A Dynamic Market Mechanism for Markets with Shiftable Demand Response ⋆

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Abstract: In this paper, we propose a dynamic market mechanism that converges to the desired market equilibrium. Both locational marginal prices and the schedules for generation and consumption are determined through a negotiation process between the key market players. In addition to incorporating renewables, this mechanism accommodates both consumers with a shiftable Demand Response and an adjustable Demand Response. The overall market mechanism is evaluated in a Day Ahead Market and is shown in a numerical example to result in a reduction of the cost of electricity for the consumer, as well as an increase in the Social Welfare.

Keywords: Smart grids, Power systems stability, Demand Response, Shiftable demand, Electricity market, Day Ahead Market

1. INTRODUCTION

Two dominant features of a smart grid are a high penetration of renewable energy resources (RERs) and Demand Response (DR), the flexibility to adjust power consumption (see Annaswamy et al. [2013]). The introduction of both renewable energy sources as well as efforts to integrate DR-compatible consumption brings in a set of dynamic interactions between the major components of a smart grid. Electricity markets, the entity that carries out power balance by scheduling power using bids from various generating companies, are crucial components that can facilitate such dynamic interactions. This paper proposes a dynamic market mechanism in a smart grid that includes the behavior of key market players such as generators, consumers, and Independent System Operator (ISO), together with DR-compatible participants.

Demand Response consists of systems, services, and strategies that enable demand resources to adjust their consumptions in response to economic signals from a competitive wholesale market. Given these varied flexibilities, DR can be grouped into two major categories, $\mathrm{DR_a}$, which corresponds to DRs that adjust their consumption (see for example, Sioshansi, Short [2009], Staff [2008]) and $\mathrm{DR_s}$ that shift their consumption (see for example, Roscoe, Ault [2010]), both in response to wholesale energy prices. Both types of consumption can provide relief from capacity constraints and promote more economically efficient uses of electrical energy. In Kiani, Annaswamy [2011], a market model was proposed that only included $\mathrm{DR_a}$. In this paper, an extension is proposed to include the second category $\mathrm{DR_s}$, where the demand is shiftable.

Various methods have been proposed in the literature to determine market models, based on market equilibrium, Equilibrium Programming with Equilibrium Constraints (EPECs), Variational and Complementarity Problem (CP) formulation, and game theory (see for example, Yao et al. [2008], Ruiz, Conejo [2009], Hu, Ralph [2007], Morales et al. [2009], Klemperer, Meyer [1989], Visudhiphan, Ilic [1999], Alvarado et al. [2000], Cunningham et al. [2002]). In contrast to the above, a dynamic approach has been proposed in Kiani, Annaswamy [2011], Kiani, Annaswamy [2012] where iterative negotiations are proposed between the key market players so as to arrive at the desired market equilibrium. Similar to, Arrow et al. [1958], the idea is to use gradient approach to determine these iterative strategies. And similar to Alvarado et al. [2000], price is used as a feedback control input for power balance.

The approach in, Kiani, Annaswamy [2011], Kiani, Annaswamy [2012] included Demand Response that was assumed to be directly responsive to prices; that a proportional increase or decrease was assumed to occur for a decrease or increase in the price. This class is expanded much further in this paper to include DR that is shiftable over a 24-hour interval rather than adjustable at each instant. This introduces a significant challenge of integral constraints. A two-step approach is proposed to address this challenge; in the first, a desired demand profile is generated that accommodates the integral constraint over

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a period of interest and in the second, a dynamic market mechanism is proposed, similar to Kiani, Annaswamy [2011], which suitably allows the flexible demand to converge to the desired demand profile. Using the resulting model, conditions of stable convergence are defined.

This paper has been organized as follows: In Section 2 the desirable demand response profile for $\mathrm{DR_s}$ is presented. In Section 3, a wholesale energy market structure is introduced, and the underlying dynamic model is presented including both $\mathrm{DR_a}$ and $\mathrm{DR_s}$. In Section 3.6 stability properties are derived and the region of attraction is established. In Section 4 numerical studies of an IEEE 4-bus are reported and finally in Section 5, we provide summary and concluding remarks.

2. A DESIRABLE DEMAND PROFILE

The main goal of electricity markets is to ensure efficient power balance. A dominant factor that may impede this is the variation in demand. Given the paradigm shift that is occurring at present with Demand Response, and the possibility of a shiftable demand (see Roscoe, Ault [2010]), we discuss the optimal demand profile in this section, and introduce constraints to represent the shiftable component.

Denoting the shiftable and non-shiftable demand components as P_{Ds} and P_{Df}, respectively, we represent the shiftable demand at a consumption unit using the following constraints:

$$\sum_{t\in T} P_{Ds_t} = P_{Ds}^{\mathrm{tot}} \tag{1a}$$

$$0 \leq P_{Ds_t} \leq P_{Ds_t}^{\max}, \qquad \forall t \in T \tag{1b}$$

where P_{Ds_t} is the shiftable demand at time t, T denotes the time period of interest, P_{Ds}^{tot} is the total shiftable demand over T, and $P_{Ds_t}^{\max}$ is the maximum allowable shiftable demand at time t.

