Distributed Coordination of Energy Storage with Distributed Generators

Tao Yang, Di Wu, Anton A. Stoorvogel, and Jakob Stoustrup

***Abstract*—With a growing emphasis on energy efficiency and system flexibility, a great effort has been made recently in developing distributed energy resources (DER), including distributed generators and energy storage systems. This paper first formulates an optimal DER coordination problem considering constraints at both system and device levels, including power balance constraint, generator output limits, and storage operational constraints such as energy and power capacity and charging/discharging efficiencies. An algorithm is then proposed to dynamically and automatically coordinate DERs in a distributed manner. With the proposed algorithm, the coordination agent at each DER only maintains a local incremental cost and updates it through information exchange with a few neighbors. Simulation results are used to illustrate and validate the proposed algorithm.**

***Index Terms*—Distributed control, distributed energy resources, distributed generators, energy storage.**

Nomenclature

d_t	Total demand of period t.
E_s	Battery energy capacity.
E_0	Initial value of energy stored in the battery.
E_T	End value of energy stored in the battery.
E_t	Energy stored in the battery at the end of period t.
N	The number of generators in the power system.
$p_{i,t}$	Power generation of generator i at period t.
$p_i^{\min}, p_i^{\max}$	Lower and upper bound of the power generation of generator i, respectively.
s_t	Power exchange between the storage device and grid (measured at the grid connection point) during period t, which is positive when injecting power into grid, i.e., using generator convention.
s_t^{batt}	Rate of change of energy stored in the storage device at the end of period t, which is positive when the storage device is discharged.
$s^{\min}, s^{\max}$	Lower and upper bound of the power generation of the storage device, respectively.
T	The number of periods.
ΔT	The duration of the period.
η^+, η^-	Discharging and charging efficiency of the storage device, respectively, including components such as conductor, power electronics, and battery.

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I. Introduction

A smart grid integrates advanced sensing and communication technologies as well as control methods into existing power systems at both transmission and distribution levels. Focusing on distribution system, a great effort has been made in developing distributed generation and energy storage technology. These resources are commonly referred as distributed energy resources (DERs). DERs are small and highly flexible compared with conventional large-scale power plants, and are playing an increasingly important role in the nation's power system. The deployment of DERs can help not only avoid (or defer) investment in power system infrastructure, but also meet additional reserve requirement from intermittent renewable generation.

In order to effectively deploy DERs, proper coordination and control need to be designed. One solution to this problem can be achieved through a centralized control strategy where each distributed resource is commanded from a single control center that gathers information from and provides control signals to the entire system. This centralized control framework may be subjected to performance limitations, such as high communication requirement and cost, substantial computational burden, limited flexibility, and disrespect of privacy. To overcome these limitations and accommodate various resources in future smart grid, it is desirable to develop an alternative distributed control strategy, where the agent at each DER maintains a set of variables and updates them through information exchange with a few neighbors. During last few years, various distributed coordination strategies have been proposed for distributed generators (DGs) at a single period. The authors of [1] propose a ratio consensus distributed algorithm for optimal DG dispatch problem. In [2], the authors present a strategy based on the local replicator equation to define functions and tasks assigned to each agent in a connected network and apply the method to economic dispatch of DGs. Other algorithms that can be applied to DER coordination include leader-follower consensus algorithm [3], consensus based algorithm where agents collectively learn the system imbalance [4], distributed algorithm based on consensus and bisection method [5], and minimum-time consensus algorithm [6], just to name a few.

Recent developments and advances in energy storage (ES) technology are making its application a viable solution for increasing flexibility and improving reliability and robustness of power systems [7]. It is desirable to take the advantage of ES and utilize it with DG to serve the smart distribution systems. Realizing such as a need, the authors of [8] propose a distributed algorithm based on consensus + innovation method to coordinate DG and losses ES over multiple time periods within a microgrid. As shown in [9] and other existing studies, the optimal charging/discharging operation and the corresponding benefits from a storage device could vary significantly with its efficiencies. Therefore, in this paper, losses during the charging and discharging operation are considered. In addition, we exclude the cost function of storage from the objective function, because there is no fuel cost associated with discharging a battery, and the cost of obtaining power and energy from a battery has already been captured in generators' cost when the battery is charged. Due to these two changes in problem formulation, existing distributed coordination algorithms cannot be applied. This paper develops a novel distributed DER coordination strategy, where no cost function for storage is required and charging/discharging losses are handled.

