a model, which is obtained by keeping some of the elementary subsystems of the actual MIMO process and ignoring the rest. For example, assume that one of the ordinary methods for pairing is used and a decentralized control is synthesized. If the inputs and outputs are re-labeled, one only needs to design m independent SISO controller loops, for elementary diagonal subsystems. In this case:

$$Struc(\Pi_o) = \begin{bmatrix} * & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & * \end{bmatrix}_{m\times m}. \tag{23}$$

For the designed controller C we have

$$Struc(C) = \begin{bmatrix} * & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & * \end{bmatrix}_{m\times m}. \tag{24}$$

The elements of the PM show which elementary subsystems are significant and should be considered in the nominal model. When ψ_{ij} is small, the associated elementary subsystem to the pair (i, j) is either hard to control or hard to observe. This shows that this subsystem does not have any significant effect in the actual input–output relation and could be ignored in the nominal model. When ψ_{ij} is larger than $1/m^2$, some states in the elementary system with output y_j and input u_j are easy to control and easy to observe and therefore Π_{ij} is a good candidate to be kept in the nominal system. The suitability of the pairing and the performance of the controller structure highly depend on how close the sum of the chosen ψ_{ij} elements is to one. When the sum of the chosen ψ_{ij} elements is close to one, the nominal and the actual model are close to each other and the error is not significant. The complexity of the selected controller structure depends on the number of the ψ_{ij} elements. In the completely decentralized control, which is the least complicated controller structure, the number of the chosen elements would be m.

For example consider a 3×3 process model with PM:

$$\Psi = \begin{bmatrix} 0.1833 & 0.1685 & 0.0861\\ 0.1200 & 0.0445 & 0.1783\\ 0.0639 & 0.0691 & 0.0863 \end{bmatrix}.$$

To pair inputs and outputs for decentralized control structure, we have to select one element per row and one element per column. $\psi_{11}, \psi_{12}, \psi_{21} > 1/m^2$, therefore their associated elementary subsystems are good candidates to be involved in the nominal model. However, the best paring for a decentralized controller can be obtained with (u_1, y_1), (u_2, y_3), (u_3, y_2) which are associated with

$$\Sigma = \psi_{11} + \psi_{23} + \psi_{32} = 0.4307.$$

The structure of the nominal model is

$$Struc(\Pi_o) = \begin{bmatrix} * & 0 & 0\\ 0 & 0 & *\\ 0 & * & 0 \end{bmatrix}.$$

A simple controller structure for selection is the structure of Π_o^{-1}:

$$Struc(C) = Struc(\Pi_o^{-1}) = \begin{bmatrix} * & 0 & 0\\ 0 & 0 & *\\ 0 & * & 0 \end{bmatrix}.$$

If practically it is possible to use more complicated control structures than completely decentralized control, y_1 could be commanded from u_2 and then we will have

$$\Sigma = \psi_{11} + \psi_{12} + \psi_{23} + \psi_{32} = 0.599.$$

The structure of the nominal model then will be

$$Struc(\Pi_o) = \begin{bmatrix} * & * & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{bmatrix}.$$

A simple controller structure to select:

$$Struc(C) = Struc\left(\Pi_o^{-1}\right) = \begin{bmatrix} * & 0 & * \\ 0 & 0 & * \\ 0 & * & 0 \end{bmatrix}.$$

In this case the structure is partially decentralized.

4 Pairing and the controller structure for non-square systems

Non-square systems are multivariable systems with an unequal number of inputs and outputs. Non-square systems are divided into two classes: systems with more inputs than outputs and systems with more outputs than inputs. Although non-square plants are often encountered in many engineering disciplines, the theory for analysis and control of non-square plants even for linear systems is not well developed.

The main strategy for control of non-square processes is the squaring strategy. That is, the necessary number of outputs or inputs are added or deleted from the system to obtain a square plant. Then, the well-established control methods can be used for the non-square processes. However, this has its own problems. Introducing some more outputs and inputs results in more costs and maintenance issues and ignoring some of the inputs will certainly degrade the degrees of freedom for achieving the desired response. If we delete some outputs, it degrades the reliability of our measurements. For non-square linear systems with more inputs than outputs, the control considering the non-square system structure leads to higher performance compared with the squared case [31, 32]. For non-square systems with more outputs than inputs, the notion of perfect control in the least squares sense similar to the one proposed in [32] can be derived. However, a natural extension of our proposed interaction measure to non-square bilinear system is presented in the sequel.

