Lumped Thermal Household Model

Benjamin Biegel,
Palle Andersen, and
Jakob Stoustrup
Department of Electronic Systems
Aalborg University, Denmark
Email: {bbi,jakob,pa}@es.aau.dk

Mathias Bækdal Madsen
Neogrid Technologies
Aalborg, Denmark
E-mail: mbm@neogrid.dk

Lars Henrik Hansen
DONG Energy A/S
Copenhagen, Denmark
E-mail: larha@dongenergy.dk

***Abstract*—In this paper we discuss two different approaches to model the flexible power consumption of heat pump heated households: individual household modeling and lumped modeling. We illustrate that a benefit of individual modeling is that we can overview and optimize the complete flexibility of a heat pump portfolio. Following, we illustrate two disadvantages of individual models, namely that it requires much computational effort to optimize over a large portfolio, and second that it is difficult to accurately model the houses in certain time periods due to local disturbances. Finally, we propose a lumped model approach as an alternative to the individual models. In the lumped model, the portfolio is seen as baseline consumption superimposed with an ideal storage of limited power and energy capacity. The benefit of such a lumped model is that the computational effort of flexibility optimization is significantly reduced. Further, the individual disturbances will smooth out as the number of houses in the portfolio increases.**

I. Introduction

The use of heat pumps is expected to increase in the foreseeable future throughout the developed world, due to its high efficiency and ability to utilize the inexhaustible and renewable ambient ground or air heat. In the future Danish electricity system it is expected that domestic heat pumps will play an important role as flexible consumption: already now, around $27,000$ heat pumps are installed in Denmark [1], and potentially $205,000$ households can benefit from replacing oil-fired boilers with heat pumps in the coming years [2]. It is therefore most relevant to consider how to aggregate and control this flexibility towards the electricity markets.

Control of smaller flexible consumers to support grid stability has been discussed as early as the 1980s [3]. Since, the topic of demand-side management has received much attention from a research perspective including control of heat pumps [4], [5]. In particular, optimization of heat pumps has received much attention in Denmark the last few years, see [6], [7], [8], [9], [10].

Adequate flexibility models are a crucial element in the control of flexible devices. In many works, such flexibility models are used to design appropriate control strategies for controlling heat pumps, see e.g. [11], [7], [9], [12]. Several studies demonstrate how to construct such household models: In [13], an individual heat pump model is successfully constructed based on real life experiments conducted in a laboratory setting. In [14], a custom built house is modeled successfully with a linear model for an inhabited household. Other works construct household models based on inhabited households, see [8], [15].

In this work, we argue that an alternative to such individual household models is to utilize a lumped model that represents an *entire portfolio* of households. Two main arguments for proposing this lumped model are as follows. The first reason is that a lumped flexibility model has the clear advantage that the computational efforts of flexibility optimization decreases drastically by comparison with individual models. The second reason is that the many disturbances to some extent will cancel out as the number of households in the portfolio increases, reducing the disturbances seen in the lumped model. Such a lumped model approach is described for example in [16], [17], [18], but is only dealt with via simulations. In this work we use real life data to motivate the use of such a lumped model.

First, we use data from inhabited Danish houses heated with heat pumps to illustrate that local disturbances at the households can be large and that this may result in poor modeling results at certain times. Following, we present the lumped model as an alternative approach. The basic concept is to consider a portfolio of households as an ideal storage of a given volume. Combined with the baseline consumption of the portfolio, this model can be utilized for flexibility optimization.

The paper is structured as follows. First in Sec. II we describe the heat pump platform where the data is taken from; following in Sec. III, we introduce the concept of flexibility modeling and optimization. In Sec. IV we present the individual modeling approach and show the benefits and limitations associated with this method, similarly in Sec. V, a lumped model is presented and the benefits and limitations of this method is illustrated. Finally in Sec. VI, a discussion of the two methods is presented and in Sec. VII we conclude the work.

II. Real Life Heat Pump Demonstration Setup

In this section, we describe the platform of households with heat pumps used as data source in this study.

A. Heterogeneous Households Portfolio

The platform called *Styr din varmepumpe* (meaning: Control your heat pump) consists of 300 households with heat pump heating [8]. The houses are all real life inhabited houses located in different locations in Denmark. The houses vary from smaller houses with a total area of 100 m^2 to larger houses with an area of 400 m^2. Further, the houses vary in type: some are old houses constructed in the 1850s while other houses are newly constructed.

