Distributed Optimal Coordination for Distributed Energy Resources in Power Systems

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Abstract—Driven by smart grid technologies, distributed energy resources (DERs) have been rapidly developing in recent years for improving reliability and efficiency of distribution systems. Emerging DERs require effective and efficient coordination in order to reap their potential benefits. In this paper, we consider an optimal DER coordination problem over multiple time periods subject to constraints at both system and device levels. Fully distributed algorithms are proposed to dynamically and automatically coordinate distributed generators with multiple/single storages. With the proposed algorithms, the coordination agent at each DER only maintains a set of variables and updates them through information exchange with a few neighbors. We show that the proposed algorithms with properly chosen parameters solve the DER coordination problem as long as the underlying communication network is connected. Simulation results are used to illustrate and validate the proposed method.

Note to Practitioners—This paper was motivated by the problem of coordinating distributed energy resources (DERs) in order to increase the reliability and efficiency of distribution systems. Existing approaches are centralized, where a single control center gathers information from and provides control signals to the entire system. This centralized control framework may be subjected to performance limitations, such as high communication requirement and cost, substantial computational burden, limited flexibility, and disrespect of privacy. To overcome these limitations and accommodate various resources in the future smart grid, in this paper, we develop an alternative distributed control strategy, where each DER only communicates with its neighbors, without a need for a central coordinator. In this paper, only distributed generators and energy storage devices are considered. In future research, we will extend the proposed algorithms to include other types of flexible resources, such as thermostatically controlled loads, plug-in electric vehicles, and deferrable loads.

Index Terms—Consensus and gradient algorithm, distributed coordination, energy storage, multi-agent systems, multi-step optimization, smart grid.

NOMENCLATURE

D_t	Total demand of period t.
$E_i^{\max}$	Energy capacity of storage device i.
$E_{i,t}$	Energy stored in storage i at time period t.

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N	Number of distributed generators.
M	Number of distributed storage devices.
$p_{i,t}$	Power from generator or storage i during period t. The power from storage is measured at the grid coupling point, and is positive when injecting power into grid, i.e., using generator convention.
$p_i^{\min}, p_i^{\max}$	Lower and upper bound of the power limits of device i, respectively.
$p_{i,t}^{\text{batt}}$	Rate of change of energy stored in storage device i at the end of period t, which is positive when storage device is discharged.
T	Number of time periods.
$\Delta \underline{p}_i, \Delta \overline{p}_i$	Lower and upper bound of ramping rates of generator i, respectively.
ΔT	Time step size.
η_i^+, η_i^-	Discharging and charging efficiency of storage device i, respectively, including components such as conductor, power electronics, and battery.

I. INTRODUCTION

A smart grid integrates advanced sensing and communication technologies as well as control methods into existing power systems at both transmission and distribution levels. Distributed generation (DG) and energy storage (ES) are important ingredients of the emerging smart grid paradigm. For ease of reference, these resources are often referred to as distributed energy resources (DERs) [1]–[3]. Much of the DERs emerging under the smart grid are targeted at the distribution level. They are small and highly flexible compared with conventional large-scale power plants. The deployment of DERs will not only defer infrastructure investment, but also meet additional reserve requirement from intermittent renewable generation. As the electric power grid continues to modernize, DERs can help facilitate the transition to a future smart grid [4], [5].

In order to effectively deploy DERs, proper coordination and control need to be designed. One solution to this problem can be achieved through a completely centralized control strategy, where a single control center gathers information from and provides control signals to the entire system. This centralized control framework may be subjected to performance limitations, such as high communication requirement and cost, substantial computational burden, limited flexibility, and disrespect of privacy [6]–[8]. To overcome these limitations and accommodate various resources in the future smart grid, it is desirable to develop an alternative distributed

control strategy, where the agent at each DER maintains a set of variables and updates them through information exchange with a few neighbors. During the past few years, various distributed coordination strategies have been proposed for DGs for a single period. The authors of [9] propose a distributed ratio consensus based algorithm. This distributed algorithm is later robustified against potential packet drops in communication networks and applied to DG coordination in [10]. In [11], a strategy based on the local replicator equation is presented. In [12], an efficient decentralized algorithm called Combinatorial Optimization Heuristic for Distributed Agents is designed to optimally coordinate distributed resources in a fully decentralized manner. Other algorithms that can be applied to DG coordination include a leader-follower consensus algorithm [13], a consensus based algorithm where agents collectively learn the system imbalance [14], a distributed algorithm based on the consensus and bisection method [15], a minimum-time consensus algorithm [16], and a distributed algorithm based on the push-sum and gradient method [17], just to name a few.

Recent developments and advances in ES technology are making its application a viable solution for increasing flexibility and improving reliability and robustness of power systems [18]. In [19], a distributed coordination algorithm is proposed to utilize batteries of plug-in electric vehicles to shift load and therefore minimize the energy cost. However, the utilization of DGs and their coordination with storage devices are not considered. In order to fully explore the benefits of DERs, the authors of [20] propose a distributed algorithm based on the consensus + innovation method to coordinate ES and DG over multiple time periods in a microgrid. In the formulation of the optimal coordination problem, a quadratic generation cost function of ES is included in the objective function. With such an assumption, the objective function is strictly convex and thus charging/discharging power can be determined for a given marginal cost during the iteration process, similarly as for generators. However, unlike generators, there is no fuel cost associated with discharging a storage. The cost of power and energy discharged from storages has already been captured in generators' cost when the storage is charged. Moreover, the charging and discharging efficiencies are not modeled. As shown in [21], [22] and other existing studies, the optimal charging/discharging operation and the corresponding benefits from a storage device could vary significantly with its efficiencies.

To overcome these two shortcomings, we have previously developed a distributed DER coordination strategy, where no artificially assigned cost function is needed for storage and charging/discharging losses are modeled [23]. In that work, we considered DER coordination with a single storage and proposed a distributed algorithm based on the leader-follower consensus algorithm with the leader to be the storage. This paper extends our previous work to the multi-storage case. Moreover, the proposed algorithms in this paper do not require any leader and therefore are fully distributed. In the proposed algorithms, each agent only maintains a few variables and updates them through information exchange with neighboring DERs. The algorithms are based on the consensus and gradient strategy for the local incremental cost update, where the consensus part ensures that the incremental cost at all agents asymptotically approach the same value based on only local information exchange, and the gradient part ensures that the power balance condition is met. We show that the proposed algorithms with appropriately chosen parameters solve DER coordination as long as the underlying communication network is connected.

