Primary Control by ON/OFF Demand-Side Devices

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***Abstract*—We consider an aggregator managing a portfolio of ON/OFF demand-side devices. The devices are able to shift consumption in time within certain energy limitations; moreover, the devices are able to measure the system frequency and switch ON and OFF accordingly. We show how the aggregator can manage the portfolio of devices to collectively provide a primary reserve delivery in an unbundled liberalized electricity market setting under current regulations. Furthermore, we formulate a binary linear optimization problem that minimizes the aggregator's cost of providing a primary reserve delivery of a given volume, and demonstrate this method on numerical examples.**

***Index Terms*—Demand response, liberalized electricity market, primary reserve, smart grids.**

I. Nomenclature

A. Indices

i	Index of devices.
j	Index of frequency deviation.
k	Index of time sample number.

B. Parameters

a	[W/Hz] Droop curve slope;
J_{prim}	[−] Cost of act. devices $\mathcal{I}_{\text{prim}}$.
$f_{\text{db}}, f_{\max}, f_{\text{tol}}$	[Hz] Droop curve parameters;
f_{nom}	[Hz] Nominal system frequency;
$f_{\text{sys}}(k)$	[Hz] System frequency;
K	[−] Samples in a delivery period;
m	[−] Number of frequency intervals;
n	[−] Number of devices
p	[W] Nominal power consumptions
$p_{\text{ctrl}}(k)$	[W] Primary control reference;
$p_{\text{prim}}(k)$	[W] Primary reserve volume;
$p_{\text{prim}}^{\max}, p_{\text{prim}}^{\min}$	[W] Up./lower primary res. limit;
T	[s] Duration of delivery period;
t_i	[Hz] Trigger frequency of device i;
$t_i^{\min}, t_i^{\max}$	[Hz] Min/max trig. freq. for dev. i;
$\overline{t}, \underline{t}$	[Hz] Frequency interval vectors;
T_s	[s] Sampling time;
$u(k)$	[W] Device power consumptions;
v	[W] Device drain rates;
$x(k)$	[J] Device energy storage levels;
$x^{\min}, x^{\max}$	[J] Up./lower energy limits;
$x_0(k)$	[J] Initial device energy levels;
$\Delta f(k)$	[Hz] Frequency deviation;
π	[−] Device activation costs.

C. Sets

$\mathcal{I}$	Devices index set.
$\mathcal{I}_{\text{prim}}$	Devices activated for primary reserve.
$\mathcal{I}_{\text{up}}, \mathcal{I}_{\text{up}}$	Upward/downward regulation devices.
$\mathcal{J}$	Frequency deviation index set.
$\mathcal{K}$	Sample number index set.

D. Variables

$\overline{X}$	Frequency allocation matrix for upward reg.
$\underline{X}$	Frequency allocation matrix for downward reg.

Throughout the nomenclature, the notation [−] is used to denote a dimensionless parameter. Notice that the costs are assumed normalized and hence described as dimensionless.

II. Introduction

With an increasing focus on climate-related issues and rising fossil fuel prices, the penetration of renewable energy sources is likely to increase in the foreseeable future throughout the developed world [1]. Many actions have been taken from a political point to increase the penetration of renewables: in the US, almost all states have renewable portfolio standards or goals that ensure a certain percentage of renewables [2]. Similarly, the commission of the European Community has set a target of 20% renewables by 2020 [3], while China has doubled its wind power production every year since 2004 [4].

Manuscript received October 01, 2012; revised February 07, 2013; accepted April 01, 2013. Date of publication April 25, 2013; date of current version November 25, 2013. The work is completed as a part of the project iPower and supported in part by the Danish government via the DSR-SPIR program 10-095378.

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Digital Object Identifier 10.1109/TSG.2013.2257892

In Denmark, the 2020 goals are 35% sustainable energy over all energy sectors and 50% wind power in the electrical energy sector [5].

A major challenge arises when replacing central power plants with renewable energy sources: the central power plants do not only deliver power but also provide ancillary services to ensure a reliable and secure electrical power system. This includes frequency stability support, power balancing, voltage control, etc. When the conventional power plants are replaced with renewables such as wind turbines and photovoltaics, the ability to provide ancillary services in the classical sense disappears; the renewable energy sources will often fully utilize the available power and thus not be able to provide balancing ancillary services. Furthermore, conventional fossil fuel power plant generators are synchronous with the grid and therefore provide rotating inertia that supports the system frequency against changes [6]. As renewable energy sources typically interface with the grid via power electronics, they do not directly provide inertia to the grid as the conventional synchronous generators [7], which further increases the balancing challenges. Although recent works suggest that wind turbines can provide synthetic and artificial inertia by regulating the active power output of the generator according to the system frequency [8], [9], this type of control is generally not implemented in the wind power plants of today. Moreover, many renewable sources are characterized by highly fluctuating power generation: they can suddenly increase or decrease production depending on weather conditions. These rapid production changes are not always predictable and can therefore imply severe consequences for grid stability [10].

It is therefore evident that in a grid with high penetration of renewables, the need for balancing ancillary services will increase [11], [12]. As conventional power plants are phased out gradually, alternative sources of ancillary services must be established. One of the approaches to obtaining alternative ancillary services is the *smart grid* concept, where demand-side devices with flexible power consumption take part in the balancing effort [13], [14]. The basic idea is to let an *aggregator* manage a portfolio of flexible demand-side devices and utilize the accumulated flexibility in the unbundled electricity markets on equal terms with conventional generators [15].

Flexible demand-side devices have significantly different characteristics than conventional generators: while conventional generators are able to provide more or less energy by adjusting the fuel consumption, demand-side devices will on average roughly consume the same amount of energy. An electrical vehicle may for example be able to consume energy in one hour and deliver the energy back in the following hour; however, over the course of a year, the net energy consumption will roughly be the same independent of how the flexibility is utilized. On the other hand, many demand-side devices can be switched ON and OFF almost instantaneously enabling them to react faster than most conventional generators. These characteristics make demand-side devices ideal for primary frequency control, as this type of reserve demands rapid up and down power regulation abilities but generally does not require actual energy deliveries.

Another benefit of primary frequency control in this context is that the delivery of reserves depends on local system frequency measurements; hence, no expensive near real-time communication from aggregator to devices is necessary. Furthermore, primary reserves are generally the most expensive deliveries, as they require fast control action. This increases the attractiveness of enabling demand-side devices to participate in the primary reserve market.

Demand-side management by controlling smaller appliances to support grid stability has been discussed as early as the 1980s [16]. Since, the topic of demand-side management has received much attention from a research perspective. See, e.g., [17]–[19]. Currently, demand-side programs are in operation in many systems, for example in the UK and the US systems [20]–[22]; moreover, a growth is seen in the volume of these programs. As an example of this growth, New England has experienced an increase in demand-side programs from contracts on 200 MW in 2003 to more than 2,000 MW in 2009 [23].

Recent works have discussed the use of demand-side management to provide primary reserve. A few examples are: refrigeration systems that adjust the power consumption according to the system frequency deviation [24], [25], thermal systems that respond when the system frequency drops below a certain value [26], and primary frequency control of flexible domestic consumption devices activated through a local smart meter [27]. While these works discuss methods for providing primary reserves, they do not consider these services sold through the current liberalized market system. In other words: the cited works show how to deliver primary reserve for grid support but do, however, not design the control strategies such that the accumulated response of the demand-side devices satisfy the regulatory requirements for primary reserve deliveries.