For a non-square MIMO bilinear system with representation (1), we have

$$\begin{aligned} B &= [b_1 \quad b_2 \quad \dots \quad b_m], \\ C^* &= [c_1 \quad c_2 \quad \dots \quad c_p]. \end{aligned} \tag{25}$$

If we associate a set of elementary SISO systems to this MIMO system such that each SISO system has a single input $u_j(t)$ and single output $y_i(t)$, the state-space representation of each elementary system will be

$$\Pi_{ij} : \begin{cases} x(k+1) = Ax(k) + N_j x(k) u_j(k) + b_j u_j(k), \\ y_i(k) = c_i^* x(k). \end{cases}$$

Assume that the controllability and the observability gramians for these subsystems are P_j and Q_i.

The non-square PM is defined as

$$\Psi = [\psi_{ij}] \in \mathbb{R}^{m \times p}, \tag{26}$$

where

$$\psi_{ij} = \frac{trace[P_j Q_i]}{\sum_{i=1}^{p} \sum_{j=1}^{m} trace[P_j Q_i]}. \tag{27}$$

The criterion for the control structure selection based on this PM is similar to the one of the square system. The only difference is that if the structure of the nominal model is $Struc(\Pi_o)$, a simple non-square suggested structure for the controller C will be

$$Struc(C) = Struc\left(\Pi_o^{+}\right), \tag{28}$$

where "+" denotes the Moore–Penrose pseudo inverse. The procedure is similar for continuous-time bilinear systems.

5 Pairing and the controller structure selection: illustrative examples

In this section, three numerical examples are used to illustrate the proposed controller structure selection method. The first example is a square continuous-time bilinear model, the second is a non-square continuous-time bilinear model and the last one is a discrete-time bilinear model of a multi-motor hydraulic system.

5.1 Continuous-time bilinear model

Consider a continuous-time bilinear model with the following standard state-space representation:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{j=1}^{3} N_j x(t) u_j(t) + Bu(t), \\ y(t) = Cx(t). \end{cases} \tag{29}$$

Here

$$A = \begin{bmatrix} -30 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -15 \end{bmatrix},$$

$$N_1 = N_2 = \begin{bmatrix} 0 & -0.07 & 0 \\ 30 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} 0 & 0 & -0.07 \\ 0 & 0 & 0 \\ 30 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 50 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Solving the Lyapunov equation for the subsystems, the PM is computed as

$$\Psi = \begin{pmatrix} 0.00133 & 0.219 & 0.744 \\ 0.000000133 & 0.00219 & 0.0000595 \\ 0.000000648 & 0.0000857 & 0.033 \end{pmatrix}.$$

The structure of nominal model which this PM suggests for a decentralized control is

$$Struc(\Pi_o) = \begin{bmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & 0 \end{bmatrix}.$$

This structure is associated to $\Sigma = \psi_{22} + \psi_{31} + \psi_{13} = 0.7461$ and a simple controller which is suggested for this is

$$Struc(C) = \begin{bmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & 0 \end{bmatrix}.$$

A more complicated model structure is

$$Struc(\Pi_o) = \begin{bmatrix} 0 & * & * \\ 0 & * & 0 \\ * & 0 & 0 \end{bmatrix}$$

which is associated to $\Sigma = \psi_{22} + \psi_{31} + \psi_{13} + \psi_{12} = 0.9655$ and a simple controller which is suggested for this is

$$Struc(C) = \begin{bmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & * & 0 \end{bmatrix}.$$

5.2 Non-square bilinear model

Consider a non-square bilinear model with three inputs and two outputs and the following standard state-space representation:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{j=1}^{3} N_j x(t) u_j(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{30}$$

where

$$A = \begin{bmatrix} -30 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -15 \end{bmatrix},$$

$$N_1 = N_2 = \begin{bmatrix} 0 & -0.07 & 0 \\ 30 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} 0 & 0 & -0.07 \\ 0 & 0 & 0 \\ 30 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 50 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0.1 & 0 \end{bmatrix}.$$

The non-square PM for this system is

$$\Psi = \begin{pmatrix} 0.000274 & 0.0452 & 0.16 \\ 0.0011 & 0.181 & 0.613 \end{pmatrix}.$$