The remainder of the paper is organized as follows: In Section II, some preliminaries on graph theory and notations are introduced. Section III presents the formulation of multi-step optimal DER coordination problem. In Section IV, a fully distributed DER coordination algorithm with multiple storages is first developed, and a simplified distributed DER coordination algorithm is proposed for the single storage case. Section V presents case studies and simulation results. Finally, concluding remarks are drawn in Section VI. Appendix A provides the technical proof of Theorem 2.

II. PRELIMINARIES

We first recall some basic concepts from graph theory [24]. A *weighted directed graph* $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \ldots, n\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the weighting matrix, with $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$ and $a_{ii} = 0$. Each pair in $\mathcal{E}$ is called an *edge*. The graph is *undirected* if $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$ and moreover $a_{ij} = a_{ji}$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \ldots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. An undirected graph $\mathcal{G}$ is connected if there exists a path between any pair of distinct nodes. For a graph $\mathcal{G}$, a matrix $L = [\ell_{ij}]$ with $\ell_{ii} = \sum_{j=1}^{n} a_{ij}$ and $\ell_{ij} = -a_{ij}$ for $j \neq i$, is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. If the undirected weighted graph $\mathcal{G}$ is connected, then the Laplacian matrix L has a simple eigenvalue at zero with corresponding right eigenvector $\mathbf{1}$ and all other eigenvalues are strictly positive.

Given a matrix A, A' denotes its transpose and $\|A\|$ denotes its induced norm. We denote by $A \otimes B$ the Kronecker product between matrices. I_n denotes the identity matrix of dimension $n \times n$.

III. PROBLEM FORMULATION

We consider a distribution system including N DGs and M storage devices, where the first N systems are generators and the last M systems are storage devices. The objective of optimal coordination is to minimize the total production cost on the premise that all the DGs and storage devices collectively provide a required amount of power within individual generation and storage capacity. Since there is only limited energy that can be stored in a storage device, the operation of storages in different time steps is interdependent. Thus it is indispensable to optimize over multiple time steps concurrently. In order to take into account these inter-temporal

constraints and to make efficient use of storage devices, we formulate the following multi-step optimization problem:

$$\textbf{P:} \quad \min_{p_{i,t}, p_{i,t}^{\text{batt}}, E_{i,t}} \sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_{i,t}), \tag{1a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N+M} p_{i,t} - D_t = 0 \qquad \forall t \in \mathcal{T} \tag{1b}$$

$$\Delta \underline{p}_i \le p_{i,t} - p_{i,t-1} \le \Delta \overline{p}_i \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \tag{1c}$$

$$p_i^{\min} \le p_{i,t} \le p_i^{\max} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{L} \tag{1d}$$

$$p_{i,t}^{\text{batt}} = \begin{cases} \dfrac{p_{i,t}}{\eta_i^+}, & \text{if } p_{i,t} \ge 0 \\ p_{i,t}\eta_i^-, & \text{if } p_{i,t} < 0 \end{cases} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{1e}$$

$$E_{i,t} = E_{i,t-1} - p_{i,t}^{\text{batt}}\Delta T \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{1f}$$

$$0 \le E_{i,t} \le E_i^{\max} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{1g}$$

$$E_{i,T} = E_{i,0} \qquad \forall i \in \mathcal{M} \tag{1h}$$

where $\mathcal{T} = \{1, \ldots, T\}$, $\mathcal{N} = \{1, \ldots, N\}$, $\mathcal{M} = \{N+1, \ldots, N+M\}$, and $\mathcal{L} = \{1, \ldots, N+M\}$. Note that $p_{i,0}$ and $E_{i,0}$ are initial values prior to coordination, which therefore become parameters in the optimization problem. The objective function and constraints are described below:

- The objective expressed in (1a) includes the total production cost within the look-ahead window of T. $C_i(p_{i,t})$ is the cost function of generator i for period t, which is dominated by fuel cost. A quadratic function is used to represent generation cost as a function of power output [25], which is given by

$$C_i(p_{i,t}) = a_i p_{i,t}^2 + b_i p_{i,t} + c_i, \tag{2}$$

where $a_i > 0$. The operating and maintenance cost associated with energy storage is assumed to be fixed, and therefore excluded from the objective function. When discharging the storage, there is no need to consider the cost of power/energy obtained from storage, because the stored energy is essentially obtained from generators and has been already included in the generators' production cost. In addition, cost of energy losses associated with charging/discharging operation of storage has also been taken into account in production cost of generators through the energy/power balancing constraint and modeling of storage charging/discharging efficiency.

- Constraint (1b) corresponds to the power balance at each time step, i.e., the total output from generators and storage devices must be equal to the system load at each period. In power system operation, it is typically required that the load can be served solely by generators in order to ensure the system reliability, i.e.

$$\sum_{i=1}^{N} p_i^{\min} \le D_t \le \sum_{i=1}^{N} p_i^{\max}. \tag{3}$$

Constraint (1c) enforces ramping up/down constraints for generators. The ramping rates of storage devices are usually very high and can be excluded from the constraints. Constraint (1d) corresponds to power output limits of each device. Constraint (1e) expresses rate of change of energy stored in storage devices. Constraint (1f) captures the

dynamics of energy stored in storage device. Constraint (1g) restricts the energy stored in the storage device to be between its lower and upper bounds. Constraint (1h) specifies the energy stored in a storage device at the end of the scheduling period. It is equal to the initial energy state in this formulation but can be set to other feasible values.

It should be noted that the cost of discharging energy and the corresponding energy losses in storage devices are captured by the cost functions in (1a) through the power balance constraint in (1b) and charging/discharging efficiency model in (1e). For example, charging 1 kWh energy during an off-peak hour into storage i only yields $\eta_i^+ \eta_i^-$ kWh energy discharged during peak hours. Because of the power balance constraint in (1b), that 1 kWh charging energy must be provided by generators during the off-peak charging hour. The corresponding cost of 1 kWh charging energy (including $\eta_i^+ \eta_i^-$ for discharging and $1 - \eta_i^+ \eta_i^-$ for losses) is captured in the generators' cost in that hour.

In this paper, we assume information exchange between DGs and storage devices is described by an undirected connected graph composed of $N + M$ nodes, where the first N nodes correspond to generators and the last M nodes correspond to storage devices. Our objective is to solve the global optimization problem (1) in a distributed fashion.