The main contribution of this work is to show how an aggregator can manage a portfolio of ON/OFF demand-side devices to collectively provide a delivery of primary reserve that comply with the current regulations in the European electric power system. This allows the aggregator to enter the primary frequency control market and thus compete with the conventional generators as is desired in a liberalized market setting [15].

The paper is structured as follows. In Section III, we present a system architecture where an aggregator manages a portfolio of ON/OFF device. Following, in Section IV, we describe how these ON/OFF devices can be managed to provide frequency reserves complying with current regulations. In Section V, we present a method for minimizing the cost of a reserve delivery, and in Section VI this method is applied to a numerical example. Finally, in Section VII, we conclude the work.

III. System Architecture

In this section, we describe the overall relation between consumers, aggregator and the primary reserve market.

A. Aggregator as Player in the Electricity Markets

We consider an unbundled liberalized electricity system architecture. In this setup, the Transmission System Operators (TSOs) are responsible for secure and reliable system operation and must consequently ensure balance between production and consumption. Generally, in an unbundled electricity system, TSOs do not own production units and must therefore procure

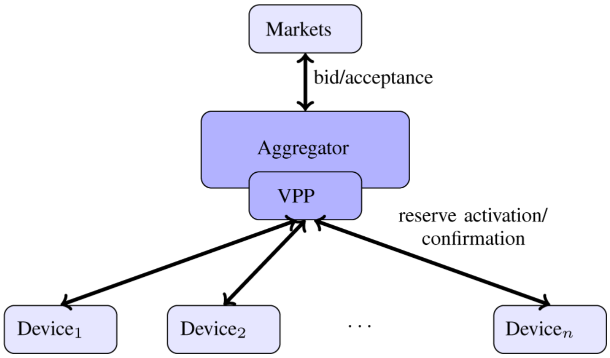

Fig. 1. Aggregator bidding in the electricity markets by managing n devices through a VPP.

ancillary services in the electricity markets to ensure system stability.

The aggregator is a legal entity able to enter into flexibility contracts with consumers. These contracts allow the aggregator to manage the consumers' flexible consumption; hereby the aggregator is able to utilize the accumulated consumer flexibility to participate in the electricity markets. The flexible devices are managed by the aggregator through a technical unit often referred to as a Virtual Power Plant (VPP) as illustrated in Fig. 1. In this work, the aggregator utilizes the consumer flexibility to participate in the primary reserve market.

B. Primary Reserve

Primary reserve is an automatic control used in frequency control. A main target for the primary control is to stabilize the system frequency in case of major outages of either loads or suppliers until the primary control reserve is relieved by secondary control [28]. The activation time for primary control is in the magnitude of 1–180 seconds depending on the system [29].

Primary frequency control must be provided according to the deviation from the nominal system frequency. Basically, a local control loop must assure that upward regulation is provided at frequencies below the nominal frequency, and similarly, downward regulation is provided at frequencies above the nominal frequency. For the sake of consistency we only consider *symmetric* primary reserve deliveries where equal volumes of upward and downward regulation must be delivered. It is, however, straightforward to extend the methods presented in this work to asymmetric deliveries.

Primary reserve is critical to grid stability. Therefore, the local control loop must rely on a *local* system frequency measurement. This makes the primary reserve delivery independent from communication links etc.

In the liberalized electricity market, the TSOs will ensure grid stability by procuring sufficient volumes of primary reserve from the suppliers. Typically, each day is divided into a number of primary reserve delivery periods, for example 24 one-hour periods. The suppliers can place sales offers for primary reserves in each delivery period of the day. Following, the TSOs will purchase the cheapest of these bids according to the need for primary reserve. We assume that each bid is either full accepted or not accepted at all as is the case in for example the Nordic electricity system [29].

C. Demand-Side Devices as Primary Reserve

The aggregator manages a portfolio of ON/OFF devices with flexible power consumption: the power consumption of each device is not continuously adjustable; rather, it is either turned ON or OFF. This covers large class of devices, for example thermal devices such as heat pumps, refrigeration systems, water heaters, etc.

In order for such consumption devices to provide ancillary services, they must be separated from and independent of ordinary consumption and must be approved by a TSO as consumption that can be used as regulation reserves [30]. In this work, we assume that the portfolio of devices under the jurisdiction of the aggregator indeed is approved by a TSO. Moreover, we assume that the devices are able to measure the system frequency with the required accuracy, typically in the range of few mHz, and that they can apply the control action as fast as required, typically in the range of few seconds.

Note that this setup requires very little communication between devices and VPP: the devices respond to local system frequency measurements and therefore do not need external control signals. The only communication needed is before the start of each reserve delivery period where each device will send state information to the VPP after which the VPP will send primary reserve activation commands to the devices. Hence, no near real-time communication link is needed. This is an attractive feature of the presented architecture and greatly lowers the overall communication costs.

IV. Primary Reserve Via ON/OFF Devices

In this section we examine the primary frequency control requirements and describe how ON/OFF devices collectively can fulfill these requirements.

A. ON/OFF Consumers

The VPP manages a portfolio of n flexible consumption devices represented by the index set $\mathcal{I} = \{1, \ldots, n\}$. We assume that these devices can be modeled as energy storages with a certain drain rate. Further, we assume that the drain rates can be assumed constant within each primary reserve delivery period. This assumption allows us to clearly show the main message of this work: that an aggregator can manage a portfolio of small ON/OFF devices to collectively provide primary frequency reserve on market terms. Note, however, that modeling consumers as having a constant drain rate may be a crude assumption: in many cases, the drain rate will be characterized by stochastic behavior depending on user behavior, weather conditions, etc. An extenuating circumstance is that the assumption only has to hold for the delivery period which is in the order of an hour—hereafter the model can be updated.

Let the energy storage levels of the flexible consumers be denoted $x(k) \in \mathbf{R}^n$, the power consumptions $u(k) \in \mathbf{R}^n$, and the drain rates $v \in \mathbf{R}^n$, where k is the sample number. We model device i is as

$$x_i(k+1) = x_i(k) + T_s\left(u_i(k) - v_i\right), \quad i \in \mathcal{I} \qquad (1)$$

$$x_i(0) = x_i^0, \quad i \in \mathcal{I} \qquad (2)$$

where $x^0 \in \mathbf{R}^n$ is the initial energy storage level and T_s is the chosen sampling time. The notation x_i is used to denote element number i of the vector x, i.e., $x = [x_1, \ldots, x_n]^T$; this notation is used throughout the work. Let $p \in \mathbf{R}^n$ denote the nominal power consumptions of the n ON/OFF devices, hence

$$u_i(k) = \begin{cases} p_i & \text{if device } i \text{ is ON} \\ 0 & \text{ifdevice } i \text{ is OFF} \end{cases}, i \in \mathcal{I} \tag{3}$$

as each device is only able to be turned ON or OFF. In this work, we do not include any penalty for the number of switches per device. It might, however, be a useful extension to include switching costs as rapid switching may cause damage or reduce lifetime depending on the type of device. Each energy storage is limited in size which we describe by the limit vectors $x^{\min}, x^{\max} \in \mathbf{R}^n$; hence we have the requirement that

$$x_i^{\min} \le x_i(k) \le x_i^{\max}, \quad i \in \mathcal{I}. \tag{4}$$

The interpretation of these limitations depends on the type of device. For space heating systems, space cooling systems, water heating systems, etc., the limits could represent a desired temperature band [31]. Therefore, we refer to the constraints (4) as *comfort constraints* in the sequel.