The remainder of the paper is organized as follows: Section II presents the formulation of multi-period optimal DER coordination problem. A distributed coordination method is then proposed in Section III. Section IV presents case studies and simulation results. Finally, concluding remarks are offered in Section V.

II. Problem Formulation

For simplicity, this paper considers a distribution system including N DGs and a storage device, but the idea can be extended to cases with multiple storages. The objective of the optimal coordination problem is to minimize the total production cost on the premise that DGs and the ES cooperatively serve the demand within individual generation and storage capacity. Since there is only limited energy that can be stored in ES, the operation of the storage system in different time steps is interdependent. Thus, the coordination needs to be made over multiple time steps concurrently, and a multi-step optimization problem is formulated in (1).

$$\textbf{PP:} \min_{p_{i,t}, s_t} \quad \sum_{t=1}^{T}\sum_{i=1}^{N} C_i(p_{i,t}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} p_{i,t} + s_t = d_t, \qquad \forall t = 1,\dots,T, \tag{1b}$$

$$p_i^{\min} \le p_{i,t} \le p_i^{\max}, \qquad \forall t = 1,\dots,T, \tag{1c}$$

$$s^{\min} \le s_t \le s^{\max}, \qquad \forall t = 1,\dots,T, \tag{1d}$$

$$s_t^{\text{batt}} = \begin{cases} \frac{s_t}{\eta^+} & \text{if } s_t \ge 0, \\ s_t \eta^- & \text{if } s_t < 0, \end{cases} \qquad \forall t = 1,\dots,T, \tag{1e}$$

$$E_t = E_{t-1} - s_t^{\text{batt}} \Delta T, \qquad \forall t = 1,\dots,T, \tag{1f}$$

$$0 \le E_t \le E_s, \qquad \forall t = 1,\dots,T, \tag{1g}$$

$$E_T = E_0. \tag{1h}$$

The objective expressed in (1a) is the total production cost within the look-ahead window of T, and $C_i(p_{i,t})$ is the cost function of generator i for period t, which is assumed to be quadratic as

$$C_i(p_{i,t}) = a_i p_{i,t}^2 + b_i p_{i,t} + c_i, \tag{2}$$

where $a_i > 0$. The O&M cost associated with ES is assumed to be fixed, and therefore excluded from the objective function. When discharging the battery, there is no need to consider the cost of power/energy discharged from battery, because the stored energy is essentially obtained from generators and has been already included in generators' production cost. In addition, cost of energy losses associated with battery charging/discharging has also been taken into account in production cost of generators through energy/power balancing constraint and modeling of battery charging/discharging efficiency.

The constraints are described as follows. Constraint (1b) corresponds to the power balance requirement, where d_t denotes the demand in time period t. Constraint (1c) restricts power output from generators to be within the lower ($p_i^{\min}$) and upper bounds ($p_i^{\max}$). Constraints (1d) restricts the power exchange between ES and grid to be within its charging and discharging power capacity. Constraint (1e) expresses rate of change of energy stored in storage device. Constraint (1f) models the dynamics of energy stored in ES. Constraint (1g) restricts the energy stored in the storage device to be within its lower and upper bounds. Constraint (1h) restricts the end value of the energy stored in ES is the same as the initial value.

A. Centralized Lagrangian Approach

In order to develop a distributed coordination algorithm, we dualize problem (1) with respect to constraint (1b), which couples the operation of all DERs. The other constraints are not relaxed because there is no coupling among devices. Due to the structure of the problem, there is zero duality gap. We can solve the primal problem in (1) by considering its dual problem.