A. Modified Equivalent Problem

For $\forall i \in \mathcal{M}$ (storage), let $\Omega_{p,i}$ be the set of all $p_i \in \mathbb{R}^T$ for which (1d)–(1h) are satisfied, where

$$p_i = \left(p_{i,1}, p_{i,2}, \ldots, p_{i,T} \right)'. \tag{4}$$

Due to non-convex constraint (1e), the set $\Omega_{p,i}$ is in general not convex, and thus the original problem (1) is difficult to solve even in a centralized manner. Hence, we need to convert the original problem to its convex equivalency. To do so, define

$$p_{i,t} = p_{i,t}^+ - p_{i,t}^-, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{5}$$

where

$$0 \le p_{i,t}^+ \le p_i^{\max}, \ 0 \le p_{i,t}^- \le -p_i^{\min}, \ t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{1i*}$$

and replace constraint (1e) by

$$p_{i,t}^{\text{batt}} = \frac{1}{\eta_i^+} p_{i,t}^+ - \eta_i^- p_{i,t}^-. \tag{1e*}$$

Hence, the original non-convex problem in (1) is equivalent to

$$\textbf{P}' : \quad \min_{p_{i,t}, p_{i,t}^+, p_{i,t}^-, p_{i,t}^{\text{batt}}, E_{i,t}} \sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_{i,t}), \tag{6}$$

subject to (1b), (1c), (1d), (1e*), (1f), (1g), (1h), and (1i*). It should be noted that we can express some decision variables as functions of other decision variables using some equality constraints, and eliminate the decision variables and constraints from $\textbf{P}'$. Nevertheless, herein, we keep all the constraints because each of them has its own physical meaning, and list all the decision variables to differentiate them from parameters in the optimization problem.

Since a physical storage device cannot be charged and discharged at the same time, we need to ensure that either $p_{i,t}^+$ or $p_{i,t}^-$ needs to be zero, i.e.,

$$p_{i,t}^+ p_{i,t}^- = 0, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M}. \tag{7}$$

However, when η_i^+ or η_i^- is strictly less that 1, there is no need to add (7) to $\mathbf{P}'$, because the *optimal* solution of $\mathbf{P}'$ automatically satisfies (7) as shown in the following theorem.

Theorem 1: Any solution with a pair of $p_{i,t}^+$ and $p_{i,t}^-$ to be nonzero cannot be an optimal solution of $\mathbf{P}'$ when $\eta_i^+ \eta_i^- < 1$.

Proof : We prove this theorem by contradiction. Suppose $\{p_i^\diamond\}$ is an optimal solution of (6), in which there exist a period t_1 and a storage $i_1 \in \mathcal{M}$ such that

$$p_{i_1,t_1}^{\diamond+} \, p_{i_1,t_1}^{\diamond-} \neq 0 \,.$$

According to (1e*), we have

$$p_{i_1,t_1}^{\diamond\text{batt}} = \frac{1}{\eta_{i_1}^+} p_{i_1,t_1}^{\diamond+} - \eta_{i_1}^- p_{i_1,t_1}^{\diamond-}. \tag{8}$$

Let $\{p_i^\dagger\}$ be another set of variables where operations of all generators and storages are the same as those in $\{p_i^\diamond\}$ for all the periods except t_1, as expressed in (9)

$$p_{i,t}^\dagger = p_{i,t}^\diamond, \quad \forall t \in \mathcal{T} \setminus t_1. \tag{9}$$

In addition, for time t_1, let i_2 be the index of a generator whose power output can be reduced without violating the ramping constraints or its lower bound. In the unlikely scenario that such i_2 does not exist, we can use the extra energy which is wasted at time t_1 earlier at some time t_2 to reduce one of the generators and therefore the associated cost and use a storage device to carry energy over unless there is some time $t_2 \leq t_3 \leq t_1$ where the storage device is completely empty. If that is not possible we can use the energy at some later time t_4 to reduce one of the generators and therefore the associated cost and use a storage device to carry energy over unless there is some time $t_1 \leq t_5 \leq t_4$ where the storage device is completely full. We note that the only way the above fails if there exists some interval $[t_3, t_5]$ where we cannot reduce any generator while at the same time the storage devices are filled from completely empty to completely full. In order words, there exists some t, when $p_{i,t} = p_i^{\min}$ for $\forall i \in \mathcal{N}$ and $p_{i,t} < 0$ (charging) for $\forall i \in \mathcal{M}$. To ensure the feasibility of the problem, constraint (1b) must be met. Hence, we have $D_t = \sum_{i=1}^{N+M} p_{i,t} < \sum_{i=1}^{N} p_i^{\min}$, which contradicts to (3).

Let us return to the original case where we can reduce one of the generators at time t_1. Next, we choose the variables for all generators except i_2 and all storages except i_1 to be the same as those in $\{p_i^\diamond\}$, as expressed in (10)

$$p_{i,t_1}^\dagger = p_{i,t_1}^\diamond, \quad \forall i \in \mathcal{L} \setminus \{i_1, i_2\}. \tag{10}$$

For storage i_1, we set

$$p_{i_1,t_1}^{\dagger+} = p_{i_1,t_1}^{\diamond+} - \eta_{i_1}^+ \eta_{i_1}^- \epsilon \quad \text{and} \quad p_{i_1,t_1}^{\dagger-} = p_{i_1,t_1}^{\diamond-} - \epsilon. \tag{11}$$

where ϵ is an arbitrarily small positive constant. Therefore,

$$\begin{aligned} p_{i_1,t_1}^\dagger = p_{i_1,t_1}^{\dagger+} - p_{i_1,t_1}^{\dagger-} &= (p_{i_1,t_1}^{\diamond+} - p_{i_1,t_1}^{\diamond-}) + \epsilon(1 - \eta_{i_1}^+ \eta_{i_1}^-) \\ &= p_{i_1,t_1}^\diamond + \epsilon(1 - \eta_{i_1}^+ \eta_{i_1}^-). \end{aligned} \tag{12}$$

For generator i_2, we set

$$p_{i_2,t_1}^\dagger = p_{i_2,t_1}^\diamond - \epsilon(1 - \eta_{i_1}^+ \eta_{i_1}^-). \tag{13}$$

Note that it follows from (1e*), (11) and (8) that

$$\begin{aligned} p_{i_1,t_1}^{\dagger\text{batt}} &= \frac{1}{\eta_{i_1}^+} p_{i_1,t_1}^{\dagger+} - \eta_{i_1}^- p_{i_1,t_1}^{\dagger-} \\ &= \frac{1}{\eta_{i_1}^+} p_{i_1,t_1}^{\diamond+} - \eta_{i_1}^- p_{i_1,t_1}^{\diamond-} \\ &= p_{i_1,t_1}^{\diamond\text{batt}}. \end{aligned}$$

Thus, $\{p_i^\dagger\}$ satisfies all the constraints in (6) and is a feasible solution.