Also the heat pumps are different; more than 50 different heat pump designs are present. Moreover, the heating systems vary much in the different houses: all the houses have a heat pump but some of the houses use underfloor heating while other have radiators. Additionally, some of the houses are equipped with other heating sources than the heat pump, for example a wood stove or solar heating. Consequently, we are dealing with a realistic real life heterogeneous household portfolio representative of typical Danish households.

B. Controlling and Monitoring the Households

The households included in this platform have all installed the heat pumps before being a part of this project. Various sensor equipment has therefore been subsequently installed. These sensors include power measurements of the heat pump, a single indoor thermometer, and an outdoor thermometer. In this project it has not been possible to remotely control the heat pumps.

The sensor data is transmitted over an Internet connection to a server. The sampling time of the communication link between heat pump and the server is 5 minutes.

III. Modeling and Optimization of Flexible Consumption

In this section, we briefly describe the purpose of a heat pump flexibility model, how such a model can be utilized, and motivate why it is interesting to examine a lumped flexibility model approach.

A. Flexibility Optimization

Heat pumps are flexible consumption devices due to the inherent thermal capacity of the houses. Consequently, it is possible to aggregate and optimize the consumption of a portfolio of heat pumps towards some given objective. Examples could be to optimize the consumption based on a price signal or based on predictions of the spot price, or it could be to provide ancillary services.

B. Flexibility Model

A flexibility model is required to perform flexibility optimization, i.e. we must know to what extent the consumption can be shifted without violating the comfort limits of the inhabitants. Such flexibility models are constructed on household level in many works, meaning that a flexibility model of each house is constructed, see e.g. [11], [7], [9], [12].

In this work we are argue that when dealing with real life households equipped with a single indoor thermometer, disturbances can in certain time periods be so severe and the available sensing is so limited, that individual flexibility modeling is difficult. Therefore we propose an alternative approach: Instead of modeling each house separately, we consider the portfolio as one entity and construct a model of the combined flexibility, i.e., we consider a lumped model. The benefit of such a lumped model is that the many disturbances will cancel out as the number of houses comprising the portfolio increases.

IV. Individual Modeling of Households

In this section, we show the concept of an individual household model and how such a model can be used for flexibility optimization. Further, we illustrate the difficulties in utilizing such a flexibility model.

A. Individual Model and Flexibility Optimization

Let i index the households, let I be the total number of houses in the portfolio, and let $\mathcal{I} = \{1, \dots, I\}$ represent the entire portfolio. A linear nth-order individual household model can be expressed as

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + C_i v_i(k) \tag{1}$$

where $x_i(k) \in \mathbf{R}^n$ is the state vector, $u_i(k) \in \mathbf{R}$ is the power input from the heat pump, and $v_i(k) \in \mathbf{R}^m$ is the disturbance inputs such as outdoor temperature, solar irradiation, wind, etc. The matrices $A_i \in \mathbf{R}^{n\times n}, B_i \in \mathbf{R}^{n\times 1}, C_i \in \mathbf{R}^{n\times m}$ represent the household dynamics. Further, the state and input limitations are modeled as follows

$$x_i(k) \in \mathcal{X}_i, \quad u_i(k) \in \mathcal{U}_i, \quad i \in \mathcal{I} \tag{2}$$

where sets $\mathcal{X}_i$, $\mathcal{U}_i, i \in \mathcal{I}$ describe the system limitations such as the power limitations of the heat pump and the thermal comfort limitations. Many of the works using individual heat pump models rely on such linear models.

A simple version of this thermal model is a first order model where the state x_i is the indoor temperature and where $\mathcal{X}_i$ represent the lower and upper allowable temperature and where u_i is the electrical power and $\mathcal{U}_i$ describe the power limitations for house number i. This simple model can be extended for example to include a state for the floor temperature or for separate rooms, etc.

We construct a small example to illustrate how such a model can be utilized for flexibility optimization. Assume we have purchased electricity at the spot market for example for the following day. We denote the purchased electricity $p_{\text{spot}}(k), k \in \mathcal{K}$ where $\mathcal{K} = \{k_1, \dots, k_2\}$ represent our horizon. Further, assume our objective is to consume what we have purchased at the spot market to avoid imbalance and thus avoid trading balancing power at possibly unfavorable prices. We can formulate this as an optimization problem

$$\begin{array}{ll} \text{min.} & \displaystyle\sum_{k\in\mathcal{K}} \left| p_{\text{spot}}(k) - \sum_{i\in\mathcal{I}} u_i(k) \right| \\ \text{s.t.} & x_i(k+1) = A_i x_i(k) + B_i u_i(k) + C_i \hat{v}_i(k) \\ & x_i(k+1) \in \mathcal{X}_i, \quad u_i(k) \in \mathcal{U}_i \\ & k \in \mathcal{K}, i \in \mathcal{I} \end{array} \tag{3}$$

where the variables are $u_i(k), x_i(k+1), i \in \mathcal{I}, k \in \mathcal{K}$ and the data is current state $x(k)$, the purchased electricity and predictions of the disturbance inputs $p_{\text{spot}}(k), \hat{v}_i(k), k \in \mathcal{K}$.