A *flexibility contract* between aggregator and consumer will specify the payment the consumer must receive for being activated by the aggregator for a primary frequency reserve delivery. The payment could for example be *flex rate* with a certain payment each time the device is activated or it could be *flat rate* with an annual payment or electricity discount independent of how often the device is activated. The type of contract will depend on the aggregator/consumer setup [32]. For example, a heating system could be sold with a given discount; in return an aggregator is allowed to utilize the device for primary reserve provisions as long as the comfort limits are honored. This is an example of a flat rate contract, where the consumer does not get any activation payment but instead a one-off payment (in the form of a discount). Such a contract will be relevant if the aggregator is willing to take all the risk. If the consumer is willing to take more risk, a flex rate contract can be established with a given payment per activation which may generate higher profit for the consumer in the long run. The consumer's willingness to take risks will therefore affect what type of contract is signed. Further, the flat rate or flex rate payment will depend on a number of parameters including the energy and power capacity of the consumer and how often the consumer allows activations for primary reserve deliveries.

In this work, we represent the costs by a vector $\pi \in \mathbf{R}_+^n$ where π_i is the payment the aggregator must pay consumer i for activation of a primary frequency reserve delivery. This means that if the aggregator constructs a primary reserve bid based on the devices $\mathcal{I}_{\text{prim}} \subseteq \mathcal{I}$, he will face an expense given by

$$J_{\text{prim}} = \sum_{i \in \mathcal{I}_{\text{prim}}} \pi_i \tag{5}$$

if the bid is accepted for that given delivery period. Later, these costs will be further elaborated.

Notice that a number of other constraints and conditions can be included in the flexibility contracts such as constraints on when the devices will allow activations [32]. Certain devices may only allow activation during certain hours of the day, certain days in the week, only certain seasons, etc. Such constraints are not included in this work. Further, the flexibility contract must describe the penalty for non-compliance. In this case where we deal with primary reserve which is crucial for grid stability, non-compliance should be associated with a large penalty such as financial penalty and termination of the contract. The regulations in the Nordic electricity systems specify that in case the sold delivery of primary reserve cannot be delivered, the reserve must be re-esablished within 30 minutes after the incident [29]. If the aggregator detects that a device does not deliver as required, the aggregator must then exclude this device and redistribute his portfolio to re-establish the sold delivery.

B. Frequency Control Specifications

Frequency control depends on the system frequency deviation $\Delta f(k) \in \mathbf{R}$ which is the difference between the system frequency $f_{\text{sys}}(k) \in \mathbf{R}$ and the nominal system frequency $f_{\text{nom}} \in \mathbf{R}$:

$$\Delta f(k) = f_{\text{sys}}(k) - f_{\text{nom}}. \tag{6}$$

Let $p_{\text{prim}} \in \mathbf{R}$ denote a symmetric delivery of primary reserve. An entity activated for a delivery p_{prim} must deliver power according to the measured frequency deviation $\Delta f(k)$: between the frequency deviations $\pm f_{\max}$, the sold reserve p_{prim} must be provided proportionally with $\Delta f(k)$ except for a dead band of $\pm f_{\text{db}}$; moreover, a controller tolerance of $\pm f_{\text{tol}}$ is allowed. In the ENTSO-E grid, the parameters of this droop curve are $f_{\max} = 200$ mHz, $f_{\text{db}} = 20$ mHz, and $f_{\text{tol}} = 10$ mHz [33] resulting in a primary frequency control droop curve as illustrated in Fig. 2.

Let $p_{\text{ctrl}}(\Delta f(k))$ denote the primary reserve that must be delivered at sample number k when the system frequency deviation is $\Delta f(k)$ and the delivery is p_{prim}. Then we have (sample number k is omitted to ease the notation):

$$p_{\text{ref}}(\Delta f) = \begin{cases} p_{\text{prim}} & \text{if } \Delta f < -f_{\max} \\ a(\Delta f + f_{\text{db}}) & \text{if } -f_{\max} \le \Delta f < -f_{\text{db}} \\ 0 & \text{if } -f_{\text{db}} \le \Delta f \le f_{\text{db}} \\ a(\Delta f - f_{\text{db}}) & \text{if } f_{\text{db}} < \Delta f \le f_{\max} \\ -p_{\text{prim}} & \text{if } \Delta f > f_{\max} \end{cases} \tag{7}$$

$$p_{\text{ctrl}}(\Delta f) \in [p_{\text{ref}}(\Delta f) + a f_{\text{tol}},\ p_{\text{ref}}(\Delta f) - a f_{\text{tol}}] \tag{8}$$

where $p_{\text{ref}}(\Delta f) \in \mathbf{R}$ is the reference that the primary frequency control should track, $a = p_{\text{prim}}/(f_{\text{db}} - f_{\max})$ is the slope of the primary reserve droop curve, and $\pm a f_{\text{tol}}$ specifies the control tolerance band.

C. Delivery of Primary Reserves

In this section we illustrate how the accumulated response of a portfolio of ON/OFF consumption devices can comprise a primary reserve delivery.

1) Local Frequency Measurements and Local Control: Primary frequency control must be provided based on local fre-

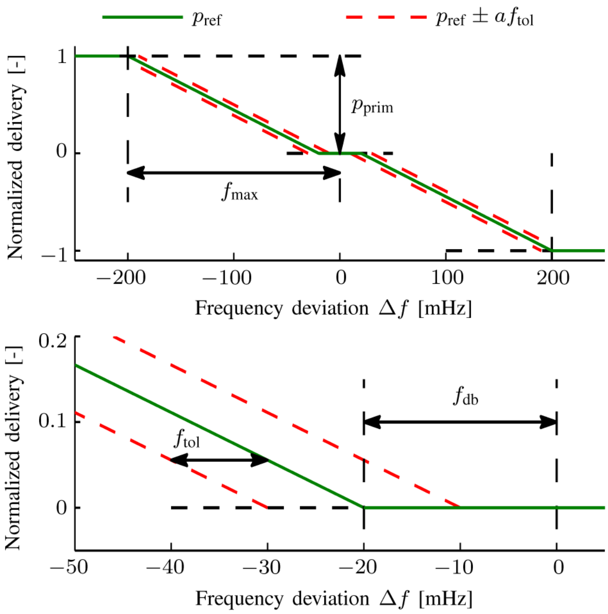

Fig. 2. Primary frequency control droop curve with parameters from the ENTSO-E grid illustrating the power reference p_{ref} and the allowed tolerance bands for a normalized delivery.

quency measurements. As each device is only able to be either ON or OFF, the local control law of each device is on the form

$$u_i(\Delta f(k)) = \begin{cases} p_i & \text{if} \quad \Delta f(k) \geq t_i \quad \text{(device ON)} \\ 0 & \text{if} \quad \Delta f(k) < t_i \quad \text{(device OFF)} \end{cases} \tag{9}$$

for $i \in \mathcal{I}_{\text{prim}} \subseteq \mathcal{I}$ where $\mathcal{I}_{\text{prim}}$ is an index set of the ON/OFF devices activated for a primary reserve delivery. Here $t_i \in \mathbf{R}, i \in \mathcal{I}_{\text{prim}}$ are predefined frequency deviation thresholds for each of the devices that comprise the delivery. In the following, we refer to the threshold t_i as the *trigger frequency* of device i.