Since $C_{i_2}(\cdot)$ is strictly convex,

$$C_{i_2}(p_{i_2,t_1}^\dagger) < C_{i_2}(p_{i_2,t_1}^\diamond). \tag{14}$$

This together with (9) and (10) implies

$$\sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_{i,t}^\dagger) < \sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_{i,t}^\diamond),$$

which contradicts with our assumption that $\{p_i^\diamond\}$ is an optimal solution of (6). ∎

Let $\tilde{\Omega}_{\mathcal{M},i}$ be the set of all $p_i^+, p_i^- \in \mathbb{R}^T$ for which (1e*), (1f)–(1h), and (1i*) are satisfied, where $i \in \mathcal{M}$,

$$p_i^+ = (p_{i,1}^+, p_{i,2}^+, \ldots, p_{i,T}^+)'$$

and

$$p_i^- = (p_{i,1}^-, p_{i,2}^-, \ldots, p_{i,T}^-)'.$$

We also denote $\Omega_{\mathcal{N},i}$ as the set of all $p_i \in \mathbb{R}^T$ for which (1c) and (1d) are satisfied, where $i \in \mathcal{N}$ and

$$p_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,T})'.$$

It is clear that both $\tilde{\Omega}_{\mathcal{M},i}$ and $\Omega_{\mathcal{N},i}$ are convex polytopes since all constraints are linear.

IV. DISTRIBUTED COORDINATION APPROACH

In this section, we first propose a fully distributed algorithm to solve the modified equivalent problem $\mathbf{P}'$. It is shown that the proposed algorithm with appropriately chosen parameters is convergent. We then show that the algorithm can be simplified for the case with a single storage device.

In problem $\mathbf{P}'$, both sets $\Omega_{\mathcal{N},i}$ and $\tilde{\Omega}_{\mathcal{M},i}$ are convex polytopes which contain local constraints for generators and storage devices, respectively. In addition, the local cost function $\sum_{t=1}^{T} C_i(p_{i,t})$ which needs to be minimized is also convex. Motivated by the recent development in the area of distributed optimization in [26]–[28], we can thus solve these $N + M$ optimization problems locally via a Lagrangian method with only power balance constraint (1b), i.e., each node runs the local optimization with an estimate of the optimal dual variable λ_i, which is the marginal cost of bus i. These estimates are updated using the consensus and gradient strategy: i) the consensus part ensures that all estimates (consensus variables λ_i) asymptotically approach the same value based on only local information exchange, because it is necessary for an

Algorithm 1 Multiple storage devices–Part I

1: Initialize $k = 0$ and $\lambda_i(0) = 0 \in \mathbb{R}^T$ for $\forall i \in \mathcal{L}$.
2: **repeat**
3:   **procedure** LOCAL OPTIMIZATION
4:     **for** each node $i = 1, \ldots, N$ **do**
5:       $p_i(k) = \underset{p_i \in \Omega_{\mathcal{N},i}}{\arg\min} \sum_{t=1}^{T} C_i(p_{i,t}) - \lambda_i(k)' p_i$
6:     **end for**
7:     **for** each node $i = N+1, \ldots, N+M$ **do**
8:       $\{p_i^+(k), p_i^-(k)\} = \underset{\{p_i^+, p_i^-\} \in \bar{\Omega}_{\mathcal{M},i}}{\arg\min} \lambda_i(k)' \left(p_i^- - p_i^+\right)$
9:       $p_i(k) = p_i^+(k) - p_i^-(k)$
10:     **end for**
11:   **end procedure**
12:   **procedure** CONSENSUS AND GRADIENT
13:     **for** each node $i = 1, \ldots, N+M$ **do**
14:       $\lambda_i(k+1)$
15:       $= \lambda_i(k) - \beta \underbrace{\sum_{v=1}^{N+M} \ell_{iv} \lambda_v(k)}_{\text{consensus part}} - \underbrace{\alpha_k \left(p_i(k) - D^i\right)}_{\text{gradient part}}$
16:     **end for**
17:   **end procedure**
18:   $k = k+1$
19: **until** Error small enough
20: **for** each node $i = 1, \ldots, N$ **do**
21:   $p_i^{\text{sol}} = p_i(k-1)$
22: **end for**

Algorithm 2 Multiple storage devices–Part II

1: Initialize $m = 0$ and $\lambda_i(0) = 0$ for $\forall i \in \mathcal{L}$.
2: **repeat**
3:   **procedure** LOCAL OPTIMIZATION
4:     **for** each node $i = 1, \ldots, N$ **do**
5:       $p_i(m) = p_i^{\text{sol}}$
6:     **end for**
7:     **for** each node $i = N+1, \ldots, N+M$ **do**
8:       $\{p_i^+(m), p_i^-(m)\}$
9:       $= \underset{\{p_i^+, p_i^-\} \in \bar{\Omega}_{\mathcal{M},i}}{\arg\min} \|p_i^+ - p_i^-\|^2 - \lambda_i(m)' \left(p_i^+ - p_i^-\right)$
10:       $p_i(m) = p_i^+(m) - p_i^-(m)$
11:     **end for**
12:   **end procedure**
13:   **procedure** CONSENSUS AND GRADIENT
14:     **for** each node $i = 1, \ldots, N+M$ **do**
15:       $\lambda_i(m+1)$
16:       $= \lambda_i(m) - \beta \underbrace{\sum_{v=1}^{N+M} \ell_{iv} \lambda_v(m)}_{\text{consensus part}} - \underbrace{\alpha_m \left(p_i(m) - D^i\right)}_{\text{gradient part}}$
17:     **end for**
18:   **end procedure**
19:   $m = m+1$
20: **until** Error small enough
21: **for** each node $i = N+1, \ldots, N+M$ **do**
22:   $p_i^{\text{sol}} = p_i(m-1)$
23: **end for**

optimal solution to have the same marginal cost; ii) the gradient part guarantees that the power balance is satisfied. The iterations of these two procedures yield the optimal dual variable. This naturally leads to Algorithm 1 and 2, where α_k, β are weight parameters, $\lambda_i = \left(\lambda_{i,1}, \ldots, \lambda_{i,T}\right)'$, $D^i = \left(D_1^i, \ldots, D_T^i\right)'$ for $i \in \mathcal{L}$, p_i^{sol} is the final solution obtained from the algorithm. These algorithms terminate when the error is small enough, in the sense that $\|\lambda_i(k) - \lambda_i(k-1)\| < \epsilon_1$ and $\max_{i,j \in \mathcal{V}} \|\lambda_i(k) - \lambda_j(k)\| < \epsilon_2$, where ϵ_1 and ϵ_2 are small constants depending on the desired accuracy. In order to implement Algorithm 1 and 2, each generator or storage agent i needs to be aware of part of the load D_t^i for $t = 1, \ldots, T$ such that $\sum_{i=1}^{N+M} D_t^i = D_t$.