We notice two things in Problem 3: First we see that this method is able to deal with each household individually and therefore will handle the energy optimization optimally within the horizon provided the models are true and the disturbance predictions are perfect. Second, we observe that the computationally complexity grows rapidly with the number of houses I indicating that this method might not be suitable when dealing with thousands of heat pumps.

B. Individual Modeling of Inhabited Household

In the above subsection, we illustrated how household models can be utilized to optimize the flexibility towards some objective. In this subsection, we illustrate some of the difficulties of making individual flexibility models.

We use data from 40 of the houses in the available platform and attempt to fit different models including 1st and 2nd order linear models. The heat pump power is taken as input and the indoor temperature as output. Different disturbance inputs are included in the model: the outdoor temperature, the solar irradiation, and a daily consumer load pattern. As presented in [15], [8], it is possible to capture the main dynamics of the households. However in this study we also conclude, that in certain time periods, it is difficult to capture the house dynamics presumably because of local disturbances. We illustrate this with a concrete example where we fit a first order model that takes power and outdoor temperature as inputs and the indoor temperature as output. The prediction error method is utilized based on observations of the last 7 days to predict the behavior the 24 hours of the following day; this is repeated each day. The result is illustrated in Fig. 1 showing both the predicted indoor temperature when the future temperature and future power consumption is known and the actual indoor temperature realization. As the figure shows, it is in this case not possible to make a good model fit. Similar results are obtained also when including other inputs such as solar irradiation and also for higher order models.

To illustrate the difficulties that the individual modeling of inhabited households can have we observe the figure more closely. In the afternoon of the 25th of March (indicated with a vertical dashed black line) the outdoor temperature is dropping, the sun is setting (not visible from plot) and the heat pump power is approximately constant; however, surprisingly the indoor temperature is rising. The reason could be the use of a wood-stove; however, we do not have access to this kind of information. Consequently, we cannot capture this in the system identification resulting in poor modeling for this particular example.

C. Sub-conclusion on Individual Household Modeling

In this section we have showed that flexibility optimization of heat pump flexibility based on individual models can be computationally heavy. Further we have showed that individual modeling of real inhabited households can be done, see for example [15], [8]; however, large disturbances makes it difficult to capture the house dynamics in certain time periods. These disturbances are believed to be the effect of opening/closing

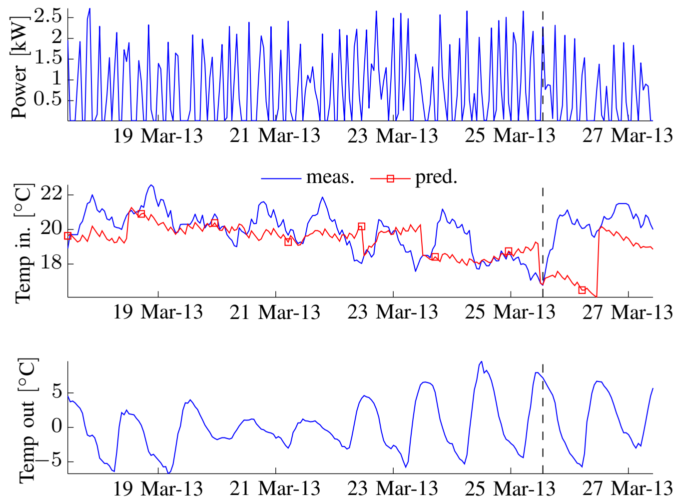

Fig. 1. Data from 10 days in March 2013 showing the heat pump power consumption, the indoor temperature and predicted indoor temperature, and finally the outdoor temperature. The jumps in the predicted indoor temperature every 24 hours occur because we predict for one full day at a time. The vertical black dashed line represents a time instance where the behavior of the household is particularly inexplicable: the indoor temperature is rising although the outdoor temperature is decreasing and the power is constant.

of windows and doors, wood stove or other alternative heating sources, use of electronic devices generating heat such as oven, computers, etc.

We conclude that it is reasonable to consider if there are alternatives to the individual modeling approach.

V. Lumped Modeling of Households

In this section, we show the concept of a lumped model of a portfolio of heat pump heated houses and show how such a model can be utilized for flexibility optimization. We are not able to conduct the necessary experiments to verify the proposed flexibility model altogether, instead we illustrate the benefit of such a model by examining some historical heat pump data.