Let $\Omega_{p,i}$ be the set of all $p_{i,t}$ for which (1c) is satisfied, and Ω_s be the set of all s_t for which (1d)–(1h) are satisfied. Define the Lagrangian

$$\begin{aligned} &L(p_{i,t}, s_t, \lambda_t) \\ &= \sum_{t=1}^{T}\left\{ \sum_{i=1}^{N} C_i(p_{i,t}) - \lambda_t \left(\sum_{i=1}^{N} p_{i,t} + s_t - d_t \right) \right\} \\ &= \sum_{t=1}^{T}\left\{ \sum_{i=1}^{N} [C_i(p_{i,t}) - \lambda_t p_{i,t}] - \lambda_t s_t + \lambda_t d_t \right\}. \end{aligned}$$

The dual function is

$$\begin{aligned} &D(\lambda_t) \\ &= \min_{\substack{p_{i,t}\in\Omega_{p,i} \\ s_t\in\Omega_s}} L(p_{i,t}, s_t, \lambda_t) \end{aligned}$$

$$= \min_{\substack{p_{i,t}\in\Omega_{p,i}\\ s_t\in\Omega_s}} \sum_{t=1}^{T}\left\{\sum_{i=1}^{N}[C_i(p_{i,t})-\lambda_t p_{i,t}]-\lambda_t s_t+\lambda_t d_t\right\}$$

$$= \underbrace{\min_{\substack{p_{i,t}\in\Omega_{p,i}\\ s_t\in\Omega_s}} \sum_{t=1}^{T}\sum_{i=1}^{N}[C_i(p_{i,t})-\lambda_t p_{i,t}]-\lambda_t s_t}_{\Phi(\lambda_t)}+\sum_{t=1}^{T}\lambda_t d_t.$$

The dual problem is

$$\textbf{DP:}\ \max_{\lambda_t\geq 0} D(\lambda_t) := \max_{\lambda_t\geq 0}\left[\Phi(\lambda_t)+\sum_{t=1}^{T}\lambda_t d_t\right]. \quad (3)$$

Notice that $\Phi(\lambda_t)$ is a minimization problem across multiple DERs and multiple periods, which can be decoupled as

$$\Phi(\lambda_t) = \sum_{t=1}^{T}\sum_{i=1}^{N}\min_{p_{i,t}\in\Omega_{p,i}}\Big(C_i(p_{i,t})-\lambda_t p_{i,t}\Big)-\min_{s_t\in\Omega_s}\sum_{t=1}^{T}\lambda_t s_t. \quad (4)$$

On the right hand side of (4), in the first term, there is no coupling between generators or between time periods. The second term is a minimization problem which only involves ES over multiple time periods. Therefore, for any given λ_t, the minimizer $p_{i,t}$ and s_t can be obtained in a decentralized manner.

We can then apply the gradient method (5) [10] to solve the dual problem in (3):

$$\lambda_t(k+1)=\lambda_t(k)+\gamma(k)\Delta_t(k), \qquad \forall t\in\mathcal{T}, \quad (5a)$$

$$p_{i,t}(k+1)=\nabla C_i^{-1}(\lambda_t(k+1)), \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, \quad (5b)$$

$$\mathbf{s}(k+1)=\arg\max_{s_t\in\Omega_s}\sum_{t=1}^{T}\lambda_t(k+1)s_t, \quad (5c)$$

$$\Delta_t(k+1)=d_t-\sum_{i=1}^{N}p_{i,t}(k+1)-s_t(k+1), \quad \forall t\in\mathcal{T}, \quad (5d)$$

where

$$\nabla C_i^{-1}(\lambda_t(k+1)) = \min(\max(\frac{\lambda_t(k+1)-b_i}{2a_i}, p_i^{\min}), p_i^{\max}),$$

$\mathcal{T}=\{1,\ldots,T\}$, $\mathcal{N}=\{1,\ldots,N\}$, $\mathbf{s}=[s_1,s_2,\ldots,s_T]^{\intercal}$, and $\gamma(k)>0$ is the step-size at time step k. The mismatch $\Delta_t(k)$ is the gradient of the dual function $D(\lambda_t(k))$. The update of the dual variable (incremental cost) is according to (5a). The update of DG generation is according to (5b), which is the solution of the first part on the right hand side of (4). The update of power output from ES is according to (5c), which is the solution of the second part on the right hand side of (4). This centralized algorithm will be implemented in a distributed fashion in the next section.

III. Distributed Coordination for DER

In this section, a consensus-based algorithm is proposed to implement the gradient method in (5). To do so, we assign each DER a coordination agent. Information exchange among the agents is described by an undirected connected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$, where $\mathcal{V}=\{1,\ldots,N+1\}$ is an agent set, whose first N agents correspond to DGs and the last agent corresponds to the ES, and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is a set of edges indicating communication links among agents. In particular, an edge $(j,i)\in\mathcal{E}$ denotes that agents i and j can obtain each other's information mutually. The $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{(N+1)\times(N+1)}$ is the weighting matrix, with $a_{ij}>0$ if and only if $(i,j)\in\mathcal{E}$ and $a_{ii}=0$. We also define a Laplacian matrix associated with the communication network as $L=[\ell_{ij}]$ with $\ell_{ii}=\sum_{j=1,j\neq i}^{N+1}a_{ij}$ and $\ell_{ij}=-a_{ij}$ for $j\neq i$.