Remark 1: Since the objective function in (6) is strictly convex with respect to the power of each DG, the power of DG is uniquely determined according to Algorithm 1. However, the objective function is only convex but not strictly convex with respect to power of each ES. Thus in general the optimal solution for storage may not be unique. Our proposed Algorithm 2 is able to find one of the optimal solutions.

We next present our main result which shows that Algorithm 1 and 2 with properly chosen parameters solve the DER coordination problem. The proof of this theorem is rather technical and will be presented in Appendix A.

Theorem 2: Algorithm 1 and 2 solve the optimization problem (6) if

$$0 < \beta < \frac{2}{\mu_{N+M}} \tag{15}$$

where μ_{N+M} is the largest eigenvalue of the Laplacian matrix L and $\alpha_k > 0$ is a decreasing sequence such that

$$\sum_{k=0}^{\infty} \alpha_k = \infty, \quad \sum_{k=0}^{\infty} \alpha_k^2 < \infty. \tag{16}$$

In particular, Algorithm 1 yields

$$\lim_{k \to \infty} p_i(k) = p_i^*, \qquad \forall i \in \mathcal{N}$$

and Algorithm 2 yields

$$\lim_{m \to \infty} p_i(m) = p_i^*, \qquad \forall i \in \mathcal{M}$$

provided that $p_i^{\text{sol}} = p_i^*$ for all $i \in \mathcal{N}$, where p_i^* for all $i \in \mathcal{L}$ is the centralized optimal solution of the optimization problem (6).

Remark 2: The typical choice for a sequence α_k satisfying (16) is

$$\alpha_k = \frac{a}{k+b},$$

where $a > 0$ and $b \geq 0$.

A. Special Case with Single Storage

In the case of a single storage device at node $N + 1$, the uniqueness of p_{N+1}^* actually follows much easier from the power balance constraint (1b). In order to solve the optimal coordination for this case, we therefore replace Algorithm 2 with a simplified Algorithm 3. The procedures in Algorithm 1 remain the same.

Algorithm 3 Single storage device–Part II

1: **procedure** INITIALIZATION
2:   **for** each node $i = 1, \ldots, N$ **do**
3:     $v_i(0) = (N + 1)(D^i - p_i^{\text{sol}})$
4:   **end for**
5:   $v_{N+1}(0) = (N + 1)D^{N+1}$
6:   $m = 0$
7: **end procedure**
8: **repeat**
9:   **for** each node $i = 1, \ldots, N + 1$ **do**
10:     $v_i(m + 1) = v_i(m) - \beta \sum_{j=1}^{N+1} \ell_{ij} v_j(m)$
11:   **end for**
12:   $m = m + 1$
13: **until** Error small enough
14: $p_{N+1}^{\text{sol}} = v_{N+1}(m - 1)$

Corollary 1: In case of single storage device, if the parameters β and α_k are chosen as those given in Theorem 2, Algorithm 1 and 3 solve the optimization problem (6).

Proof : As shown in the proof of Theorem 2, running Algorithm 1 yields

$$\lim_{k \to \infty} \|p_i(k) - p_i^*\| = 0, \ \forall i \in \mathcal{N}.$$

It thus remains to show that Algorithm 3 yields

$$\lim_{m \to \infty} \|v_{N+1}(m) - p_{N+1}^*\| = 0. \tag{17}$$

We first note that according to the consensus theory [29], [30]

$$\lim_{m \to \infty} v_{N+1}(m) = \frac{1}{N + 1} \sum_{i=1}^{N+1} v_i(0)$$

$$= D^{N+1} + \sum_{i=1}^{N} \left(D^i - p_i^{\text{sol}} \right).$$

This together with the fact that $p_i^{\text{sol}} = p_i(k - 1) \to p_i^*$ as $k \to \infty$ and that the optimal solution satisfies the balance equation (1b) yields the required result (17). ∎

V. CASE STUDIES

In this section, a case study is presented in order to illustrate and validate the proposed algorithms. Due to space limitations, we only present the results for multi-storage case. The IEEE 6-bus system shown in Fig. 1 is used as a test system, where Buses 1–4 are connected with distributed generators and Bus 5 and 6 are connected to energy storage devices. The parameters of DGs and ESs are adopted from [31], [32], as listed in Table I, and Table II, respectively. In this example, the topology of the communication network is assumed to be the same as the physical system. In general, the communication and physical layers do not necessarily have the same topology, and the only requirement on the communication network is that its associated graph must be connected. Herein, all edge weights a_{ij} are set to be equal to 1. These values are used to
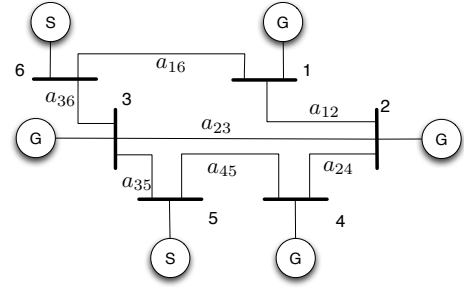

Fig. 1. IEEE six-bus power system.

TABLE I
GENERATOR PARAMETERS

Bus	a_i (kW^{2}h)	b_i ($/kWh)	c_i ($/h)	Range (kW)
1	0.00024	0.0267	0.38	[30,60]
2	0.00052	0.0152	0.65	[20,60]
3	0.00042	0.0185	0.4	[50,200]
4	0.00031	0.0297	0.3	[20,140]

TABLE II
STORAGE PARAMETERS

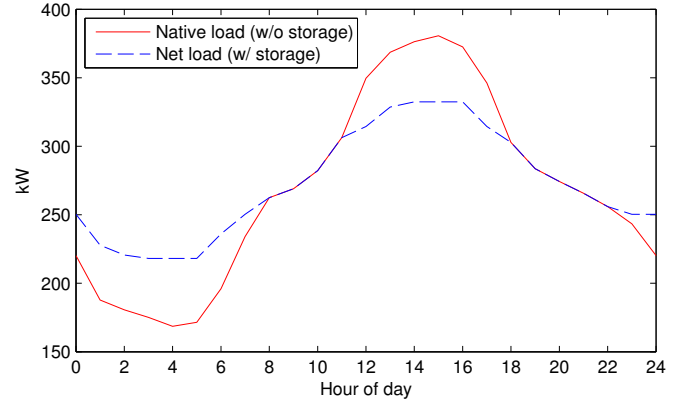| Bus	E_s (kWh)	$p_{\min}$ (kW)	$p_{\max}$ (kW)	η_+	η_-
5	500	-50	50	0.8	0.8
6	400	-40	40	0.88	0.88

Fig. 2. Native load vs. Net load.

determine ℓ_{ij} of the Laplacian matrix used in Algorithm 1 and 2. Note that the convergence speed of the algorithm partially depends on β and α_k.