2) Combined Delivery: The basic idea in this work is as follows: to assign trigger frequencies t_i to a subset of devices $\mathcal{I}_{\text{prim}}$ such that they collectively can provide a delivery of p_{prim} at the lowest possible cost J_{prim}. This means that the activated devices $\mathcal{I}_{\text{prim}}$ must satisfy

$$p_{\text{ctrl}}(\Delta f(k)) = p_{\text{base}} - \sum_{i \in \mathcal{I}_{\text{prim}}} u_i(\Delta f(k)) \tag{10}$$

where $p_{\text{base}} \in \mathbf{R}$ is a chosen baseline consumption of the devices $\mathcal{I}_{\text{prim}}$ and where $p_{\text{ctrl}}(\Delta f(k))$ satisfy the primary frequency requirements as specified by (8). We comment further on the baseline p_{base} in the following section. Graphically, (10) corresponds to fitting the staircase shaped combined response of the devices $i \in \mathcal{I}_{\text{prim}}$ between the primary frequency control droop curve tolerance bands shown in Fig. 2.

3) Symmetric Delivery: As described, we consider a symmetric delivery where we provide equal volumes of upward and downward reserve according to Fig. 2. This can be expressed as follows: the set of devices $\mathcal{I}_{\text{prim}}$ that comprise the symmetric delivery consists of devices that provide upward regulation $\mathcal{I}_{\text{up}}$ and devices that provide downward regulation $\mathcal{I}_{\text{down}}$ where $\mathcal{I}_{\text{prim}} = \mathcal{I}_{\text{up}} \cup \mathcal{I}_{\text{down}}, \mathcal{I}_{\text{up}} \cap \mathcal{I}_{\text{down}} = \emptyset$. The devices $i \in \mathcal{I}_{\text{up}}$ that provide upward regulation have trigger frequencies $t_i < 0$ and comprise the left half of the droop curve in Fig. 2 while the devices $i \in \mathcal{I}_{\text{down}}$ that provide downward regulation have trigger frequencies $t_i > 0$ and comprise the right half of the droop curve. This means that at zero frequency deviation $\Delta f(k) = 0$, no frequency reserve is to be delivered; consequently, all upward regulation devices $i \in \mathcal{I}_{\text{up}}$ will be ON while all downward regulation devices $i \in \mathcal{I}_{\text{down}}$ will be OFF, hence the baseline consumption is $p_{\text{base}} = \sum_{i \in \mathcal{I}_{\text{up}}} p_i$.

To further illustrate the concept, we can describe the setup as follows. If the frequency deviation gradually decreases from 0 to $-f_{\max}$, the devices $i \in \mathcal{I}_{\text{up}}$ will gradually switch from ON to OFF as the frequency deviation becomes lower than the individual trigger frequencies. Hereby the portfolio will provide upward regulation. A similar argument can be made when the frequency deviation increases from 0 to $f_{\max}$ for the devices $i \in \mathcal{I}_{\text{up}}$; hence, the combined response will comprise a symmetric primary reserve delivery. Note that we assume that the aggregator is free to choose any baseline p_{base} for the devices activated for primary reserve.

4) Primary Reserve Volume: In the following we consider an upper and a lower bound on the volume of primary frequency control that the portfolio can deliver and neglect the activation costs π.

Optimistic Case: Consider an optimistic case where we completely ignore the comfort constraints (4) and regardless of the activation costs π activate the whole portfolio for primary reserve. The smallest consumption of the portfolio is 0 while the maximum consumption of the total portfolio is $\mathbf{1}^T p$, where $\mathbf{1}$ is a vector with all components one; hence, we are able to deliver at most a symmetric primary reserve bid of

$$p_{\text{prim}}^{\max} = \mathbf{1}^T \frac{p}{2}. \tag{11}$$

This optimistic example is not meant as an implementable method as the comfort constraints are ignored, but provides an upper bound on the volume we can bid as primary reserve.

Conservative Case: Now consider a conservative strategy where we only utilize the devices that are fully flexible, again independent of the costs π. The fully flexible devices are those that will not violate the comfort constraints (4), no matter if they are turned ON or OFF for the whole primary reserve delivery period. In this case we are able to deliver at most a symmetric primary reserve bid of

$$p_{\text{prim}}^{\min} = \sum_{i \in \mathcal{I}^{\min}} \frac{p_i}{2}, \tag{12}$$

$$\mathcal{I}^{\min} = \{i \in \mathcal{I} | x_i^0 - T v_i \geq x_i^{\min}, \; x_i^0 + T(p_i - v_i) \leq x^{\max}\}. \tag{13}$$

Here T is the duration of a delivery period such that $x_i^0 - T v_i$ is the end state of device i if it is turned OFF the whole period T; similarly, $x_i^0 + T(p_i - v_i)$ is the end state of device i if it is turned ON the whole period T – hereby $\mathcal{I}^{\min}$ corresponds to the devices that do not violate the comfort constraints for any input sequence $u_i(k)$ throughout the whole period T. This delivery $p_{\text{prim}}^{\min}$ serves as a lower bound on the volume we can bid as primary reserve.

Probabilistic Approach: In this work, we propose an alternative to the optimistic and the conservative methods. In our method we require that the comfort constraints (4) should be honored with probability α:

$$\mathbf{Prob}(x_i^{\min} \leq x_i(k) \leq x_i^{\max}, \forall k = 1, \ldots, K) \geq \alpha, \quad i \in \mathcal{I}_{\text{prim}} \tag{14}$$

where $K = T/T_s$ is the total number of samples in a delivery period. Hereby we will be able to utilize the portfolio to a far greater extent than the conservative case as illustrated in the numerical example in the end of this work.

This setup requires that the flexibility contract states that the comfort constraints might be violated when the consumer is activated for reserve deliveries; in return, the consumer achieves the activation payment specified by π_i. By choosing the parameter α sufficiently high, the aggregator will ensure that the consumer rarely will experience discomfort thereby making it attractive for consumers to be part of the portfolio.

V. Controller Synthesis

In this section we describe how to construct bids for the primary frequency reserve market based on the portfolio of ON/OFF devices. The basic idea is simple: we find the set of devices $\mathcal{I}_{\text{prim}}$ with the lowest total cost J_{prim} that collectively can provide a symmetric reserve p_{prim} that is kept within the tolerance bands $p_{\text{ref}} \pm af_{\text{tol}}$ while honoring the comfort constraints with a desired certainty.

A. Problem Variables and Parameters

In the following, we define the variables and parameters needed to formulate the problem of minimizing the cost J_{prim} of providing a primary reserve bid of volume p_{prim}. Due to the discontinuity of the primary control droop curve caused by the dead band, we separate the problem formulation into an upward regulation part and a downward regulation part. Consequently, we will distinguish between the parameters associated with upward regulation and the parameters associated with downward regulation. We indicate upward regulation parameters with an upper bar and downward regulation parameters with a lower bar.