One can then generate the desired demand profile by constructing an objective function

$$\sum_{t\in T}(P_{Ds_t} + P_{Df_t})^2, \tag{2}$$

where P_{Df_t} is the fixed demand component at time t, and then pose the overall problem as the minimization of (2) subject to constraints in (1). The resulting solution, denoted as $P_{Ds_t}^{\mathrm{ref}}$, can be seen to be one where the demand is more uniform over T.

Fig. 1 illustrates a desired demand profile using typical consumption data[1] over a 24-hour period where it is assumed that 10%[2] of the power consumption is shiftable at each hour t. That is, $P_{Ds_t} = 0.1(P_{Ds_t} + P_{Df_t})$ for $t \in [1, 24]$. With the constraints presented in (1) chosen as $P_{Ds}^{\mathrm{tot}} = 117.1$ MWh and $P_{Ds_t}^{\max} = 60$ MWh, the solutions $P_{Ds_t}^{\mathrm{ref}}$ (blue) and $P_{Df_t}^{\mathrm{ref}}$ (black) at each t of (2) subject to (1) are determined. For $t \in [9, 23]$ peak reductions (gray) are achieved.

[1] From ISO New England at http://www.iso-ne.com/markets/ with a scaling factor of 0.01.

[2] Such a percentage is within projected DR penetration found in studies of demand (Milligan, Kirby [2010]).

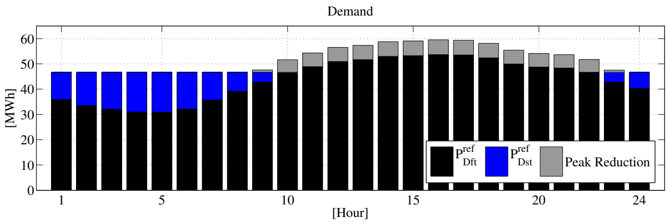

Fig. 1. 90% of hourly demand distribution (black), remaining 10% optimally distributed as shiftable consumption (blue) and peak reduction (gray) from the original distribution in NEMASSBOST on 07/19/13.

3. WHOLESALE ENERGY MARKET STRUCTURE

The electricity market that we consider in this paper is wholesale and is assumed to function as follows: First, each consumer (ConCo) submits the bidding stacks of each of its demands to the pool. Similarly, each generating company (GenCo) submits the bidding stacks to the pool. Then, the ISO clears the market using an appropriate market-clearing procedure resulting in prices and consumption and production schedules. In what follows, we model each of the components (ConCo, GenCo and ISO) together with their constraints and the overall optimization goal.

In the modeling, the following nomenclature will be used. N, N_t, N_D, N_{Gc} and N_{Gw} denote the total number of buses, transmission lines, consuming units, dispatchable generating units and non-dispatchable generating units, respectively. Further, θ_n, ϑ_n and ϕ_n denote the set of indices of dispatchable generating units, non-dispatchable generating units and consuming units at node n, respectively. Ω_n denote the set of indices of nodes connected to node n and finally, D_q, G_c and G_w denote the set of indices of consuming units, dispatchable generating units and non-dispatchable generating units, respectively.

3.1 Consumer Modeling

For consumer company $j \in D_q = \{1, 2, \ldots, N_D\}$ the consumption is divided into three classes: fixed, adjustable and shiftable, denoted P_{Df_j}, P_{Da_j} and P_{Ds_j}, respectively. Each consumer company is assumed to consist of one unit of each class of consumption and the total consumption $P_{D_j} = P'_{Da_j} + P_{Ds_j}$ where $P'_{Da_j} = P_{Df_j} + P_{Da_j}$. The value of using each class of electricity for the consumer is represented by the associated utility functions in which the marginal utilities are decreasing linear functions of power consumption. The utility functions are

$$U_{Da_j}(P'_{Da_j}) = b_{Da_j} P'_{Da_j} + \frac{c_{Da_j}}{2} P'^2_{Da_j} \tag{3a}$$

$$U_{Ds_j}(P_{Ds_j}) = b_{Ds_j} P_{Ds_j} + \frac{c_{Ds_j}}{2} P^2_{Ds_j}, \tag{3b}$$

where b_{Da_j}, b_{Ds_j}, c_{Da_j} and c_{Ds_j} are consumer utility coefficients. The utility of the total consumption $U_{D_j}(P_{D_j}) = U_{Da_j}(P'_{Da_j}) + U_{Ds_j}(P_{Ds_j})$.

The consumption values are constrained; P'_{Da_j} and P_{Ds_j} must evolve such that in equilibrium, they reach a value no smaller than the derived reference and reach the derived reference, respectively, where both references are determined as in Section 2.

3.2 Generator Modeling

The generating companies are separated into conventional dispatchable units e.g. coal plants, and non-dispatchable renewable energy resource units e.g. wind energy, and modeled separately. It is assumed that all GenCos bid their marginal cost, rather than performing strategic biddings. The non-dispatchable generator companies are furthermore assumed to submit their bids to the market like conventional GenCos.

Conventional Dispatchable GenCos Each dispatchable generator company $i \in G_c = \{1, 2, \ldots, N_{Gc}\}$ is assumed to consist of one generating unit and the production of each generator company is denoted P_{Gc_i}. The costs of operation are assumed quadratic functions of generated power, implying a linear marginal cost i.e. cost of changing production one unit. The operating cost of dispatchable GenCo i is given by

$$C_{Gc_i}(P_{Gc_i}) = b_{Gc_i} P_{Gc_i} + \frac{c_{Gc_i}}{2} P_{Gc_i}^2, \quad P_{Gc_i}^{\min} \leq P_{Gc_i} \leq P_{Gc_i}^{\max} \tag{4}$$

where b_{Gc_i} and c_{Gc_i} are generator cost coefficients and $P_{Gc_i}^{\min}$ and $P_{Gc_i}^{\max}$ are lower and upper bounds on GenCo i, respectively. Rate constraints, startup and shutdown costs on the dispatchable generation are not included in this model.