The demand to be supplied by these DERs is plotted in red in Fig. 2. We have applied Algorithm 1 and 2 to coordinate four DGs with two storages over a 24-hour period. It was found that the obtained solution agrees with the centralized one. The resulting net load (load minus storage) is plotted in blue in Fig. 2. The power output and state of charge (SOC) for both storages are provided in Fig. 3.

As can be seen, two storage devices are coordinated to cut the peak and fill the valley, i.e., they are discharged during peak hours when the energy price is high and charged during off-peak hours when energy price is low. For each

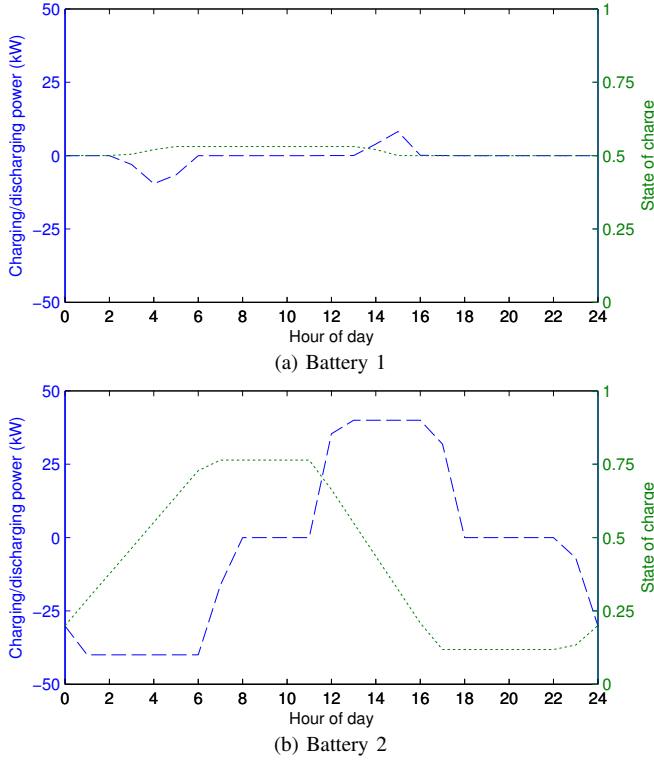(a) Battery 1

(b) Battery 2

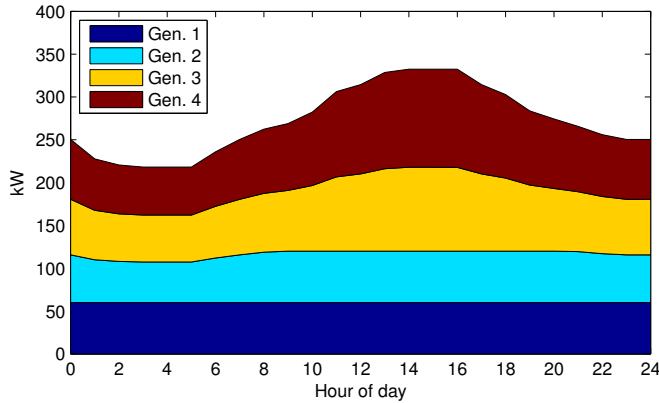Fig. 3. Charging (negative) and discharging (positive) power and state of charge.

Fig. 4. Generations from DGs.

storage device, SOC is the same at the beginning and end of the scheduling period, but the total charging energy (area between the negative blue curve and x-axis) is more than the discharging energy (area between the positive blue curve and x-axis) because of losses. The storage devices are idle when the energy price is not high (or low) enough to make the discharging (or charging) profitable considering the round-trip efficiency. In other words, the generator output levels are not increased (or decreased) to charge (or discharge) storage because cycling energy using storage in these hours is not efficient due to energy losses. Storage 1 is idle all the time except Hour 3-5 and 14-15 because of its low efficiency, while Storage 2 is engaged more often due to its higher efficiency.

The coordination of DGs is visualized in Fig. 4. In particu-

lar, Generator 1 is at its upper bound of the power output all the time because it is the cheapest among all DGs and therefore generates as much as possible. Generator 2 is at its maximal output from Hour 9 to Hour 20 of the day. The remaining net load is supported by Generator 3 and 4. In each hour, the marginal costs are the same for all DGs. It is not difficult to see that the top boundary of red area in Fig. 4 matches the blue curve in Fig. 2, which means the total generation from all generators is equal to the net load.

VI. CONCLUSIONS

This paper considered the optimal coordination problem of DERs, including distributed generators and energy storage devices. Storage charging/discharging efficiencies were explicitly modeled. In addition, the cost of energy and power discharged from storage is captured through power balance constraints and efficiencies. We first showed that the DER coordination problem can be modified to an equivalent problem that is convex. For this modified problem, we then proposed fully distributed algorithms based on the consensus and gradient strategy for multiple/single storages. We showed that the proposed algorithms with properly chosen parameters converge to the centralized solution. The proposed algorithms have been illustrated and validated by case studies. An interesting direction is to extend the proposed algorithms to the optimal coordination problem including other types of flexible resources, such as thermostatically controlled loads, plug-in electric vehicles, and deferrable loads.

APPENDIX A
PROOF OF THEOREM 2

We first define the average process

$$\bar{\lambda}(k) = \frac{1}{N+M} \sum_{i=1}^{N+M} \lambda_i(k). \tag{18}$$

It then follows from the update for $\lambda_i(k)$ and the property of the Laplacian matrix that $\bar{\lambda}(k)$ satisfies

$$\bar{\lambda}(k+1) = \bar{\lambda}(k) - \frac{\alpha_k}{N+M} \bar{r}(k), \tag{19}$$

where

$$\bar{r}(k) = \sum_{i=1}^{N+M} p_i(k) - D, \tag{20}$$

with $D = (D_1, \dots, D_T)'$.