First, let us define two vectors describing the frequency ranges associated with upward and downward regulation denoted $\overline{t}$ and $\underline{t}$, respectively. Each range is divided into m equidistant intervals:

$$\overline{t} = (-f_{\text{db}}, \ldots, -f_{\max}) \in \mathbf{R}^m$$
$$\underline{t} = (f_{\text{db}}, \ldots, f_{\max}) \in \mathbf{R}^m$$

where $(f_{\max} - f_{\text{db}})/(m-1)$ is the quantization of the two frequency vectors. This quantization can for example be chosen as the accuracy of the frequency measurement equipment. Furthermore, we define two binary matrices $\overline{X}, \underline{X} \in \mathbf{R}^{m \times n}$ where

$$\overline{X}_{ji} = \begin{cases} 1 & \text{if device} i \text{ has threshold } \overline{t}_j \\ 0 & \text{else,} \end{cases} \tag{15}$$

$$\underline{X}_{ji} = \begin{cases} 1 & \text{if device} i \text{ has threshold } \underline{t}_j \\ 0 & \text{else} \end{cases} \tag{16}$$

for $i \in \mathcal{I}$ and $j \in \mathcal{J} = \{1, \ldots, m\}$. These matrices describe the trigger frequencies of the devices activated for upward regulation $i \in \mathcal{I}_{\text{up}}$ and downward regulation $i \in \mathcal{I}_{\text{down}}$. The sets $\mathcal{I}_{\text{up}}$ and $\mathcal{I}_{\text{down}}$ can be expressed in terms of $\overline{X}$ and $\underline{X}$ as:

$$\mathcal{I}_{\text{up}} = \{i \in \mathcal{I} | (\overline{X}^T \mathbf{1})_i = 1\}, \; \mathcal{I}_{\text{down}} = \{i \in \mathcal{I} | (\underline{X}^T \mathbf{1})_i = 1\}. \tag{17}$$

B. Problem Objective and Constraints

1) Objective: The objective is to minimize the total cost J_{prim} of providing a primary reserve bid of volume p_{prim} for a delivery period T, as specified by (5). We can express J_{prim} in terms of $\overline{X}, \underline{X}$ as

$$J_{\text{prim}} = \mathbf{1}^T (\overline{X} + \underline{X}) \pi \tag{18}$$

as $\left(\mathbf{1}^T (\overline{X} + \underline{X})\right)_i = 1$ if and only if device i is activated for upward or downward regulation and as the associated cost is π_i.

2) Reference Tracking: The devices comprising a bid of primary reserve $i \in \mathcal{I}_{\text{prim}}$ must collectively track the power reference p_{ref} within the given control tolerance bands as described by (8). This is equivalent to allocating the trigger frequencies of the devices $i \in \mathcal{I}_{\text{prim}}$ such that the combined upward and downward regulation lie within the tolerance bands. In the following we describe how to constrain $\overline{X}, \underline{X}$ such that this is achieved. We illustrate this first for upward regulation.

Let $R \in \mathbf{R}^{m \times m}$ serve as a cumulative sum operator by having zeros on all elements above the diagonal and ones in all elements on and below the diagonal. The power provision between a frequency deviation $\overline{t}_j$ to $\overline{t}_{j+1}$ can thus be described as $(R\overline{X}p)_j$. To honor the control tolerance bands it is necessary that

$$a(\overline{t}_{j+1} + f_{\text{db}} + f_{\text{tol}}) \leq (R\overline{X}p)_j \leq a(\overline{t}_j + f_{\text{db}} - f_{\text{tol}}) \tag{19}$$

for $j \in \mathcal{J} \backslash m$ due to the allowed control tolerance $\pm f_{\text{tol}}$. Further we must assure that we deliver the required reserve p_{prim} when the system frequency deviation reaches $\overline{t}_m = -f_{\max}$ which can be described as

$$(R\overline{X}p)_m \geq p_{\text{prim}}. \tag{20}$$

The requirements (19) and (20) can be rearranged and written in compact form as a constraint to the allocation matrix $\overline{X}$ as follows

$$\begin{aligned} \overline{X} \in \overline{\mathcal{X}}_{\text{ref}} = \{X \in \mathbf{R}^{m \times n} | & \overline{t}_j - f_{\text{tol}} \\ & \leq \frac{(RXp)_j}{a} - f_{\text{db}} \\ & \leq \overline{t}_{j+1} + f_{\text{tol}}, \\ & (RXp)_m \geq p_{\text{prim}}, \forall j \in \mathcal{J} \backslash m\}. \end{aligned} \tag{21}$$

By a similar set of arguments, we can make a compact formulation of the requirements for the downward regulation to honor the tolerance bands and deliver the full reserve $-p_{\text{prim}}$ at frequency deviation $\underline{t}_m = f_{\max}$. Hereby we obtain

$$\underline{X} \in \underline{\mathcal{X}}_{\text{ref}} = \{X \in \mathbf{R}^{m \times n} | \underline{t}_j + f_{\text{tol}}$$

$$\leq \frac{(RXp)_j}{a} + f_{\mathrm{db}}$$
$$\leq \underline{t}_{j+1} - f_{\mathrm{tol}},$$
$$(RXp)_m \leq -p_{\mathrm{prim}}, \forall j \in \mathcal{J} \backslash m\}. \quad (22)$$

3) Assure Comfort: As described in (14), we must assure that comfort is maintained for the devices activated for upward and downward regulation with probability α or greater. The key idea in assuring this comfort is to use historical system frequency measurements to determine probabilities for how long time a device will be ON and OFF respectively when assigned with a given trigger frequency. Hereby we can determine the trigger frequencies that with a given probability will not cause violations of the comfort constraints.

In Appendix A, we present a method for mapping the device parameters $\{x_i^0, p_i, v_i, x_i^{\min}, x_i^{\max}\}$ into upper and lower limits $\{t_i^{\min}, t_i^{\max}\}$ on the trigger frequency of device i for $i \in \mathcal{I}$. The mapping is based on historical system frequency measurements and is constructed such that if device i is activated for upward or downward regulation according to the control law (9) with trigger frequency t_i, then the largest allowable trigger frequency band that assures comfort with probability at least α is $t_i^{\min} \leq t_i \leq t_i^{\max}$. Hence, sufficient comfort is assured if

$$t_i^{\min} \leq t_i \leq t_i^{\max}, \quad i \in \mathcal{I}_{\mathrm{prim}} \quad (23)$$

which can be expressed in terms of $\overline{X}, \underline{X}$ as

$$\mathbf{diag}(t^{\min})\overline{X}^T \mathbf{1} \preceq \overline{X}^T \overline{t} \preceq \mathbf{diag}(t^{\max})\overline{X}^T \mathbf{1} \quad (24)$$
$$\mathbf{diag}(t^{\min})\underline{X}^T \mathbf{1} \preceq \underline{X}^T \underline{t} \preceq \mathbf{diag}(t^{\max})\underline{X}^T \mathbf{1}, \quad (25)$$

where $\mathbf{diag}(x)$ denotes a diagonal matrix with diagonal entries $x_1, \ldots, x_n$ and where $\preceq$ represents componentwise inequality. Constraint (24) can be explained as follows: $(\mathbf{diag}(t^{\min})\overline{X}^T \mathbf{1})_i$ and $(\mathbf{diag}(t^{\max})\overline{X}^T \mathbf{1})_i$ are the minimum and maximum allowable trigger frequencies for device i if activated for upward regulation $i \in \mathcal{I}_{\mathrm{up}}$; otherwise it is zero. Similarly, $(\overline{X}^T \overline{t})_i$ is the trigger frequency of device i if activated for upward regulation $i \in \mathcal{I}_{\mathrm{up}}$; otherwise it is zero. Hereby, constraint (24) ensures that device i will have a trigger frequency within the allowable range $[t_i^{\min},\ t_i^{\max}]$ if it is activated for upward regulation. Similarly for the downward regulation inequality (25).