We are now ready to present the proposed leader-follower consensus algorithm:

$$\lambda_{i,t}(k+1)=\sum_{j\in\mathcal{N}_i}\beta_{ij}\lambda_{j,t}(k)+\varepsilon_i\gamma(k)\Delta_t(k), \forall i\in\mathcal{V}, \forall t\in\mathcal{T} \quad (6a)$$

$$p_{i,t}(k+1)=\nabla C_i^{-1}(\lambda_{i,t}(k+1)), \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \quad (6b)$$

$$\mathbf{s}(k+1)=\arg\max_{s_t\in\Omega_s}\sum_{t=1}^{T}\lambda_{N+1,t}(k+1)s_t, \quad (6c)$$

where $\mathcal{N}_i=\{j\in\mathcal{V}|(j,i)\in\mathcal{E}\}$ is the neighbor set of agent i,

$$\beta_{ij}=\frac{|\ell_{ij}|}{\sum_{j=1}^{N+1}|\ell_{ij}|}, \quad (7)$$

$\varepsilon_i=1$ if agent i is the leader and $\varepsilon_i=0$ otherwise, and $\Delta_t(k)$ is given in (5d) and only evaluated at the leader.

The step-size $\gamma(k)>0$ satisfies the following conditions:

$$\sum_{k=0}^{\infty}\gamma(k)=\infty, \quad \sum_{k=0}^{\infty}\gamma^2(k)<\infty,$$
$$\gamma(k_1)\leq\gamma(k_2) \text{ for all } k_1>k_2\geq 0. \quad (8)$$

In the proposed algorithm in (6), each agent has a variable $\lambda_{i,t}$, which is the local incremental cost of agent i at period t. This variable is updated according to (6a) based on the information received from its neighboring agents and possible the mismatch depends on whether the agent is an leader. Since $\mathbf{B}=[\beta_{ij}]$ is a row stochastic matrix and the communication network is connected, all $\lambda_{i,t}(k)$ converge to a common value according to classical consensus theory [11]. In addition to the consensus term, there is a mismatch term in (6a) in order to ensure that the power balance constraint can be met. The update equations (6b) and (6c) are distributed implementations of (5b) and (5c), respectively.

Remark 1: The optimization problem in (6c) is generally piecewise linear because of (1e). Nevertheless, this problem can be converted into a standard linear programming problem by introducing some auxiliary variables. Please refer to [9] for details.

The proposed distributed algorithm solves the primal problem in (1), i.e., $\lim_{k\to\infty}p_{i,t}(k) = p_{i,t}^*$ and $d_t - \sum_{i=1}^{N}\lim_{k\to\infty}p_{i,t}(k)=s_t^*$, where $p_{i,t}^*$ and s_t^* are respectively the optimal generation of DG i and the optimal power output of ES during period t.

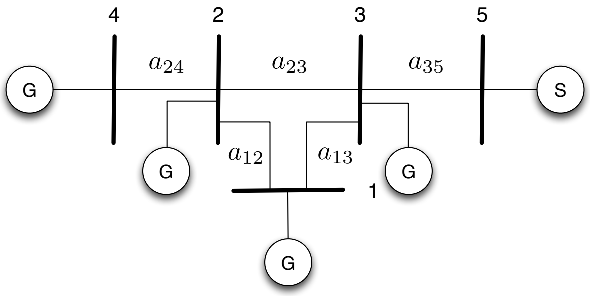

Fig. 1. IEEE five-bus power system.