Non-Dispatchable RER GenCos Each non-dispatchable generating company $l \in G_w = \{1, 2, \ldots, N_{Gw}\}$ is assumed to consist of only one generating unit and the production of each generator company is denoted P_{Gw_l}. The costs of operating are given by

$$C_{Gw_l}^{\text{tot}}(P_{Gw_l}) = C_{Gw_l}(P_{Gw_l}) + C_{w_l}^r(\Delta_{w_l}), \tag{5}$$

where

$$C_{Gw_l}(P_{Gw_l}) = b_{Gw_l} P_{Gw_l} + \frac{c_{Gw_l}}{2} P_{Gw_l}^2 \tag{6}$$

$$C_{w_l}^r(\Delta_{w_l}) = b_{w_l} \Delta_{w_l} + \frac{c_{w_l}}{2} \Delta_{w_l}^2. \tag{7}$$

Coefficients b_{Gw_l} and c_{Gw_l} are generator cost coefficients, b_{w_l} and c_{w_l} are reserve cost coefficients. $C_{Gw_l}(P_{Gw_l})$ denotes the traditional generation cost, which is very close to zero. $C_{w_l}^r(\Delta_{w_l})$ denotes the cost of committing specific generators as reserves due to the wind uncertainty and Δ_{w_l} is given by

$$\Delta_{w_l} = P_{Gw_l} \Delta_{Gw_l}, \tag{8}$$

where $0 < \Delta_{Gw_l} < 1$ corresponds to overestimated wind energy which implies the assumption that the power can be purchased elsewhere or demand can be adjusted. Underestimation is present when $-1 < \Delta_{Gw_l} < 0$ which implies a surplus in power that is assumed to be handled alternatively.

The non-dispatchable generation is constrained by

$$0 \leq P_{Gw_l} \leq P_{Gw_l}^{\max}, \tag{9}$$

where $P_{Gw_l}^{\max}$ is the maximum achievable generation of non-dispatchable unit l, limited by external factors e.g. wind speeds. Rate constraints, startup and shutdown costs on the non-dispatchable generation are not included and the reserve market is assumed cleared independently of the energy market.

3.3 ISO Market-Clearing Model

The ISO clears the electricity market optimizing a cost function subject to network constraints. Generally, the most significant constraints in the network are due to network losses and line capacity limits. Technical limitations are constraining the power flow through lines, which is said to be congested when approaching these upper limits. Congestion is explicitly included in the model, whereas ohmic losses are not modeled.

The market-clearing procedure acts on behalf of ConCos and GenCos by maximizing the utility of consumption for the ConCos and minimizing the cost of generation for the GenCos. The cost function to optimize is often termed Social Welfare and is defined as

$$S_W = \sum_{j \in D_q} U_{D_j}(P_{D_j}) - \sum_{i \in G_c} C_{Gc_i}(P_{Gc_i}) - \sum_{l \in G_w} C_{Gw_l}^{\text{tot}}(P_{Gw_l}). \tag{10}$$

Expressed as an optimization problem, the market-clearing procedure is given by

$$\text{Maximize } S_W = \text{Minimize } -S_W \tag{11}$$

subject to

$$-\sum_{i \in \theta_n} P_{Gc_i} - \sum_{l \in \vartheta_n} (P_{Gw_l} + \Delta_{w_l}) + \sum_{j \in \phi_n} P_{D_j} + \sum_{m \in \Omega_n} B_{nm}[\delta_n - \delta_m] = 0; \rho_n' \quad \forall n \in N \tag{12a}$$

$$B_{nm}[\delta_n - \delta_m] \leq P_{nm}^{\max}; \gamma_{nm} \quad \forall n \in N, \forall m \in \Omega_n \tag{12b}$$

$$P_{Ds_j} - P_{Ds_j}^{\text{ref}} = 0; \lambda_j \quad \forall j \in D_q \tag{12c}$$

$$P_{Df_j}^{\text{ref}} \leq P_{Da_j}'; \zeta_j \quad \forall j \in D_q \tag{12d}$$

$$P_{Gw_l} \leq P_{Gw_l}^{\max}; \xi_l \quad \forall l \in G_w \tag{12e}$$

where δ_n is the voltage angle of node n, B_{nm} is the susceptance of line n to m, $P_{nm}^{\max}$ is the power capacity limit of line n to m and $P_{Ds_j}^{\text{ref}}$ and $P_{Df_j}^{\text{ref}}$ are the references for shiftable and fixed demand, respectively, defined in Section 2. Power balance and capacity limits are respected through constraint (12a) and (12b), respectively. Constraint (12c) and (12d) dictates the desired profile for the shiftable and fixed demand, respectively. Constraint (12e) corresponds to the limit on RERs. The associated Lagrange multipliers, ρ_n', γ_{nm}, λ_j, ζ_j and ξ_l are indicated for each constraint.