A. Lumped Model and Flexibility Optimization

In the previous section it was concluded that it makes sense to examine alternatives to the individual modeling approach. Therefore, we propose to consider a much simpler approach: namely to model an entire heat pump portfolio using a lumped model. Obviously such a model will have its limitations as it is not able to capture the different dynamics and constraints of the individual heat pumps in the portfolio; on the contrary, it will capture the main flexibility and make it possible to optimize this flexibility towards a given objective. The advantage of such a lumped model is the simplicity and the low computational effort of optimizing the portfolio flexibility; further, the many disturbances affecting the individual heat pumps will to a large extend cancel out as the number of houses increases.

The main idea is to consider the heat pump portfolio as two parts: a baseline consumption (consumption when heat pumps operate in default mode) superimposed with an ideal storage. Let $p(k)$ be the accumulated consumption of the heat pump portfolio at time k, i.e. $p(k) = \sum_{i\in\mathcal{I}} u_i(k)$. Now assume that the accumulated consumption $p(k)$ consists of a baseline consumption $\overline{p}(k)$ and a flexible consumption part $\tilde{p}(k)$; finally, let $x(k)$ denote the energy stored in the ideal storage. We can write this flexibility model as

$$p(k) = \overline{p}(k) + \tilde{p}(k), \quad p_{\min} \leq p(k) \leq p_{\max} \tag{4}$$
$$x(k+1) = x(k) + T_s\alpha\tilde{p}(k), \quad x_{\min} \leq x(k) \leq x_{\max} \tag{5}$$

where $p_{\min}, p_{\max}$ and $x_{\min}, x_{\max}$ are power and energy limitations, T_s is the sampling time, and α is a parameter that performs a desired scaling of the power to energy. One method of implementing this is to use the individual indoor temperatures as a measure of $x(k)$ and the individual temperature comfort limits to find $x_{\min}$ and $x_{\max}$, see [19]. In this case, the parameter α will describe the households' thermal parameters and the heat pumps' COP. The power limitations $p_{\min}, p_{\max}$ can be set to the minimum and maximum power consumption of the entire portfolio, possibly adjusted by some margin.

Obviously, the simple model presented in (4), (5) have many limitations. An example is that the ideal storage model will predict that the energy loss to the ambient is independent of the indoor temperature. This obviously conflicts with the physics, as the energy loss will increase with increasing indoor temperature. It is, however, the authors' opinion that the presented model is a solid starting point when performing real life optimization of the flexibility of inhabited houses. The reason is that when dealing with real life inhabited households, the local disturbances are so severe that what is needed is a rough estimation of the available flexibility and not a high fidelity model. For example, the disturbances illustrated in Fig. 1 will be much larger than the increased loss to the ambient that will occur if we increase the indoor temperature one or two degrees from the set-point.

We consider the same small problem as presented in the previous section to illustrate how this model can be used for flexibility optimization. With the lumped model, we can formulate the power tracking optimization problem as follows

$$\begin{array}{ll} \text{min.} & \sum_{k\in\mathcal{K}} \left| p_{\text{spot}}(k) - \tilde{p}(k) - \hat{\overline{p}}(k) \right| \\ \text{s.t.} & p_{\min} \leq \hat{\overline{p}}(k) + \tilde{p}(k) \leq p_{\max}, \quad k \in \mathcal{K} \\ & x(k+1) = x(k) + T_s\tilde{p}(k), \quad k \in \mathcal{K} \\ & x_{\min} \leq x(k) \leq x_{\max}, \quad k \in \mathcal{K} \end{array} \tag{6}$$

where the variables are $x(k+1), \tilde{p}(k), k \in \mathcal{K}$ while the data is the current storage level $x(k)$ and the purchased electricity and the baseline consumption predictions $p_{\text{spot}}(k)$, $\hat{\overline{p}}(k)$.

Notice that solving Problem (6) can be done with low computational effort independent on the number of households due to the lumped model, i.e. we can easily optimize over thousands of heat pumps. Further notice, that the solution $\tilde{p}^\star(k), k \in \mathcal{K}$ will show the *accumulated* flexible consumption over time. Therefore a so-called *dispatcher* must dispatch the total consumption among the individual heat pumps, i.e. the dispatcher must translate $\overline{p}(k) + \tilde{p}^\star(k)$ into $u_1(k), \ldots, u_I(k)$. Such dispatch strategies can be implemented based on sorting algorithms and thus require low computational effort and easily handle thousands of units. For details on how this can be achieved, see for example [19], [20].