TABLE I
GENERATOR PARAMETERS

Bus	a_i (kW^{2}h)	b_i (\$/kWh)	c_i (\$/h)	Range (kW)
1	0.00024	0.0267	0.38	[30,60]
2	0.00052	0.0152	0.65	[20,60]
3	0.00042	0.0185	0.4	[50,200]
4	0.00031	0.0297	0.3	[20,140]

TABLE II
STORAGE PARAMETERS

Bus	E_s (kWh)	$s_{\min}$ (kW)	$s_{\max}$ (kW)	η_+	η_-
5	960	-80	80	0.85	0.85

Remark 2: Note that for the case without ES and in a single period, our proposed algorithm (6) reduces to the leader-follower consensus algorithm in [3]. For the case with ES, we need to choose the storage as the leader, i.e., $\varepsilon_{N+1} = 1$ and $\varepsilon_i = 0$ for $i \in \{1, \ldots, N\}$. Nevertheless, the proposed strategy can also be used to incorporate ES in other leaderless distributed algorithms in a similar way, such as two-level consensus based algorithm [12] and consensus + innovation consensus [8]. We have omitted the details due to the space limitation.

IV. CASE STUDY

In this section, a case study is presented in order to illustrate and validate the proposed algorithm. The IEEE 5-bus system [13] shown in Fig. 1 is used as a test system, where buses 1, 2, 3, and 4 are connected with distributed generators and bus 5 is connected to energy storage. The parameters of DGs and ES are given in Table I, and Table II, respectively. In this example, the topology of communication network is assumed to be the same as the physical system. In general, the communication and physical layers do not necessarily to have the same topology, and the only requirement on the communication network is that its associated graph must be connected. Herein, all edge weights a_{ij} are set to be equal to 1. These values are used to determine ℓ_{ij} of the Laplacian matrix and then coefficients β_{ij} used in the update (6a) according to

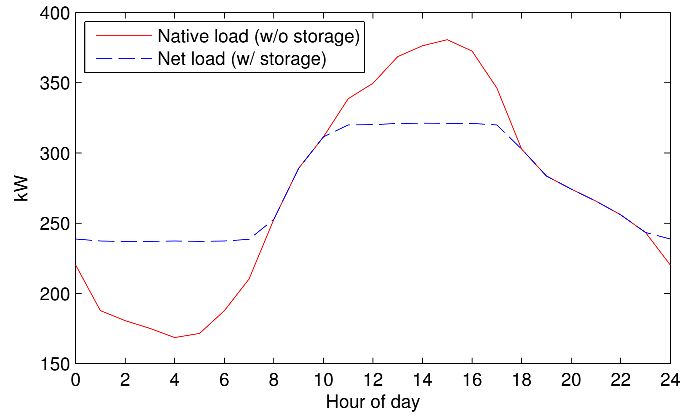

Fig. 2. Native load vs. Net load.

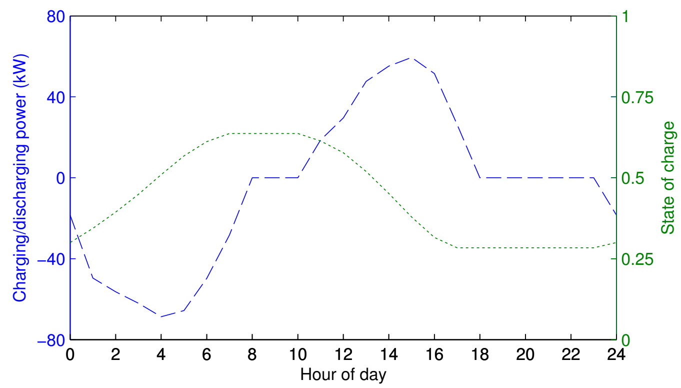

Fig. 3. Charging (negative) and discharging (positive) power and state of charge.

(7). Note that the convergence speed of the algorithm partially depends on the eigenvalue of the matrix $\mathbf{B} = [\beta_{ij}]$.

The demand to be supplied by these DERs is plotted in red in Fig. 2. We apply the proposed distributed algorithm (6) with $\varepsilon_i = 0$ for $i = 1, 2, 3, 4$, $\varepsilon_5 = 1$, and the step-size $\gamma(k) = \frac{0.2}{k}$, to solve the optimal coordination problem. It is found that the obtained solution agrees with the centralized one. The resulted net load (load minus storage) is plotted in blue in Fig. 2. The power output and state of charge (SOC) of ES are provided in Fig. 3. As can be seen, ES cuts the peak and fills the valley, i.e., ES is discharging during peak hours when energy price is high and charging during off-peak hours when energy price is low. The SOC are the same at the beginning and end of the scheduling period, but the total charging energy (area between the negative blue curve and x-axis) is more than the discharging energy (area between the positive blue curve and x-axis) because of losses. There are some time periods when ES is idle (with zero output). This is because that the system marginal cost is not high (or low) enough considering the round-trip efficiency to make the discharging (or charging) profitable.