4) ON/OFF Behavior: The devices are only able to be turned ON or OFF which can be formulated as

$$\overline{X}, \underline{X} \in \mathcal{X}_{\mathrm{bin}} \quad (26)$$
$$\mathcal{X}_{\mathrm{bin}} = \{X \in \mathbf{R}^{m \times n} | X_{ji} \in \{0,1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}\}. \quad (27)$$

Furthermore, we must construct $\overline{X}, \underline{X}$ such that each device is associated with at most one trigger frequency. This requirement can be expressed as

$$(\overline{X} + \underline{X})^T \mathbf{1} \preceq \mathbf{1}. \quad (28)$$

C. Optimization Problem

Based on the objective and constraints, we can formulate the problem that minimizes the cost of providing a delivery p_{prim} of primary reserve:

$$\begin{array}{ll} \text{minimize} & J_{\mathrm{prim}} = \mathbf{1}^T(\overline{X} + \underline{X})\pi \\ \text{subject to} & \overline{X} \in \overline{\mathcal{X}}_{\mathrm{ref}}, \quad \underline{X} \in \underline{\mathcal{X}}_{\mathrm{ref}}, \quad \overline{X}, \underline{X} \in \mathcal{X}_{\mathrm{bin}} \\ & (\overline{X} + \underline{X})^T \mathbf{1} \preceq \mathbf{1} \\ & \mathbf{diag}(t^{\min})\overline{X}^T \mathbf{1} \preceq \overline{X}^T \overline{t} \preceq \mathbf{diag}(t^{\max})\overline{X}^T \mathbf{1} \\ & \mathbf{diag}(t^{\min})\underline{X}^T \mathbf{1} \preceq \underline{X}^T \underline{t} \preceq \mathbf{diag}(t^{\max})\underline{X}^T \mathbf{1} \end{array} \quad (29)$$

where the variables are $\overline{X}, \underline{X} \in \mathbf{R}^{m \times n}$. The data to the problem is the activation costs $\pi \in \mathbf{R}_+^n$, the primary frequency delivery specification described by the sets $\overline{\mathcal{X}}_{\mathrm{ref}}, \underline{\mathcal{X}}_{\mathrm{ref}}$, the ON/OFF behavior set $\mathcal{X}_{\mathrm{bin}}$, the upward and downward frequency ranges $\overline{t}, \underline{t} \in \mathbf{R}^m$, and the upper and lower trigger frequency limits $t^{\min}, t^{\max} \in \mathbf{R}^n$. The optimal value $J_{\mathrm{prim}}^\star$ of the optimization problem (29) is the minimum cost associated with a delivery p_{prim} of primary reserve under the specified comfort constraints throughout the delivery period T. The optimal solution $\overline{X}^\star, \underline{X}^\star$ specifies which devices should be activated for this delivery and the associated trigger frequencies according to (15) and (16).

Problem 29 is a linear mixed integer optimization problem and resembles a unit commitment problem. See, e.g., [34]. Generally, this type of program is hard and can only be solved for a smaller number of devices (up to hundreds) using commercial optimization tools. For a larger number of devices, alternative methods are needed such as decomposition techniques [35], [36] or heuristics [37], [38]. In Appendix B, we present a very simple and straightforward heuristic method that is able to handle large numbers of devices and approximately solve the binary optimization problem. Note that the heuristic method is presented mainly to illustrate that Problem 29 can be approximately solved with reasonable performance via heuristic methods, which is useful when the number of devices is large; it is, however, beyond the scope of this work to develop more sophisticated heuristics or to conclude proofs of the performance of the presented heuristic.

Algorithm

Based on the previous sections, we present an algorithm for utilizing a portfolio of ON/OFF devices to provide primary reserve. The algorithm must be executed before the bidding deadline of each primary reserve delivery period.

1) Collect state information of the portfolio of ON/OFF devices $\{x_i^0, p_i, v_i, x_i^{\min}, x_i^{\max}\}, i \in \mathcal{I}$.
2) Map the state information into upper and lower allowable trigger frequency limits $\{t_i^{\min}, t_i^{\max}\}, i \in \mathcal{I}$ according to Appendix A.
3) Solve the binary linear program (29) or approximately solve it using the heuristic method presented in Appendix B based on a desired delivery volume p_{prim}. If feasible, denote the resulting values of the binary matrices $\overline{X}^+, \underline{X}^+$ and the associated cost J_{prim}^+.
4) Place a bid of p_{prim} in the primary reserve market at price J_{prim}^+.
5) If the bid is accepted, find $\mathcal{I}_{\mathrm{up}}, \mathcal{I}_{\mathrm{down}}$ according to (17) and activate by assign trigger frequencies $(\overline{t}^T \overline{X}^+)_i$ to devices

$i \in \mathcal{I}_{\text{up}}$ and trigger frequencies $(\underline{t}^T \underline{X}^+)_i$ to devices $i \in \mathcal{I}_{\text{down}}$.

A natural extension to the above algorithm is to repeat step 3 with varying primary reserve volumes to find the associated costs. This information can be used to place several bids into the reserve market allowing the aggregator to become a more competitive player. Note that bidding the marginal cost as in step 4 is just meant as an example of a bidding strategy—alternative strategies can be applied as well.

VI. Numerical Examples

We consider two numerical examples: a small-scale example with 100 ON/OFF devices and a large scale example with 10,000 ON/OFF devices. We assume a primary reserve delivery period of 1 hour, a sampling time of 10 s, a frequency resolution of 2 mHz, and a comfort constraint certainty of $\alpha = 0.99$. The following parameters are used:

$$x_i^{\min} = 0, x_i^{\max} \in [0, 6], x_i^0 \in [0, x_i^{\max}] \quad [\text{kWh}],$$
$$p_i \in [2, 5], v_i \in [0, p_i] \quad [\text{kW}] \tag{30}$$

for $i \in \mathcal{I}$. The parameters are uniformly distributed within the given intervals. An interpretation of this portfolio could be water heaters with tanks between 0 and 250 L where each heater allows the water temperature to vary within a band of $50 \pm 10\,^\circ$C. The nominal power consumption of each water heater lies in the interval from 2 kW to 5 kW. It is assumed that one quarter of the consumers have signed flat rate contracts and receive no additional payment per activation while the remaining three quarters of the consumers have flex rate contracts causing a cost per activation:

$$\pi_i = 0, i = 1, \ldots, \frac{n}{4}$$
$$\pi_i \in [0, 1], i = \frac{n}{4} + 1, \ldots, n, \tag{31}$$

where the flex rate costs are assumed uniform in the given interval.

A. Small-Scale Example

In this example, the portfolio consists of $n = 100$ ON/OFF devices. The maximum power consumption of the entire portfolio is 360 kW. The upper and lower bounds on the primary reserve volume are

$$p_{\text{prim}}^{\max} = 180\text{ kW}, p_{\text{prim}}^{\min} = 17\text{ kW}, \tag{32}$$

remembering that the upper bound corresponds to completely ignoring the comfort constraints while the lower bound corresponds to guaranteeing no violated comfort constraints.

We use the algorithm presented in Section V-D to find the minimum cost of providing a primary frequency delivery of respectively 50 kW and 100 kW. The binary optimization problem (29) is used whereby we obtain

$$J^{\star}_{\text{prim},50\text{ kW}} = 2.1, \quad J^{\star}_{\text{prim},100\text{ kW}} = 15.0. \tag{33}$$

This shows that we are able to construct a bid of 50 kW almost solely relying on the flat rate consumers while we are able to construct a bid of additionally 50 kW at an additional

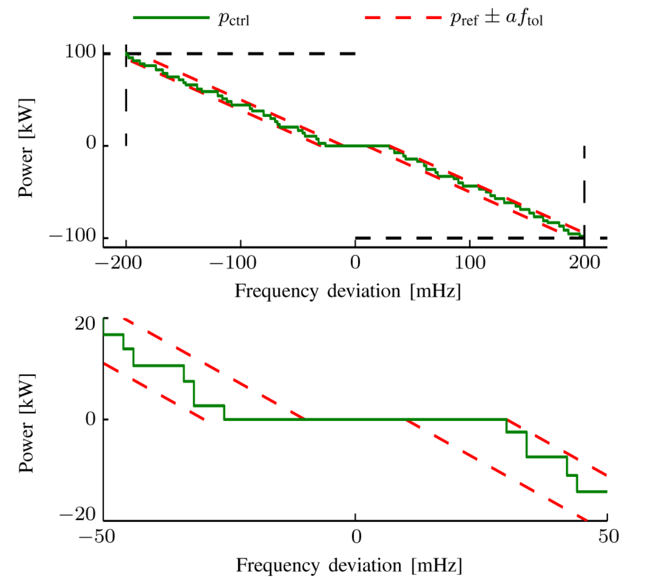

Fig. 3. Allocation of ON/OFF devices that maximizes the delivery p_{prim}.