3.4 Game-Theoretical Dynamic Model of Wholesale Market

As argued in Kiani, Annaswamy [2011], an optimization problem as in (11)-(12) behaves as a game between the ConCos, GenCos and the ISO where each player seek maximization of their own benefit. The solution to this state-based game can be arrived at using gradient play, which leads to differential equations describing the path of (P_{Gc_i}, P_{Gw_l}, P_{Da_j}', P_{Ds_j}, δ_n, ρ_n', λ_j, γ_{nm}, ζ_j, ξ_l) from a perturbed state to the equilibrium ($P_{Gc_i}^*$, $P_{Gw_l}^*$, $P_{Da_j}'^*$, $P_{Ds_j}^*$, δ_n^*, $\rho_n'^*$, λ_j^*, γ_{nm}^*, ζ_j^*, ξ_l^*) and is given by

$$\tau_{Gc_i} \dot{P}_{Gc_i} = \rho_{n(i)}' - c_{Gc_i} P_{Gc_i} - b_{Gc_i} \tag{13a}$$

$$\tau_{Gw_l} \dot{P}_{Gw_l} = \rho_{n(l)}' - (c_{Gw_l} + c_{w_l} \Delta_{Gw_l}^2) P_{Gw_l} - (b_{Gw_l} + b_{w_l} \Delta_{Gw_l}) - \xi_l \tag{13b}$$

$$\tau_{Da_j} \dot{P}_{Da_j}' = c_{Da_j} P_{Da_j}' + b_{Da_j} - \rho_{n(j)}' + \zeta_j \tag{13c}$$

$$\tau_{Ds_j}\dot{P}_{Ds_j} = c_{Ds_j}P_{Ds_j} + b_{Ds_j} - \rho'_{n(j)} - \lambda_j \tag{13d}$$

$$\tau_{\delta_n}\dot{\delta}_n = -\sum_{m\in\Omega_n} B_{nm}\left[\rho'_n - \rho'_m + \gamma_{nm} - \gamma_{mn}\right] \tag{13e}$$

$$\begin{aligned}\tau_{\rho'_n}\dot{\rho}'_n = &-\sum_{i\in\theta_n} P_{Gc_i} - \sum_{l\in\vartheta_n} P_{Gw_l}(1+\Delta_{Gw_l}) \\ &+ \sum_{j\in\phi_n} P_{D_j} + \sum_{m\in\Omega_n} B_{nm}\left[\delta_n - \delta_m\right]\end{aligned} \tag{13f}$$

$$\tau_{\lambda_j}\dot{\lambda}_j = P_{Ds_j} - P^{\text{ref}}_{Ds_j} \tag{13g}$$

$$\tau_{\gamma_{nm}}\dot{\gamma}_{nm} = \text{Proj}_{\gamma_{nm}}\left(B_{nm}\left[\delta_n - \delta_m\right] - P^{\max}_{nm}, d_{\gamma_{nm}}, \epsilon\right) \tag{13h}$$

$$\tau_{\zeta_j}\dot{\zeta}_j = \text{Proj}_{\zeta_j}\left(P^{\text{ref}}_{Df_j} - P'_{Da_j}, d_{\zeta_j}, \epsilon\right) \tag{13i}$$

$$\tau_{\xi_l}\dot{\xi}_l = \text{Proj}_{\xi_l}\left(P_{Gw_l} - P^{\max}_{Gw_l}, d_{\xi_l}, \epsilon\right), \tag{13j}$$

where all τ's are associated time constants. In (13) an interpretation of λ_j as the incentive price for reaching the shiftable consumption reference, ζ_j as the incentive price for reaching the fixed consumption reference and ξ_l as the shadow price for keeping the non-dispatchable power generation below maximum becomes evident. The solution $P^*_{Gc_i}$ is the power to be generated by dispatchable generator i, $P^*_{Gw_l}$ is the power to be generated by the non-dispatchable generator l, $P'^*_{Da_j}$ is the adjustable and fixed consumption by consumer j, $P^*_{Ds_j}$ is the shiftable consumption by consumer j, and γ^*_{nm} is the congestion price. The slack variable ρ'_n is defined as a manipulated locational marginal price (LMP),

$$\rho'_n = \rho_n \mu_t \tag{14}$$

where ρ_n is the true LMP and μ_t is a scalar that will be suitably defined later. Additionally, with $d'_y = d_y - \epsilon$,

$$\text{Proj}_y(f(x), d_y, \epsilon) = \begin{cases} \dfrac{d_y^2 - y^2}{d_y^2 - (d'_y)^2} f(x) & \text{if } \begin{bmatrix} d'_y \le y \le d_y, \\ f(x) > 0 \end{bmatrix} \\ \dfrac{y^2}{\epsilon^2} f(x) & \text{if } \begin{bmatrix} 0 \le y \le \epsilon, \\ f(x) < 0 \end{bmatrix} \\ f(x), & \text{otherwise} \end{cases} \tag{15}$$

ensures non-negativity of γ_{nm}, ζ_j, and ξ_l as required by the KKT conditions and bounds them by $d_{\gamma_{nm}}$, d_{ζ_j}, and d_ξ, respectively, as stated in Lemma 1.

Lemma 1. If $\dot{y} = \text{Proj}_y(f(x), d_y, \epsilon)$ then $0 \le \|y(t_0)\| \le d_y$ implies $0 \le \|y(t)\| \le d_y$ for all $t \ge t_0$.