The coordination of DGs are visualized in Fig. 4. In partic-

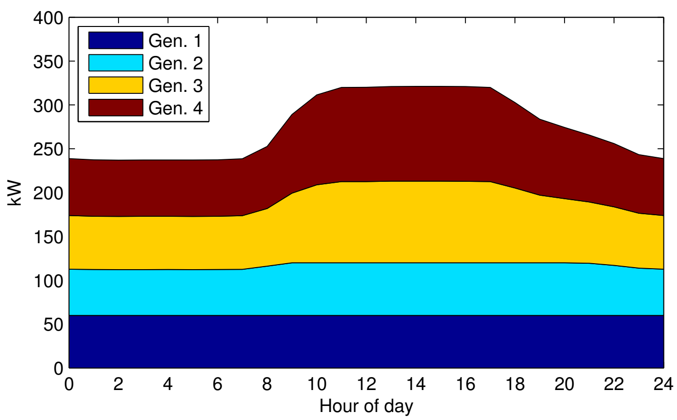

Fig. 4. Generations.

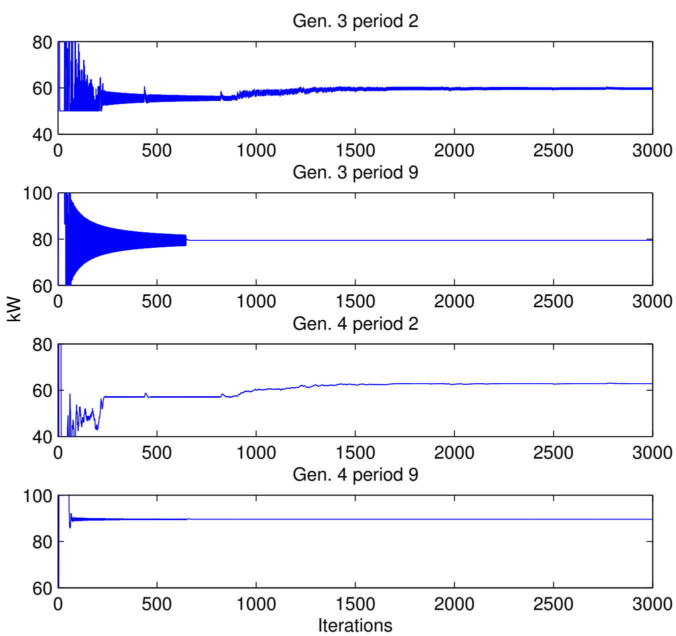

Fig. 5. Iteration processes for generators 3 and 4 during hours 2 and 9.

ular, generator 1 is at its upper bound of power output all the time because it is the cheapest among all DGs and therefore generates as much as possible. Generator 2 is at its maximal output from hour 9 to 20 of the day. The remaining net load is supported by generator 3 and 4. In each hour, the marginal costs are the same for all DGs that are not at their upper or lower bounds. It is not difficult to see that the top boundary of red area in Fig. 4 matches the blue curve in Fig. 2, which means the total generation from all generators is equal to the net load.

The iteration dynamics varies with DG and time period. As an example, Fig. 5 plots the iteration processes for generators 3 and 4 for hours 2 and 9. For both generators, the optimal generations in hour 9 is obtained earlier than hour 2. This is because ES is idle in hour 9, and the demand is supported by generators only. Hence, there is no temporal interdependent constraints on generator coordination and the output of generators can be determined independently.

V. Conclusions

This paper considers the optimal coordination problem for distributed generators and energy storage system. In the problem formulation, battery charging/discharging efficiencies are explicitly considered. The cost of energy and power discharged from battery is captured through power balance constraints and battery efficiencies, rather than some virtual cost functions. A distributed DER coordination strategy has been developed, and a leader-follower consensus algorithm has been proposed using such a strategy. The proposed algorithm has been illustrated and validated by a case study. An interesting direction is to extend the proposed algorithm to the optimal coordination problem for other types of DER, such as thermostatically controlled loads, plug-in electric vehicles, and residential deferrable loads.

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