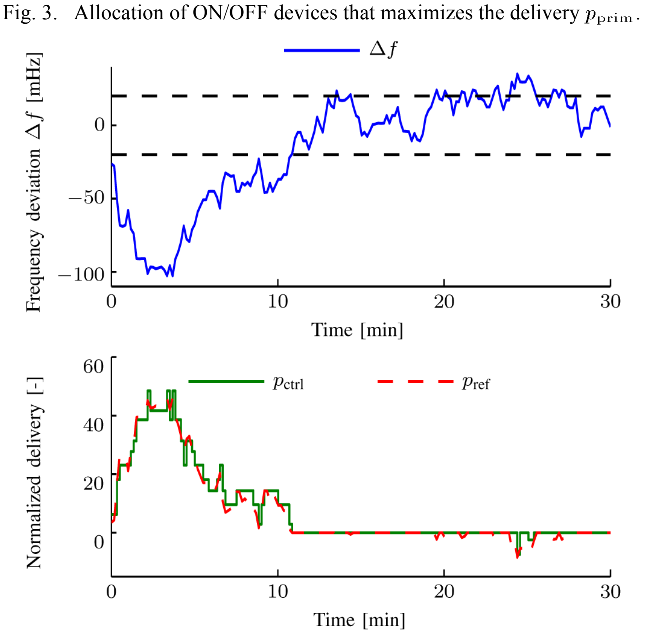

Fig. 4. Transient response of the portfolio of ON/OFF devices to a given sequence of frequency deviations $\Delta f(k)$.

cost of 12.9. The maximum volume of primary reserve we are able to construct using the binary optimization is 117 kW corresponding to 65% of the maximum possible $p_{\text{prim}}^{\max}$ and 6.7 times as much as the conservative bid $p_{\text{prim}}^{\min}$. The resulting droop curve for $p_{\text{prim}} = 100$ kW is presented in Fig. 3.

The performance of the portfolio is examined by evaluating 300 sequences of duration 1 hour based on frequency measurements from the ENTSO-E grid. The first 30 minutes of one such sequence is illustrated in Fig. 4 to show the behavior of the portfolio. Through these 300 simulations, less than 1% of the devices experience comfort constraint violations as expected.

For comparison, the optimization problem is approximately solved using the heuristic method yielding

$$J^{+}_{\text{prim},50\text{ kW}} = 2.3 \quad J^{+}_{\text{prim},100\text{ kW}} = 18.4. \tag{34}$$

This shows that the heuristic method also is able to deliver 50 kW of primary reserve relying almost solely on the flat rate con-

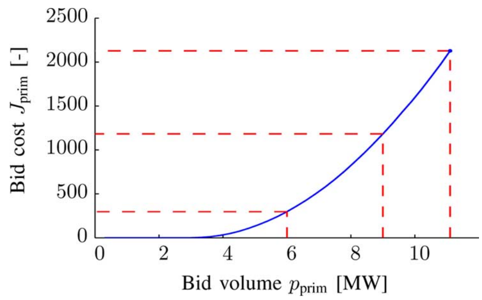

Fig. 5. Bid costs J_{prim} as a function of the bid volume p_{prim}.

sumers while the delivery of 100 kW is 23 % more expensive than the optimal cost. The maximum volume of primary reserve we are able to deliver using the heuristic method is 103 kW corresponding to 12% less than when solved using the binary optimization.

B. Large Scale Example

We consider a portfolio of $n = 10{,}000$ ON/OFF devices with the same distribution as in the previous example. The maximum power consumption of the entire portfolio is 35.1 MW and the upper and lower bounds on the primary reserve volume are

$$p_{\text{prim}}^{\max} = 17.5 \text{ MW}, \quad p_{\text{prim}}^{\min} = 1.8 \text{ MW}. \tag{35}$$

We cannot solve the binary optimization problem using commercial solvers due to the high number of devices. Therefore, we use the heuristic method described in Appendix B to approximately solve the problem. This is illustrated by Fig. 5 where the cost J_{prim} at different primary reserve volumes p_{prim} is illustrated. Four of these bids and associated costs are

$$\begin{aligned} J^{+}_{\text{prim,3 MW}} &= 0, \quad J^{+}_{\text{prim,6 MW}} = 297 \\ J^{+}_{\text{prim,9 MW}} &= 1.183, \quad J^{+}_{\text{prim,11.1 MW}} = 2.354 \end{aligned} \tag{36}$$

illustrating that the flat rate consumers allow the aggregator to construct regulating power reserve bids associated with very low costs; however, as the volume increases, the associated costs increase drastically. The pairs of different volumes of primary reserve and associated costs allow the aggregator to place multiple bids with different costs and hereby increase the competition with the conventional generators.

The maximum volume of primary reserve we are able to construct using the heuristic method is 11.1 MW corresponding to 63 % of the upper bound $p_{\text{prim}}^{\max}$ and 6.2 times as much as the conservative bid $p_{\text{prim}}^{\min}$. Note that 11.1 MW corresponds to more than 40 % of the entire need for primary reserve in Western Denmark.

VII. Conclusion

In this work we showed how a portfolio of ON/OFF devices with flexible power consumption is able to collectively provide a delivery of primary reserve. We described how to minimize the cost of a given primary reserve delivery while honoring device comfort constraints with a given certainty. Through numerical examples, we illustrated the ability of this method to mobilize a large fraction of a portfolio for primary reserve even when only a small fraction of the portfolio possessed full flexibility throughout the delivery period.

Appendix A
Map

This appendix describes how we can perform a mapping from device characteristics $\{x_i^0, p_i, v_i, x_i^{\min}, x_i^{\max}\}$ to upper and lower limits $\{t_i^{\min}, t_i^{\max}\}$ on the trigger frequency for device i. The mapping is constructed as follows: $t_i^{\min} \leq t_i \leq t_i^{\max}$ is the largest trigger frequency interval where the comfort constraints are honored at least with probability α. We determine this mapping based on a large set of system frequency measurement sequences taken from the ENTSO-E grid and assume that these sequences are representative for the system frequency characteristics.