Proof. Let $V(y) = \frac{1}{2}y^2$ such that $\dot{V}(y) = yf(x)$. It is easily seen that if $f(x) \le 0$ then $\dot{V} \le 0$ and if $f(x) > 0$ then (15) ensures that $\dot{V}$ will gradually decrease towards zeros when y approaches d_y. ■

One can view the dynamics in (13) as a market mechanism with the evolution of the differential equations representing a continuous negotiation process between market players. The time it takes to complete this negotiation is shown in Fig. 2 with an overall market mechanism time scale where T_{neg} denotes the interval over which negotiations take place and T_{dpc} denotes the period over which the desired demand profile is calculated. The premise of our dynamic market mechanism is that once the desired demand profile is calculated within the time frame T_{dpc}, continued negotiations take place between the market players over a period T_{neg}, with these periods chosen such that $T_{\text{dpc}} + T_{\text{neg}} < T$, the period of interest. For example, $T = 24$ hours in a Day Ahead Market (DAM) and $T = 5$ minutes in a Real-Time Market (RTM). This implies that the time-scales of the negotiations are faster than other events and hence we represent the underlying interactions using a differential equation as in (13).

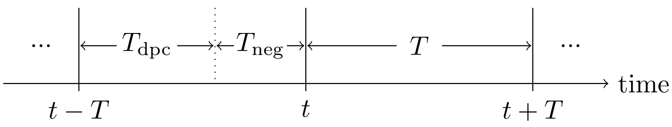

Fig. 2. Overall market mechanism structure where T specifies the time period of interest as in Section 2, T_{dpc} is the desired demand profile calculation time slot and T_{neg} is the time for the overall negotiation of the market equilibrium.

The remaining problem that should be addressed is if these negotiations are well-behaved, i.e. if the underlying dynamic model is stable and all solutions converge to the desired equilibrium. This is analyzed in the next section.

3.5 Equilibrium of Wholesale Market

Using the market mechanism proposed in (13), a dynamic model can be written compactly as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \left[\begin{array}{c|c} A_{11}\ A_{12} & A_2 \\ \hline 0 & 0\end{array}\right]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}[b_{11}\ b_{12}]^T\\ b_2\end{bmatrix} \tag{16}$$

where

$$x_1(t) = \left[P_{Gc_i}\ P_{Gw_l}\ P'_{Da_j}\ P_{Ds_j}\ \delta_n\ \rho'_n\ \lambda_j\right]^T \tag{17}$$

$$x_2(t) = \left[\gamma_1\ \cdots\ \gamma_{N_t}\ \zeta_j\ \xi_l\right]^T \tag{18}$$

$$A_{11} = \begin{bmatrix} -\tau_{gc}^{-1}c_{gc} & 0 & 0 & 0 \\ 0 & -\tau_{gw}^{-1}(c_{gw} + c_w\Delta^2_{gw}) & 0 & 0 \\ 0 & 0 & \tau_{da}^{-1}c_{da} & 0 \\ 0 & 0 & 0 & \tau_{ds}^{-1}c_{ds} \\ 0 & 0 & 0 & 0 \\ -\tau_{\rho'}^{-1}A_{gc} & -\tau_{\rho'}^{-1}A_{gw}(I + \Delta_{gw}) & \tau_{\rho'}^{-1}A_d & \tau_{\rho'}^{-1}A_d \\ 0 & 0 & \tau_\lambda^{-1} & 0 \end{bmatrix} \tag{19}$$

$$A_{12} = \begin{bmatrix} 0 & \tau_{gc}^{-1}A_{gc}^T & 0 \\ 0 & \tau_{gw}^{-1}A_{gw}^T & 0 \\ 0 & -\tau_{da}^{-1}A_d^T & 0 \\ 0 & -\tau_{ds}^{-1}A_d^T & -\tau_{ds}^{-1} \\ 0 & -\tau_\delta^{-1}A_r^T B_{line}A & 0 \\ \tau_{\rho'}^{-1}A^T B_{line}A_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{20}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & -\tau_\delta^{-1}B_{line}A_r & 0 & 0 \\ 0 & 0 & \tau_{da}^{-1} & 0 & 0 & 0 & 0 \\ 0 & -\tau_\xi^{-1} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{21}$$

$$b_{11} = \left[-b_{gc}^T\tau_{gc}^{-1}\ \ -(b_{gw} + b_w\Delta_{gw})^T\tau_{gw}^{-1}\ \ b_{da}^T\tau_{da}^{-1}\ \ b_{ds}^T\tau_{ds}^{-1}\right] \tag{22}$$

$$b_{12} = \left[0\ 0\ -P^{\text{ref}}_{ds}\tau_\lambda^{-1}\right] \tag{23}$$

$$b_2 = \begin{bmatrix} \tau_\gamma^{-1}\text{Proj}_{\gamma_{nm}}\left(B_{nm}\left[\delta_n - \delta_m\right] - P^{\max}_{nm}, d_{\gamma_{nm}}, \epsilon\right) \\ \tau_\zeta^{-1}\text{Proj}_{\zeta_j}\left(P^{\text{ref}}_{Df_j} - P'_{Da_j}, d_{\zeta_j}, \epsilon\right) \\ \tau_\xi^{-1}\text{Proj}_{\xi_l}\left(P_{Gw_l} - P^{\max}_{Gw_l}, d_{\xi_l}, \epsilon\right) \end{bmatrix}. \tag{24}$$