Further notice that this optimization problem clearly illustrates some of the limitations of the lumped model: The fixed limits on the power consumption assumes that none of the heat pumps are saturated such that they all are available for regulation. This is obviously a simplification and may cause a performance loss. A solution is to reduce the limits $p_{\min}, p_{\max}$ with a given margin. Again we remind the reader that we do not seek a high fidelity model; rather, we seek the most simple model that can be used for flexibility optimization.

B. *Lumped Modeling of Inhabited Households*

In the subsection above we presented a lumped model of a heat pump portfolio consisting of a baseline consumption combined with an ideal storage. In the following, we show how the power baseline $\overline{p}(k)$ can be estimated for a 24 hour horizon. In this study we do, however, not estimate the energy and power limits $x_{\min}, x_{\max}$ as it requires active control of the heat pumps, which is not possible in this study.

The baseline prediction is constructed as follows: The hourly energy consumption of the heat pump portfolio and the hourly outdoor temperature is collected for the previous 7 days and an affine transformation is made relating the observed outdoor temperature and energy consumption. Other parameters such as solar irradiation could be included, but for simplicity this is left out in this study. Now, meteorological predictions of the outdoor temperature the following day can be converted to predicted energy consumption based on the affine transformation.

This simple method is one out of many: higher order models could have been used, additional inputs could have been included, etc. However, we have implemented this very simple model to emphasize how easily such baseline estimation can be made. We test this method on data from 40 households from 1st of January until end of May 2013. The result is an average prediction error of 140 W per heat pump corresponding to an average prediction error of less than 11 % as the average heat pump consumption is 1.3 kW. Again, 10 days data are presented showing this predictor's ability to capture the hourly consumption of the portfolio, see Fig. 2.

Notice that the fit in Fig. 2 only shows the ability to predict the hourly portfolio consumption; hence, it does not show any dynamics of the portfolio and therefore does not validate the proposed flexibility model an and can therefore not be compared to the individual model fig in Fig. 1. For this reason, Fig. 2 does not validate the proposed lumped model; rather, it validates that we can predict the portfolio baseline consumption and motivates our argument that the disturbances will cancel out as number of households increases.

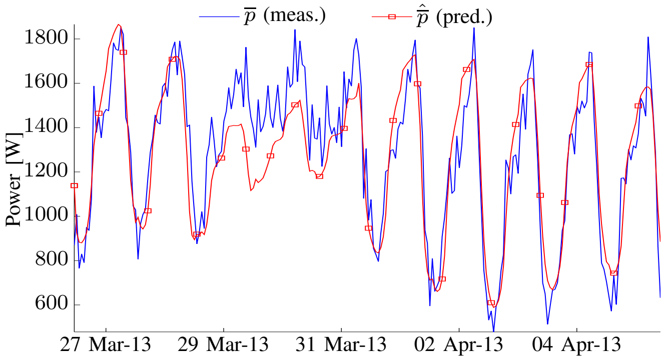

Fig. 2. Predictions made every midnight for the 24 hours of the following day for a total of 10 days for a portfolio of 40 heat pumps. The power is scaled with a factor 40 and thus represents and "average" heat pump.

C. Sub-conclusion on Lumped Household Modeling

We illustrated that the lumped modeling approach reduced the computational effort of flexibility optimization radically. We were, however, not able to verify the proposed model altogether as this would require extensive experiments. Instead, we used historical data to illustrate the benefit of the lumped modeling approach: that the disturbances on the individual houses to a large extend will cancel out which enables us to predict the baseline consumption with acceptable performance.

VI. Discussion

A much debated issue within the smart grid community is the use of flexible consumers to resolve grid congestion issues. Here we notice that we cannot use a lumped model to resolve local grid congestion issues: we do not know the geographical location of the individual heat pumps as they are all lumped into one model. One way to extend the presented method to cover this is to construct a lumped model for each feeder with issues; consequently we will have a number of lumped models for example with hundreds of heat pumps in each. Another approach is to incorporate the congestion alleviation mechanism in the dispatcher.

VII. Conclusion

In this paper we discussed two different approaches of modeling a heat pump portfolio: individual modeling and lumped modeling. We proposed a simple lumped model approach where an entire heat pump portfolio was modeled all together. This lumped model consisted of a baseline consumption superimposed with an ideal storage of limited energy capacity and with given power constraints. A clear benefit of the lumped model was that low computational effort required for flexibility optimization. Another advantage of the lumped model was the smoothing of individual household disturbances. We were not able to verify the mode altogether but motivated the benefit of the approach by showing that the portfolio baseline consumption could be predicted 24 hours ahead with an acceptable accuracy.

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