Next define

$$\lambda_e(k) = (\lambda_1(k) - \bar{\lambda}(k), \dots, \lambda_{N+M}(k) - \bar{\lambda}(k))'. \tag{21}$$

It is easy to show that $\lambda_e(k)$ satisfies

$$\lambda_e(k+1) = (I - \beta(L \otimes I_T))\lambda_e(k) - \alpha_k r(k), \tag{22}$$

where

$$r(k) = \begin{pmatrix} p_1(k) - D^1 - \frac{1}{N+M}\bar{r}(k) \\ \vdots \\ p_{N+M}(k) - D^{N+M} - \frac{1}{N+M}\bar{r}(k) \end{pmatrix}. \tag{23}$$

Since (15) is satisfied, the matrix $W = I - \beta(L \otimes I_T)$ has T eigenvalues in 1, which are maximal in modulus. Moreover, the corresponding eigenvectors are $x_i = \mathbf{1} \otimes e_i$, where $\mathbf{1}$ is a vector in $\mathbb{R}^{N+M}$ consisting of only ones while e_i denotes the i'th basis vector in $\mathbb{R}^T$. Note that it follows from (20) and (23) that $x_i' r(k) = 0$ for all $i \in \mathcal{L}$ and all k. Therefore,

$$\|W^j r(k)\| \leq \bar{\mu}^j \|r(k)\|, \tag{24}$$

where

$$\bar{\mu} = \max_i \left\{ |\bar{\mu}_i| \mid \bar{\mu}_i \neq 1 \right\}$$

while $\bar{\mu}_1, \ldots, \bar{\mu}_{N+M}$ are the eigenvalues of W.

Since the constraints (1d) are satisfied, we know that there exists M_1 such that $\|r(k)\| < M_1$ for all k. Note that it follows from $\lambda_i(0) = \mathbf{0}$ that $\lambda_e(0) = \mathbf{0}$. Thus from (22), we have

$$\lambda_e(k) = -\sum_{j=0}^{k-1} \alpha_j W^{k-j-1} r(j) \tag{25}$$

and therefore

$$\alpha_k \|\lambda_e(k)\| \leq M_1 \sum_{j=0}^{k-1} \alpha_k \alpha_j \bar{\mu}^{k-j-1}. \tag{26}$$

It then follows from (16) that

$$\sum_{k=0}^{\infty} \alpha_k \|\lambda_e(k)\| < \infty. \tag{27}$$

We now proceed our analysis by considering two cases. We show that Case 1 will lead to a contradiction while Case 2 will yield the required convergence to the optimal value.

- Case 1: We have that $\sum_{i=1}^{N+M} p_i(k)$ is bounded away from D. In that case choose λ^* to be the optimal dual variable for the problem.
- Case 2: If $\{\bar{\lambda}(k)\}, \{p_1(k)\}, \ldots, \{p_{N+M}(k)\}$ has a convergent subsequence with limit $\lambda^*, p_1^*, \ldots, p_{N+M}^*$ such that $\sum_{i=1}^{N+M} p_i^* = D$.

Note that we have

$$p_i^* = \arg\min_{p_i \in \Omega_{\mathcal{N},i}} \sum_{t=1}^T C_i(p_{i,t}) - (\lambda^*)' p_i \tag{28}$$

for distributed generator $i \in \mathcal{N}$ while for storage $i \in \mathcal{M}$,

$$(p_i^{*+}, p_i^{*-}) \in \arg\min_{\{p_i^+, p_i^-\} \in \tilde{\Omega}_{\mathcal{M},i}} (\lambda^*)' \left(p_i^- - p_i^+ \right) \tag{29}$$

where $p_i^{*+} - p_i^{*-} = p_i^*$. Because of lack of *strict* convexity, p_i^{*+} and p_i^{*-} are solutions of the above optimization problem but are in general not uniquely determined (in case of a single storage device, uniqueness follows from the power balance equation (1b)).

Since $p_i^* \in \Omega_{\mathcal{N},i}$ for $i = 1, \ldots, T$, we conclude that

$$(2a_i p_i^* + b_i - \lambda^*)' (p_i(k) - p_i^*) \geq 0 \tag{30}$$

for $i = 1, \ldots, N$ while

$$- \lambda^* (p_i(k) - p_i^*) \geq 0 \tag{31}$$

for $i = N+1, \ldots N+M$. Here we have used the standard fact that if x^* is the minimizer of some convex function $F(x)$ over

a convex set X, and $F(\cdot)$ is differentiable, then $\nabla F(x^*)'(x - x^*) \geq 0$ for all $x \in X$, where ∇F is the gradient of F, see, e.g., [33, Proposition 3.1].

Recall that

$$p_i(k) = \arg\min_{p_i \in \Omega_{\mathcal{N},i}} \sum_{t=1}^T C_i(p_{i,t}) - \lambda_i(k)' p_i$$

for $i = 1, \ldots N$ and

$$(p_i^+(k), p_i^-(k)) \in \arg\min_{\{p_i^+, p_i^-\} \in \tilde{\Omega}_{\mathcal{M},i}} \lambda_i(k)' \left(p_i^- - p_i^+ \right)$$

for $i = N+1, \ldots N+M$. Since $p_i(k) \in \Omega_{\mathcal{N},i}$ for $i = 1, \ldots, T$, applying the similar analysis as for obtaining (30) and (31) yields

$$(2a_i p_i + b_i - \lambda_i(k))' (p_i^* - p_i(k)) \geq 0 \tag{32}$$

for $i = 1, \ldots, N$ while

$$- \lambda_i(k)' (p_i^* - p_i(k)) \geq 0 \tag{33}$$

for $i = N + 1, \ldots N + M$. By adding (30) with (32) and by adding (31) with (33) respectively, we obtain that

$$2a_i \|p_i(k) - p_i^*\|^2 \leq (\lambda_i(k) - \lambda^*)' (p_i(k) - p_i^*) \tag{34}$$

for distributed generator $i \in \mathcal{N}$, and

$$0 \leq (\lambda_i(k) - \lambda^*)' (p_i(k) - p_i^*) \tag{35}$$

for the storage device $i \in \mathcal{M}$.