A denotes the $N_t \times N$ bus incidence matrix and A_r is the $N_t \times N - 1$ reduced bus incidence matrix with the column corresponding to the reference bus removed. A_d

denotes the $N \times N_D$ consumers incidence matrix, where $A_{d_{ij}} = 1$ if the j^{th} consumer is connected to the i^{th} bus and $A_{d_{ij}} = 0$ if the j^{th} consumer is not connected to the i^{th} bus. Similarly, A_{gc} denotes the $N \times N_{Gc}$ dispatchable generator incidence matrix and A_{gw} denotes the $N \times N_{Gw}$ non-dispatchable generator incidence matrix. B_{line} denotes the $N_t \times N_t$ diagonal line admittance matrix, $P^{\max}$ is the vector of maximum capacity limits $P_{nm}^{\max}$, P_{df}^{ref} is the vector of fixed demand references $P_{Df_j}^{\text{ref}}$, $P_{Gw}^{\max}$ is the vector of maximum available non-dispatchable power $P_{Gw_l}^{\max}$ and P_{ds}^{ref} is the vector of shiftable power references $P_{Ds_j}^{\text{ref}}$. I denotes a $N_{Gw} \times N_{Gw}$ identity matrix, Δ_{gw} denotes the diagonal matrix of wind uncertainties Δ_{Gw_l} and let R_1, R_2 and R_3 denote rotation matrices which make $R_1 x_1 = [\delta_1 \ \cdots \ \delta_{N-1}]^T$, $R_2 x_1 = [P'_{Da_1} \ \cdots \ P'_{Da_{N_D}}]^T$ and $R_3 x_1 = [P_{Gw_1} \ \cdots \ P_{Gw_{N_{Gw}}}]^T$. Further b's denote vectors of corresponding b coefficients and c's denote diagonal matrices of corresponding c coefficients. Finally, τ's denote diagonal matrices of corresponding time constants. Let $[A_{11} \ A_{12}]$ be referred to as A_1, $[b_{11} \ b_{12}]^T$ as b_1 and denote the columns of A_2 as A_{21}, A_{22} and A_{23}, respectively.

Defining the equilibrium set of the wholesale market under the game stated in (13) as

$$E = \{(x_1, x_2) | A_1 x_1 + A_2 x_2 + b_1 = 0 \wedge b_2 = 0\}, \quad (25)$$

then (x_1^*, x_2^*) is an equilibrium point if $(x_1^*, x_2^*) \in E$.

3.6 Stability of Wholesale Market

The stability property of the equilibrium is now established with the Lyapunov approach. It is assumed that strong duality holds and $(x_1^*, x_2^*) \in E$ exists.

We first introduce a few definitions. Let P be the symmetric solution of the equation $A_1^T P + P A_1 = -I$. Let $\|PA_{21}\| \leq \beta_1$, $\|PA_{22}\| \leq \beta_2$, $\|PA_{23}\| \leq \beta_3$, $y_2 = [y_{21} \ y_{22} \ y_{23}]^T$, $\Omega_{\max} \equiv \{(y_1, y_2) \mid \|y_{21}\| \leq d_\gamma, \|y_{22}\| \leq d_\zeta, \|y_{23}\| \leq d_\xi\}$, $\Omega_{\min} \equiv \{(y_1, y_2) \mid V(y_1) \leq \lambda_{min}(P)\alpha^2, \|y_{21}\| \leq d_\gamma, \|y_{22}\| \leq d_\zeta, \|y_{23}\| \leq d_\xi\}$, and

$$\alpha = 2d_\gamma \beta_1 + 2d_\zeta \beta_2 + 2d_\xi \beta_3. \quad (26)$$

With $y_1 = x_1 - x_1^*$, $y_{21} = \gamma - \gamma^*$, $y_{22} = \zeta - \zeta^*$ and $y_{23} = \xi - \xi^*$, Lemma 1 establishes boundedness of y_2 and Theorem 2 establishes the stability and region of attraction around the equilibrium $(x_1^*, x_2^*) \in E$.

Theorem 2. Let strong duality hold. Then the equilibrium $(x_1^*, x_2^*) \in E$ of the market defined by the dynamic game in (16) is stable for all initial conditions in $\Omega_{\max}$ if A_1 is Hurwitz. In addition all trajectories will converge to $\Omega_{\min}$.

Proof. Differentiating a positive definite Lyapunov function $V(y_1) = y_1^T P y_1$ along its trajectories we get

$$\dot{V} \leq y_1^T (PA_1 + A_1^T P) y_1 + y_1^T P \big(A_{21} y_{21} + A_{22} y_{22} + A_{23} y_{23}\big) + \big(A_{21} y_{21} + A_{22} y_{22} + A_{23} y_{23}\big)^T P y_1. \quad (27)$$