The nominal model which this PM suggests for a decentralized control is

$$Struc(\Pi_o) = \begin{pmatrix} 0 & * & 0 \\ 0 & 0 & * \end{pmatrix},$$

and therefore a simple controller which is suggested for this is

$$Struc(C) = Struc(\Pi_o^+) = \begin{pmatrix} 0 & 0 \\ * & 0 \\ 0 & * \end{pmatrix}.$$

A more complicated model structure according to PM is

$$Struc(\Pi_o) = \begin{pmatrix} 0 & * & 0 \\ 0 & * & * \end{pmatrix}$$

and therefore a simple controller which is suggested for this is

$$Struc(C) = Struc(\Pi_o^+) = \begin{pmatrix} 0 & 0 \\ * & 0 \\ * & * \end{pmatrix}.$$

5.3 Discrete-time bilinear model

In the sequel, the proposed interaction measure is used for pairing and the controller structure selection for a discrete-time bilinear model for a hydraulic multi-motor rotary system. In general, hydraulic systems are highly non-linear dynamical systems; see, e.g. [27, 28]. The linear models are not sufficiently accurate to describe them and consequently the controllers which are designed based on the linear models of the hydraulic systems quite often do not give satisfying results in practice. On the other hand, due to the complexity of the highly non-linear hydraulic models, methods for analyzing them or synthesizing their controllers are not well developed and often they are difficult to apply in practice. In between the spectrum of different models to describe a hydraulic system from linear model to highly non-linear models, the bilinear models often offer an adequately accurate model with a well-developed theory for the analysis and control [23–25].

Consider a continuous-time bilinear model of the hydraulic rotary multi-motor system in [23]. The model is discretized by Euler's forward discretization method with the sampling time $T_s = 0.001$. One should note that in general we do not have to discretize the model as we have developed the method for the continuous-time case in previous sections. However, to illustrate the method and to see how the method works for discrete-time systems, discretization is performed to obtain a discrete-time example. The discrete-time bilinear model is

$$\Pi : \begin{cases} x(k+1) = Ax(k) + \sum_{j=1}^{3} N_j x(k) u_j(k) \\ \qquad\qquad + Bu(k), \\ y(k) = Cx(k), \end{cases} \tag{31}$$

where

$$A = \begin{bmatrix} 0.99997 & 0 & 0 \\ 0 & -0.99997 & 0 \\ 0 & 0 & 0.99997 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0 & -0.00007 & 0 \\ 0.03 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} 0 & 0 & -0.00007 \\ 0 & 0 & 0 \\ 0.03 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 10 \end{bmatrix}.$$

The participation matrix (PM) for this bilinear system is obtained using the proposed method:

$$\Psi = \begin{pmatrix} 0.0985 & 0.000399 & 0.00111 \\ 0.443 & 0.0018 & 0.00499 \\ 0.443 & 0.0018 & 0.00501. \end{pmatrix}$$

The ψ_{21}, ψ_{31} entries are significant compared to other entries. The Σ associated to the best possible pairing for the decentralized control is

$$\Sigma = \psi_{33} + \psi_{12} + \psi_{21} = 0.4486.$$

The structure of the nominal model for this pairing will be

$$Struc(\Pi_o) = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & * \end{bmatrix}.$$

The suggested simple control structure for this pairing is

$$Struc(C) = Struc(\Pi_o^{-1}) = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & * \end{bmatrix}.$$

If it is allowed to use more complex controllers than decentralized control (partially decentralized), y_1 could be commanded from u_3 and then we have

$$\Sigma = \psi_{33} + \psi_{21} + \psi_{12} + \psi_{31} = 0.8918,$$

associated with

$$Struc(\Pi_o) = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ * & 0 & * \end{bmatrix}.$$

The simple control structure for this pairing is

$$Struc(C) = Struc\left(\Pi_o^{-1}\right) = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & * & * \end{bmatrix}.$$

The Σ for this structure is very close to one.

6 Conclusion

A gramian-based interaction measure for the decentralized control of bilinear processes is proposed in this paper. The proposed MIMO interaction measure is the extension of its gramian-based analogous counterpart, which has been proposed for decentralized input–output pairing as well as for the controller architecture selection for the linear processes. The proposed measure reveals more information about the ability of the channels to be controlled and to be observed and provides hints for the selection of the richer controller structures such as triangular, sparse and block diagonal.

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