Denote the system frequency deviation measurement sequences $\Delta f_l(k)$ for $k \in \mathcal{K} = \{1, \ldots, K\}$ and $l \in \mathcal{L} = \{1, \ldots, L\}$, where K is the total number of samples in a delivery period (in this example, $K = 360$ corresponding to a primary reserve delivery period of $1 \text{ hour} = 3{,}600$ s and a sampling time $T_s = 10$ s) and L is the number of examined sequences. Let $\overline{e}_{lj}(k)$ denote the *accumulated duty cycle* of a device with trigger frequency $\overline{t}_j$ when system frequency deviation measurement sequence l is applied:

$$\overline{e}_{lj}(k) = \frac{1}{k} \sum_{\kappa=1}^{k} I(\Delta f_l(\kappa), \overline{t}_j) \tag{37}$$

where

$$I(a,b) = \begin{cases} 1 & \text{if } a \geq b \\ 0 & \text{else.} \end{cases} \tag{38}$$

Hereby, $\overline{e}_{lj}(k)$ will be the accumulated duty cycle, or average duty cycle, of a device with trigger frequency $\overline{t}_j$ at time k in the case of the specific frequency realization Δf_l. By having a large set of such frequency realizations (large L), we can use the accumulated duty cycles $\overline{e}_{lj}(k)$ to examine the expected duty cycle of devices with a trigger frequency given by $\overline{t}_j$. By removing the number of outliers corresponding to the value of $1 - \alpha$, we can select the realization with the highest and lowest accumulated duty cycle among the remaining accumulated duty cycle realization. If a given device is able to be turned ON/OFF according to both these two extreme realizations, it will also be able to handle all realizations within these two extreme realizations and thus able to handle the fraction α of all the given realizations. Hence, it will be able to be associated with trigger frequency $\overline{t}_j$ given that the observed data is representable. This is described in more detail in the following.

The α-envelopes (the two extreme realizations) for the accumulated duty cycle can be found as

$$\overline{e}_j^{\max}(k) = \max_{l \in \mathcal{L} \setminus \overline{\mathcal{L}}} \overline{e}_{lj}(k), \overline{e}_j^{\min}(k) = \min_{l \in \mathcal{L} \setminus \overline{\mathcal{L}}} \overline{e}_{lj}(k), j \in \mathcal{J} \tag{39}$$

where $\overline{\mathcal{L}}$ is a set consisting of the $\lfloor L(1-\alpha) \rfloor$ largest outliers of $\overline{e}_{lj}(k)$; hereby we remove the accumulated duty cycle sequences that deviate the most from the remaining sequences.

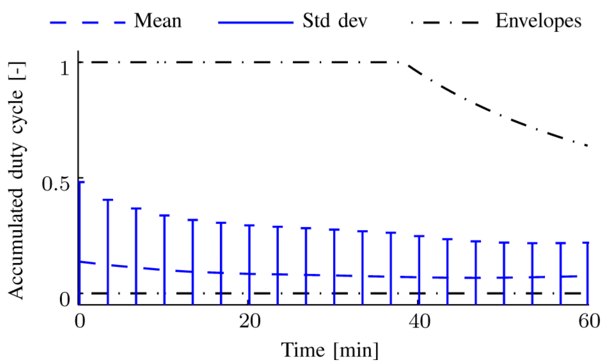

Fig. 6. Accumulated duty cycle mean, standard deviation and $\alpha = 0.99$ envelopes $\overline{e}_j^{\max}(k), \underline{e}_j^{\max}(k)$ for trigger frequency $\overline{t}_j = 20$ mHz.

The removed duty cycle sequences correspond to the most extreme frequency deviations where we are allowed to violate the comfort constraints in concordance with the parameter α. In a similar manner, we can determine the accumulated duty cycle envelopes $\underline{e}_j^{\max}, \underline{e}_j^{\min}$ for the trigger frequencies $\underline{t}_j$.

An illustration of the accumulated duty cycle is seen in Fig. 6 for a trigger frequency of 20 mHz. The figure is built according to the description above: a large number of system frequency measurements are compared to a trigger frequency $\overline{t} = 20$ mHz and a number of accumulated duty cycle sequences are generated according to (37). The outliers are removed and the envelopes (extreme realizations) are found according to (39), these extreme realizations are plotted in the figure (black dash-dotted). For comparison, the overall mean and standard deviation of the accumulated duty cycle sequences are also presented.

A number of observations can be made from the figure. The overall mean of the observed sequences illustrates that the system frequency is above 20 mHz approximately 10 % of the time. The figure further shows the accumulated duty cycle envelopes. A device with trigger frequency 20 mHz must be able to handle any duty cycle sequence within these envelopes to ensure comfort with the required probability. This means that a device with trigger frequency of 20 mHz must be fully flexible the first 40 minutes whereafter the duty cycle requirement decreases.

If instead of a trigger frequency of $\overline{t} = 20$ mHz we had taken a higher value, for example $\overline{t} = 100$ mHz, we would see different envelopes: the lower envelope would still be at 0, but the higher envelope would decrease drastically. The reason is that a device with such a high trigger frequency only rarely will be ON, as the system frequency deviation only rarely increases above 100 mHz.

Based on envelopes $\overline{e}_j^{\max}(k), \overline{e}_j^{\min}(k), j \in \mathcal{J}$, we can perform the desired mapping from device characteristics to a trigger frequency interval. Let the set $\mathcal{T}_i$ denote all feasible trigger frequencies for device i. Then we have

$$\overline{t}_j \in \mathcal{T}_i \tag{40}$$

if and only if the comfort constraints (4) holds in two cases:

1) for the upper envelope $u_i(k) = \overline{e}_j^{\max}(k),\ k \in \mathcal{K}$,
2) for the lower envelope $u_i(k) = \overline{e}_j^{\min}(k),\ k \in \mathcal{K}$.

If the comfort constraints hold for the accumulated duty cycle envelopes, the constraint will also hold for any realizations between the envelopes and consequently hold with probability α. Similarly, we can determine the necessary conditions for having $\underline{t}_j \in \mathcal{T}_i$. The resulting trigger frequency limitations are given as

$$t_i^{\max} = \max \mathcal{T}_i, \quad t_i^{\min} = \min \mathcal{T}_i. \tag{41}$$

Appendix B
Heuristic Method

In this appendix we present a simple heuristic method that approximately solves the mixed integer problem (29) for large n. The following steps describe the method at an overall level.

1) **Initialize** $\overline{X}_{ji}, \underline{X}_{ji} = 0, i \in \mathcal{I}, j \in \mathcal{J}$ and $\mathcal{I}_{\text{up}}, \mathcal{I}_{\text{down}} = \emptyset$.
2) **Loop** through all upward regulation trigger frequencies $\overline{t}_j, \quad j = 1, \ldots, m$.
3) **Repeat**
4) Determine the feasible devices for trigger frequency $\overline{t}_j$: $\mathcal{I}_{\text{feas}} := \{i \in \mathcal{I} \backslash \mathcal{I}_{\text{prim}} | t_i^{\min} \le \overline{t}_j \le t_i^{\max}\}$.
5) If $\mathcal{I}_{\text{feas}} \neq \emptyset$, assign trigger frequency $\overline{t}_j$ to the device with the lowest cost by $\overline{X}_{ji} := 1$ where $i = \arg\min_{i \in \mathcal{I}_{\text{feas}}} \pi_i$. Update $\mathcal{I}_{\text{up}}$ according to (17).
6) **Until** the error between the delivery $\overline{p}_j$ and the reference $p_{\text{ref}}(\overline{t}_j)$ increases.
7) Repeat for down-regulating frequencies.
8) Denote the final allocation matrices $\overline{X}^+, \underline{X}^+$.

This illustrates the basic idea in the method: to start from the innermost trigger frequency $\overline{t}_1$ and assign trigger frequencies to devices until we are as close as possible to the power reference, always selecting the device with the lowest activation cost. After allocating devices for the first trigger frequency $\overline{t}_1$, move outwards to the following trigger frequency $\overline{t}_2$, etc. When the algorithm has run to completion we can test that the final allocation as defined by $\overline{X}^+, \underline{X}^+$ indeed satisfy the constraints as specified in (29).

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