The right-hand side of (27) can be rewritten using the definitions of β_1, β_2, and β_3 above as

$$y_1^T (P_1 A_{21}) y_{21} \leq \beta_1 \|y_1\| \|y_{21}\| \quad (28)$$

$$y_1^T (P_1 A_{22}) y_{22} \leq \beta_2 \|y_1\| \|y_{22}\| \quad (29)$$

$$y_1^T (P_1 A_{23}) y_{23} \leq \beta_3 \|y_1\| \|y_{23}\|. \quad (30)$$

Using (28)-(30), the projection bounds d_γ, d_ζ, and d_ξ on $\|y_{21}\|$, $\|y_{22}\|$ and $\|y_{23}\|$, respectively, and since A_1 is Hurwitz, we obtain that

$$\dot{V} \leq -\|y_1\|^2 + 2\|y_1\| \big(\beta_1 d_\gamma + \beta_2 d_\zeta + \beta_3 d_\xi\big),$$

and by using (26), we can get

$$\dot{V} \leq -\|y_1\| \big(\|y_1\| - \alpha\big). \quad (31)$$

Boundedness of y_1 and the convergence of all trajectories to $\Omega_{\min}$ follow from (31). ■

Table 1. Results for 24-hour period.

	Case 1	Case 2	Unit
D_1 Consumption	2354.8	2336.8	MWh
D_2 Consumption	2913.5	2886.3	MWh
D_1 Cost	92815.4	89410.9	\$
D_2 Cost	113193.7	108610.1	\$
Total Cost	206009.1	198021.0	\$
Cost pr. Unit	72.2	70.6	\$/MWh
Social Welfare	84047.3	85067.5	\$

4. ILLUSTRATIVE EXAMPLE

We illustrate the dynamic market mechanism with a shiftable demand response proposed in Sections 2-3 using a 4-bus network (see Kiani, Annaswamy [2010]). The network consists of three generating units located at Bus 1 and 2. G_1 is a base-load dispatchable generator with low cost coefficients and slow dynamic response. G_2 is a peak-load dispatchable generator acting like a spinning reserve with high cost coefficients and fast dynamic response. G_3 is a non-dispatchable generator with very low cost coefficients and fast dynamic response. Power consumption is at Bus 3 and 4. All cost coefficients and time constants for both generating and consuming units and transmission line data, namely, susceptance B_{nm} and capacity limit $P_{nm}^{\max}$ are given in Section 4.1.

In the following, two cases are considered, named Case 1 and Case 2. Case 1 is the base case which incorporates zero DR_s i.e. includes only fixed and adjustable demand, P_{Df_j} and P_{Da_j}, respectively. Case 2 incorporates fixed demand, DR_a and DR_s i.e. P_{Df_j}, P_{Da_j} and P_{Ds_j}. As in Section 2, the amount of shiftable demand is set to 10% of the total demand. It is assumed that $P_{Ds_j}^{\text{ref}}$ and $P_{Df_j}^{\text{ref}}$ are computed as shown in Fig. 1. With this profile, and the cost and utility coefficients as in Section 4.1 the dynamic market model in (13) was simulated. A $\mu_t = [1 \ 1.02 \ 1.03 \ 1.03 \ 1.02 \ 1.01 \ 1 \ 0.90 \ 0.85 \ 0.8 \ 0.72 \ 0.66 \ 0.64 \ 0.63 \ 0.64 \ 0.66 \ 0.72 \ 0.8 \ 0.85 \ 0.90 \ 0.97 \ 1 \ 1]^T$ was chosen as this resulted in the corresponding LMP profile to match a typical LMP profile as in ISO New England (see http://www.iso-ne.com/markets/). The resulting Social Welfare and total consumer costs are shown in Table 1.

Table 1 demonstrates an increase in Social Welfare and a decrease in cost per unit of consumption and hence a positive effect of introducing DR_s. The distribution of the demand profile between fixed, adjustable, and shiftable demand is shown in Fig. 3 for each hour of a 24-hour period. This figure also show the LMPs at the consumer nodes. From Fig. 3 we see that P_{Da_j} adjusts according to the utility function and as a result, in hours with a large quantity of low-cost RERs, i.e. low LMP, the amount of adjustable power demand rises. Furthermore, it is apparent that in hours with high LMP no adjustable power is consumed. A comparison between Case 1 and Case 2 shows a more even distribution of demand which is preferable since this aids in lowering the prices for consumers as a direct result of the ability to shift power from high demand hours to low demand hours.

4.1 Coefficients

The generator cost and demand utility coefficients are $b_{Gc} = [47.2\ 48.2]^T$ \$/MWh, $c_{Gc} = [0.25\ 0.53]^T$ \$/MW²h, $b_{Gw_1} = 1$ \$/MWh, $b_{w_1} = 50$ \$/MWh, $c_{Gw_1} = 0.02$ \$/MW²h, $c_{w_1} = 0.7$ \$/MW²h, $\Delta_{Gw_1} = 0$ \$/MW²h, $b_{Da} = [67\ 67]^T$ \$/MWh, $b_{Ds} = [60\ 60]^T$ \$/MWh, $c_{Da} = [-0.21\ -0.21]^T$ \$/MW²h, $c_{Ds} = [-0.41\ -0.41]^T$ \$/MW²h, $\tau_{Gc} = [2.8\ 0.7]^T$ \$/MW, $\tau_{Gw_1} = 0.7$ \$/MW and $\tau_{Da} = \tau_{Ds} = [0.8\ 0.8]^T$ \$/MW. The transmission line coefficients are $P_{13}^{\max} = P_{24}^{\max} = 50.5$ MW, $P_{14}^{\max} = P_{23}^{\max} = 50$ MW, $B_{13} = B_{24} = 0.0372$ p.u and $B_{14} = 0.0504$ p.u, $B_{23} = 0.0336$ p.u.