Summing (34) and (35) yields

$$\sum_{i=1}^N 2a_i \|p_i(k) - p_i^*\|^2 \leq \sum_{i=1}^{N+M} (\lambda_i(k) - \lambda^*)'(p_i(k) - p_i^*). \tag{36}$$

Note that in Case 1 we considered the optimal solution which clearly satisfies the power balance, while In Case 2 by construction p_i^* satisfies the power balance. It then follows from (20) that

$$\bar{r}(k) = \sum_{i=1}^{N+M} (p_i(k) - p_i^*). \tag{37}$$

With some algebra, we get

$$\|\bar{\lambda}(k+1) - \lambda^*\|^2 - \|\bar{\lambda}(k) - \lambda^*\|^2$$
$$= 2 \left(\bar{\lambda}(k) - \lambda^* \right)' \left(\bar{\lambda}(k+1) - \bar{\lambda}(k) \right) + \|\bar{\lambda}(k+1) - \bar{\lambda}(k)\|^2$$
$$= \frac{-2\alpha_k}{N+M} \left(\bar{\lambda}(k) - \lambda^* \right)' \bar{r}(k) + \frac{\alpha_k^2 \|\bar{r}(k)\|^2}{(N+M)^2}$$
$$= \frac{-2\alpha_k}{N+M} \sum_{i=1}^{N+M} \left(\bar{\lambda}(k) - \lambda^* \right)' (p_i(k) - p_i^*) + \frac{\alpha_k^2 \|\bar{r}(k)\|^2}{(N+M)^2}$$
$$= \frac{-2\alpha_k}{N+M} \sum_{i=1}^{N+M} \left(\bar{\lambda}(k) - \lambda_i(k) \right)' (p_i(k) - p_i^*)$$
$$\quad - \frac{2\alpha_k}{N+M} \sum_{i=1}^{N+M} (\lambda_i(k) - \lambda^*)' (p_i(k) - p_i^*) + \frac{\alpha_k^2 \|\bar{r}(k)\|^2}{(N+M)^2},$$

where we have used (19) in the second equality and (37) in the third equality.

By using (21) and (36), we get

$$\|\bar{\lambda}(k+1) - \lambda^*\|^2 - \|\bar{\lambda}(k) - \lambda^*\|^2$$

$$\leq \alpha_k \|\lambda_e(k)\| M_3 - \frac{4\alpha_k}{N+M} \sum_{i=1}^{N} a_i \|p_i(k) - p_i^*\|^2 + \alpha_k^2 M_4,$$

where M_3 is such that $\|p_i(k) - p_i^*\| \leq \frac{M_3}{2}(N+M)$ for all $i \in \mathcal{L}$ and M_4 is such that $\|\bar{r}(k)\|^2 \leq M_4(N+M)^2$.

In Case 1 we note there exists δ such that

$$\sum_{i=1}^{N} a_i \|p_i(k) - p_i^*\|^2 \geq \delta$$

since p_i^* satisfies the power balance and $p_i(k)$ is by assumption never close to satisfying the power balance. This yields that

$$u_k = \frac{4\alpha_k}{N+M} \sum_{i=1}^{N} a_i \|p_i(k) - p_i^*\|^2 \tag{38}$$

is a divergent sequence, while

$$\sum_{k=0}^{\infty} \alpha_k \|\lambda_e(k)\| M_3 < \infty \text{ and } \sum_{k=0}^{\infty} \alpha_k^2 M_4 < \infty.$$

This leads to a contradiction according to the deterministic counterpart of the supermartingale convergence result, which can be found in [34, Lemma 6] and is presented as Lemma 1 in Appendix B for readers' convenience.

Hence we only need to consider Case 2. Using the deterministic counterpart of the supermartingale convergence result in Lemma 1, it follows that the sequence $\{\|\bar{\lambda}(k) - \lambda^*\|^2\}$ is convergent. In particular, this result is obtained by applying the result of Lemma 1 with

$$v(k) = \|\bar{\lambda}(k) - \lambda^*\|^2, \quad b(k) = 0,$$

$$c(k) = \alpha_k \|\lambda_e(k)\| M_3 + \alpha_k^2 M_4.$$

and $u(k)$ as in (38). Since the sequence $\{\bar{\lambda}(k)\}$ has a subsequence that converges to λ^*, we can then conclude that it must converge to λ^*.

Note that from (34), by using the Cauchy-Schwarz inequality, we obtain

$$2a_i \|p_i(k) - p_i^*\|^2 \leq \|\lambda_i(k) - \bar{\lambda}(k) + \bar{\lambda}(k) - \lambda^*\| \|p_i(k) - p_i^*\|.$$

Since $\|p_i(k) - p_i^*\| \leq \frac{M_3}{2}(N+M)$, which is bounded, and $\lim_{k \to \infty} \bar{\lambda}(k) = \lambda^*$, we have

$$\lim_{k \to \infty} \|p_i(k) - p_i^*\| \leq C \lim_{k \to \infty} \|\lambda_i(k) - \bar{\lambda}(k)\|.$$

for some constant $C > 0$. The right hand side of the above inequality is actually zero due to (21), (25), (24) and the fact that α_k is decreasing and converging to zero. Hence, we conclude that

$$\lim_{k \to \infty} \|p_i(k) - p_i^*\| = 0, \forall i \in \mathcal{N}.$$

It remains to show that the power of storage devices converges to the correct value. In the case of multiple storage devices, we only need to find a feasible solution since the storage devices do not affect the cost function. To find a feasible solution, we solve the following convex optimization problem:

$$\mathbf{P}'_s : \min_{p_{i,t}^+, p_{i,t}^-} \sum_{t=1}^{T} \sum_{i=N+1}^{N+M} (p_{i,t}^+ - p_{i,t}^-)^2 \tag{39}$$

subject to (1i*), (1b), (1e*), (1f), (1g) and (1h) with $p_{i,t} = p_i^{\text{sol}}$ for $i \in \mathcal{N}$ and $t \in \mathcal{T}$. Note that the cost function of optimization problem (39) is irrelevant since we are only looking for a feasible solution. The cost function is only helpful in the sense that it guarantees that we have a way to select a unique solution from the set of all feasible solutions. We can then follow the first part of proof with some modifications to show that $\lim_{m \to \infty} \|p_i(m) - p_i^*\| = 0$ for all $i \in \mathcal{M}$.

APPENDIX B
DETERMINISTIC COUNTERPART OF SUPERMARTINGALE CONVERGENCE THEOREM

Lemma 1: Let the sequence $\{v(k)\}$ be a non-negative scalar sequence such that

$$v(k+1) \leq (1+b(k))v(k) - u(k) + c(k) \quad \text{for all } k \geq 0,$$

where $b(k) \geq 0$, $u(k) \geq 0$ and $c(k) \geq 0$ for all $k \geq 0$ with $\sum_{k=0}^{\infty} b(k) < \infty$, and $\sum_{k=0}^{\infty} c(k) < \infty$. Then, the sequence $\{v(k)\}$ converges to some $v \geq 0$ and $\sum_{k=0}^{\infty} u(k) < \infty$.

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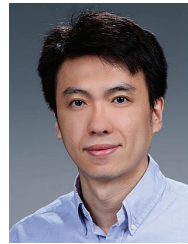
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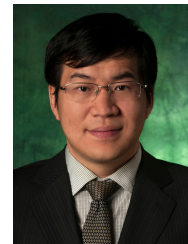
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