5. SUMMARY

Increasing demand for electrical power generation and the current energy crisis have created an urgent need in incorporating RERs into the power grid. In this paper, we begin with the model of the players including dispatchable and RER GenCos, ConCo, and ISO, together with their constraints and the optimization goal and then capture the dynamics of the RTM using an interactive framework which introduces a state-space structure to the static market. A gradient play is used to derive the dynamic evolution of the actions for players and underlying states of the game as dual variables to reach the optimum solution of the RTM. In addition to the dynamic model this paper includes the incorporation of shiftable demand response evaluated over 24 hours. The stability of the resulting dynamical model of the RTM is investigated and the region of attraction around the equilibrium is established.

Numerical results are included that validate the theoretical results using a IEEE 4-bus system. The simulation results show that for a 10% shiftable demand, an 4% cost reduction to the consumer and a 1% increase in the Social Welfare is possible.

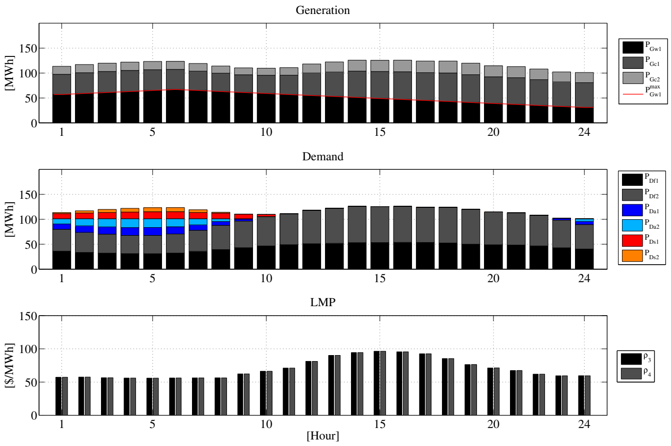

Fig. 3. Case 2 results: Generation, Demand and LMPs.

REFERENCES

F. Alvarado, J. Meng, W. Mota and C. DeMarco. "Dynamic coupling between power markets and power systems." *IEEE Transactions on Power Systems,* vol. 4, pp. 2201-2205, (2000).

A.M. Annaswamy (Project Lead), Eds. M. Amin, C. DeMarco and T. Samad. "Vision for Smart Grid Control: 2030 and Beyond." *IEEE Standards Publication,* (2013).

K. J. Arrow, L. Hurwicz and H. Uzawa. Stanford University Press Standford, California. *Studies In Linear and Non-Linear Programming,* (1958).

L. B. Cunningham, R. Baldick and M. L. Baughman. "An empirical study of applied game theory: transmission constrained cournot behaviour." *IEEE Transactions on Power Systems,* vol. 17, pp. 166-172, (2002).

X. Hu and D. Ralph. "Using EPECs to Model Bilevel Games in Restructured Electricity Markets with Locational Prices." *Operations Research,* pp. 809-827, (2007).

A. Kiani and A. M. Annaswamy. "Pertubation Analysis of Market Equilibrium in the Presence of Renewable Energy Resources and Demand Response." *IEEE Innovative Smart Grid Technologies (ISGT),* (2010).

A. Kiani and A. M. Annaswamy. "Wholesale Energy Market in a Smart Grid: Dynamic Modeling and Stability." *IEEE Conference on Decision and Control,* (2011).

A. Kiani and A. M. Annaswamy. "A Hierarchical Transactive Control Architecture for Renewables Integration in Smart Grids." *IEEE Conference on Decision and Control,* (2012).

P. D. Klemperer and M. A. Meyer. "Supply function equilibria." *Econometrica,* vol. 57, pp. 1243-1277, (1989).

M. Milligan and B. Kirby. "Utilizing Load Response for Wind and Solar Integration and Power System Reliability." *WindPower 2010,* 23-26 May 2010, Dallas, Texas (2010).

J. Morales, A. Conejo and J. Prez-Ruiz. "Economic Valuation of Reserves in Power Systems With High Penetration of Wind Power." *IEEE Transactions on Power Systems,* Vol 24, pp. 900-910, (2009).

A. Nedic and A. Ozdaglar. "Subgradient methods for saddle-point problems." *Journal of Optimization Theory and Applications,* vol. 142, pp. 205-228, (2009).

A. Roscoe and G. Ault. "Supporting high penetrations of renewable generation via implementation of real-time electricity pricing and demand response." *IET Renewable Power Generation,* vol. 4, no. 4, pp. 369-382, (2010).

C. Ruiz and A. Conejo. "Pool Strategy of a Producer With Endogenous Formation of Locational Marginal Prices." *IEEE Transactions on Power Systems,* pp. 1855-1866, (2009).

R. Sioshansi and W. Short. "Evaluating the impacts of real-time pricing on the usage of wind generation." *IEEE Transactions on Power Systems,* vol. 24, no. 2, pp. 516-524, (2009).

FERC Staff. "Assessment of demand response and advanced metering." Technical Report, (2008).

P. Visudhiphan and M. D. Ilic. "Dynamic game-based modeling of electricity markets." *Proceedings of the IEEE Power Engineering Society,* (1999).

J. Yao, I. Adlar and S. Oren. "Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network." *Operations Research,* pp. 34-47, (2008).